

Section 2 – Jan 21

This Week's Big Ideas
mixed strategies, expected payoffs, and mixed-strategy NE

1. A Word on Symmetry

Just because a game seems symmetric (players all have the same payoff functions and therefore the same best responses) does **not** mean the equilibrium is automatically symmetric. In the n -player Cournot game in Problem Set 2, you must **show** $q_i = q_j$, not just state it. (Symmetric games can have very asymmetric equilibria – consider the splitting-the-sushi problem.)

2. Strict Dominance with Mixed Strategies

When we consider domination by mixed strategies, we can solve the following game by IESDS:

	L	R
U	10, 2	0, 3
M	4, 7	4, 6
D	0, 4	10, 5

Player 1's strategy M is strictly dominated by $\frac{1}{2}U + \frac{1}{2}D$. Once M is removed, R dominates L ; once L is removed, D dominates U , so (D, R) uniquely survives IESDS.

3. Rationalizability with Mixed Strategies

	L	R
U	11, 4	0, 5
M	10, 3	10, 3
D	0, 3	11, 4

M is not a best-response to any pure strategy, so last week we would have eliminated it. However, M is a best-response to many mixed strategies, such as $\frac{1}{2}L + \frac{1}{2}R$. In fact, $(M, \frac{1}{2}L + \frac{1}{2}R)$ is a mixed-strategy Nash equilibrium. Note that equilibria where only one player mixes are rare.

4. The Key To Finding Mixed-Strategy Nash Equilibria

If a player plays multiple strategies in a Nash equilibrium, they must all give him the same expected payoff.

5. Straightforward Example of Finding MSNE

	L	R
U	3, -3	-11, 11
D	-1, 1	5, -5

Find all Nash equilibria (there's only one, where both players mix), and each player's expected payoff in the equilibrium.

6. Scissors Paper Rock Again

		Player 2		
		S	P	R
Player 1	S	0, 0	1, -1	-1, 1
	P	-1, 1	0, 0	1, -1
	R	1, -1	-1, 1	0, 0

First, we ruled out equilibria where either player played a pure strategy; next we ruled out equilibria where either player mixed between only two strategies. Finally, we used each player's indifference conditions to show that the other player must be mixing between all three strategies with equal probabilities.

7. Equilibria of the Voting Problem

We reexamined the three-player voting problem from last week, where players 1 and 2 gain 10 and player 3 loses 10 if a strict majority of players who vote approve a new measure, and voting also incurs a cost of 1 (relative to staying home). We showed last week that the game has no PSNE. This week, we showed that there is no equilibrium where player 3 never votes; there *is* an equilibrium where player 3 always votes no, while players 1 and 2 both mix between voting yes and staying home. We ran out of time before we could tackle the question of equilibria where player 3 mixes.