

Econ 160: Game theory and Economic Applications

Winter 2005

Problem Set 4 - Due 2/11/2005

Question 1

Consider the following simultaneous move game that is played twice (the players observe the first period outcome prior to the second period play):

		Player 2		
		<i>L</i>	<i>C</i>	<i>R</i>
player 1	<i>T</i>	10,10	2,8	0,13
	<i>M</i>	8,2	5,5	0,0
	<i>B</i>	13,0	0,0	1,1

1. Find all the pure strategy SPE with no discounting ($\delta = 1$). Be precise in defining history contingent strategies for both players.
2. For each of the equilibria you found above, find the smallest discount factor that supports it.

Question 2

If we have a two player multi-stage game in which the first 2×2 stage-game has a unique pure strategy Nash equilibrium and the second has two pure strategy Nash equilibria then Proposition 10 implies that this game has at least 2 pure strategy SPE. Give an example of such a game in which the following multi-stage strategies are *not* a SPE: In the first stage play the stage-1 Nash equilibrium, and in the second stage play *conditional* continuations of the stage-2 Nash equilibria. (This example should convince you that extending the Proposition to include sequences of *conditional* Nash equilibria plays may not be itself a SPE).

Question 3

Suppose there are 2 firms duopolists that face the per-period demand represented by

$P(Q) = 100 - Q$ where $Q = q_1 + q_2$, and q_i is the quantity produced by firm i . Assume that the firms have no fixed or marginal costs of production, and assume that the firms face this duopolistic competition in every period $t = 1, 2, \dots, T$ given by the per-period demand curve above, with a discount factor < 1 .

1. Assume that the firms can choose quantities and not prices. Model the stage game as a Normal form game.
2. What is the unique subgame perfect equilibrium for any $T < \infty$? Briefly explain.

3. Now assume that $T = \infty$. Using a variation of the “grim trigger strategies” we used in class for the prisoner’s dilemma (“if someone deviates we play Nash forever...”), for which discount factors can you support the monopolistic quantities (joint profit maximizing) as a subgame perfect equilibrium? (Assume that the firms split the monopolistic profits).
4. Now assume that the firms can choose prices, p_1 and p_2 , and not quantities. Model the stage game as a Normal form game.
5. What is the unique subgame perfect equilibrium for any $T < \infty$? Briefly explain.
6. Now assume that $T = \infty$. Using a variation of the “grim trigger strategies”, for which discount factors can you support the monopolistic quantities (joint profit maximizing) as a subgame perfect equilibrium? (Assume that the firms split the monopolistic profits).