Fall 1998 Prof. Steve Tadelis

Econ 160 - Final Exam

DIRECTIONS (Read Carefully!!):

This exam includes 4 questions (the points of each part are clearly marked on the left margin). **It is strongly recommended to read the whole exam before you attempt to solve it** since questions may not necessarily appear in order of difficulty. Answer each question in a **separate blue-book** (you can uses parts of the blue-books as scratch-pads, but be sure to differentiate your final answer from your preliminary scribbles!). Please hand in your answers in a comprehensible format; illegible or unsupported answers will lose valuable points!

Date: Friday, 12/11/98 **Starting time:** 8:30 AM **Ending time:** 11:30 AM

GOOD LUCK!!

Question 1: Basic Concepts (24 points)

(a) Define, using formal notation (when possible), and **briefly** explain the following terms:

4 (i) Subgame perfect Equilibrium
4 (ii) Beliefs off the equilibrium path

(b) For each of the following statements, provide a proof if it is true or a counter-example if it is not:

4 (i) For any game Γ of *imperfect information*, every Subgame Perfect equilibrium is also a Nash Equilibrium.
4 (ii) For any game Γ of *perfect information*, there is always a Nash Equilibrium that is not a Subgame Perfect equilibrium.

(c) **Briefly** answer the following:

4 (i) What is the difference between a game of incomplete information and a game of imperfect information?
4 (ii) How is a game of incomplete information represented as a game of imperfect information?

Question 2: Twice is Better (28 points)

Two students received a team project from their professor. The deal is that each must simultaneously choose a research effort, and they cannot jointly commit to their effort levels. That is, student 1 chooses effort level $e_1 \geq 0$ and student 2 independently chooses effort level $e_2 \geq 0$ at the same time, and they cannot coordinate their efforts. If student *i* chooses effort level e_i then this imposes a *personal* monetary cost of $\frac{e_i^3}{3}$. Given their effort levels that are observed after the fact, the professor splits $2e_1e_2$ dollars equally between the students. Thus, the payoffs of each student *i* is given by the following utility function,

$$e_1e_2 - \frac{e_i^3}{3}$$

Assume first that this interaction is a one-shot project.

2 (a) Write this interaction as a normal (strategic) form game?

6 (b) Solve for the pure strategy Nash equilibria of this game.

4 (c) Show that if both students could commit in advance to certain effort levels (that is, they can write an "enforceable contract" with each other) then they would choose $e_1 = e_2 = 2$.

Now assume that these two students will perform two such projects sequentially (i.e., the above is repeated twice) where the outcomes of the first project are determined and observed by all before the second project is performed. Let e_{it} denote the effort of student *i* in period *t*. The total payoffs are the discounted sum of payoffs of the two periods where the discount factor is $0 < \delta \leq 1$.

10 (d) For which values of δ can effort levels $e_{11} = e_{21} = 2$ be suuported as part of a pure strategy Subgame Perfect Equilibrium (SPE)? Precisely define the SPE you are using to support the outcome, and show that it is a SPE.

6 (e) For δ=1, are there SPE which have $e_{11} = e_{21} > 2$? If no, prove it, if yes, give an example. Although this game is not an infinitely repeated game, can you relate your answer to this part to the folk theorem in repeated games?

Question 3: Prenuptial Agreements (24 points)

Two people players 1 and 2, can potentially get married. With probability ¼ Player 1 is “nice” (N), and with probability ¾ player 1 is a “jerk” (J). Player 1's “type” is known to player 1, but not to player 2. Player 1 can propose two alternative marriage agreements to player 2: either have a trust-based relationship (T), or enter a prenuptial agreement (A). Player 2 can then say yes (Y) or no (N).

If following a proposal of T, player 2 chooses N, then both players have a utility of 0. If a proposal of A is followed by N then player 2 gets 0, and player 1 bears the cost of such an agreement and thus gets -1.

If player 2 chooses Y, then given the type of player 1, and the proposition, the payoffs are given by the following *payoff matrix*:

		Player 1’s proposition	
		A	T
Type of	N	2,2	1,2
player 1	J	2,-1	-2,-1

(That is, player 2 really doesn’t like marrying a jerk, and a prenuptial agreement is a safe thing for any type of player 1. However, a jerk will suffer more in a trust-based marriage since the lack of protection will cause this type more pain through a costly divorce.)

4 (a) Draw the game-tree that represents the extensive form of this game, and identify the proper subgames.

In parts (c) and (e) be precise in describing the equilibria strategies and beliefs:

2 (b) What are the *pure strategies* of player 1 that are candidates for separating Perfect Bayesian Equilibria (PBE)?

8 (c) Of the strategies identified in (b) above, which can indeed be part of a PBE?

2 (d) What are the *pure strategies* of player 1 that are candidates for pooling PBE?

8 (e) Of the strategies identified in (d) above, which can indeed be part of a PBE?

Question 4: Teenagers...(24 points)

Two teenagers, named 1 and 2, have borrowed their parents' cars, and decided to play the game of "chicken" as follows: They both drive towards each other on a street, and just before impact they must *simultaneously* choose whether to be "chicken" (C) and move away to the side, or continue head-on (H). If both play C, then both gain no respect from their friends, but suffer no losses, thus both get a payoff of 0. If i plays H and $j \neq i$ plays C, then i gains all the respect, which is a payoff of w, and j gets no respect, which is 0. In this case they suffer no additional losses and these are the payoffs. Finally, if both play H, they "split" the respect (since respect is considered to be relative...), but an accident is bound to happen and they will be reprimanded by their parents, which imposes a personal loss of k, so the payoff to each kid is $w/2 - k$.

There is, however, a potential difference between these two youngsters: The punishment, k, depends on the type of parents they have. For each kid, parents can be either strict or lenient with equal probability, and the draws are independent. If they are strict, then they will beat the living daylight out of their child, and we denote this by the cost being $k = B$. If they are lenient, then they will give their child a long lecture of why his behavior is unacceptable, and we denote this by the cost being $k = L$. Each kid knows the type of his parents but does not know the type of his opponents parents. The distribution of types is common knowledge.

4 (a) Draw the game tree that represents this game in extensive form.

8 (b) Draw the bi-matrix that represents this game in normal form.

6 (c) Now assume that $w = 8$, $B = 16$, and $L = 0$. Solve for the Bayesian Nash equilibria of this game.

6 (d) A preacher, who knows some game theory, decided to use this model to claim that moving to a society in which all parents are lenient will have detrimental effects on the behavior of teenagers. Is this right? (Your answer should be supported with an equilibrium argument!)