

Section 5 – Feb 11

1. Repeated Games – Things to Remember

Finitely Repeated Games

- If the stage game has only one NE, the repeated game has only one SPE
- If the stage game has multiple NE, non-equilibrium behavior can occur in the early stages, using the various equilibria to “reward” or “punish”

Infinitely Repeated Games

- Even if the stage game has a unique NE, all sorts of behavior is possible in SPE
- Intuition: use “play Nash equilibrium forever” as a threat if there are any deviations
- Depends on δ – players need to care about the future
- LOTS of possible equilibria in infinitely-repeated games

2. Intuition – Folk Theorem

We constructed the convex hull of payoffs for the three-by-three matrix game in Problem 1 of Problem Set 4. We discussed, given any payoff pair that Pareto-dominates the lower-payoff Nash equilibria, how you would construct an SPE with those payoffs, using the Nash equilibrium to punish any deviation.

3. Intuition – One-Step Deviation Principle

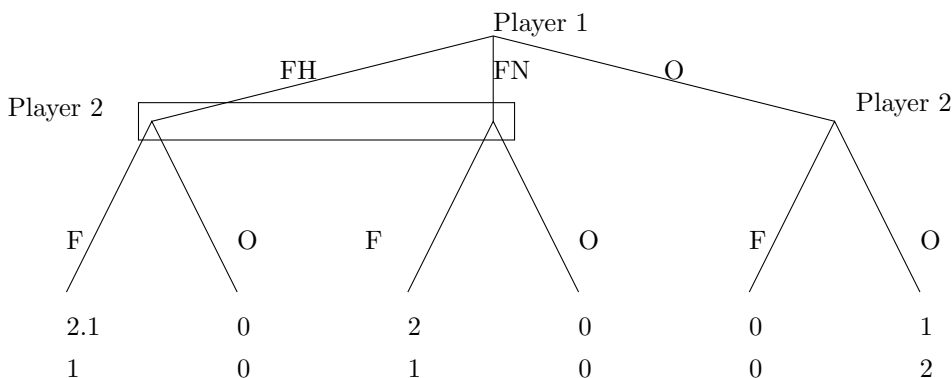
We discussed how you would construct a shorter deviation from a profitable n -step deviation, and how this proves that if a profitable n -step deviation exists, a profitable one-step deviation exists. This allows us to prove a strategy profile is an SPE by ruling out only deviations where a single action is changed.

4. Bargaining

We discussed the general bargaining setup, and solved for the unique equilibrium of a single round given the payoffs of the “continuation game” following refusal. We showed how this allows us to recursively find the SPE of any finite-step bargaining game.

5. When is SPE Not Enough?

Dynamic Battle-of-the-Sexes game. Player 1 moves first, and can go to the Football game and buy a Hot dog, go to the Football game and Not buy a hot dog, or go to the Opera. Player 2 learns where player 1 went, but not whether he bought a hot dog, and then picks Football or Opera. The game:



When player 1 goes to the football game, player 2 has a dominant strategy, which is F . However, since this information set does not start a proper subgame, $s_1 = O$ and $s_2 = OO$ is an SPE.

Subgame Perfect Equilibrium is a very good solution concept for games of perfect information, but not for games of imperfect information. Up next are solution concepts which are better suited for games of imperfect information. This will require a concept of beliefs, which will require a way to update beliefs based on new information, which is...

6. Bayes' Law

Setup for Bayes' Law. X and Y are possible "states of the world" (unobservable). a is an event (observable). The probability the state of the world is X , given that a was observed, is

$$\Pr(X|a) = \frac{\Pr(X) \Pr(a|X)}{\Pr(X) \Pr(a|X) + \Pr(Y) \Pr(a|Y)}$$

Example: I have a jar with 1000 pennies; 999 normal, one with heads on both sides. I draw one at random, flip it ten times, and get Heads each time. The probability I have the trick coin is

$$\Pr(HH|10H) = \frac{\frac{1}{1000} \times 1}{\frac{1}{1000} \times 1 + \frac{999}{1000} \times \frac{1}{1024}} \approx \frac{1}{2}$$