Spring 2006 **Steve Tadelis**

Econ 206 - Take Home Exam

Due 24 hours after you start reading the questions

Question 1:

A buyer and a seller engage in a two-period trading relationship. The buyer makes a relation-specific investment "x" at a cost $\frac{1}{2}x^2$ in the first period, and receives q units of input from the seller in the second period. The buyer's gross benefit from the input (excluding investment costs) is $q\sqrt{x}$. The seller makes no investmet but has ex post costs equal to $\frac{1}{2}cq^2$. Here c is a random variable as of period 1, whose realization will be known to both parties in period 2. The discount rate is 0 and both parties are risk neutral. The buyer's investment is observable but not verifiable.

a) Find the first-best level of q as a function of c, and the first-best level of x.

b) The parties write a contract that specifies that the seller must supply $\hat{q}$ units of input in the second period (assume that this can be enforced). The parties realize, however, that ex post efficient bargaining (under symmetric information) will lead to a q different from $\hat{q}$. Show that by a suitable choice of $\hat{q}$ the first best can be achieved:

(i) for the case where the buyer has all the bargaining power;

(ii) for the case where the seller has all the bargaining power.

Solve for $\hat{q}$ in the two cases.

c) Discuss briefly what lessons can be learned about the nature of the hold-up problem from this example.

Question 2:

Consider a relationship between a seller (S) and a buyer (B). S can produce up to one unit of a service valued by B, using asset A in production. Ex ante S is drawn from a large pool of sellers having cost c of production, and B is drawn from a large pool of buyers having valuation v for the output, with $v > c$. S can exert cost-reducing effort $x \geq 0$, which costs him $\frac{1}{2}x^2$ in utility terms. This cost reduction has an *asset-specific* component of sx and a *relationship-specific* component of σx. In other words, S's cost after such an investment will be $c - sx - \sigma x$ when he uses asset A and trades with B, and $c - sx$) when he uses asset A, but produces for another buyer. Similarly, B can undertake value-enhancing investment $y \geq 0$, which costs him $\frac{1}{2}y^2$ in utility terms. This value increase has an *asset-specific* component of by and a *relationship-specific* component of βy.

After investments have been made, S and B bargain about the optimal level of trade, dividing the surplus over their outside options equally. Each of them has the option to switch to another partner who has not made any specific investment.

a) Describe all possible deterministic (non-random) control arrangements between S and B. Which deterministic control structure is optimal?

b) Solve for the optimal stochastic (a probability of control can be allocated ex ante) control structure.

Now suppose that there exists a third party-an outsider (O), who is irrelevant for production and consumption.

c) Describe all possible control structures among S, B, and O.

d) What is the optimal deterministic/stochastic control structure now?