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Econ 160: Game theory and Economic Applications

Winter 2005

Problem Set 2 - Due 1/20/2005

1 Cournot to the limit

Suppose there are n firms in the Cournot oligopoly model. Let q_i denote the quantity produced by firm i, and let $Q = q_i + \cdots + q_n$ denote the aggregate production. Let $P(Q)$ denote the market clearing price (when demand equals Q) and assume that inverse demand function is given by $P(Q) = a - Q$ (where $Q < a$). Assume that firms have no fixed cost, and the cost of producing quantity q_i is $c \cdot q_i$ (all firms have the same marginal cost, and assume that $c < a$).

1. Model this as a Normal form game.
2. What is the Nash (Cournot) Equilibrium of the game where firms choose their quantities simultaneously?
3. What happens as n approaches infinity? What would be an "economic" interpretation of your result?

2 Split the Sushi

You and a classmate arrive at Wilbur dining, and there is only one 6-piece box of Sushi left. Carol, the service manager, agrees to give you the box for free under the following condition. Each of you must *simultaneously* announce how many pieces you would like; that is, each player $i \in \{1, 2\}$ names his desired number pieces, $0 \leq x_i \leq 6$. If $x_1 + x_2 \leq 6$ then you both get your demands (and Carol keeps any leftover pieces). If $x_1 + x_2 > 6$, then you both get nothing. Assume you each care only about how many pieces of Sushi you individually consume, and that more is better.

1. Write out or graph your best-response correspondence.[1]
2. Find all the pure-strategy Nash equilibria.
3. We say that an equilibrium s is *Pareto Inferior* to an equilibrium s' if all players get at least as high a payoff from s' as from s, and some players get a higher payoff. From the perspective of you and your friend (ignoring Carol), are there Nash equilibria that are Pareto Inferior to other equilibria? Explain!

[1]Recall that a correspondence is like a function, except that it need not be single-valued; that is, if f is a correspondence, $f(x)$ can be a single point or a set of points.

3 Coordinating High tech Investments

Two high tech firms (1 and 2) are considering a joint venture. Each firm i can invest in a novel technology, and can choose a level of investment x_i from 0 to 5 at a cost of $c_i(x_i) = \frac{x_i^2}{4}$ (think of x as how many hours to train employees, or how much capital to buy for R&D labs). The revenue of each firm depends both on its investment, and of the other firm's investment. In particular, if firm i and j choose x_i and x_j respectively, then the *gross* revenue to firm i is

$$R(x_i, x_j) = \begin{cases} 0 & \text{if } x_i < 1 \\ 2 & \text{if } x_i \geq 1 \text{and } x_j < 2 \\ x_i \cdot x_j & \text{if } x_i \geq 1 \text{and } x_j \geq 2 \end{cases}$$

1. Write down mathematically, and draw the *profit* function of firm i as a function of x_i for three cases: (i) $x_j < 2$, (ii) $x_j = 2$, and (i) $x_j = 4$

2. What is the best response function of firm i ?

3. It turns out that there are two *identical* pairs of such firms (that is, the technology above describes the situation for both pairs). One pair in Russia where coordination is hard to achieve and business people are very cautious, and the other pair in Germany where coordination is common and business people expect their partners to go the extra mile. You learn that the Russian firms are earning significantly less profits than the German firms, despite the fact that their technologies are identical. Can you use Nash equilibrium analysis to shed light on this dilemma? If so, be precise and use your previous analysis to do so.