Econ 160 | Prof. Steve Tadelis
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1. SPE vs. NE in Dynamic Games

Simple example – Battle of the Sexes with a first mover – unique SPE, multiple possible NE outcomes. How to find Nash Equilibria which aren't SPEs in dynamic games.

2. Ultimatum Game

1. Setup

- Player 1 acts first, offers player 2 some part of 100 dollars (must be a multiple of 10)
- Then Player 2 accepts or rejects
- If Player 2 accepts, they receive that split; if Player 2 rejects, they both receive 0

2. SPEs

By sequential rationality, Player 2 must accept $x > 0$. 2 can accept, reject, or sometimes reject $x = 0$. This leads to two pure strategy SPEs, one where player 2 gets 0 and one where he gets 10. Recall that when you state equilibrium strategies, you must give player 2's move at every information set, not just on the equilibrium path.

3. Other Nash Equilibria

We look for Nash equilibria with payoffs of $(100, 0)$, $(90, 10)$, $(80, 20)$, $(70, 30)$, $(60, 40)$, $(50, 50)$, $(40, 60)$, $(30, 70)$, $(20, 80)$, $(10, 90)$, and $(0, 100)$. We also look for the Nash equilibrium with payoffs $(0, 0)$.

3. How To Play Poker

Player 1's Hand	Player 2's Hand
$K\heartsuit, J\heartsuit, 6\heartsuit, 2\heartsuit, ??$	$A\diamondsuit, A\clubsuit, 7\spadesuit, 5\diamondsuit, ??$

Note that player 1 wins (with a flush) if his downcard is a $\heartsuit$, and loses if it is not. With eight cards visible, there are 44 left, including 9 hearts, so the (prior) probability he has a flush is $\frac{9}{44} \approx \frac{1}{5}$.

Player 2 checks, player 1 can bet or check, player 2 can call or fold if 1 bets. Assume 100 in the pot already, bet size is 10. What is the Nash equilibrium for how often to bluff (if you are player 1 and don't make the flush), and how often to call (if you are player 2)?

We calculate the expected payoffs to the pure strategies, assuming that player 1 will have made a flush with probability $\frac{1}{5}$:

	Call	Fold
Bet Bet	$(14, 86)$	$(100, 0)$
Bet Check	$(22, 78)$	$(20, 80)$
Check Bet	$(12, 88)$	$(100, 0)$
Check Check	$(20, 80)$	$(20, 80)$

To find the NE, we first rule out any equilibria where player 2 plays a pure strategy. Once we know that player 2 mixes, we can rule out the strategies CB and CC, as they give strictly lower expected payoffs than BB and BC, respectively, against any strictly mixed strategy. We therefore reduce the game to a 2×2 matrix, which we solve for the unique Nash equilibrium, $\left(\frac{43}{44}BC + \frac{1}{44}BB, \frac{10}{11}C + \frac{1}{11}F\right)$.

4. Behavioral vs. Mixed Strategies

Definitions of each (using poker as an example), the fact that they are equivalent in games of perfect recall. Example of a game of imperfect recall (driving home drunk) where a behavioral strategy gives a payoff that cannot be achieved using an ordinary mixed strategy.