Prof. Steve Tadelis Stanford University

Econ 160: Game theory and Economic Applications

Winter 2005

Problem Set 7 - Due 3/8/2005

1 Campaigning

There are two politicians, an incumbent (player 1) and a potential rival (player 2) that are running for the local mayoral race. The incumbent has either a broad base of support (B) or a small base of support (S), each occurring with probability $\frac{1}{2}$. The incumbent knows its level of support but the potential rival does not. The incumbent first chooses how much soft money to spend on campaign financing: a low quantity (L) or a high quantity (H) which is observed by the potential rival. The rival can then decide to run (R) or not to run (N).

If the incumbent chooses a level of campaign financing of L, then given the support base and the reaction of the potential rival, the payoffs are given by the following *payoff matrix* (these are payoffs that represent the expectations from winning, campaigning, etc. and and this is not a matrix game)

		Player 2's response	
		R	N
Support base	B	6,4	10,0
	S	4,4	6,0

The **cost in payoffs** that an incumbent incurs for choosing H instead of L is 2 if he has a broad base of support and 4 if he has a small base of support (that is, these are costs that are deducted from the payoffs in the payoff matrix that are conditional on the type). A rival that runs against an incumbent with a broad base of support who chose H will obtain a payoff of -10, while a rival that runs against an incumbent with a small base of support who chose H will obtain a payoff of -4. If the rival chooses not to run then he obtains 0 as in the payoff matrix above.

a. Draw the extensive form of this game, and identify the proper subgames.

b. Draw the matrix that represents the normal form of the extensive from you did in (a.) above. Be clear as to your choice of strategy spaces for each player.

c. If the rival could commit in advance to a certain pure strategy that he will follow regardless of the incumbent's choice of financing, anticipating that the incumbent will then choose his best response, what would this strategy be? What would be the incumbent's best response to this strategy?

d. Are the pair of strategies you found in (c.) above a Bayesian Nash Equilibrium? Explain.

e. Can the pair of strategies you found in (c.) above be part of a Perfect Bayesian Equilibrium?

f. Are there other pairs of strategies that can be part of a Perfect Bayesian Equilibrium?

2 Choose Your School

Consider the following problem of educational choice. A young adult, player 1, is deciding whether to go to Bear University (B) or to Tree University (T). The difference is that Tree University involves harder work, and imposes more "studying" costs on the player, but at the same time a player who goes to Tree University learns more and is more productive. The costs of learning and the final productivity depends on the type of player 1, who can be either really good (G) or just above average (A). Player 1 knows his type, but all other people in society only know that a proportion p of young adults are G-types. The cost and productivity from each choice is given as follows:

G-type

choice	cost	prod
B	0	4
T	2	12

A-type

choice	cost	prod
B	2	2
T	8	10

Once player 1 finishes school, he is hired by a firm (player 2) who can place player 1 in one of two jobs: low-tech (L) or high-tech (H). The wage for the L job is $w_L = 2$ and for the H job is $w_H = 6$. The payoffs to player 1 are the **wages less the cost of education**.

The firm's profits depend both on the job assignment, and on the type of employee. If the employee is assigned to a H job, the net profits to the firm are equal to **the productivity** of the employed player 1 **less the wage** he is paid. If the assignment is to a L job, the net profits are **half the productivity** of the employed player 1 **less the wage** he is paid.

(a) Draw this game in extensive form

(b) Assume that $p = \frac{1}{2}$. Represent the matrix form of the Bayesian Game.

(c) Find all the pure-strategy Bayesian Nash equilibria.

(d) Find all the pure strategy Perfect Bayesian equilibria.

(e) What is the brief intuition that explains the comparison between your results in **(d)** and **(c)** above?

3 Reap and Weep

The market for widgets has an incumbent firm. The total value from having 100% of the market is worth 10, which the incumbent receives if no one enters. A potential entrant arrives and he can be one of two types: tough (T) or weak (W). A weak entrant can choose one of three options: small entry (S), big entry (B), or exit (X). A tough entrant can only choose between S or B (it is inconceivable that it chooses X). There is no cost for a tough entrant to enter at any level. However, it costs a weak entrant 6 to enter at any level. Exit costs nothing to the entrant. The entrant knows his type, but the incumbent only knows the prior distribution: $\Pr\{T\} = \frac{1}{2}$.

In response to any level of entry, the incumbent can choose to accommodate (A) or fight (F). Accommodating an entrant imposes no costs on any firm. Independent of the entrant's type, accommodating small entry gives the incumbent 60% of the market and the entrant 40%, while accommodating big entry gives the incumbent 40% of the market and the entrant 60%. Fighting a tough entrant increases the incumbent'sss market share by 20% (relative to accommodating), but imposes a cost of 4 on the incumbent. Fighting a weak entrant that chose S increases the market share of the incumbent to 100% but imposes a cost of 2 on the incumbent.. Fighting a weak entrant that chose B increases the market share of the incumbent to 100% but imposes a cost of 8 on the incumbent..

(a) Draw this game in extensive form. (Notice the information sets!)

(b) Using a bi-matrix representation, find all the pure strategy BNE for this game.

(c) Which one of the BNE is preferred by the incumbent? Can it be supported as a PBE?

(d) Find all the PBE of this game. Be careful to define strategies and beliefs.

(e) Is it true that in this game any separating PBE imposes unique beliefs for the incumbent in all information? Explain.