

Econ 160: Game theory and Economic Applications  
Winter 2005

Problem Set 1 - Due 1/13/2005

## 1 Dominated Strategies

In the following normal-form game, which strategy profiles survive iterated elimination of strictly dominated strategies?

|          |          | Player 2 |          |          |
|----------|----------|----------|----------|----------|
|          |          | <i>L</i> | <i>C</i> | <i>R</i> |
| Player 1 | <i>U</i> | 4,6      | 0,4      | 6,0      |
|          | <i>M</i> | 6,0      | 2,2      | 7,3      |
|          | <i>D</i> | 6,8      | 2,4      | 3,5      |

## 2 The $p$ -Beauty Contest

We played a game in class in which each player chose an integer between 0 and 20, and the winner was selected randomly among the players who were closest to  $\frac{3}{4}$  of the average. This game is called the  **$p$ -beauty contest**, since unlike a standard beauty contest, you are not trying to guess what everyone else is guessing (hence, beauty is what people choose it to be), but rather guess  $p$  times the average, in this case  $p = \frac{3}{4}$ . Assume that there were  $n$  players in the class, where  $n < \infty$  (there are a finite number of students in the class).

1. Which strategy (or strategies) of each player are eliminated after one round of elimination of **strictly** dominated strategies?

A strategy  $s'_i \in S_i$  of player  $i$  is *weakly dominated* if there is some strategy  $s_i \in S_i$  such that  $u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$  for all  $s_{-i} \in S_{-i}$ .

2. How many rounds of elimination of **weakly** dominated strategies are needed to reach the set of strategies that survive iterated elimination? What is this set?
3. Does your answer to 2 above change if  $p = \frac{1}{4}$ ?

### 3 In the Neighborhood

There are  $n$  neighbors, each can choose to spend time cleaning the neighborhood where their limit is 2 hours. So, each can choose an amount of time  $0 \leq x_i \leq 2$ . After all the cleaning is done, the payoff of for each person is  $v > 0$  times the sum of all efforts of all players, less his own effort  $x_i$ . (that is, assume that the personal cost of time is  $c(x_i) = x_i$ , whereas the benefit to each player is the same, for example, they all equally enjoy a clean neighborhood.) All the efforts are chosen simultaneously (i.e., each player chooses his/her effort without observing the other players' choices).

- (i) Formulate this situation as a normal-form game.
- (ii) What is the pure-strategy Nash equilibrium of this game when  $v < 1$ . Is it Pareto optimal?
- (iii) If  $v = 0.5$  and  $n = 4$ , would the neighbors prefer an outcome different from (ii) above?
- (iv) What is the intuition for your conclusions in (ii) and (iii) above?