Section 1 - Jan 14

big concepts so far  “classic” games
normal form games  Prisoner’s Dilemma
dominant strategies  Battle of the Sexes
IESDS  Matching Pennies
best responses  Cournot Competition
rationalizability  Bertrand Competition
Nash equilibrium

1. Recap – What’s A Solution Concept

2. Why not IEWDS

\[
\begin{array}{ccc}
\text{ } & L & R \\
U & 3,4 & 4,3 \\
M & 5,3 & 3,5 \\
D & 5,3 & 4,3 \\
\end{array}
\]

Solving the game by iteratively eliminating weakly-dominated strategies gives different answers depending on the order in which strategies are removed, and tends to eliminate one of the two Nash equilibria. This is a general problem when weakly dominated strategies are removed. When only strictly dominated strategies are removed, the order does not matter, and Nash equilibria are never eliminated.

3. Win My Money

The Game: Everyone in the room writes down a whole number between 1 and 20. The smallest number wins that many dollars. If multiple players tie, they split the prize evenly.

1. Are there any strictly dominant strategies? (No.)

2. Are there any strictly dominated strategies? (If \( n \leq 20 \), no. If \( n > 20 \), then \( s_i = 1 \) dominates \( s_i = 20 \) and the game is solvable by IESDS.)

3. What strategies are best-responses? To what? What strategies are rationalizable? (\( s_i = 1 \) is the only rationalizable strategy.)

4. Is everyone happy with their move? Does anyone want to change? (Yes. This means we were not in a Nash equilibrium. Nash equilibrium is defined as being in a strategy profile where nobody can strictly improve their own payoff unilaterally.)
4. Rationalizability vs. Nash Equilibrium

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<th>S</th>
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<tbody>
<tr>
<td>Player 1</td>
<td>0,0</td>
<td>−1,1</td>
<td>1,−1</td>
</tr>
<tr>
<td>Player 2</td>
<td>1,−1</td>
<td>0,0</td>
<td>−1,1</td>
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Scissors-Paper-Rock – every strategy is rationalizable, but there are no pure-strategy Nash equilibrium. Make sure you understand why. One way to think about the distinction: in either concept, each player is playing a best-response to their beliefs about what the other players will do. For rationalizability, these beliefs must all be reasonable; for Nash equilibrium, they must be correct.

5. A Voting Game with No Pure Strategy Nash Equilibrium

Three voters voting on a new proposal. Baseline payoff is 0. The proposal gives players 1 and 2 payoffs of 10, and player 3 a payoff of −10. A strict majority of voters is required for the proposal to pass. In addition, the hassle of voting costs each player who chooses to vote 1 unit of payoff.

1. Is there a Pure-Strategy Nash Equilibrium where no one votes?
2. Is there a PSNE where one player votes?
3. Is there a PSNE where two players vote?
4. Is there a PSNE where all three players vote?

There is no PSNE to this game.
What about with no cost to voting? Multiple equilibria.