Section 1 - Apr 8/9

big concepts so far

normal form games

dominant strategies

iterated elimination

best responses

Nash equilibrium

(mixed strategies)

“classic” games

Prisoner’s Dilemma

Battle of the Sexes

Matching Pennies

Cournot Competition

Bertrand Competition

1. Review of Function Notation

What $F : \{(N, \{S_i\}, \{u_i\})\} \Rightarrow S$ means:

- Domain is the set of all games
- Range is the strategy space of each game
- Two arrows indicates a correspondence, not a function
- So a solution concept $F$ maps a game $\langle N, \{S_i\}, \{u_i\} \rangle$ to a subset of its strategy space $S$, telling you which strategy profiles are permitted

2. The Campaigning Game

Solving PS 1 Question 3 using Strictly Dominant Strategies (none), Iterated Elimination of Strictly Dominated Strategies (gives the unique prediction that both players play $N$), Rationalizability (same prediction), and Nash Equilibrium (same prediction).

3. Win Dan’s Money

The Game: Suppose $n > 20$. Everyone in the room writes down a whole number between 1 and 20. The smallest number wins that many dollars. If multiple players tie, they split the prize evenly.

1. Are there any strictly dominant strategies? (No.)

2. Are there any strictly dominated strategies? ($s_i = 20$ is dominated by $s_i = 1$ as long as $n > 20$. Once $s_i = 20$ is eliminated, $s_i = 19$ is dominated by $s_i = 1$, and so on. Only $s_i = 1$ survives iterated elimination.)

3. What strategies are best-responses? To what? What strategies are rationalizable? ($Everything$ but $s_i = 20$ is a best response to something, but $s_i = 1$ is the only rationalizable strategy.)
4. Is everyone happy with their move? Does anyone want to change? (Yes. This means we were not in a Nash equilibrium. Nash equilibrium is defined as being in a strategy profile where nobody can strictly improve their own payoff unilaterally.)

4. Nash Equilibrium versus Rationalizability

\[
\begin{array}{cc}
O & F \\
\hline
O & 1, 2 & 0, 0 \\
F & 0, 0 & 2, 1 \\
\end{array}
\]

In this Battle of the Sexes game, Nash Equilibrium predicts \((O, O)\) or \((F, F)\). Rationalizability predicts that each player plays either \(O\) or \(F\), but doesn’t predict whether they will successfully coordinate. The difference is best explained in terms of beliefs. Both rationalizability and Nash equilibrium require me to play a best-response to some belief about what the other players are going to do. Rationalizability requires that belief to be reasonable; Nash equilibrium requires that belief to be correct.

5. Why not IEWDS

\[
\begin{array}{cc}
L & R \\
\hline
U & 3, 4 & 4, 3 \\
M & 5, 3 & 3, 5 \\
D & 5, 3 & 4, 3 \\
\end{array}
\]

Solving this game by iteratively eliminating weakly-dominated strategies gives different answers depending on the order in which strategies are removed. If \(U\) is removed first, followed by \(L\), the unique prediction is \((D, R)\); if \(M\) is removed first, followed by \(R\), the unique prediction is \((D, L)\). Iterated elimination of weakly dominated strategies can also eliminate Nash equilibria. IESDS makes a prediction which is well-defined (independent of order), and cannot eliminate a Nash equilibrium.

6. An Example with No Pure Strategy Nash Equilibrium

Three voters voting on a new proposal. Baseline payoff is 0. The proposal gives players 1 and 2 payoffs of 10, and player 3 a payoff of \(-10\). A strict majority of voters is required for the proposal to pass. In addition, the hassle of voting costs each player who chooses to vote 1 unit of payoff.

- Is there a Pure-Strategy Nash Equilibrium where no one votes?
- Is there a PSNE where one player votes? Where two vote? Where all three vote?

There is no PSNE to this game.