Forthcoming:
Real Estate Economics

Mortgage Choice: What's the Point?

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This Draft: April 16, 1997

ABSTRACT

This paper shows that, in the presence of transaction costs payable by borrowers on refinancing, it is possible to construct a separating equilibrium in which borrowers with differing mobility select fixed rate mortgages (FRMs) with different combinations of coupon rate and points. We also show that, in the absence of such costs, no such equilibrium is possible. This provides a possible explanation for the large menus of FRMs typically encountered by potential borrowers, and suggests that the menu available at the time of origination should be an important predictor of future prepayment. We numerically implement the model, developing the first contingent claims mortgage valuation algorithm that can quantify the effect of self-selection on real contracts in a realistic interest rate setting. Our algorithm allows investors to account for self-selection when valuing mortgages and mortgage-backed securities. It also, for the first time, allows lenders to determine the optimal points/coupon rate schedule to offer to a specified set of potential borrowers, given the current level of interest rates.

*The authors thank Jan Brueckner, Dennis Capozza, Patric Hendershott, Dwight Jaffee, Ravi Jagannathan, Steve LeRoy, Frank Nothaft, Chester Spatt, Matt Spiegel, two anonymous referees, and seminar participants at U.C. Berkeley, HEC, UBC, the 1995 meeting of the American Real Estate and Urban Economics Association, and the 1995 Ohio State University Summer Real Estate Workshop for helpful comments and suggestions. We are grateful for financial assistance from the Berkeley Program in Finance, from the U.C. Berkeley Committee on Research, and from the Fisher Center for Real Estate and Urban Economics.

One of the most striking features of the U.S. mortgage market is the wide variety of loans available to potential borrowers. Not only are there different types of loan (e.g. fixed vs. adjustable rate), but even within any single type there are loans with many different combinations of interest rate and points. As an illustration of the extent of the selection available, Table 1 shows a sample of fixed rate mortgages (FRMs) available on February 13, 1996 from a single representative U.S. mortgage lender. Furthermore, recent empirical findings show that, for a given coupon rate, mortgages with low points tend to be prepaid more rapidly than mortgages with high points [see Brueckner (1994), and Hayre and Rajan (1995)], suggesting that differences among the behavioral characteristics of borrowers may be associated with the interest rate/points trade-off. The empirical evidence in favor of this relationship is so strong that many of the new-generation prepayment models on Wall Street have been redesigned to account for the effect of origination points on the speed of prepayment [see, for example, Hayre and Rajan (1995)].

Several explanations have been proposed for the existence of points. Dunn and McConnell (1981b) suggest that points serve to pay for the prepayment option embedded in fixed rate mortgages, although they do not explain why this payment should be made in the form of points, rather than via a higher coupon rate. Moreover, this story does not explain why we should see large menus of loans with different combinations of rates and points. Kau and Keenan (1987) suggest tax reasons for the existence of points, and one could imagine an extension of their story, in which diverse tax situations lead to a menu of different point/rate combinations. However, if this were the explanation, we should expect the relationship between points and mobility to be the opposite of what is observed, since high tax rate individuals, who have the greatest desire to deduct points up front, also tend to be the most mobile [see, for example, Borjas et al. (1992)]. Moreover, despite the significant difference in the tax treatment of points on a first loan versus a refinance,[1] we see no difference between the sets of contracts offered to new borrowers versus refinancers. Finally, if taxes were the explanation, we should have seen a narrowing in the range of points/coupon choices available when the range of possible marginal tax brackets narrowed after 1986. In fact, the reverse has happened. Another possible explanation is liquidity differences among borrowers. Again, however, it is hard to reconcile this story with the observed relationship between points and prepayment. The people with higher liquidity, who take out loans with higher points, will find it, on average, easier to refinance subsequently, and are relatively mobile [again, see Borjas et al. (1992)]. As a result, we should expect a *positive* relationship between points

[1] Points are deductible immediately on a first loan, but must be amortized over the life of the loan on a refinance.

and refinancing, the opposite of what is observed.

A mobility-based explanation for the existence of points was first informally proposed by Dunn and Spatt (1988). They suggest that borrowers who plan to move soon ought to take out loans with a high periodic interest rate and low points, whereas those who plan not to prepay (except possibly for interest rate related reasons) should take out loans with higher points and a lower periodic interest rate. The choice of contract thus serves as a self-selection device [see Rothschild and Stiglitz (1976)], allowing the lender to learn private information about potential borrowers' mobility. Not only is this intuition attractive, but it also agrees with the informal rule for mortgage choice advocated by most mortgage lenders, and with the recent empirical findings discussed above.

The first formal model to exhibit separation by mobility was that of Chari and Jagannathan (1989), in which two borrowers, with different expected times until they next move, choose different loans. In this model, however (counter to empirical realism), it is the borrowers who expect to move sooner who choose the loan with high points and a low interest rate.[2] Brueckner (1994) develops a model in which borrowers self-select into different loans, with longer term borrowers selecting loans with higher points and a lower coupon, as observed in practice. However, like Chari and Jagannathan (1989), he assumes constant interest rates, which prevents his model from being able to address the issue of voluntary, interest rate driven prepayment, an important feature of any fixed rate mortgage. LeRoy (1996) considers a world in which interest rates can take on one of two possible values, and allows borrowers to choose from a selection of infinitely lived, interest only, fixed rate mortgages. However, he finds that, when borrowers refinance optimally if interest rates fall, the points/coupon choice can at best serve only to separate the least mobile borrower type from all others. In his semi-pooling equilibrium, a) all but the least mobile borrowers choose the same loan; and b) all but the least mobile borrowers choose loans which they optimally refinance immediately.[3] Finally, Yang (1992) constructs a loan schedule which induces self-selection by multiple classes of borrower, but he allows the (non-competitive) lender to make arbitrarily large profits.[4]

[2]In this model, some individuals face an (uninsurable) risk of moving, and their expected income, conditional on moving, is higher than if they do not move. If they take out a loan with points and a below market interest rate, their average payment is high if they move and lower if they do not move. The contract thus provides partial insurance against moving (and its associated income shock).

[3]LeRoy interprets these loans as being adjustable rate mortgages (ARMs). However, in his model, borrowers refinance their mortgages before even making their first payment. Thus they effectively do not borrow at all, and the market breaks down.

[4]Indeed, he claims that it is impossible to construct a separating equilibrium.

The contribution of this paper is twofold. First, it constructs an equilibrium model of mortgage choice in which transaction costs play a critical role in determining the nature of the equilibrium combinations of coupon rates and loan points faced by borrowers.[5] We show that the failure of Yang (1992) and LeRoy (1996) to construct a stable[6] separating equilibrium is not due to any specific details of their implementation, but follows, rather, from the optimal prepayment assumption, combined with the fact that, in their models, all payments made by borrowers are received by lenders. Retaining the optimal refinancing assumption, but introducing transaction costs payable by borrowers, and *not* received by lenders (such as appraisal fees, credit reports etc.), we show that it now becomes possible to construct a separating equilibrium in which, as we observe in real life, a) different borrowers select different fixed rate loans with different combinations of coupon rate and points; and b) no borrower finds it optimal to prepay his or her loan immediately. There is thus a crucial distinction between points (which are a transfer from borrowers to lenders) and true transaction costs (which are paid to a third party).

Second, we contribute to the broader literature on mortgage valuation and prepayment [see, for example, Stanton (1995)], by numerically implementing our model. We develop the first contingent claims mortgage valuation algorithm that can quantify the effect of self-selection on real contracts in a realistic interest rate setting. Our algorithm allows investors to account for self-selection when valuing mortgages and mortgage-backed securities. Moreover, it is an equilibrium model, unlike the reduced form models used on Wall Street. It therefore, for the first time, allows lenders to determine the optimal points/coupon schedule to offer a specified set of potential borrowers, given the current level of interest rates.

The valuation algorithm we develop is of interest in its own right. Most recent contingent claims models which attempt to take into account the effect of refinancing costs on mortgage value and optimal prepayment behavior, make the simplifying assumption that these costs are paid only on the *first* refinance [see, for example, Timmis (1985) and Stanton (1995)]. This simplifying assumption means that a borrower's refinancing decision at any time depends only on the current loan, since no matter which loan is refinanced into, its initial value will be par. Our algorithm, like that proposed by Dunn and Spatt (1986) (of which ours is an extension), allows for refinancing costs to be paid on *each* refinancing. The optimal refinancing rule now depends not only on the loan being refinanced out of, but also on the

[5]Transaction costs thus play an even more fundamental role here than in (say) Stanton (1995), where they help to explain observed patterns of prepayment for a *given* mortgage.

[6]Stable, as used here, means that no borrower takes out a loan which he or she immediately finds it optimal to refinance.

value of the loans available should the borrower refinance. The algorithm therefore has to calculate, simultaneously, the value and optimal refinancing strategy for loans with all coupon rates, as well as the optimal set of contracts for lenders to offer for every possible interest rate.

The paper is organized as follows. Section lays out the model, describing lender and borrower objectives, and shows that, in the presence of transaction costs, it is possible to construct a separating equilibrium in which lenders offer fixed rate loans differing only in their combinations of points and coupon. It also shows that, in the absence of such costs, it is impossible to construct such an equilibrium. Section develops an algorithm which numerically implements the equilibrium, and presents results for several different sets of initial conditions. Section presents some concluding remarks.

The Model

In the model to be presented here, different classes of borrower, differing only in how long they expect to remain in their current home, select loans from a menu of self-amortizing, 30 year, fixed rate mortgages offered by lenders.[7] The loans on this menu differ only in their combination of points and coupon rate. Competitive lenders, in turn, make zero expected profit on each loan that is taken out, and have no incentive to deviate from the equilibrium menu of loans. Borrowers prepay their loans either in order to move, or because interest rates have fallen and it is optimal to refinance.

Borrowers and Prepayment

We assume that all borrowers are outwardly identical, and differ only in their mobility, measured by a (borrower-specific) hazard rate λ.[8] The higher the value of λ, the sooner the borrower is likely to move. We assume that there exists a due-on-sale clause so that, on moving at time t before the loan's maturity, the borrower must refinance the outstanding balance on the mortgage, $F(c, t)$, where c is the coupon rate on the loan. In addition to moving, borrowers may also decide to refinance their mortgages if interest rates have fallen

[7] We take the choice of a 30 year, fixed rate loan (the most common type of mortgage) as given.

[8] In other words, at time t, the probability of moving in the next interval of length δt is approximately $\lambda \, \delta t$. For expositional clarity, we shall treat λ as though it is a constant for each borrower. However, λ may vary with calendar time, mortgage age, or even the level of interest rates. What is important is that we can unambiguously rank borrowers by mobility, so that if one borrower is strictly more likely to move (i.e. has a higher hazard rate) than another at any particular time and interest rate, that borrower must always be (weakly) more likely to move.

sufficiently since the loan was taken out. Write $V_\lambda^L(c, r, t)$, (L for "lender") for the market value (ignoring any points paid) of a loan with time t to maturity, and coupon rate c, with monthly payments such that $F(c, 30 \text{ years}) = \$1$, held by a borrower of type λ, if the current interest rate is r. Write $V_\lambda^B(c, r, t)$ (B for "borrower") for the market value of the associated "synthetic security", also with time t to maturity, whose cash flows are equal to the payments made each month by the borrower.[9]

We assume borrowers act to minimize V_λ^B in two ways. First, they select the best loan from the set of available contracts, subject to a maximum possible level of points, p_{max}.[10] Second, having taken out a particular loan, they follow the optimal prepayment strategy for that loan. On refinancing, either to move or because interest rates have fallen, borrowers take out a new mortgage for the amount of the remaining principal, $F(c, t)$. In addition, they face a proportional transaction cost, X, which is *not* received by the lender. This cost represents the direct monetary costs of refinancing (appraisal fees, title search etc.), as well as non-monetary costs (representing, for example, the inconvenience and time involved in the refinancing process). If the borrower refinances at time t, the value of the borrower's future stream of payments is thus

$$F(c,t)\left[V_\lambda^B(c_\lambda^*(r,t),r,t)/F(c_\lambda^*(r,t),t) + p_\lambda^*(r,t) + X\right], \tag{1}$$

where $c_\lambda^*(r, t)$ is the coupon rate on the new loan taken out by the borrower (to be determined endogenously as part of the valuation procedure), and $p_\lambda^*(r, t)$ is the level of points paid (as a percentage of principal) by the borrower to the lender. The optimal rule for the borrower is to refinance if

$$V_\lambda^B(c,r,t) \geq F(c,t)\left[V_\lambda^B(c_\lambda^*(r,t),r,t)/F(c_\lambda^*(r,t),t) + p_\lambda^*(r,t) + X\right]. \tag{2}$$

When the borrower refinances, the amount received by the lender is

$$F(c,t)\left[V_\lambda^L(c_\lambda^*(r,t),r,t)/F(c_\lambda^*(r,t),t) + p_\lambda^*(r,t)\right], \tag{3}$$

[9] It is necessary to distinguish between $V_\lambda^L(c, r, t)$ and $V_\lambda^B(c, r, t)$, since borrowers pay transaction costs on refinancing which are not received by lenders. Since the cash flows paid out by borrowers exceed those received by lenders, $V_\lambda^L(c, r, t) < V_\lambda^B(c, r, t)$ [see, for example, Dunn and Spatt (1986)].

[10] Note that the construction of the separating equilibrium does *not* require the existence of such a p_{max}. This value, motivated by liquidity considerations outside the model, does not change the qualitative features of the equilibrium [see Section]. It merely allows us to calibrate the model, ensuring that the points paid by borrowers in the model are close to those we see in the market.

which is less than the value of the borrower's payments because the lender does not receive the transaction costs.

We emphasize that our model is not a full general equilibrium specification of mortgage contract structure. In particular, we value the mortgage contracts, and determine the optimal refinancing strategy of borrowers, assuming they value their loans as if they are redundant.[11] In making this simplification, we are following one of the standard approaches used in much of the recent academic literature on mortgage prepayment and pricing, including (among many others) Dunn and McConnell (1981a,b), Dunn and Spatt (1986), Brennan and Schwartz (1985) and Stanton (1995). Its major advantage is that it allows us to use the well-developed theory of contingent claims valuation to calculate explicit numerical values for realistic loans under an arbitrarily complex/realistic interest rate model, determine borrowers' optimal refinancing strategies, and (in the present case) determine the optimal menu of loans to be offered in different interest rate environments.[12]

Lenders, Adverse Selection and Equilibrium

Assume that lenders operate in a competitive market, with costless entry and exit. Lenders may know the distribution of borrowers' types, but cannot observe the type of any individual borrower. This leads to a potential adverse selection problem for lenders,[13] since the borrower's mobility has a significant impact on the value of the cash flows received by the lender. Lenders thus have an incentive to discover borrowers' types. In an attempt to discover borrowers' types, lenders may offer a menu of prepayable, 30-year fixed rate loan contracts, differing in their tradeoff between points and coupon rate.

Define $\mathcal{L} \subseteq I\!\!R^+$ to be the set of all possible borrower types. Write a mortgage menu as the set of ordered pairs $\{(c_\lambda, p_\lambda) : \lambda \in \mathcal{L}\}$. This menu defines an equilibrium at current interest

[11] In other words, taken literally, they can frictionlessly trade in marketed securities that replicate their mortgage cash flows. We therefore cannot, for example, explain why they borrow money using mortgage contracts with substantial refinancing costs.

[12] An alternative approach, followed in several previous models of mortgage choice, is to assume that lenders are risk-neutral, while borrowers are risk-averse [see, for example, Brueckner (1994) and Chari and Jagannathan (1989)]. This approach recognizes that borrowers and lenders face different constraints, and is very useful in deriving qualitative intuition for the impact of these differences. However, it is still only an approximation, since it does not attempt to solve borrowers' full optimal intertemporal consumption and investment problem, which would be necessary to derive the full relationship between the borrower's state dependent marginal utility of consumption and the likelihood of moving. Moreover, these models assume constant interest rates, preventing their being able to address the issue of interest-driven prepayment. This approach has thus not typically been used when the focus is on obtaining numerical results, rather than qualitative intuition.

[13] See Dunn and Spatt (1988) for a good discussion of asymmetric information in mortgage markets.

rate r, if

1. (Zero profit) For all $\lambda \in \mathcal{L}$, $E\left[V_l^L(c_\lambda, r, t)\right] = F(c_\lambda, t)(1 - p_\lambda)$, where the expectation is taken over all mortgage holders with type l satisfying $c_l = c_\lambda$ and $p_l = p_\lambda$. In other words, the total value of the payments received by the lender (loan payments plus points) equals the face value of the loan.[14]

2. (Incentive compatibility) For all $\lambda, l \in \mathcal{L}$,

   $$V_\lambda^B(c_\lambda, r, t)/F(c_\lambda, t) + p_\lambda \leq V_\lambda^B(c_l, r, t)/F(c_l, t) + p_l,$$

   i.e. each borrower chooses the optimal loan for his or her type.

The equilibrium is *fully separating* if, whenever $l_1, l_2 \in \mathcal{L}$ and $l_1 \neq l_2$, we have both $c_{l_1} \neq c_{l_2}$ and $p_{l_1} \neq p_{l_2}$.

We present here two preliminary results which will be important later.

Lemma 1 *For a given coupon rate, c, the value of the borrower's cash flows, $V_\lambda^B(c, r, t)$, is an increasing function of λ. It is strictly increasing in λ as long as it is currently optimal not to refinance the loan.*

Proof: See Appendix.

The intuition behind this result is that higher values of λ imply higher mobility, leading to increased prepayment when it would not otherwise be optimal. This in turn increases the likelihood that the borrower will have to pay the transaction costs associated with refinancing.

Lemma 2 *In the presence of transaction costs, and if it is not currently optimal for the borrower to refinance, the difference $\left[V_\lambda^B(c, r, t) - V_\lambda^L(c, r, t)\right]/F(c, t)$ is positive, and strictly increasing in c.*

[14] Zero profit on average across all loans is not sufficient, since entry would occur only in the profitable markets.

Proof: This result follows from Dunn and Spatt (1986), property 8.

The intuition behind this result is that a higher coupon rate implies a greater likelihood of future refinancing being optimal, and hence a greater likelihood that the borrower will have to pay the transaction costs associated with refinancing.

Now consider Figure 1. The solid line is the lender's zero-profit line for a particular borrower type, the set of (p, c) pairs that satisfy the equation (for a particular interest rate, r),

$$
\begin{aligned}
V_\lambda^L(c, r, t)/F(c, t) &= (1 - p), \qquad \text{i.e.} && (4)\\
p &= 1 - \frac{V_\lambda^L(c, r, t)}{F(c, t)}. && (5)
\end{aligned}
$$

The dashed lines are different borrower indifference curves, i.e. each is the set of (p, c) pairs satisfying the equation

$$
\begin{aligned}
V_\lambda^B(c, r, t)/F(c, t) + p &= K, \qquad \text{i.e.} && (6)\\
p &= K - \frac{V_\lambda^B(c, r, t)}{F(c, t)}, && (7)
\end{aligned}
$$

for some constant K. A key property of this figure (an immediate consequence of Lemma 2) is that the indifference curves are less steep than the zero-profit line. Intuitively, as the coupon rate on the loan increases, the likelihood of future interest rate driven refinancing increases, thus increasing the present value of future refinancing costs paid by borrowers, and increasing the difference between the value of the loan to the borrower and the value to the lender. This means that to be indifferent between a low coupon loan and a high coupon loan, the borrower will insist on lower points for the high coupon loan than the lender.

Proposition 1 *In the absence of asymmetric information, but in the presence of refinancing costs, with competition between lenders, all borrowers will choose loans with the maximum possible points, p_{max}.*

Proof: See Appendix.

In equilibrium, the value of a new borrower's liability equals the face value of the loan plus the present value of all future refinancing related transaction costs. Thus, minimizing the value of the liability is equivalent to minimizing the present value of these transaction costs. The intuition behind Proposition 1 is that the lower the coupon rate, the less likely the borrower is to want to refinance in the future, and hence the lower the present value of the transaction costs (see Lemma 2). The refinancing costs are thus minimized by taking out a loan with the highest points possible (which, in turn, leads to the lowest possible coupon rate).

Define two borrower types h and l, with borrower-specific hazards such that $\lambda_h > \lambda_l$. We shall here make one additional assumption, which is needed only to prove Proposition 2, and has no bearing on any of our other results.

A1: The difference $\left[V^B_{\lambda_h}(c, r, t) - V^B_{\lambda_l}(c, r, t)\right] / F(c, t)$ is strictly decreasing in the coupon rate, c.

Note that the truth or otherwise of this assumption depends on the particular state of the world and interest rate specification we are using. This is because increasing the coupon rate has two different, and opposing, effects:

1. As the coupon rate increases, it becomes optimal to refinance at higher and higher interest rates [see, for example, Dunn and Spatt (1986), property 3], reducing the number of possible states of the world in which the high mobility borrower might refinance when it is not optimal to do so (due to moving). This works in the direction of A1.
2. As the coupon rate increases, the remaining balance on the loan should a borrower refinance in the future also increases, due to the change in the amortization schedule. This *increases* the transaction costs paid, working against A1.

Although factor 1 will often dominate factor 2, in which case A1 will be true, it is possible even within a single model for A1 to be true for some coupon rates, and false for others. For example, consider a world in which interest rates can take on only a discrete number of values. If the coupon rate is such that any increase makes refinancing optimal at some new interest rate, say r_j, by making the increase small enough we can make factor 1 dominate factor 2, and the difference will indeed be *decreasing* in the coupon rate. On the other hand, there will be other coupon rates where we can increase the coupon by a discrete amount without making any change to the optimal refinancing strategy of either borrower. At such coupon rates, factor 2 will dominate factor 1, and the difference will therefore be *increasing* in the coupon rate.

Proposition 2 *Given assumption A1, then in the presence of asymmetric information, with or without refinancing costs, no pooling equilibrium (where all borrowers choose the same contract) can exist.*

Proof: See Appendix for a proof of this result [a standard screening result - see, for example, Rothschild and Stiglitz (1976)].

Proposition 3 *In the presence of asymmetric information, but with no refinancing costs, no stable separating equilibrium can exist.*

This result explains the failure of Yang (1992) and LeRoy (1996) to construct a stable separating equilibrium. It is in stark contrast to Rothschild and Stiglitz (1976), and is driven by the fact that, in the absence of transaction costs, borrowers and lenders both assign the same value to the same set of cash flows. To prove this proposition, suppose such an equilibrium did exist, and suppose borrowers h and l (with $\lambda_h > \lambda_l$) select loans (p_h, c_h) and (p_l, c_l), which they do not optimally choose to refinance immediately. By the zero profit condition, we must have

$$V^L_{\lambda_h}(c_h, r, t)/F(c_h, t) + p_h = V^L_{\lambda_l}(c_l, r, t)/F(c_l, t) + p_l = 1. \tag{8}$$

In the absence of transaction costs, $V^B = V^L$, since all cashflows paid out by the borrower are received by the lender. Hence

$$V^B_{\lambda_h}(c_h, r, t)/F(c_h, t) + p_h = V^B_{\lambda_l}(c_l, r, t)/F(c_l, t) + p_l = 1. \tag{9}$$

But, by Lemma 1,

$$V^B_{\lambda_l}(c_h, r, t) < V^B_{\lambda_h}(c_h, r, t). \tag{10}$$

Combining Equations (9) and (10), we obtain

$$V^B_{\lambda_l}(c_h, r, t)/F(c_h, t) + p_h < 1, \tag{11}$$

so borrower l prefers the loan that is supposed to be taken out by borrower h, contradicting the incentive compatibility equilibrium condition.

To prove our main result, which emphasizes the importance of including refinancing costs in the analysis, we need one further assumption. Let λ_h and λ_l be as above, with $\lambda_h > \lambda_l$.

A2: $V^L_{\lambda_h}(c_l, r, t) > V^L_{\lambda_l}(c_l, r, t)$, where c_l is the coupon rate on the loan taken out in equilibrium by borrower l (which does not depend on borrower h).

Note that this assumption is exactly the same as Lemma 1, except that it relates to the *lender's* valuation, rather than the borrowers'. When transaction costs are zero, $V^B \equiv V^L$, so from Lemma 1, A2 *must* be true with zero transaction costs. More generally, A2 will be true for some range of transaction costs around zero. Moreover, it will also hold for *any* interest rate and *any* level of transaction costs, as long as we allow low enough coupon rates. To see this, consider a loan with zero coupon rate. With a zero coupon rate, the scheduled payment is \$1/360 per month, and interest rate driven refinancing will never occur. The total *nominal* payment received by lenders is identical for all borrowers (\$1, since the coupon rate is zero), but the longer the horizon, the longer the average time to each payment, the lower the present value of the payments, and so the higher the points that must be paid to make the total value of points + payments equal to par. Hence A2 will always hold at a zero coupon rate, regardless of interest rate or transaction cost level.

For higher coupon rates and high enough transaction costs, assumption A2 may fail to hold if the term structure is downward sloping. For example, suppose transaction costs are infinite. Then neither borrower will ever refinance for interest rate reasons, so we can regard both loans as nonprepayable. Because the term structure is downward sloping, long term interest rates are lower than short term rates. Borrower l on average makes payments for longer than borrower h, so for a no point loan, borrower h ought to pay a higher coupon rate than borrower l. Hence, at a given coupon rate between these two values, $V^L_{\lambda_h} < V^L_{\lambda_l}$, contradicting A2.

Proposition 4 *In the presence of both asymmetric information* and *refinancing costs, it is possible to construct a stable mortgage schedule which separates borrowers h and l, as long as Assumption A2 is satisfied.*

We shall prove this by constructing the equilibrium. Figure 3 shows the zero profit lines for the two borrower types. The zero profit line for borrower l lies to the right of that of borrower h, by assumption A2. The dashed line is borrower l's indifference curve through the first-best contract, i.e. the set of (p, c) pairs that satisfy the equation

$$V_{\lambda_l}^B(c, r, t)/F(c, t) + p = V_{\lambda_l}^B(c^*, r, t)/F(c^*, t) + p^*. \tag{12}$$

The intersection of this indifference curve and borrower h's zero-profit line is guaranteed by Lemma 2 and the fact that eventually, as we move far enough to the left, both zero profit lines will meet on the zero point axis.[15] The point at which the two lines cross is the loan selected by borrower h, which satisfies both the lender's zero profit condition and the borrower's incentive compatibility condition.[16]

Extending this construction to more than two classes of borrower is straightforward, as long as a condition analogous to A2 holds for each successive pair of borrowers. For example, Figure 4 shows the construction for three borrowers, h (high mobility/short horizon), m (medium mobility/horizon), and l (low mobility/long horizon). The construction for borrowers m and l is exactly as above. The loan for borrower h lies at the intersection of the lender's zero profit line for borrower h and the indifference curve of borrower m that passes through borrower m's equilibrium contract.

Numerical Implementation

Interest Rates

To implement the model we need to make assumptions about movements in interest rates. We assume interest rate movements are described by the Cox, Ingersoll and Ross (1985) one-factor model.[17] In this model, the instantaneous risk-free interest rate r_t satisfies the stochastic differential equation

$$dr_t = \kappa(\mu - r_t)\, dt + \sigma\sqrt{r_t}\, dz_t. \tag{13}$$

[15]For a high enough coupon rate, it is optimal for both borrowers to refinance immediately [see, for example, Dunn and Spatt (1986), property 2], resulting in an immediate return of principal to the lender. To make zero profit, the lender must therefore charge zero points to either borrower.

[16]It is possible (as in Rothschild and Stiglitz (1976)) that this schedule might not in fact represent an equilibrium, if a single contract that is preferred by both borrower types can be introduced. We can rule out such contracts, however, using Riley's (1979) definition of a reactive equilibrium.

[17]This is one of the most commonly used interest rate models. It has been applied to the valuation of mortgages by, among others, Dunn and McConnell (1981a,b) and Stanton (1995).

This equation says that, on average, the interest rate r converges toward the value μ. The parameter κ governs the rate of this convergence. The volatility of interest rates is $\sigma\sqrt{r_t}$. One further parameter, q, which measures the market price of interest rate risk, is needed to price interest-rate dependent assets. The parameter values used in this paper are those estimated by Pearson and Sun (1989):

$$\begin{aligned}
\kappa &= 0.29368, \\
\mu &= 0.07935, \\
\sigma &= 0.11425, \\
q &= -0.12165.
\end{aligned}$$

The long-run mean interest rate is 7.9%. Ignoring volatility, the time required for the interest rate to drift half way from its current level to the long-run mean is $\ln(1/2)/(-\kappa) \approx 2.4$ years.

Given this model for movements in r_t, we can now calculate the value of the mortgage using the fact that $V(r_t, t)$, the value of any interest rate contingent claim paying coupons or dividends at rate $C(r_t, t)$, satisfies the partial differential equation

$$\frac{1}{2}\sigma^2 r V_{rr} + [\kappa\mu - (\kappa + q)r]\, V_r + V_t - rV + C = 0. \tag{14}$$

Solving this equation, subject to a payout rate $C(r_t, t)$ and boundary conditions appropriate to the asset being valued,[18] yields the asset value $V(r_t, t)$.

Valuation and Optimal Prepayment Strategy

Natural boundaries for the interest rate, r, are 0 and ∞. Rather than solving Equation (14) directly, we use the transformation

$$y = \frac{1}{1 + \gamma r}, \tag{15}$$

[18]For example, for a zero coupon bond the payout rate, C, is zero, and its value at maturity is \$1. For a mortgage, there are constant scheduled monthly payments, the terminal value is zero, and we need in addition an optimal exercise condition for the embedded prepayment option.

for some constant $\gamma > 0$,[19] to map the infinite range $[0, \infty)$ for r onto the finite range $[0, 1]$ for y. The inverse transformation is

$$r = \frac{1-y}{\gamma y}. \tag{16}$$

Equation (15) says that $y = 0$ corresponds to "$r = \infty$" and $y = 1$ to $r = 0$. Next, rewrite Equation (14) using the substitutions

$$\begin{aligned} U(y,t) &\equiv V(r(y),t), && (17)\\ V_r &= U_y \frac{dy}{dr}, && (18)\\ V_{rr} &= U_y \frac{d^2y}{dr^2} + U_{yy}\left(\frac{dy}{dr}\right)^2, && (19) \end{aligned}$$

to obtain

$$\frac{1}{2}\gamma^2 y^4 \sigma^2 r(y) U_{yy} + \left(-\gamma y^2\left[\kappa\mu - (\kappa + q) r(y)\right] + \gamma^2 y^3 \sigma^2 r(y)\right) U_y + U_t - r(y)U + C = 0. \tag{20}$$

To value a single mortgage, we use the Crank-Nicholson finite difference algorithm[20] to solve Equation (20). Using this algorithm involves replacing the derivatives that appear in Equation (20) with equations involving the differences between the values of the asset at neighboring points on a discrete grid of y and t values. For convenience we use a time interval of one month, yielding a total of 360 intervals in the time dimension. The algorithm works backward one period at a time to calculate the value of the mortgage liability conditional on the prepayment option remaining unexercised, $V_u^B(r,t)$.[21] The value of the mortgage liability if the prepayment option is exercised is the amount repaid, including transaction costs,

$$F(c,t)\left[V_\lambda^B(c_\lambda^*(r,t), r, t)/F(c_\lambda^*(r,t), t) + p_\lambda^*(r,t) + X\right].$$

[19] We solve Equation (14) numerically on a rectangular grid of interest rate and time values. The finer the grid, the better will be our approximation to the solution of Equation (14). However, the processing time is proportional to each grid dimension. For a given grid size in the y direction, the denser the implied r values are in the range corresponding to observed interest rates (say 4% to 20%), the better will be our approximation. We can affect this density by our choice of the constant γ. The larger the value of γ, the more points on a given y grid correspond to values of r less than 20%. Conversely, the smaller the value of γ, the more points on a given y grid correspond to values of r greater than 4%. As a compromise between these two objectives, $\gamma = 12.5$ was used. The middle of the range, $y = 0.5$, corresponds to $r = 8\%$.

[20] See McCracken and Dorn (1969) for a discussion of this algorithm.

[21] Subscript u for "unexercised".

It is optimal to refinance the mortgage if

$$V_u^B(c,r,t) \geq F(c,t)\left[V_\lambda^B(c_\lambda^*(r,t),r,t)/F(c_\lambda^*(r,t),t) + p_\lambda^*(r,t) + X\right].$$

Otherwise it is optimal not to refinance. For notational clarity, define

$$\pi = 1 - e^{-\lambda/12}, \tag{21}$$

the probability of exogenous prepayment this month. The value of the mortgage liability is then

$$V^B(c,r,t) = \begin{cases} F(c,t)\left[V_\lambda^B(c_\lambda^*(r,t),r,t)/F(c_\lambda^*(r,t),t) + p_\lambda^*(r,t) + X\right] & \text{if this } \leq V_u^B, \\ (1-\pi)V_u^B + \pi F(c,t)\left[V_\lambda^B(c_\lambda^*(r,t),r,t)/F(c_\lambda^*(r,t),t) + p_\lambda^*(r,t) + X\right] & \text{otherwise.} \end{cases} \tag{22}$$

To determine the value of the lender's asset, V^L, the process is similar. When the prepayment option is exercised, the security owner receives the remaining principal balance on the mortgage, F_t. The value of the mortgage to the lender is thus

$$V^L(c,r,t) = \begin{cases} F(c,t) & \text{if } F(c,t)\left[V_\lambda^B(c_\lambda^*(r,t),r,t)/F(c_\lambda^*(r,t),t) + p_\lambda^*(r,t) + X\right] \leq V_u^B, \\ (1-\pi)V_u^L + \pi F(c,t) & \text{otherwise.} \end{cases} \tag{23}$$

This parallels Equation (22) above with each V^B replaced by V^L, and with a different payoff if the mortgage is prepaid. Note that this is simpler than the expression for the lender's payoff given in Equation (3), since in equilibrium

$$F(c,t)\left[V_\lambda^L(c_\lambda^*(r,t),r,t)/F(c_\lambda^*(r,t),t) + p_\lambda^*(r,t)\right] = F(c,t). \tag{24}$$

Determining the Contract Rate and Points on a Newly Issued Loan

Equations (22) and (23) involve the coupon rate and points (and value) on a newly issued loan, c^* and p^*. These are determined endogenously as part of the valuation procedure. For each interest rate, we start with the least mobile borrower, and determine the newly issued contract for each borrower in turn, in increasing order of mobility.

For the least mobile borrower (borrower l), we know the points, p_{max}, so we just need to find the coupon rate. To do this, we calculate the lender's value, $V^L(c, r, t)$, for every possible coupon rate.[22] The coupon rate on a newly issued loan, c^*, is the coupon rate at which $V^L + p_{max}$ equals par.[23] This gives us both $c^*(r, t)$ and $V^B(c^*, r, t)$.[24]

For the next least mobile borrower (borrower h), we need to calculate both the coupon rate *and* the points on a newly issued loan. The new loan offered to borrower h lies at the intersection of the lender's zero profit line (for loans issued to borrower h) and borrower l's indifference curve through the contract we just found.[25] Starting at the smallest coupon rate and working upwards, for each rate calculate the number of points required to ensure that the loan lies on borrower l's indifference curve,

$$p = V^B(c^*) + p_{max} - V^B(c). \tag{25}$$

The coupon rate on a newly issued loan is determined by the value of c at which $V^L + p$ first drops *below* par,[26] where the number of points, p, is given by Equation (25). For subsequent borrowers, we repeat this process, using borrowers m and l, then h and m, etc.

This algorithm is based on one proposed by Dunn and Spatt (1986), which also values loans taking into account the possibility of multiple rounds of refinancing. However, the situation here is substantially more complex. First, whereas Dunn and Spatt value only a single loan, assuming zero points, we value multiple loans, determining both the coupon rate *and* points for each. Second, Dunn and Spatt do not include exogenous refinancing in their model (determined by λ here). This means that they can be sure that, whenever a borrower refinances, he or she is refinancing into a loan with a strictly *lower* coupon rate than the current loan. Hence, they can value the lowest coupon loan first (which will never be refinanced), then the next lowest (which might be refinanced, but only into the lowest coupon loan), and so on, valuing the loans one at a time in increasing order of coupon rate. We cannot do this here. Because borrowers may move when interest rates are high, they

[22] Ignoring the possibility of early exercise. This will not cause any problems as it is never optimal to refinance a newly issued loan immediately, due to the presence of transaction costs.

[23] Since we are using a discrete set of possible coupon rates, in practice we find the two consecutive coupon rates, c_i and c_{i+1} such that $V^L(c_i, r, t) + p_{max}$ is below par, and $V^L(c_{i+1}, r, t) + p_{max}$ is above par, then interpolate to obtain the newly issued coupon rate.

[24] $V^L(c^*, r, t)$ is, of course, equal to $1 - p_{max}$.

[25] See Figure 3.

[26] Note that this is the opposite of the first borrower. From Figure 3, for the first borrower we followed the line $p = p_{max}$ vertically upwards, crossing the zero profit line from *below*. For borrower l, we are following borrower h's indifference curve, which crosses the zero-profit line for borrower l from *above*.

may refinance into a loan with a coupon rate *higher* than that of the current loan. We can thus no longer value the loans one at a time, as in Dunn and Spatt (1986). The values and optimal prepayment policies for loans with all coupon rates, as well as the optimal set of contracts for lenders to offer for every possible interest rate, all need to be determined simultaneously.

Numerical Results

The algorithm described above was used to value 30-year fixed-rate mortgages, assuming various initial yield curves, chosen by three classes of borrower with speed-of-moving parameters $\lambda_l = .05$, $\lambda_m = .067$, and $\lambda_h = .1$.[27] The transaction cost payable on refinancing, X, is 5% of remaining principal, and is the same for all borrower classes. The maximum number of points, p_{max}, is set to 0.1 (10 points).

Figure 5 depicts three possible yield curves: upward sloping, flat and downward sloping. Figure 6 shows the results for the upward sloping yield curve. As before, the solid lines are the lender's zero profit lines and the dashed lines are borrower indifference curves. The solid line furthest to the right is the zero profit curve for a lender issuing a thirty-year mortgage to the longest horizon borrower ($\lambda_l = .05$). As above, the longest horizon borrower chooses a loan with the lowest possible coupon rate and highest possible points. 10 points corresponds to a coupon rate of 6.9% for this borrower. The medium and short horizon borrowers select progressively higher coupon and lower point combinations, all three contracts yielding zero profit to the lender. The spread between the coupon rates on the loan taken by the longest borrower and that of the shortest borrower is approximately 200 basis points, and there is a corresponding difference of 11 points.

Figure 7 shows corresponding results for the flat yield curve depicted in Figure 5. The general pattern is similar, but the differences between the contracts are less marked than for the upward sloping yield curve. The difference between the coupon rates of the loans selected by the shortest and longest horizon borrowers is again roughly 200 basis points, but there is only a 9 point difference. The zero profit lines are closer together because the option value for the longer class of borrower falls due to the lower likelihood of non-interest rate driven refinancing, and the lower forward rates at long maturities. These differences reveal very important information about the characteristics of the borrowers taking out the loans. Ignoring this may lead to significant errors in predicting prepayment, and hence to errors in

[27]In other words, the expected time until the three borrowers next move is $1/\lambda = 20$ years, 15 years and 10 years respectively.

valuing and hedging mortgages.[28]

Figure 8 shows the results for the downward sloping yield curve depicted in Figure 5. Here again the general pattern is similar, but the differences between the contracts are even less marked than for the flat yield curve. The difference between the coupon rates of the loans selected by the shortest and longest horizon borrowers is again about 200 basis points, but there is now only a 7.7 point difference.

Table 2 presents simulation results for four possible yield-curve environments, ranging from upward sloping to steeply downward sloping (corresponding to short rates of 4.5%, 8%, 11.5% and 15%). The table compares the equilibrium points/coupon choices of high and low mobility borrowers for various restrictions on the number of points paid, transaction cost levels, X, and mobility parameters, λ_h and λ_l. Each equilibrium points/coupon combination generates zero profit to the lender. In the base case, the high mobility borrower has an expected horizon of 15 years ($\lambda_h = .067$), and the low mobility borrower has an expected horizon of 20 years ($\lambda_l = .05$). The maximum points level in the market is set to 10% of the initial balance, and transaction costs are assumed to be 5% of the remaining balance. These parameter values are the same as those used in Figures 6–8, and the numerical results are similar. The coupon rates paid by both borrowers are monotonically increasing in the riskless interest rate, as is the number of points paid by the high mobility borrower, which ranges from 1% at a short rate of 4.5% (upward sloping) to 6% at a short rate of 15% (downward sloping).

The second and third sets of simulations in Table 2 consider the impact of changing the maximum number of points paid by the low mobility borrower. In the second set of simulations, no maximum is imposed ($p_{max} = \infty$). The low mobility borrower therefore chooses a loan with a zero coupon rate, to avoid interest rate driven refinancing completely. All other assumptions about transaction costs and mobility are the same as in the base case. The high mobility borrower pays slightly more points than in the base case, but much lower than those paid by the low mobility borrower (ranging from 1.65 to 14.9 points). Thus relaxing the assumption of a maximum level of points paid by the low mobility borrower still leads to a separating equilibrium with realistic combinations of points and coupon selected by lower mobility borrowers. In the third set of simulations, the maximum level of points is set to 5% of the initial balance. Both borrowers now pay a somewhat higher coupon rate and lower points than in the base case.

[28] This was first pointed out by Dunn and Spatt (1988).

The transaction costs are increased to 10% and 20% of the remaining balance, respectively, in the next two sets of simulations. The cap on points is returned to the base case level of 10%. The results indicate that the points/coupon trade-off is relatively insensitive to transaction cost increases in all four yield-curve scenarios. As the cost of refinancing increases, the coupon rate paid by the long horizon borrower decreases slightly (keeping points constant), due to the reduced value of the prepayment option. For the high mobility borrower, on the other hand, the coupon rate increases at low interest rates, but decreases at higher interest rates. The points paid by the high mobility borrower do the opposite, decreasing at low interest rates, and increasing at higher interest rates. Note that in the upward sloping yield curve environment with a 4.5% short rate, high mobility borrowers actually require negative points in equilibrium. However, points paid plus transaction costs always exceeds zero. Borrowers never get "cash back".[29]

The last three sets of simulations demonstrate the effect of differences in the relative mobility of the low and high mobility borrowers when all other base case assumptions hold. When the horizon of the high mobility borrowers is decreased from 15 to 10 years ($\lambda_h = .1$), the high mobility borrower consistently chooses lower levels of equilibrium points and a higher coupon rate than in the base case. When the high mobility borrower's horizon is further reduced to 5 years ($\lambda_h = .2$), we see even lower points, and even higher coupon rates. When the long horizon borrower's decreases to 10 years ($\lambda_l = 0.1$), keeping the short horizon borrower's horizon at 5 years (last set of simulations), the long horizon borrower's coupon rate goes down compared with the previous set of results, due to the reduced value of the prepayment option. At the same time, the high mobility borrower's points increase slightly, and the coupon rate decreases.

From these results it is clear that the mobility of each class of borrower, the shape of the yield curve, and (to a lesser extent) the level of transaction costs are all important in determining the optimal menu of contracts offered by lenders. Our model allows lenders, for the first time, to quantify the impact of all of these factors in a realistic interest rate environment.

[29] This is ruled out by arbitrage arguments [see, for example, Dunn and Spatt (1986), property 9]. To see this, suppose in equilibrium a borrower with transaction costs of 5% selected a loan with points of -6%. Refinancing immediately would result in (6% - 5%) = 1% cash back, and the same loan as he or she started with. Doing this repeatedly would yield an arbitrarily large arbitrage profit.

Summary and Concluding Remarks

One of the most striking features of the U.S. mortgage market is the wide choice of fixed rate mortgages (FRMs), differing in their points/coupon tradeoff, available to potential borrowers. Dunn and Spatt (1988) suggested informally that the existence of such menus of FRMs could serve as a self-selection device [see Rothschild and Stiglitz (1976)], allowing the lender to learn private information about potential borrowers' mobility. This is consistent with recent empirical evidence that, for a given coupon rate, mortgages with low points tend to be prepaid more rapidly than mortgages with high points. However, previous attempts to model this formally, while at the same time allowing interest rate driven refinancing [see, for example, LeRoy (1996) and Yang (1992)], have been unable to construct an equilibrium in which multiple classes of borrower select different fixed rate loans. We show that this is not a result of the specific implementation of these previous models, but follows, rather, from the optimal prepayment assumption, combined with the fact that, in these models, all payments made by borrowers are received by lenders. Retaining the optimal refinancing assumption, but introducing transaction costs payable by borrowers, and *not* received by lenders (such as appraisal fees, credit reports etc.), we show that it now becomes possible to construct a separating equilibrium in which different borrowers select fixed rate loans with different combinations of coupon rate and points. There is thus a crucial distinction between points (which are a transfer from borrowers to lenders) and true transaction costs (which are paid to a third party).

We contribute to the broader literature on mortgage valuation and prepayment [see, for example, Stanton (1995)], by numerically implementing our model. We develop the first contingent claims mortgage valuation algorithm that can quantify the effect of self-selection on real contracts in a realistic interest rate setting. Our algorithm allows investors to account for self-selection when valuing mortgages and mortgage-backed securities. Moreover, it is an equilibrium model, unlike the reduced form models used on Wall Street. It therefore, for the first time, allows lenders to determine the optimal points/coupon schedule to offer a specified set of potential borrowers, given the current level of interest rates. Our numerical simulations show that the mobility of each class of borrower, the shape of the yield curve, and (to a lesser extent) the level of transaction costs are all important in determining the optimal menu of contracts offered by lenders.

Finally, while our model clearly shows that offering a menu of contracts with differing point/coupon combinations can provide a mechanism for lenders to learn private information about borrower mobility, this is not achieved costlessly. All but the longest horizon borrowers refinance more often than they would in the absence of asymmetric information, incurring the deadweight costs associated with such refinancing. If all borrowers knew *exactly* when they would move, lenders could persuade them to self-select costlessly by offering a menu of contracts with maturities equal to their horizons, rather than different points. Even with uncertainty about the actual time of moving (as in our model), the likelihood of a long horizon borrower having to refinance a short horizon loan is still higher than the likelihood of a short horizon borrower refinancing the same loan, making such a loan relatively less attractive to the long horizon borrower. The recent proliferation of loans with many different horizons[30] may thus represent an attempt by lenders to persuade borrowers to self-select with lower deadweight costs.[31]

[30]For example, it is now possible to take out a fixed rate loan, amortized using a 30 year schedule, but with a balloon payment due in 3, 5, 7 or 10 years, rather than the traditional 30 years.

[31]Consistent with this story is the fact that in Canada, where mortgages typically have prepayment penalties (unlike U.S. mortgages), banks tend to offer menus of loans distinguished primarily by term, rather than points. The existence of prepayment penalties makes refinancing more expensive in Canada, thus increasing the incentive for lenders to separate borrowers by an alternative mechanism to the points/coupon tradeoff.

Proofs

Proof of Lemma 1

Consider two borrowers h and l, with $\lambda_h > \lambda_l$. To prove the first part of the proposition, note that the low mobility borrower, l, can exactly imitate the high mobility borrower by

1. Randomly refinancing with a hazard rate $\lambda_h - \lambda_l$ (in addition to the exogenous hazard rate λ_l).
2. When choosing whether to refinance, following exactly the optimal strategy of borrower h.

This results in cash flows that are indistinguishable from the optimal policy of borrower h, and hence a value of $V^B_{\lambda_h}(c, r, t)$. This is one feasible policy for borrower l, so the optimal policy for borrower l must result in a value at least this low, hence

$$V^B_{\lambda_l}(c, r, t) \leq V^B_{\lambda_h}(c, r, t). \tag{26}$$

For clarity, we shall prove the strict inequality assuming that exogenous prepayment occurs only at discrete times, rather than continuously.[32] For example, assume that exogenous prepayment can occur on one occasion per month, with probability π equal to the total probability of exogenous prepayment at any time during a month given a hazard rate λ,

$$\pi = 1 - e^{-\lambda/12}.$$

Consider the value of the prepayment strategies immediately prior to such a date, V^{B-}, as a function of the values immediately after, V^{B+}, and assume that borrower l will exactly imitate borrower h after the current date, but will not deliberately increase his or her hazard rate this period. Thus

$$V^{B-}_{\lambda_h} = (1 - \pi_h) V^{B+}_{\lambda_h} + \pi_h F \left[V^{B+}_{\lambda_h}(c^*, r, t)/F^* + p^* + X \right], \tag{27}$$

$$\widehat{V}^{B-}_{\lambda_l} = (1 - \pi_l) V^{B+}_{\lambda_h} + \pi_l F \left[V^{B+}_{\lambda_h}(c^*, r, t)/F^* + p^* + X \right]. \tag{28}$$

[32] Essentially the same proof works in continuous-time, writing the value of an asset as the sum of its expected discounted payoffs under a "risk-neutral" probability distribution.

Since it is not optimal for borrower h to refinance, we must have

$$V_{\lambda_h}^{B+} < F\left[V_{\lambda_h}^{B} + (c^*, r, t)/F^* + p^* + X\right]. \tag{29}$$

Thus, since $\pi_h > \pi_l$,

$$V_{\lambda_l}^{B-} \leq \widehat{V}_{\lambda_l}^{B-} < V_{\lambda_h}^{B-}. \tag{30}$$

If the asset's value is strictly lower immediately prior to an exogenous refinance date, it must (to prevent arbitrage) be strictly lower at all prior dates, provided there is a positive probability of reaching such a date.

Proof of Proposition 1

Since lenders can observe a borrower's type, we can, without loss of generality, consider a single borrower of type λ. In equilibrium, the lender must make zero profit from the loan selected by the borrower, so the points and coupon rate, p and c, must satisfy

$$V_{\lambda}^{L}(c, r, t)/F(c, t) + p = 1. \tag{31}$$

The value of the borrower's total cash flows per dollar of principal is

$$V_{\lambda}^{B}(c, r, t)/F(c, t) + p = 1 + \frac{V_{\lambda}^{B}(c, r, t) - V_{\lambda}^{L}(c, r, t)}{F(c, t)}. \tag{32}$$

Since the right hand side of this equation is increasing in c, by Lemma 2, the lower the coupon rate (and hence the higher the points) the better off is the borrower. In any suggested equilibrium in which the lender offers the borrower a loan with points below p_{max} (such as contract Z^0 in Figure 1), another lender could offer another loan (such as contract Z^* in Figure 1) with higher points that would be preferred by the borrower, and would make the lender a positive profit.

Proof of Proposition 2

Assume such a pooling equilibrium does exist, with contract Z^p the contract chosen by borrowers h and l, as shown in figure 2. Lenders are making a profit on type h borrowers, and an offsetting loss on type l borrowers. However, now consider contract Z^*. This contract is preferred to Z^p by type h borrowers, but is less attractive than Z^p to type l borrowers.[33] As a result, if a new lender were to enter the market offering this loan, only type h borrowers would take it, the lender would make positive profits on the loan, and the old lender(s) would make a loss on the old loan. This argument extends to multiple borrower types.

[33]The divergence of the indifference curves follows from assumption A1.

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Table 1: Loans available, February 1996

Loan Life	Interest Rate	Points (Conforming)	Points (Jumbo)
30 yr	6.750%	1.470%	–
	6.875%	1.360%	–
	7.000%	0.870%	–
	7.125%	0.380%	–
	7.250%	-0.110%	1.920%
	7.375%	-0.600%	1.295%
	7.500%	-1.090%	0.795%
	7.625%	-1.500%	0.325%
	7.750%	-1.920%	-0.145%
	7.875%	-2.330%	-0.520%
	8.000%	–	-0.830%
	8.125%	–	-1.150%
	8.250%	–	-1.365%
15 yr	6.250%	1.470%	–
	6.750%	-0.010%	1.875%
	6.875%	-0.410%	1.500%
	7.000%	-0.820%	1.185%
	7.125%	-1.140%	0.845%
	7.250%	-1.460%	0.500%
	7.375%	-1.780%	0.250%
	7.500%	-2.100%	0.030%
	7.625%	–	-0.220%
	7.750%	–	-0.440%
	7.875%	–	-0.625%
	8.000%	–	-0.815%
	8.125%	–	-1.000%

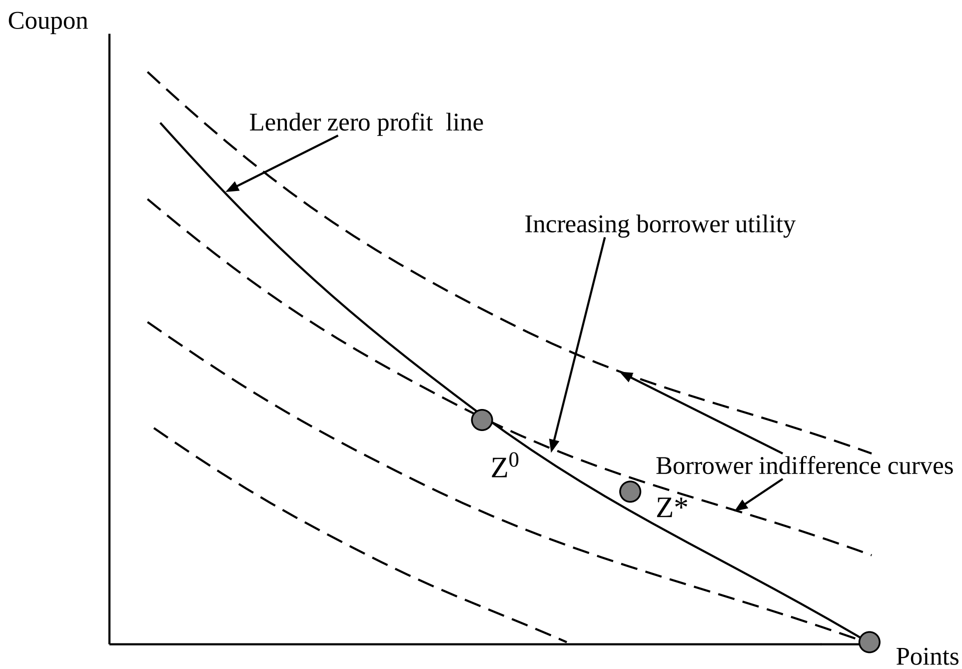
A selection of the loans available from a representative U.S. mortgage lender, February 13, 1996.

Table 2: Separating mortgage schedules

		Low Mobility		High Mobility	
	r	Points	Coupon	Points	Coupon
Base Case	4.5%	0.1	0.0696	0.0103	0.0845
	8.0%	0.1	0.0856	0.0437	0.0971
	11.5%	0.1	0.1047	0.0561	0.1161
	15%	0.1	0.1249	0.0612	0.1368
$p_{max} = \infty$	4.5%	0.4742	0.0	0.0165	0.0829
	8.0%	0.5216	0.0	0.0680	0.0905
	11.5%	0.5638	0.0	0.1132	0.0987
	15%	0.5996	0.0	0.1490	0.1077
$p_{max} = 0.05$	4.5%	0.05	0.0780	0.0056	0.0859
	8.0%	0.05	0.0976	0.0201	0.105
	11.5%	0.05	0.1197	0.0287	0.1262
	15%	0.05	0.1431	0.0346	0.1487
Cost = 10%	4.5%	0.1	0.0696	-0.0111	0.0870
	8.0%	0.1	0.0846	0.0453	0.0944
	11.5%	0.1	0.1034	0.0617	0.1119
	15%	0.1	0.1238	0.0656	0.1333
Cost = 20%	4.5%	0.1	0.0696	-0.0674	0.0951
	8.0%	0.1	0.0842	0.0051	0.1000
	11.5%	0.1	0.1001	0.0633	0.1069
	15%	0.1	0.1194	0.0743	0.1252
$\lambda = 0.05, 0.1$	4.5%	0.1	0.0696	-0.0151	0.0898
	8.0%	0.1	0.0856	0.0112	0.1053
	11.5%	0.1	0.1047	0.0222	0.1266
	15%	0.1	0.1249	0.0340	0.1463
$\lambda = 0.05, 0.2$	4.5%	0.1	0.0696	-0.0312	0.0942
	8.0%	0.1	0.0856	-0.0136	0.1132
	11.5%	0.1	0.1047	-0.0036	0.1361
	15%	0.1	0.1249	0.0048	0.1583
$\lambda = 0.1, 0.2$	4.5%	0.1	0.0610	-0.0217	0.0877
	8.0%	0.1	0.0782	-0.0011	0.1049
	11.5%	0.1	0.0983	0.0091	0.1278
	15%	0.1	0.1192	0.0180	0.1497

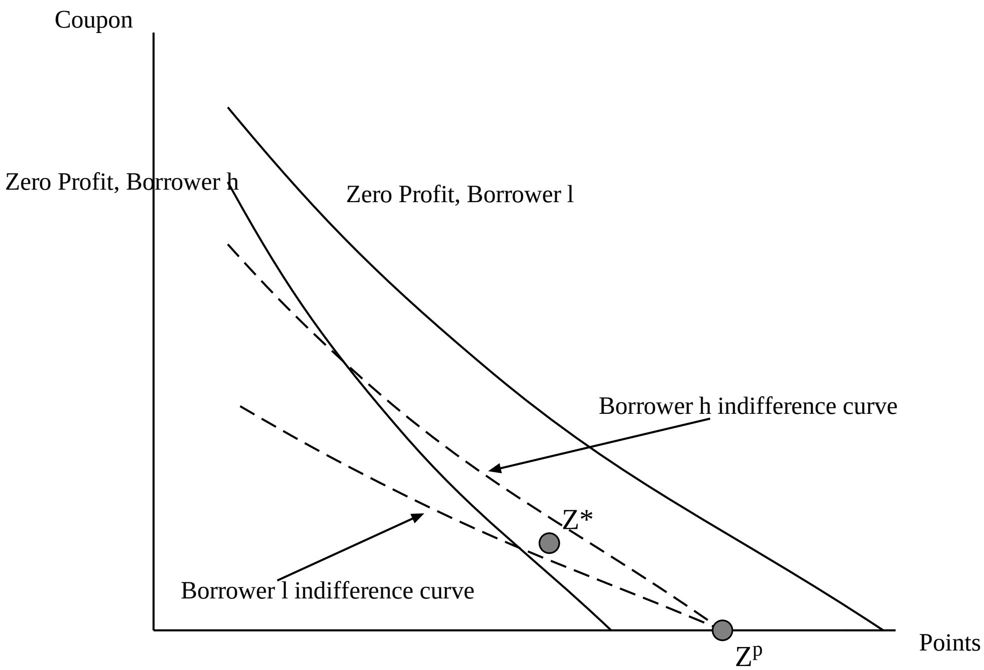
Separating loan schedules for different initial conditions. In the base case, the mobility parameters for the two borrowers are 0.05 and 0.067 (corresponding to an expected horizon of 20 and 15 years respectively), the transaction cost payable on refinancing is 5% of the remaining principal, and there is a cap of 10% on the maximum number of points payable. For every other set of results, one parameter is varied relative to the base case, keeping the other two the same.

Figure 1: Loan choice without asymmetric information



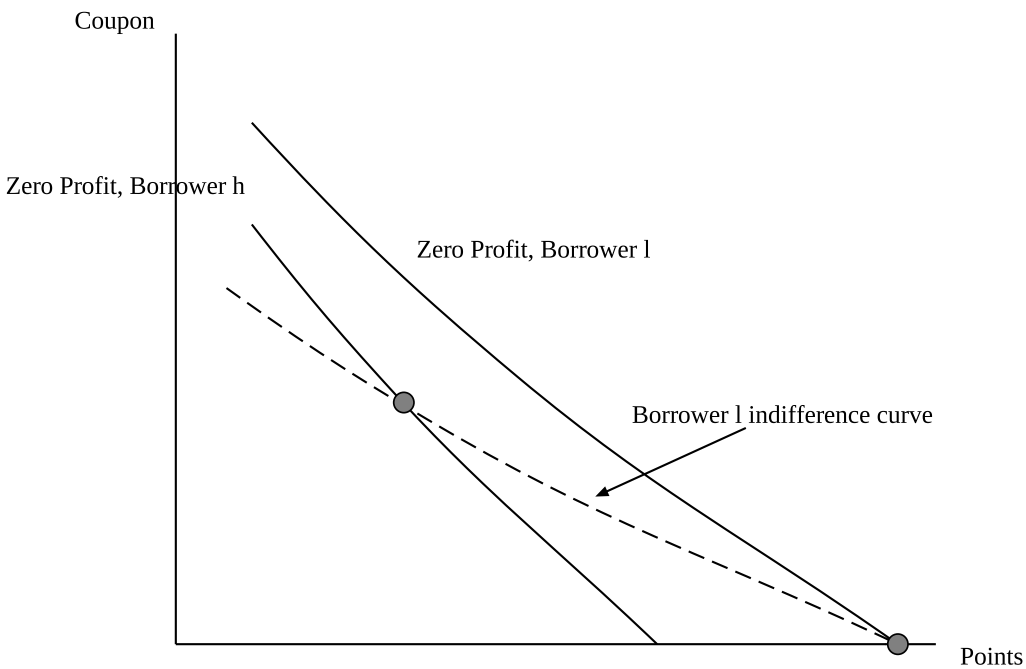
Solid line shows contracts yielding zero profit to the lender. Dashed lines are borrower indifference curves. Both lender and borrower prefer contract Z^* to contract Z^0.

Figure 2: Non-existence of pooling equilibrium



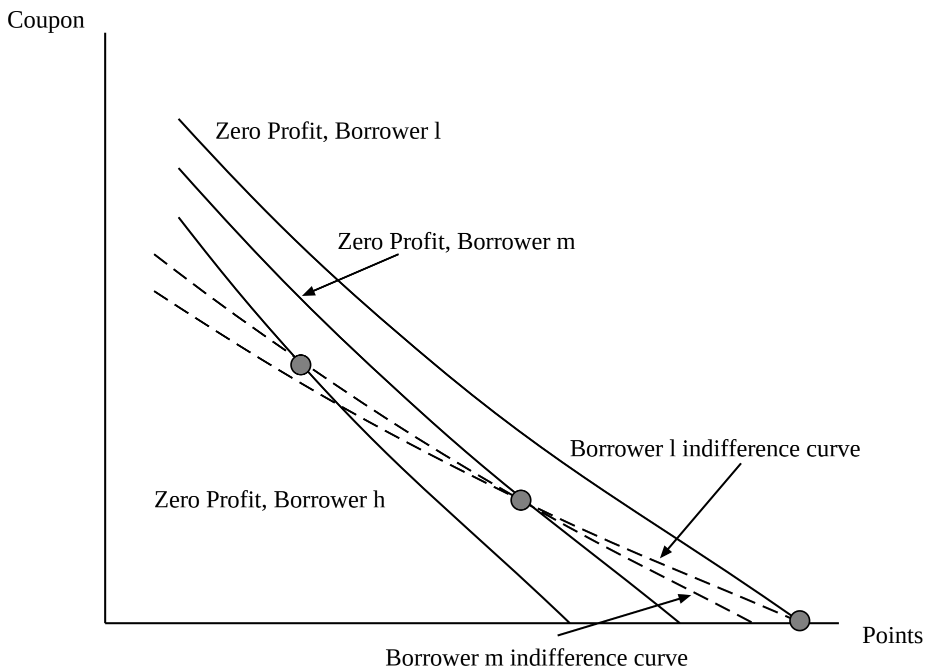
Solid lines show contracts yielding zero profit to the lender when taken out by borrower h (high mobility/short horizon) and borrower l (low mobility/long horizon) respectively. Dashed lines are borrower indifference curves. If pooling contract Z^p is offered, offering contract Z^* will attract only the (profitable) high mobility borrowers.

Figure 3: Separation with two borrowers



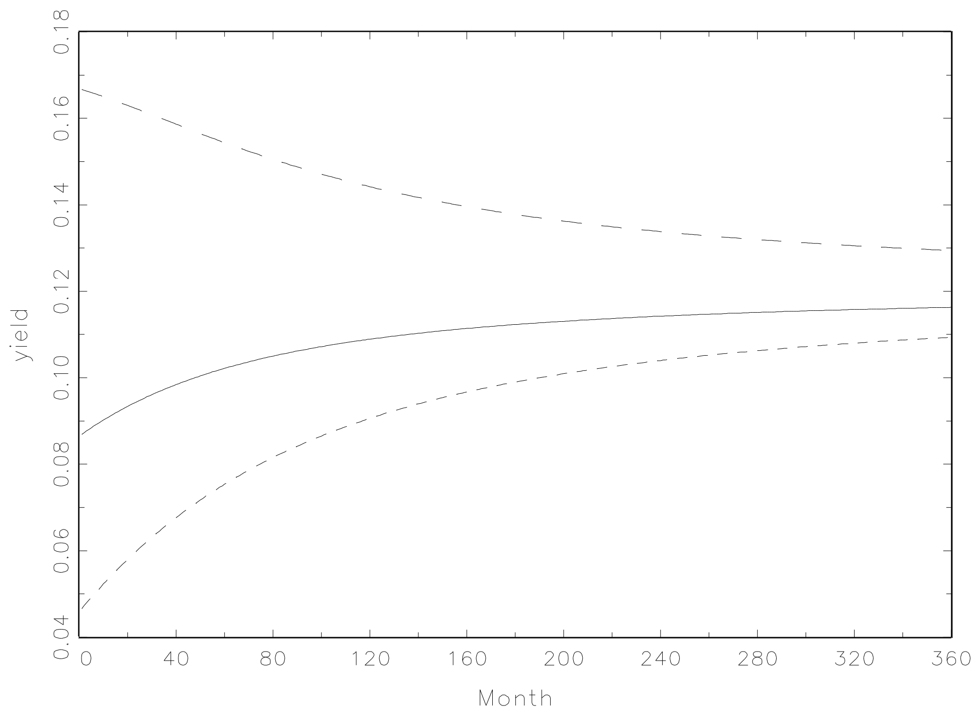
Solid lines show contracts yielding zero profit to the lender when taken out by borrower h (high mobility/short horizon) and borrower l (low mobility/long horizon) respectively. Dashed line is borrower l's indifference curve. The high mobility borrower (borrower h) selects loan that lies on intersection of lender's zero-profit line and borrower l's indifference curve through borrower l's first-best contract.

Figure 4: Separation with three borrowers



Solid lines shows contracts yielding zero profit to the lender when taken out by borrowers h (high mobility/short horizon), m (medium mobility/horizon), and l (low mobility/long horizon) respectively. Dashed lines are borrower indifference curves. The medium mobility borrower (borrower m) selects loan that lies on intersection of lender's zero-profit line and borrower l's indifference curve through borrower l's first best contract. The high mobility borrower (borrower h) selects loan that lies on intersection of lender's zero-profit line and borrower m's indifference curve through borrower m's equilibrium contract.

Figure 5: Yield Curves

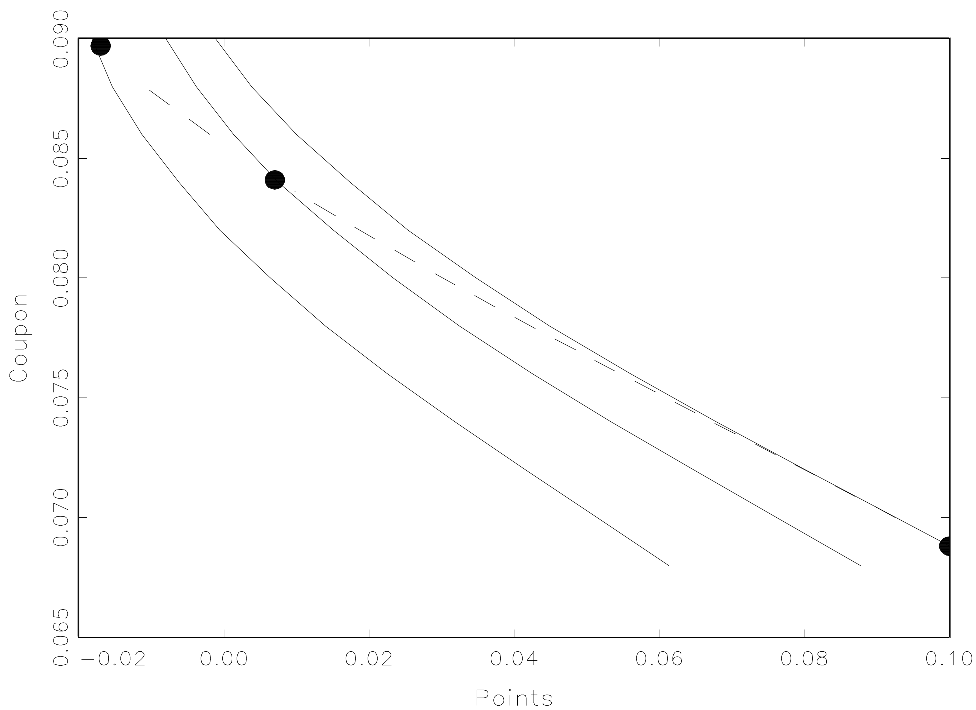


Flat, upward and downward sloping zero-coupon yield curves generated using the Cox, Ingersoll and Ross (1985) interest rate model,

$dr_t = \kappa\,(\mu - r_t)\,dt + \sigma\sqrt{r_t}dZ_t,$

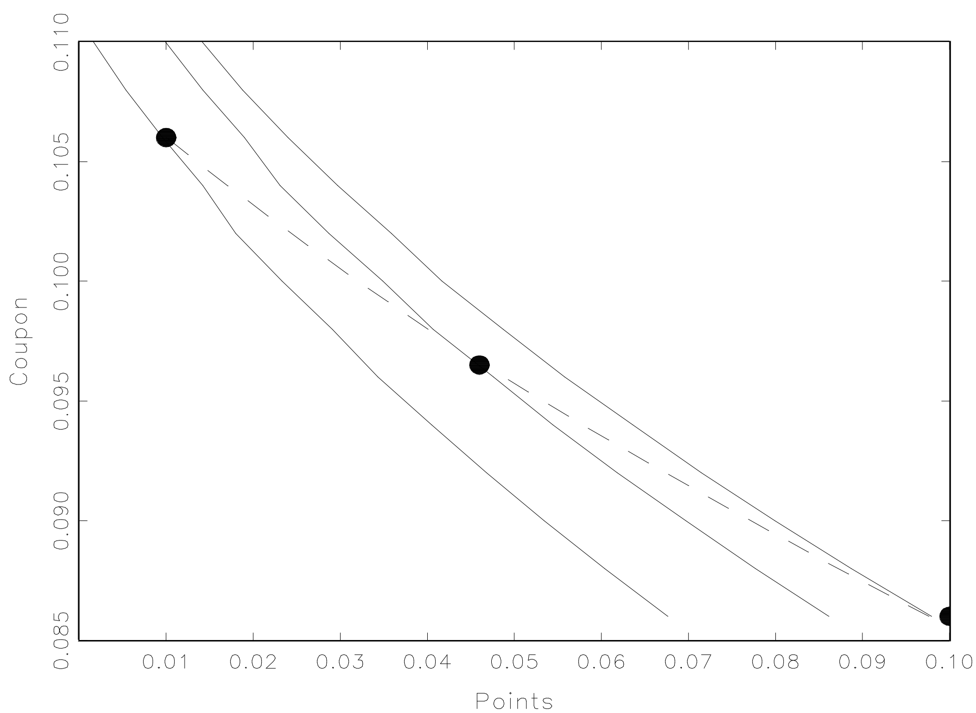
with parameters $\kappa = 0.29368$, $\mu = 0.07935$, $\sigma = 0.11425$, and risk aversion parameter $q = -0.12165$.

Figure 6: Separating equilibrium, upward sloping yield curve



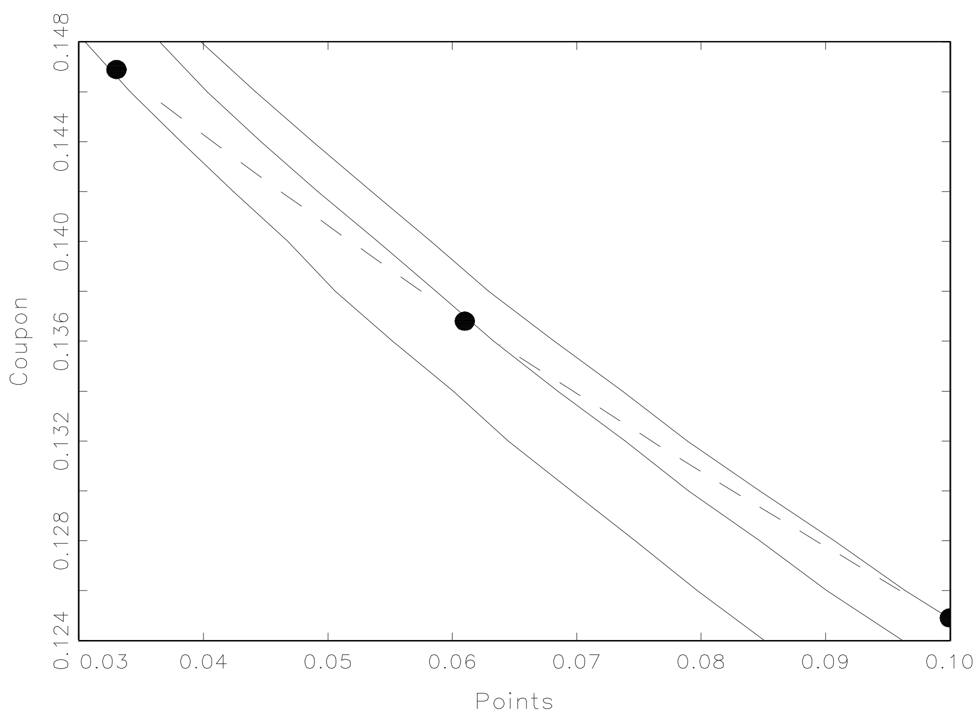
Separating loan schedule for three classes of borrower, with moving governed by hazard rates 0.05, 0.067, and 0.1 (corresponding to an expected horizon of 20, 15 and 10 years respectively). For all three borrowers, the transaction cost payable on refinancing is 5% of the remaining principal on the loan.

Figure 7: Separating equilibrium, flat yield curve



Separating loan schedule for three classes of borrower, with moving governed by hazard rates 0.05, 0.067, and 0.1 (corresponding to an expected horizon of 20, 15 and 10 years respectively). For all three borrowers, the transaction cost payable on refinancing is 5% of the remaining principal on the loan.

Figure 8: Separating equilibrium, downward sloping yield curve



Separating loan schedule for three classes of borrower, with moving governed by hazard rates 0.05, 0.067, and 0.1 (corresponding to an expected horizon of 20, 15 and 10 years respectively). For all three borrowers, the transaction cost payable on refinancing is 5% of the remaining principal on the loan.