Econ 160 | Prof. Steve Tadelis
Winter 2005 | T.A.: Dan Quint

Section 4 – Feb 4

1. Problem 3 From the Midterm

We solved for the SPE of the two-stage Cournot game from the midterm. The key step: once you've found (for arbitrary c_1 and c_2) the equilibrium quantities, price, and payoffs of the second-stage Cournot competition, you can consider these as the payoffs of the pure strategies in the first-stage game. (This only works because the second-stage game has a unique equilibrium.) So for example, if firm 1 invests and firm 2 does not, then in the second-stage game, firm 1 will produce $q_1 = 11$ and have marginal cost $c_1 = 0$; firm 2 will produce $q_2 = 8$ and have marginal cost $c_2 = 3$; price will be $P = 30 - q_1 - q_2 = 11$; firm 1 will have profit of $11(11 - 0) = 121$ minus the cost k of investment, and firm 2 will have profit of $8(11 - 3) = 64$. We put the payoffs from the four strategy profiles into the following matrix:

		Player 2: 0	Player 2: 3
Player 1	0	$100 - k, 100 - k$	$121 - k, 64$
	3	$64, 121 - k$	$81, 81$

We then plug in $k = 20$ and $k = 50$ and find the Nash equilibrium of the first-stage game.

2. Two-Stage Prisoner's Dilemma

We considered a two-stage game consisting of two repetitions of a Prisoner's Dilemma problem:

		Player 2: M	Player 2: F
Player 1	M	$3, 3$	$-2, 4$
	F	$4, -2$	$-1, -1$

We sketched the extensive-form representation of the two-stage game, and used backward induction to show that the unique SPE of the two-stage game is for both players to play F at every information set. We discussed how this result generalizes: in a finite-stage game where every stage has a unique Nash equilibrium, the game has a unique SPE consisting of the stage-game NE at every stage regardless of history.

We then considered a two-stage game consisting of the Prisoner's Dilemma, followed by a game with two pure-strategy equilibria, one of which Pareto-dominated the other. We showed that the "bad" equilibrium could be played in the second stage as punishment, and the "good" equilibrium used as a reward, to enforce first-stage behavior other than (F, F). We constructed an SPE in this game where (M, M) was played in the first stage, and another where (M, F) was played in the first stage.

3. One-Step Deviation Principle

I glossed over this very quickly, but I hope you caught it; when checking whether something is an SPE, we only need to look at deviations where a single action is changed. We'll discuss this again later.

4. Multiplicity of Equilibria in Multi-Stage Games

I made the point that in multi-stage or repeated games, there are often so many SPEs that it is unreasonable to try to find or categorize all of them; instead, we will typically try to construct equilibria that fit certain characteristics. This is even more true with infinitely-repeated games; we will see a theorem that says that in an infinitely-repeated game, just about anything can happen on the equilibrium path of an SPE.