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Econ 160: Game theory and Economic Applications

Winter 2005

Problem Set 6 - Due 2/29/2005

Question 1: Trade

Two players, 1 and 2, each own a house. Each player i values her own house at v_i. The value of player i's house to the other player, i.e. to player $j \neq i$, is however $\frac{3}{2}v_i$. Each player i knows the value v_i of her own house to herself, but not the value of the other players

house. The values v_i are drawn from the interval [0, 1] with uniform distribution.

1. Suppose players announce simultaneously whether they want to exchange their houses. If both players agree to an exchange, the exchange takes place. Otherwise, no exchange takes place. Find a (Bayesian) Nash equilibrium of this game in pure strategies in which each player i accepts an exchange if and only if the value v_i does not exceed some threshold t_i.
2. How would your answer to (1) change if player j's valuation of player i's house were: $\frac{5}{2}v_i$?
3. Bonus: try to explain why any Bayesian Nash equilibrium of the game described in part (1) must involve threshold strategies of the type postulated in part (1).

Question 2: Harsanyi again

Consider the matching pennies game where player 1 wins 1 (and player 2 loses 1) if the pennies match, and player 2 wins 1 (and player 1 loses 1) if they don't match. Assume that each player has an additional preference beyond the monetary payoff so that if player i plays "heads", there is an added utility of ε_i, and no additional utility from playing "tails". However, to capture variation in the preferences for heads or tails some players prefer heads with $\varepsilon_i = 0.1$ and some prefer tails with $\varepsilon_i = -0.1$. This "type" of player i is only known to himself, but every player knows that types are chosen randomly and independently with $\Pr\{\varepsilon_i = 0.1\} = 0.5$.

1. Draw the extensive form of this Bayesian Game.
2. Write down the normal form matrix of this Bayesian game.
3. Find a pure strategy Bayesian Nash equilibrium of the game. Is it unique?
4. Harder: Assume that instead of $\varepsilon_i \in \{-0.1, 0.1\}$ we have $\varepsilon_i \in \{-\varepsilon, \varepsilon\}$ with equal probability. What happens to the set of pure strategy Bayesian Nash equilibria as $\varepsilon \to 0$?

Question 3: Strategic Battles

Consider the following strategic situation: Two opposed armies are poised to seize an island. Each army's general can choose either to "attack" (A) or to "not attack" (N). In addition, each army is either "strong" (S) or "weak" (W) with equal probability, and the realizations for each army are *independent*. Furthermore, the type of each army is known only to that army's general.

An army can capture the island if either (i) it attacks and its rival does not, or (ii) it and its opponent attack, but it is strong and the rival is weak. If both attack and are of equal strength then neither captures the island. As for payoffs, The island is worth m if captured and each army has a cost of fighting equal to s if it is strong and w if it is weak, where $s < w$. If an army attacks but its rival does not, no costs are bared by either side.

Identify all the pure-strategy Bayesian Nash Equilibria of this game for the following two cases, and briefly (!) describe the intuition for your results:

1. $m = 3, w = 2, s = 1$.
2. $m = 3, w = 4, s = 2$.