PROF. STEVE TADELIS STANFORD UNIVERSITY

Econ 160: Game theory and Economic Applications

Winter 2005

Problem Set 3 - Due 1/27/2005

1 The Dean's Dilemma

A student stole the DVD from the Cyber-café at Stern Dining. The dean of students (player 1) suspects that the student (player 2) and engages in evidence collection. However, evidence collection is a random process, and concrete evidence will be available to the dean only with probability $\frac{1}{2}$. The student knows the evidence generating process, but does not know whether the dean received evidence or not.

The game proceeds as follows: The dean realizes if he has evidence or not, and then can choose his action, whether to Accuse the student (A), or Bounce the case (B) and forget it. Once accused, the student has two options: he can either Confess (C) or Deny (D).

Payoffs are realized as follows: If the dean bounces the case then both players get 0 utils. If the dean accuses the student, and the student confesses, the dean gains 2 utils and the student loses 2 utils. If the dean accuses the student and the student denies, then payoffs depend on the evidence: If the dean has no evidence then he loses face which is losing 4 utils, while the student gains glory which gives him 4 utils. If, however, the dean has evidence then he is triumphant and gains 4 utils, while the student is put on probation and loses 4 utils.

(a) Draw the game-tree that represents the extensive form of this game, and identify the proper subgames.

(b) Write down the matrix that represents the normal form of the extensive form you did in (a) above.

(c) Solve for the Nash Equilibria of the game.

(d) Can you find a Nash equilibrium that is not subgame perfect? Explain.

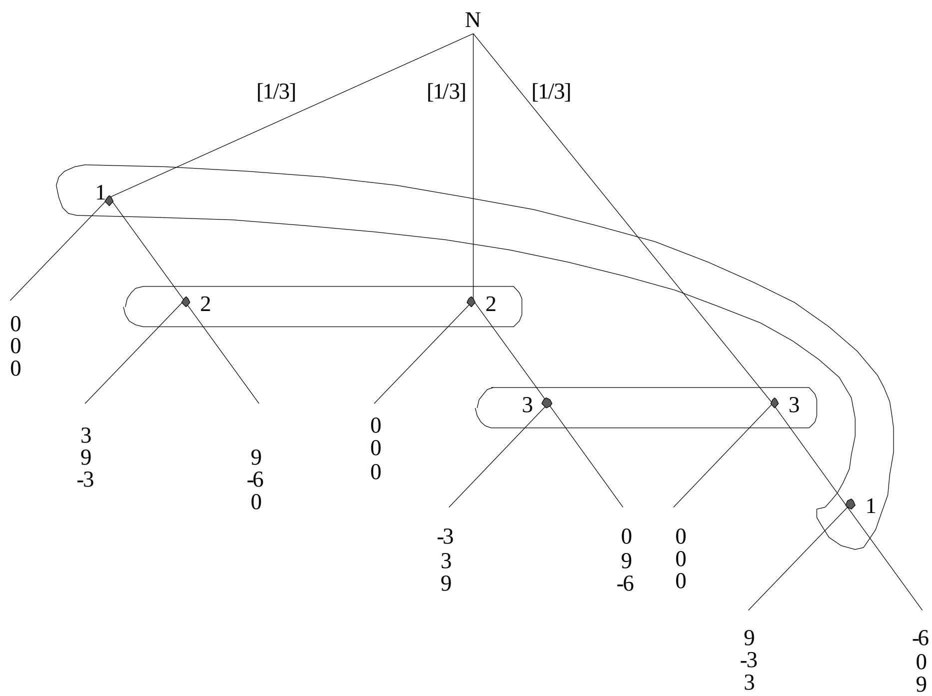
2 Competition with Multiple Entrants

Three oligopolists operate in a market with inverse demand given by $P(Q) = a - Q$, where $Q = q_1 + q_2 + q_3$, and q_i is the quantity produced by firm i. Each firm has a constant marginal cost of production, c, and no fixed cost. Firm 1 is the incumbent with the first mover advantage, so the firms choose their quantities dynamically as follows: (1) Firm 1 chooses $q_1 > 0$; (2) Firms 2 and 3 observe q_1 and then simultaneously choose q_2 and q_3 , respectively.

1. What is the subgame perfect equilibrium of this game?
2. Find a Nash equilibrium that is not Subgame Perfect

3 Who is the First Mover?

Consider a game in which Nature randomly chooses one of three players to move. The chosen player can end the game, or not, in which case a second player will end the game with one of two choices. Each player can be a first or second mover, and they don't know which of the two events occurred. this is captured by the following game tree:



1. Model this as a Normal form game (Hint: exclude Nature from the normal form representation)

2. Find a Nash equilibrium of the normal form game. Is it unique?

3. Find a Nash equilibrium of the extensive form game. Is it unique?

4 All Pay Auctions

In class we saw a version of an **all pay auction** that worked as follows: each bidder first submits a bid. The highest bidder gets the good, but **all bidders pay there bids**. Consider such an auction for a good worth \$1 to each of the two bidders. Each bidder can choose to offer a bid from the interval [0,1] (continuous, not discrete!). Players only care about the expected value they will end up with at the end of the game (i.e., if a player bids \$0.4 and expects to win with probability 0.7 then his payoff is $0.7 \times 1 - 0.4$).

1. Model this auction as a normal-form game.
2. Show that this game has no pure strategy Nash Equilibrium.
3. Show that this game cannot have a Nash Equilibrium in which each player is randomizing over a finite number of bids.
4. Consider mixed strategies of the following form: Each player i chooses and interval, $[\underline{x}_i, \overline{x}_i]$ with $0 \leq \underline{x}_i < \overline{x}_i \leq 1$ together with a cumulative distribution $F_i(x)$ over the interval $[\underline{x}_i, \overline{x}_i]$. (Alternatively you can think of each player choosing $F_i(x)$ over the interval $[0, 1]$, with two values $\underline{x}_i$ and $\overline{x}_i$ such that $F_i(\underline{x}_i) = 0$ and $F_i(\overline{x}_i) = 1$.)
   (a) Show that if two such strategies are a mixed strategy Nash equilibrium then it must be that $\underline{x}_1 = \underline{x}_2$ and $\overline{x}_1 = \overline{x}_2$.
   (b) Show that if two such strategies are a mixed strategy Nash equilibrium then it must be that $\underline{x}_1 = \underline{x}_2 = 0$.
   (c) Using the above, argue that if two such strategies are a mixed strategy Nash equilibrium then both players must be getting an expected utility of zero.
   (d) Show that if two such strategies are a mixed strategy Nash equilibrium then it must be that $\overline{x}_1 = \overline{x}_2 = 1$.
   (e) Show that $F_i(x)$ being uniform over $[0, 1]$ is a symmetric Nash equilibrium of this game.