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ARM Wrestling: Valuing Adjustable Rate Mortgages Indexed to the Eleventh District Cost of Funds

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ABSTRACT

This paper analyzes adjustable rate mortgages (ARMs) based on the Eleventh District Cost of Funds Index (EDCOFI). We study the behavior of EDCOFI over the period 1981–1993, and find that adjustments in this index lag substantially behind term structure fluctuations. We also find that the seasonality and days-in-the-month effects noted by previous authors are really symptoms of a "January effect".

Due to the lag in EDCOFI, if interest rates fall, mortgage holders may want to refinance their mortgage loans to avoid paying a coupon rate that exceeds the market rate. We develop a finite difference valuation algorithm which accounts for all usual ARM contractual features, in addition to the dynamics of EDCOFI. The advantage of our pricing algorithm over commonly used simulation strategies is that it allows us to determine endogenously the optimal prepayment strategy for mortgage holders, and hence the value of their prepayment options. We find that the dynamics of EDCOFI give significant value to this option, typically around .5% of the remaining principal on the loan. Our algorithm permits issuers and investors in ARMs based on EDCOFI to quantify the effects of the many interacting contract features, such as reset margin, coupon rate caps and reset frequency, that determine mortgage value.

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Adjustable rate mortgages (ARMs) based on the Eleventh District Cost of Funds Index (EDCOFI) are a significant factor in the primary and secondary mortgage markets. In the U.S., the estimated stock of EDCOFI based ARMs is approximately $150 to $200 billion. In California, more than 50% of ARM originations in 1991 were indexed to EDCOFI.[1] At the end of January 1993, nearly a quarter of all outstanding Agency[2] stock of ARM collateralized mortgage backed securities and Collateralized Mortgage Obligations (CMOs) were indexed to EDCOFI (approximately $26 billion in outstanding book value).[3] In addition, there are another $6 billion in CMO tranches that float off EDCOFI.[4]

Despite the popularity of EDCOFI, no existing study adequately addresses the valuation and hedging of ARMs based on this index. Movements in EDCOFI lag substantially behind shifts in the term structure.[5] The slower the adjustment of the underlying index, the more an ARM resembles a fixed rate mortgage (FRM). In particular, just as with FRMs, if interest rates fall sharply, holders of ARMs based on EDCOFI may find it optimal to refinance (presumably into an ARM based on another index, or an FRM), to avoid paying above market rates on their loan.[6] Previous ARM valuation studies have assumed that the index underlying the ARM resets with the contemporaneous term structure.[7] There is simulation evidence that EDCOFI ARMs have longer duration than Treasury indexed ARMs (Ott 1986), and lower present value in the context of a non-option adjusted present value model (Passmore 1993). However, neither of these studies considers the effects of EDCOFI dynamics on mortgage holder prepayment behavior or the value of the mortgage holder's prepayment option.

In this paper, we extend previous empirical studies of the time series properties of EDCOFI, and consistently embed our results in a contingent claims valuation model. This allows us to determine the impact of index behavior and contract terms on prepayment behavior and the value of the mortgage and its embedded prepayment option. Rather than valuing the mortgages using Monte Carlo simulation, we use a finite difference technique to solve the pricing equation, taking into account all the main contractual features of the ARM, plus the dynamics of EDCOFI. The valuation method-

[1] Source: Private communication, Federal Home Loan Bank of San Francisco.

[2] The Agencies are the Federal National Mortgage Association (FNMA) and the Federal Home Loan Mortgage Corporation (FHLMC). Statistics are not available on ARM mortgage-backed security stock for private issuers.

[3] Source: Lehman Brothers.

[4] Source: First Boston.

[5] See, for example, Cornell (1987), Crockett, Nothaft and Wang (1991), Hayre, Lodato and Mustafa (1991), Nothaft (1990), Nothaft and Wang (1992), Passmore (1993), and Roll (1987) for analyses of the dynamics of EDCOFI.

[6] Previous authors, such as Kau, Keenan, Muller and Epperson (1990), have noted that some ARM prepayment may occur because the coupon rate on ARMs adjusts only at discrete intervals, typically annually. The prepayment we focus on, induced by the behavior of the index, will still occur, even if the coupon rate on the mortgage adjusts every month to the prevailing index level.

[7] See, for example, Kau, Keenan, Muller and Epperson (1990), and McConnell and Singh (1991).

ology builds upon techniques used by Kau, Keenan, Muller and Epperson (1990) and Kishimoto (1990). Like Monte Carlo simulation, we may, if we desire, embed an empirical prepayment model into the algorithm. However, unlike Monte Carlo simulation, our valuation strategy alternatively allows us to determine endogenously the optimal prepayment policy for mortgage holders. This yields explicit values for the mortgage holder's prepayment option, and hence for the mortgage.

We find that the slow adjustment in EDCOFI gives significant value to the prepayment option embedded in EDCOFI based ARMs. For a simple loan, with no caps, and annual coupon reset, the minimum value of the prepayment option is about 0.5% of the remaining principal on the loan. More generally, the size of the reset margin, coupon rate caps, and reset frequency all interact with the behavior of EDCOFI to determine the exact value of the option, and hence of the mortgage. Our algorithm permits issuers and investors in ARMs based on EDCOFI to quantify these effects. The same algorithm could also be used to price other assets with embedded options, such as interest rate swaptions, whose cash flows depend on both contemporaneous and lagged interest rates.

Modeling EDCOFI

The Eleventh District Cost-of-funds index (EDCOFI) is computed from the book values of liabilities for all insured savings and loan (S&L) institutions in the Eleventh District (institutions in California, Nevada, and Arizona). The index is the ratio of the month end total interest expenses on savings accounts, advances, and purchased funds to the average book value of these liabilities from the beginning to the end of the month. The ratio is adjusted with an annualizing factor so that the interest expenses are comparable across months. The index is thus:

$$\mathrm{EDCOFI}_t = \frac{\text{Total interest, month } t}{\left[\mathrm{BVL}_{t-1} + \mathrm{BVL}_t\right]/2} \times \frac{365}{d}, \tag{1}$$

where BVL_t is the book value of liabilities at the end of month t, and d is the number of days in month t. The use of book values to compute the rates on the liabilities introduces a lag in the index because the rates cannot change until the individual liabilities mature or are withdrawn. The lag in EDCOFI is clearly evident in figure 1, which compares EDCOFI with rates on 3 Month T-Bills, 1 Year T-Bills, and 5 Year Treasury Notes from July 1981 to May 1993.[8] EDCOFI never reaches the highs and lows of the other three series, lags them by several months, and is much less volatile.

[8]EDCOFI rates were obtained from the Federal Home Loan Bank Board of San Francisco. The treasury rates were obtained from the Federal Reserve Statistical Release H15, monthly auction averages.

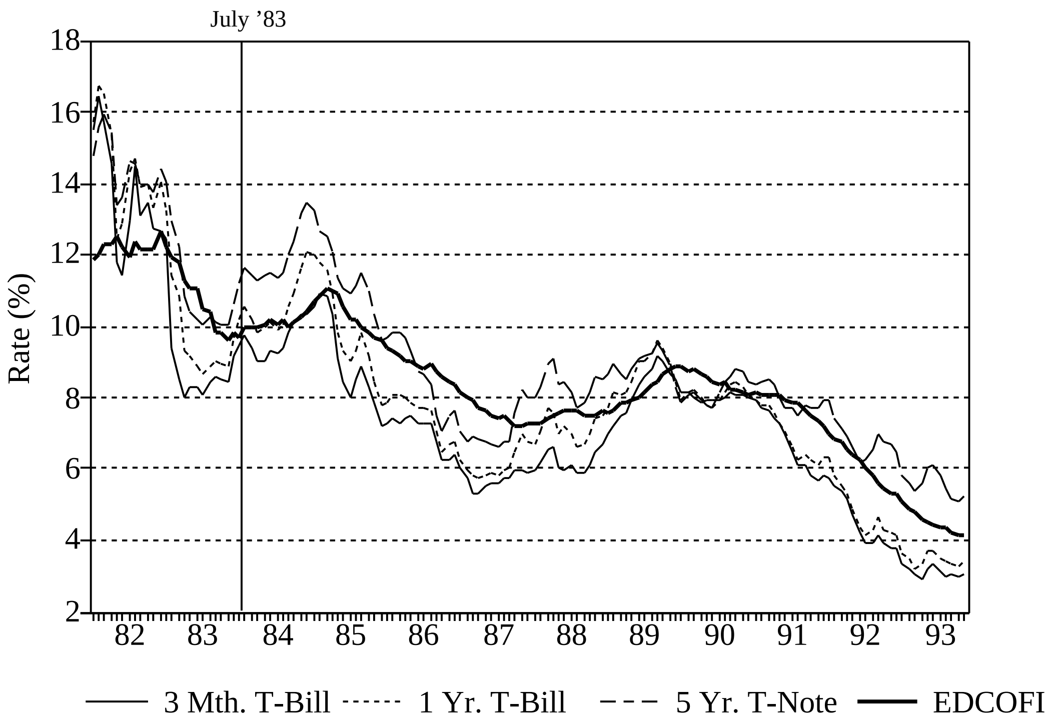

Figure 1: EDCOFI, 3 month T-Bill, 1 year T-Bill, and 5 year T-Note rates, July 1981 to May 1993. Source: Federal Home Loan Bank of San Francisco.

A Simple Model for EDCOFI

Assume that the rate set on newly issued liabilities of every maturity is a function of the prevailing term structure. This implies that EDCOFI is a weighted average of functions of the entire history of the term structure. Suppose that in period t, an institution issues an amount $N_{\tau,t}$ of new liabilities with maturity date $t + \tau$, whose liability rate, $L_{t,\tau}$, equals the yield on a riskless bond of the same maturity, plus an additive constant, plus a maturity independent error term u_t. If we assume for simplicity that the term structure only makes parallel shifts, the liability rate $L_{t,\tau}$ can then be written in the form

$$L_{t,\tau} = \mathrm{TR}_t + \Delta_\tau + u_t, \tag{2}$$

where TR_t is the current short term riskless interest rate, and Δ_τ is a maturity specific constant. Assuming the proportion of different maturities issued each period remains constant, the average liability rate for all newly issued liabilities is then

$$\begin{aligned} L_t^0 &= \frac{1}{N_t}\sum_{\tau=1}^{\infty} N_{t,\tau} L_{t,\tau}, & (3)\\ &= \mathrm{TR}_t + K_0 + u_t, & (4) \end{aligned}$$

for some constant K_0 (a weighted average of the Δ_τ), and where N_t is the total value of newly issued liabilities,

$$N_t = \sum_{\tau=1}^{\infty} N_{t,\tau}.$$

Now suppose that liabilities of different maturities expire at known rates, which potentially differ across different maturities,[9] but are the same for each issue date. Then the average liability rate for outstanding liabilities issued j periods ago is

$$L_t^j = \mathrm{TR}_{t-j} + K_j + u_{t-j}, \tag{5}$$

for some constant K_j. If we also assume the total amount of newly issued liabilities, N_t, is constant each period, and that the total outstanding balance of liabilities with each prior issue date decreases

[9] In particular, after one month all one-month liabilities expire; after two months all two-month liabilities expire, etc.

at a constant geometric rate β,[10] then the current value of EDCOFI can be written as

$$\begin{aligned} \text{EDCOFI}_t &= (1-\beta)(\text{TR}_t + K_0 + u_t) + \beta(\text{TR}_{t-1} + K_1 + u_{t-1}) \\ &\quad + \beta^2(\text{TR}_{t-2} + K_2 + u_{t-2}) + \ldots \qquad (6) \\ &= \alpha_0 + \alpha_1\,\text{EDCOFI}_{t-1} + \alpha_2\,\text{TR}_t + \epsilon_t, \qquad (7) \end{aligned}$$

for some constants α_0, α_1 and α_2, and where $\epsilon_t = (1-\beta)u_t$. This specification corresponds to the partial adjustment model considered by Cornell (1987), Roll (1987), and Passmore (1993), and provides theoretical justification for its use.

Estimation Results

We here test for the effects of seasonality in EDCOFI,[11] and estimate a model based on equation 7 using data from July 1983 to May 1993.[12] Columns 2–3 of table 1 model seasonality, simultaneously nesting Nothaft and Wang's (1992) days-in-the-month seasonality correction, by using a dummy variable for all months except February and December. February is handled with two dummy variables, one for months with 28 days, and another for months with 29 days. With the exception of January, the separate monthly dummies are not individually different from zero. A likelihood ratio test of the joint null hypothesis that all the monthly dummies, except January, are zero fails to reject the null. We conclude that the observed seasonality is in fact a January effect, rather than a days-in-the-month effect. This may arise from balance sheet adjustments following deposit withdrawals during the holidays.

The specifications reported in column 2–5 of table 1 show that EDCOFI exhibits serial correlation.[13] This is consistent with the simple model above. To correct for first order serial correlation, in columns 6–7 of table 1 we estimate the partial adjustment model defined in equation 7, using the 3 month T-Bill rate as the short term riskless rate.[14] Since there is a significant January effect in

[10] I.e. the total outstanding balance drops each period to β times its previous balance, though the balances for different maturities will in general fall at different rates.

[11] See, for example, Nothaft and Wang (1992) and Passmore (1993).

[12] The start date of July 1983 reflects the regime test results reported by Nothaft and Wang (1992). It is also close to the October 1983 starting date used by Passmore (1993).

[13] The autoregressive parameter for the twelve month seasonal is statistically insignificant, but the first and second order serial correlation terms in the first regression are statistically significant. The autoregressive parameters for the second regression reported in columns 4–5 indicate significant first order serial correlation. However, second order serial correlation is no longer statistically significant.

[14] Augmented Dickey-Fuller (1979) tests of the form

$$\Delta x_t = \mu + \gamma^*\, x_{t-1} + \sum_{j=1}^{p-1} \phi_j\, \Delta x_{t-j} + \epsilon_t$$

Independent Variables	Effect of Month on Seasonal Variation in EDCOFI (Dep. Var. = $EDCOFI_t$)				Partial Adjustment Model with Jan./Feb. effects (Dep. Var. = $EDCOFI_t$)	
	12 dummies		Jan. dummy			
	Est.	t-stat.	Est.	t-stat.	Est.	t-stat.
Intercept	7.1	(.93)	7.2*	(1.9)	.056	(.87)
January	−.065*	(−1.8)	−.086***	(−5.8)	−.047	(−1.5)
February					.11***	(2.6)
Feb. (29 days)	.019	(.32)				
Feb. (28 days)	.022	(.34)				
March	−.074	(−1.0)				
April	−.064	(−.82)				
May	−.063	(−.75)				
June	−.018	(−.21)				
July	−.042	(−.50)				
August	−.016	(−.20)				
September	.041	(.59)				
October	.002	(.04)				
November	.022	(.59)				
$EDCOFI_{t-1}$					.89***	(35)
3 Month $T\text{-}Bill_t$					.11***	(5.2)
Chow test					.943	
Breusch-Pagan (χ^2_5)					9.2	
ARCH (χ^2_1)					1.0	
White's test (χ^2_{11})					17	
R^2	.984		.980		.997	
Akaike	−224		−199		−166	

* Significant at the 10% level.
** Significant at the 5% level.
*** Significant at the 1% level.

Table 1: Estimation results. Columns 2–5 show vector autoregressions of EDCOFI to investigate seasonality and serial correlation structure. Columns 6–7 show results of estimating partial adjustment model for EDCOFI using Newey-West correction.

the EDCOFI series, and because a lagged value of EDCOFI appears on the right hand side of the regression, we estimate the model with two dummy variables, one for January and one for February. The instrumental variables technique suggested by Cumby, Huizinga and Obstfeld (1983) was used, with the Newey and West (1987) covariance matrix, to correct for any possible heteroscedasticity and first order serial correlation.[15] The χ^2 test for parameter constancy, the Chow test, compares parameter estimates excluding the last twenty four observations to those based on the entire sample. It is not statistically significant indicating that the parameter estimates for this period are stable. The other reported test statistics indicate that the residuals from the partial adjustment model are neither predictable from lagged residuals nor predictable from other variables in the model. All three χ^2 tests are insignificantly different from zero at the .05 level or better.[16] The parameter estimates, reported in the sixth column of table 1, show there is considerable lag in the response of EDCOFI to movements in the 3 month T-Bill rate.[17] Figure 2 shows the expected response of EDCOFI to a jump in r_t from 7.5% to 8.5%,[18] where EDCOFI is assumed to move according to the model given in the last column of table 1 (ignoring monthly dummies),

$$\text{EDCOFI}_t = .056 + .889\,\text{EDCOFI}_{t-1} + .112\,\text{TR}_t + \epsilon_t. \tag{8}$$

When r_t jumps, EDCOFI also rises, but more slowly. It takes almost 6 months to get about half way to its long run value of 9.1%. This lag needs to be recognized in a properly specified ARM valuation model.

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were performed on EDCOFI and the 3 month T-Bill series using twelve lagged differences to control for possible seasonality effects. We were unable to reject the null hypothesis that there are unit roots in the series. Phillips and Perron (1988) nonparametric unit root testing procedures were also applied, with the same result. Tests for the cointegration of EDCOFI and the Treasury series, using Johansen (1988), showed that they are not cointegrated. However, because our interest rate series are relatively short, and it is well known that the low power of standard unit roots tests often leads to acceptance of the null hypothesis of a unit root in many economic time series (Kwiatkowski, Phillips, Schmidt and Shin 1992; Faust 1993), we rely on our strong priors that our interest rate series are mean reverting rather than explosive, and undertake all our estimation in levels of interest rates.

[15] As a further check on the specification for the partial adjustment model, we ran a twelfth order autoregression on the residuals using maximum likelihood. Only the twelfth order term was statistically significant at the .05 level. A Lagrange Multiplier test on the joint null hypothesis that the coefficients on all twelve lags were zero rejected the null at the .05 level, indicating that there may still remain some serial correlation.

[16] Although not reported here, we also tested a number of other specifications for EDCOFI using the model comparison methods of Hendry and Richard (1982) and Passmore (1993). The partial adjustment model performed as well as or better than the other specifications considered (using various lags and moving averages of Treasury rates). We also found that including the very volatile period from late 1981 to mid 1982 led to some differences across models. This result is consistent with the finding of regime shifts in Nothaft and Wang (1992).

[17] Regressions using other maturities yield very similar results.

[18] EDCOFI is assumed initially to take the value 8.1%. Inserting this value for EDCOFI_{t-1} into equation 8 shows that, if r_t stays constant at 7.5%, EDCOFI stays constant at 8.1%.

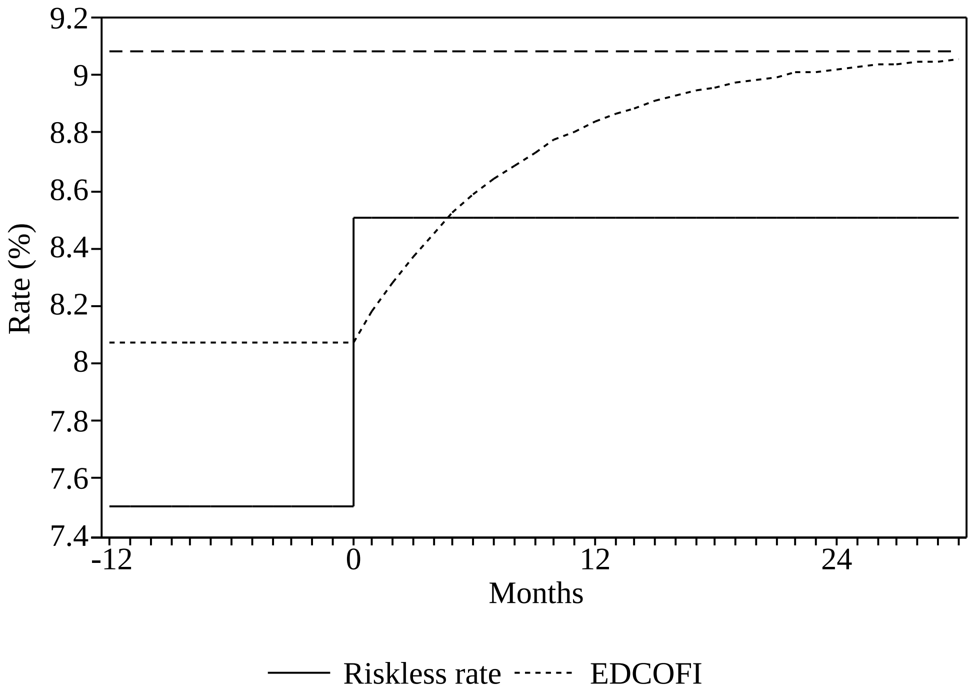

Figure 2: Example of the lag in EDCOFI's response to movements in the term structure. The short term interest rate jumps from 7.5% to 8.5%. The graph shows the resulting movement in the value of EDCOFI.

Valuation

Holders of EDCOFI based ARMs may find early prepayment of their loans optimal if market interest rates fall sharply. Like other ARMs, the coupon rate on an EDCOFI based ARM usually adjusts only discretely, typically once per year. The longer the time between coupon resets, the more the ARM resembles an FRM. If interest rates fall substantially while there is still a long time until the next coupon reset date, the mortgage holder may prefer to avoid several months of above market interest payments on his or her loan by refinancing into another loan (either ARM or FRM), with a lower current coupon rate. Unlike other ARMs, holders of EDCOFI based ARMs also have another reason why they might want to prepay their loans. EDCOFI moves slowly in response to shifts in market interest rates. As a result, if interest rates fall suddenly, it will take some time before the coupon rate drops to market levels, even if the coupon adjusts every month to the current index level. Again, borrowers may find it preferable to avoid paying above market interest rates by refinancing.

This section of the paper develops an algorithm for valuing EDCOFI based ARMs. The algorithm can handle all of the important features of the contract, including the model for movements in EDCOFI developed above. We shall assume that EDCOFI moves according to the process

$$\mathrm{EDCOFI}_t = .056 + .889\,\mathrm{EDCOFI}_{t-1} + .112\,\mathrm{TR}_t. \tag{9}$$

In other words, EDCOFI is a deterministic function of the entire past history of interest rates, whose movements are locally perfectly correlated with movements in interest rates.[19] In principle, we could use an empirically derived prepayment function in the algorithm. However, our algorithm can also derive endogenously the optimal prepayment strategy for mortgage holders, and hence calculate the value of their prepayment option. This latter strategy has the advantage that it is robust to possible changes in the economic environment, such as changes in the interest rate process. Such changes would have an unquantifiable effect on an empirical prepayment function. Using the optimal prepayment strategy allows us to determine an upper bound for the value of the prepayment option possessed by mortgage holders. The algorithm is an extension of techniques described by Kau, Keenan, Muller and Epperson (1990), and Kishimoto (1990). We first describe the features of the contract that we wish to capture, then the algorithm itself.

[19]From table 1, 99.4% of the monthly variation in EDCOFI is explained by movements in r_t. Our assumption of perfect correlation is thus very close to the truth. It may, in fact, not be an approximation at all, if we assume that we are seeing monthly observations on truly continuous processes. Two continuous processes with perfect local correlation will not in general exhibit 100% correlation when sampled discretely.

Our implementation of this algorithm applies to a world in which movements in EDCOFI and interest rates are locally perfectly correlated, and where interest rates are described by a particular one factor model. It could, however, be extended to more complex worlds. For example, we could add an additional state variable to correspond to unexpected changes in EDCOFI, and/or another interest rate variable. It would still be possible to solve the resulting partial differential equation for the mortgage value.[20] This would be more time-consuming, and the numerical results would certainly differ, but the intuition behind the value of the embedded prepayment option is identical to that described here.

Main Features of an ARM Contract

Coupon rate, C_t. The coupon rate on an ARM changes at each "reset date". The coupon determines the monthly cash flows on the mortgage until the next reset date. The monthly cash flow equals that on a fixed rate mortgage with the same time to maturity, same remaining principal balance, and same coupon rate as the ARM.

Underlying Index, I_t. The adjustment rule for the coupon rate specifies an underlying index to which the rate is tied. In this paper, we are considering mortgages based on EDCOFI, but adjustable rate mortgages exist with other underlying indices, such as the 1 year T-Bill rate and LIBOR.

Margin, m. At each coupon reset date, the new rate is set by adding a margin, m (e.g. 2%), to the prevailing level of the underlying index (subject to certain caps, discussed below).

"Teaser" rate, C_0. It is common for the initial coupon rate to be lower than the "fully indexed" rate given by adding the margin to the initial level of the index. The initial rate, C_0, is often referred to as a "teaser" rate.

Annual cap, Δ. ARM contracts usually specify a maximum adjustment in the coupon rate at each reset period (e.g. 2% per year).

Lifetime caps, $\overline{C}$ and $\underline{C}$. ARM contracts usually specify an overall maximum coupon rate over the life of the loan, $\overline{C}$ (e.g. the initial rate plus 6%), and a minimum coupon rate over the life of the loan, $\underline{C}$.

Reset Frequency. The coupon rate on an ARM contract adjusts at prespecified intervals. This

[20] Partial differential equations in more than one state variable plus time may be solved by use of methods such as the hopscotch algorithm (Gourlay and McKee 1977).

interval is usually every 6 months or one year. In this paper, we assume yearly adjustment. If month t is a coupon reset date, the new coupon rate is given by

$$C_t = \max\left[\underline{C},\, C_{t-1} - \Delta,\, \min\left[I_t + m,\, C_{t-1} + \Delta,\, \overline{C}\right]\right] \tag{10}$$

Interest Rates

To value the mortgage, we need to make assumptions about the process governing interest rate movements. We use the Cox, Ingersoll and Ross (1985) one-factor model. In this model, the instantaneous risk-free interest rate, r_t, satisfies the stochastic differential equation

$$dr_t = \kappa(\mu - r_t)\, dt + \sigma\sqrt{r_t}\, dz_t. \tag{11}$$

This equation says that, on average, the interest rate r converges toward the value μ. The parameter κ governs the rate of this convergence. The volatility of interest rates is $\sigma\sqrt{r_t}$. One further parameter, λ, which summarizes risk preferences of the representative individual, is needed to price interest rate dependent assets.

The parameter values used here are those estimated by Pearson and Sun (1989), using data from 1979–1986. These values are

$$\begin{aligned}
\kappa &= 0.29368, \\
\mu &= 0.07935, \\
\sigma &= 0.11425, \\
\lambda &= -0.12165.
\end{aligned}$$

The long run mean interest rate is 7.9%. Ignoring volatility, the time required for the interest rate to drift half way from its current level to the long run mean is $\ln(1/2)/(-\kappa) \approx 2.4$ years.

Other factors affecting ARM value

The value of an ARM depends not only on the current interest rate, r_t, but on the whole path of interest rates since its issue. This determines the current coupon rate, C_t, the current level of the underlying index, I_t (which in turn determines future movements in the coupon rate), and the current remaining principal balance, F_t. These three variables summarize all relevant information about the history of interest rates. By adding these as extra state variables, we return to a Markov setting (where all prices can be written as a function only of the current values of a set of underlying state variables).

Write B_t for the value of a non-callable bond which makes payments equal to the promised payments on the ARM. The mortgage holder's position can be decomposed into a short position in B_t (the scheduled payments on the mortgage) plus a long position in a call option on B_t, with (time varying) exercise price F_t. Writing M_t for the market value of the mortgage, and O_t for the value of the prepayment option, we have

$$M_t = B_t - O_t \tag{12}$$

Since B_t does not depend on the mortgage holder's prepayment decision, minimizing his or her liability value is equivalent to maximizing the value of the prepayment option, O_t. Write

$$\begin{aligned} B_t &\equiv B(r_t, I_t, C_t, F_t, t), && (13) \\ O_t &\equiv O(r_t, I_t, C_t, F_t, t). && (14) \end{aligned}$$

All values are homogeneous of degree one in the current remaining principal amount, F_t. Thus, if each month we value a mortgage with \$1 remaining principal, we can scale up or down as necessary for different principal amounts. Define normalized asset values (values per \$1 of remaining principal) by

$$\begin{aligned} \widehat{B}_t &= B_t/F_t, && (15) \\ &\equiv \widehat{B}(r_t, I_t, C_t, t). \\ \widehat{O}_t &= O_t/F_t, && (16) \\ &\equiv \widehat{O}(r_t, I_t, C_t, t). \end{aligned}$$

Valuation with one State Variable

Given the interest rate model defined by equation 11, write $V(r, t)$ for the value of an asset whose value depends only on the current level of r_t and time, and which pays coupons or dividends at a rate $\delta(r_t, t)$. This value satisfies the partial differential equation[21]

$$\frac{1}{2}\sigma^2 r V_{rr} + [\kappa\mu - (\kappa + \lambda)r]\, V_r + V_t - rV + \delta = 0, \tag{17}$$

[21] We need to assume some technical smoothness and integrability conditions (see, for example, Duffie (1988)).

which can be solved for V, subject to appropriate boundary conditions.

Natural boundaries for the interest rate, r, are 0 and ∞. Rather than working directly with r, define the variable y by

$$y = \frac{1}{1+\gamma r}. \tag{18}$$

for some constant $\gamma > 0$,[22] The infinite range $[0, \infty)$ for r maps onto the finite range $[0, 1]$ for y. The inverse transformation is

$$r = \frac{1-y}{\gamma y}. \tag{19}$$

Equation 18 says that $y = 0$ corresponds to "$r = \infty$" and $y = 1$ to $r = 0$. Next, rewrite equation 17 using the substitutions

$$U(y,t) \equiv V(r(y),t), \qquad \text{so} \tag{20}$$

$$V_r = U_y \frac{dy}{dr}, \tag{21}$$

$$V_{rr} = U_y \frac{d^2y}{dr^2} + U_{yy}\left(\frac{dy}{dr}\right)^2, \tag{22}$$

to obtain

$$\frac{1}{2}\gamma^2 y^4 \sigma^2 r(y) U_{yy} + \left(-\gamma y^2 \left[\kappa\mu - (\kappa+\lambda) r(y)\right] + \gamma^2 y^3 \sigma^2 r(y)\right) U_y + U_t - r(y)U + \delta = 0. \tag{23}$$

We can solve equation 23 using a finite difference algorithm. Finite difference algorithms replace derivatives with differences, and approximate the solution to the original partial differential equation by solving the set of difference equations that arise. We use the Crank-Nicholson algorithm, described in the appendix.

Represent the function $U(y,t)$ by its values on the finite set of points,

$$y_j = j\,\Delta y, \tag{24}$$

$$t_k = k\,\Delta t, \tag{25}$$

[22]The larger the value of γ, the more points on a given y grid correspond to values of r less than, say, 20%. Conversely, the smaller the value of γ, the more points on a given y grid correspond to values of r greater than, say, 4%. We are most interested in values of r in an intermediate range. Therefore, as a compromise between these two competing objectives, we choose $\gamma = 12.5$. The middle of the range, $y = 0.5$, then corresponds to $r = 8\%$.

for $j = 0, 1, \ldots, J$, and for $k = 0, 1, \ldots, K$. Δy and Δt are the grid spacings in the y and t dimensions respectively. $\Delta y = 1/J$, and Δt is chosen for convenience to be one month, making a total of 360 intervals in the time dimension. Write

$$U_{j,k} \equiv U(y_j, t_k), \tag{26}$$

for each (j, k) pair. The Crank-Nicholson algorithm rewrites equation 23 in the form

$$MU_k = D_k, \tag{27}$$

where M is a matrix, U_k is the vector $\{U_{0,k}, U_{1,k}, \ldots, U_{I,k}\}$, and D_k is a vector whose elements are functions of $U_{j,k+1}$. This system of equations relates the values of the asset for different values of y at time t_k to its possible values at time t_{k+1}. To perform the valuation, we start at the final time period, when all values are known, and solve equation 27 repeatedly, working backwards one period at a time.

Extension to multiple state variables

In general, when asset prices depend on more than one state variable plus time, solution of the resultant partial differential equation becomes numerically burdensome. In this case, the additional variables, I_t and C_t, are functions of the path of interest rates, and so they introduce no additional risk premia. This allows us to extend the Crank-Nicholson finite difference algorithm to handle the multiple state variable case. The extensions required are to:

1. Allow values to depend on C_t and I_t as well as r_t and t, allowing for dependence between the processes governing movements in these variables.
2. Scale values to correspond to $1 remaining principal.
3. Handle caps, floors and teaser rates.

In addition to the finite sets of values for y and t defined above, define a finite set of values for I and C by

$$I_l = l\,\Delta I, \tag{28}$$
$$C_m = m\,\Delta C, \tag{29}$$

for $l = 0, 1, \ldots, L$, and for $m = 0, 1, \ldots, M$. ΔI and ΔC are the grid spacings in the I and C dimensions respectively. We are now solving for values on the points of a 4-dimensional grid, whose elements are indexed by the values of (j, k, l, m). Write the value of an asset whose cash flows depend on these state variables as

$$U_{j,k,l,m} \equiv U(y_j, t_k, I_l, C_m), \tag{30}$$

for each (j, k, l, m). I and C are functions of the path of interest rates. Over the next instant, the movement in r completely determines the movements in both I and C. Assume that movements in EDCOFI are described by the equation

$$I_{t+1} = g(I_t, r_{t+1}), \tag{31}$$

so that EDCOFI this month is a deterministic function of EDCOFI last month, plus the short term riskless rate this month (the models estimated above are of this type). Define l^* by

$$I_{l^*_{1,j,l,m}} \approx g(I_l, r_{j+1}), \tag{32}$$

$$I_{l^*_{0,j,l,m}} \approx g(I_l, r_j), \tag{33}$$

$$I_{l^*_{-1,j,l,m}} \approx g(I_l, r_{j-1}). \tag{34}$$

In words, l^* gives the closest index to the value of I next period given the current values of r, I and C, and three possible values of r next period (up, the same, and down). Assuming that next month is a coupon reset date (since otherwise, the coupon rate next month will just be the same as the coupon rate this month), define m^* similarly, to give the index of C next period given the current values of r, I and C, and the value of r next period. m^* is determined by the interplay between the current coupon C_t, the index I_t, the margin m, and the caps $\overline{C}$, $\underline{C}$ and Δ. Note that the effects of caps, floors and teaser rates are all automatically captured in this definition of m^*.

We can now generate a set of finite difference equations for each pair (l, m). For example, the approximation for the time derivative given in equation 40 in the appendix now becomes

$$U_t(y_j, t_k, I_l, C_m) \approx \left(U_{j,k+1,l^*_{0,j,l,m},m} - U_{j,k,l,m} \right) / \Delta t, \tag{35}$$

if t_{k+1} is *not* a coupon reset period, and

$$U_t(y_j, t_k, I_l, C_m) \approx \left(U_{j,k+1,l^*_{0,j,l,m},m^*_{0,j,l,m}} - U_{j,k,l,m} \right) / \Delta t, \tag{36}$$

if t_{k+1} *is* a coupon reset period. This allows us to write down one set of systems of equations like equation 27 for each (l, m) pair. These equations are independent of each other, so we can solve them for each (l, m) pair in turn, looping over l and m to calculate values at every grid point at time t_k.

The final step in the process is to deal with the normalization of asset prices to correspond to a remaining principal balance of $1. This is possible because, at any time, we know exactly how much principal will be repaid over the next one month. Given a coupon rate C_t and a current remaining principal F_t, the usual amortization formula tells us the value of F_{t+1}, regardless of any possible movements in r_t, I_t or C_t. The values stored in the grid for next period correspond to $1 in remaining principal *next* period. These need only to be multiplied by F_{t+1}/F_t (a function only of C_t) to make them correspond to $1 of remaining principal today.

Valuation Results without Caps

The algorithm described above was used to value 30 year EDCOFI based ARMs. Starting in month 360, the algorithm works backward to solve equation 23 one month at a time, calculating the normalized bond value, $\widehat{B_t}$.[23] For the option, the same process gives the value conditional on its remaining unexercised for the next month. This value must then be compared with the option's intrinsic value (max$[0, \widehat{B}_t - 1]$) to determine whether exercise (prepayment) is optimal. $\widehat{O}_t$ is set to the higher of these two values, and the mortgage value is calculated from the relationship

$$\widehat{M_t} = \widehat{B_t} - \widehat{O_t}.$$

The algorithm was applied first to a mortgage with no caps, whose coupon rate adjusts annually to the prevailing value of EDCOFI. For this mortgage, the value of the prepayment option can only be a function of the lag in the index, plus the annual reset frequency. Figure 3 shows the value per $100 of remaining principal of the underlying bond (the stream of promised payments, with no prepayment option), and the market value of the mortgage, for different values of the instantaneous riskless interest rate, r, and the index, EDCOFI. For every point plotted, the current coupon rate is assumed to equal the current value of EDCOFI. Figure 4 shows the value of the mortgage holder's prepayment option. These figures show that, for certain combinations of r and EDCOFI, this

[23]The values of the bond and the option in month 360 are 0, since all principal has been repaid. Given known values for the assets at month $t + 1$, for all possible combinations of r_{t+1}, I_{t+1} and C_{t+1}, the algorithm calculates their values for every combination of these variables at month t by discounting back a weighted average of their possible values at time $t + 1$, plus any coupon payments made at time $t + 1$. This is analogous to the "binomial tree" option pricing algorithm. For a detailed discussion of the relationship between binomial methods, discounted expected values, explicit and implicit finite difference methods for the valuation of contingent claims, see Brennan and Schwartz (1978).

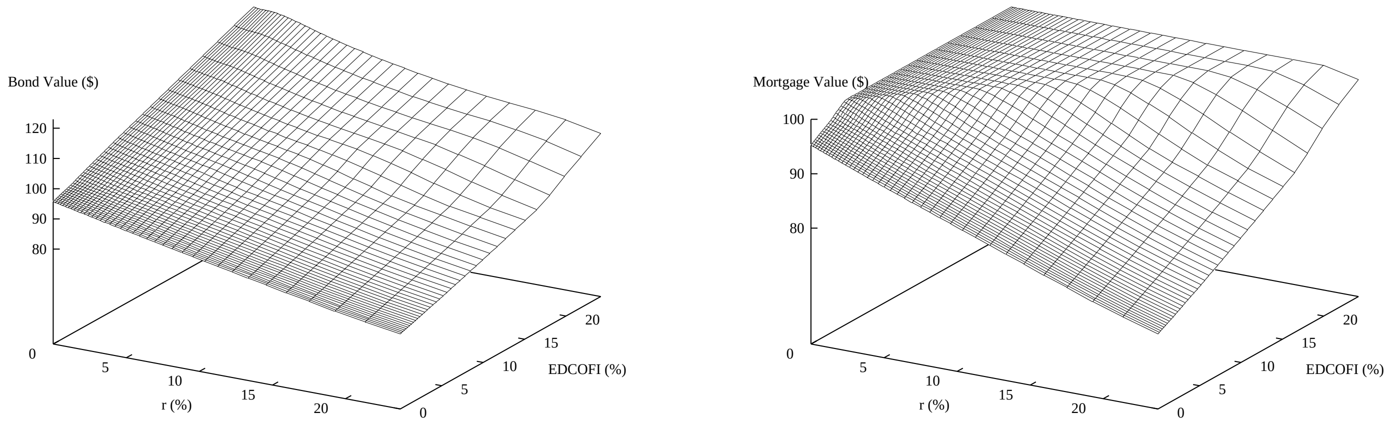

Figure 3: Bond and mortgage values for different values of r, EDCOFI. Coupon rate currently equals value of EDCOFI, and resets annually to the prevailing value of EDCOFI. There are no caps on coupon movements. Remaining principal is $100.

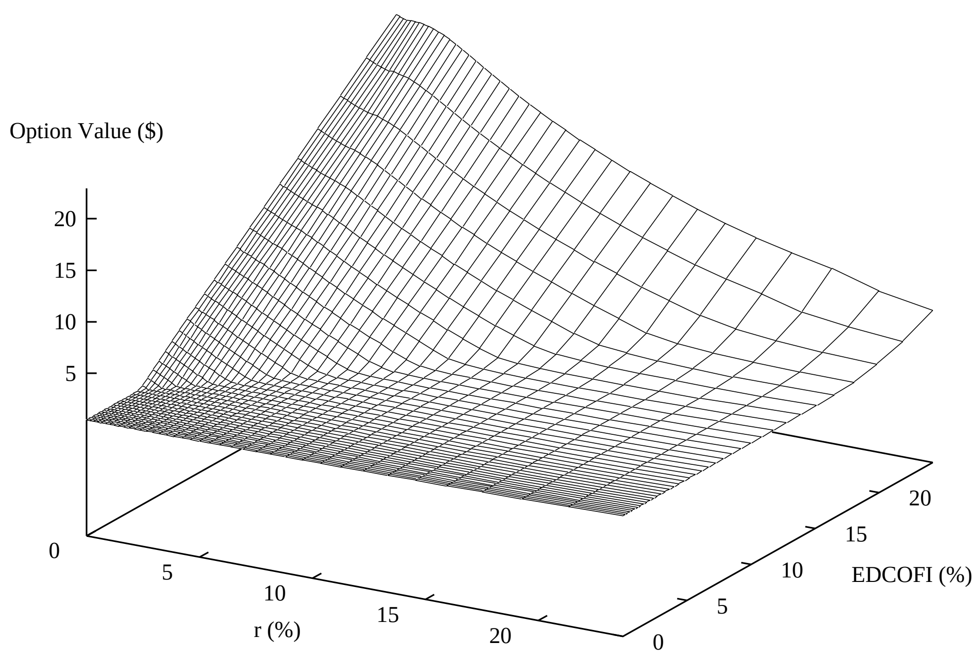

Figure 4: Value of prepayment option for different values of r, EDCOFI. Coupon rate currently equals value of EDCOFI, and resets annually to the prevailing value of EDCOFI. There are no caps on coupon movements. Remaining principal is $100.

option is extremely valuable. However, most of these combinations (high EDCOFI together with low r) are unlikely to occur in practice.

To see the value of the prepayment option in more likely interest rate environments, and to see how this value would be affected by jumps in the level of interest rates, figure 5 shows the values of the bond, the mortgage, and the prepayment option for different values of the interest rate r, with the current value of EDCOFI set to 8.5% (the average value of EDCOFI over the period 1981 – 1993).[24] For r between 0% and about 5%, the value of the option drops from almost 6% of remaining principal to approximately 0.5%. The graph of bond and mortgage values shows that, in this region, it is optimal for mortgage holders to prepay immediately. As r increases from 5%, the option value *increases* again. This initially seems counterintuitive, since the value of the underlying asset decreases as r increases. However, this is more than offset by the fact that the higher r is today, the faster EDCOFI will rise in the near future. Mean reversion in r, combined with the lagging behavior of EDCOFI means that the higher r is today, the more likely we are to see EDCOFI (and hence the coupon rate on the mortgage) rise, and then get left behind as r drops towards its long run mean, making prepayment optimal. If we were to use an interest rate model which did not exhibit mean reversion, this increase might not occur. Figure 6 shows the values of the bond, the mortgage, and the prepayment option for different values of EDCOFI, with the riskless interest rate equal to 7.5% (the sample mean over the period studied).[25] The higher the current value of EDCOFI (and the current coupon rate on the mortgage), the higher the value of the underlying instrument and the value of the prepayment option. These results show that the prepayment option embedded in an EDCOFI based ARM may have significant value.

The Effect of Caps

Another important feature of most ARM contracts is the existence of caps on movements in the coupon rate. It is possible that much of the option value found above stems from the possibility that the coupon rate may become very high at some time in the future. This is not permitted for almost all existing EDCOFI based ARM contracts. To examine this, figures 7 and 8 differ from figures 5 and 6 in one respect. The coupon rate on the mortgage now has a lifetime cap of 13.5%. This change significantly reduces the value of the prepayment option. In most regions of figure 7, the line for the bond value and the line for the option value are almost indistinguishable.

While this at first seems to argue that the prepayment option is insignificant after all, one other feature of most ARM contracts has been ignored, the margin over the underlying index that is used

[24] This corresponds to cutting a vertical slice, parallel to the r axis, through the graphs in figures 3–4.

[25] This corresponds to cutting a vertical slice, parallel to the EDCOFI axis, through the graphs in figures 3–4.

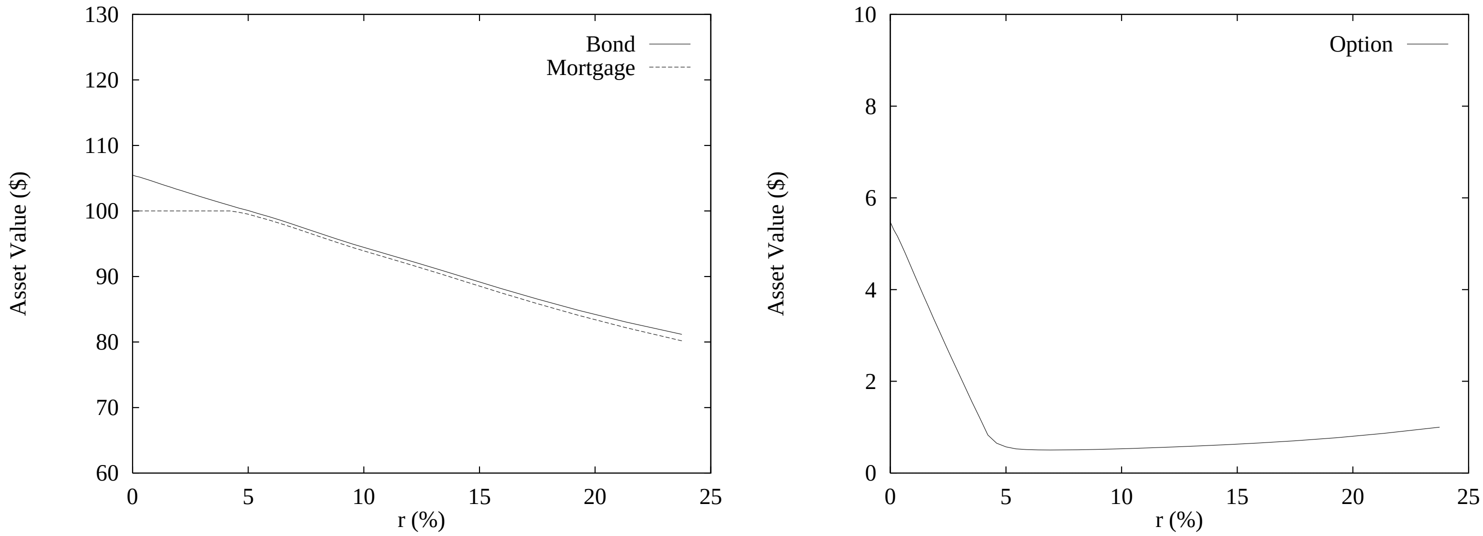

Figure 5: Bond and option values for different values of r. Current EDCOFI is 8.5%. Coupon rate currently equals 8.5%, and resets annually to the prevailing value of EDCOFI. There are no caps on coupon movements. Remaining principal is $100.

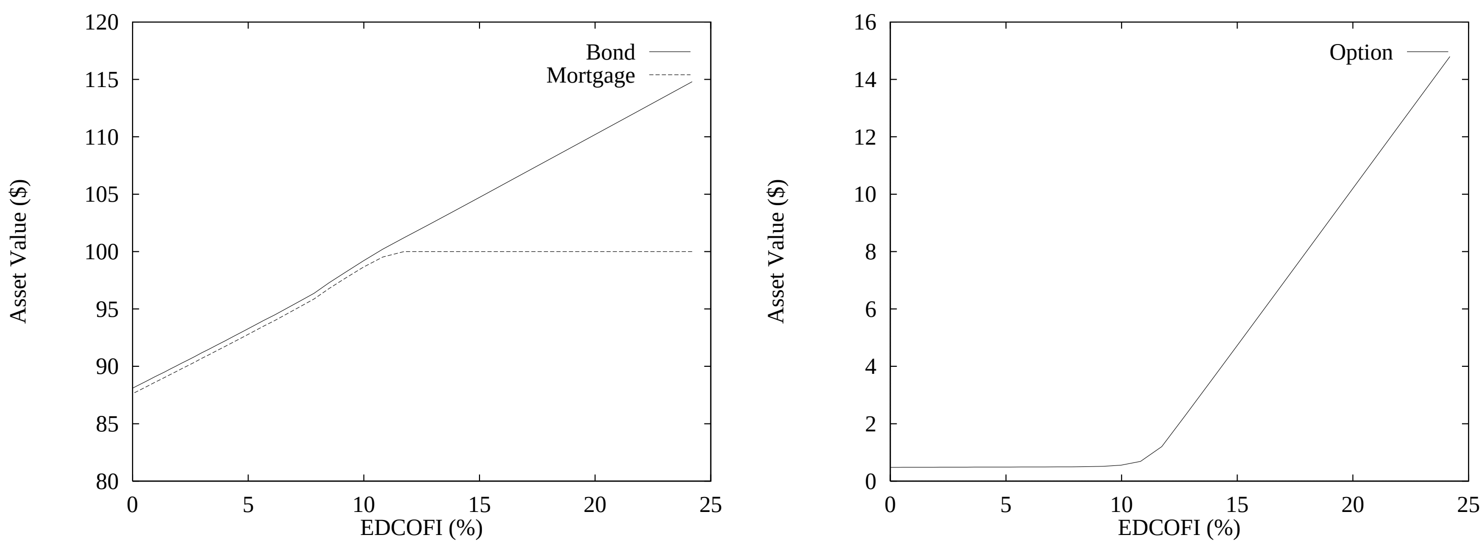

Figure 6: Bond and option values for different values of EDCOFI. Current short term riskless interest rate is 7.5%. Coupon rate equals current value of EDCOFI, and resets annually to the prevailing value of EDCOFI. There are no caps on coupon movements. Remaining principal is $100.

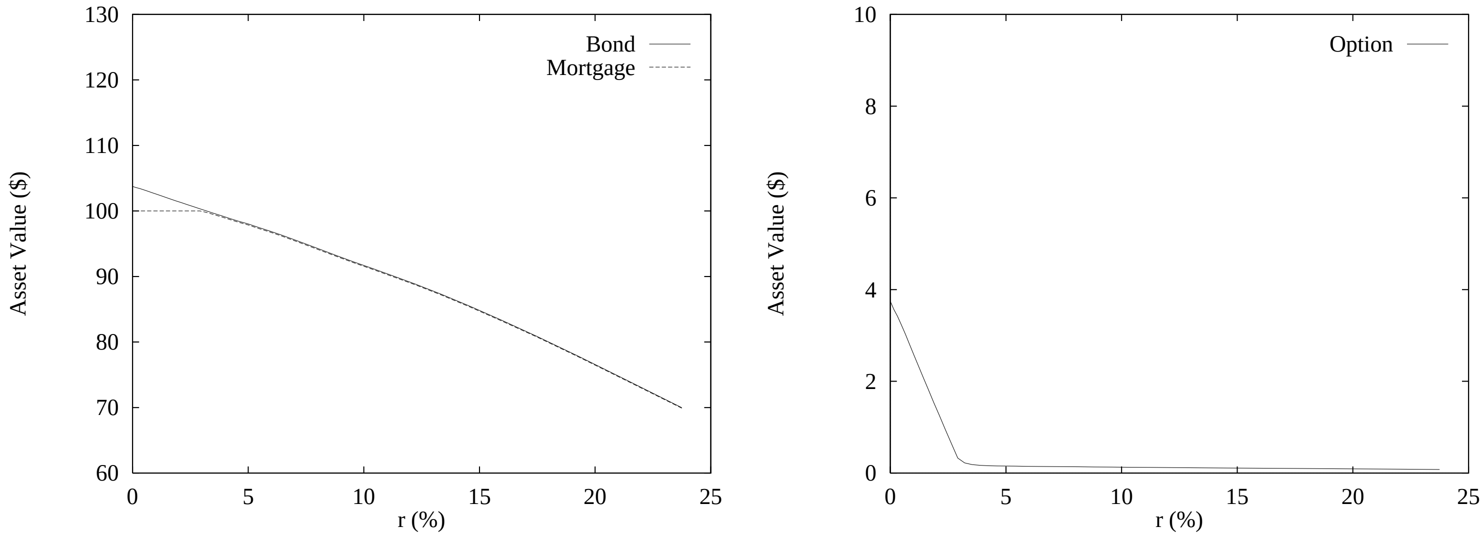

Figure 7: Bond and option values for different values of r. Current EDCOFI is 8.5%. Coupon rate currently equals 8.5%, and resets annually to the prevailing value of EDCOFI. Coupon rate has a lifetime cap of 13.5%. Remaining principal is \$100.

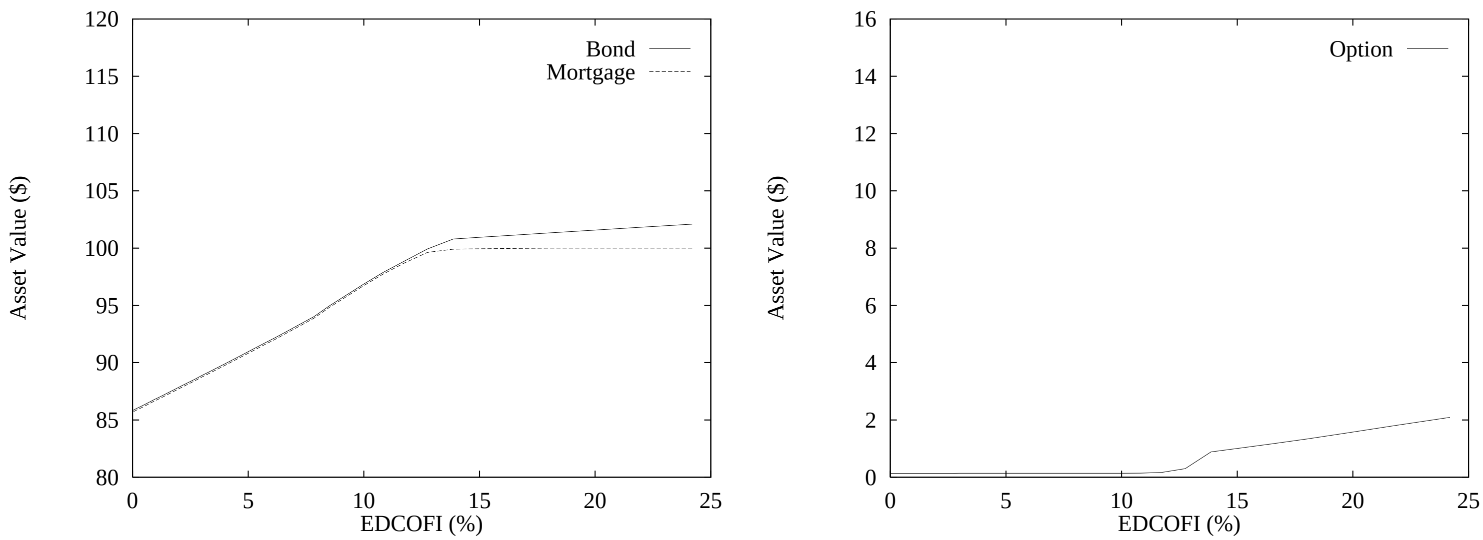

Figure 8: Bond and option values for different values of EDCOFI. Current short term riskless interest rate is 7.5%. Coupon rate equals current value of EDCOFI, and resets annually to the prevailing value of EDCOFI. Coupon rate has a lifetime cap of 13.5%. Remaining principal is \$100.

to determine the new coupon rate at each reset date. We have assumed a margin of 0% above EDCOFI, which is lower than is commonly observed in practice. Figures 9 and 10 differ from figures 7 and 8 in having a reset margin of 0.5% instead of 0%. This has a strong effect on the value of the prepayment option, which is now worth approximately 1% of the remaining principal amount over a wide range of values for the interest rate, r. In figures 11 and 12, the margin is increased to 1%, and figures 13 and 14 depict mortgages with a 2% reset margin (a fairly typical value in practice). These figures show that the value of the prepayment option is critically dependent on the interaction between contract features, especially caps and margin levels.

We know that some of the prepayment option value we have found may derive from the discrete coupon adjustment frequency. To show that this is not the only source of value, and that index dynamics play a significant role, we also valued an EDCOFI based ARM with no caps, 0% margin, and monthly coupon reset period. The value of the prepayment option was approximately 0.3% of remaining principal, compared with 0.5% with annual coupon reset. Thus, approximately 60% of the option value comes from the dynamics of the underlying index.

Given realistic values for contract features such as caps and reset levels, the mortgage holder's prepayment option has a significantly greater value than it would if (as assumed by previous authors) the underlying index adjusted instantaneously to movements in the term structure. The lags inherent in EDCOFI mean that ARMs based on this index look more like FRMs than has been previously realized.

Summary

This paper analyzes Adjustable Rate Mortgages (ARMs) based on the Eleventh District Cost of Funds Index (EDCOFI). Unlike the theoretical indices used in previous ARM valuation models, movements in EDCOFI lag substantially behind shifts in the term structure. We examine this relationship, and find that a partial adjustment model, with a contemporaneous interest rate variable and a single lagged value of EDCOFI, accurately describes the dynamics of EDCOFI.

The lag in the index means that, if interest rates suddenly fall substantially, mortgage holders may want to refinance their loans to avoid paying above market interest rates (even when the coupon rate adjusts every month to the prevailing index level). Previous methods for valuing adjustable rate mortgages are inadequate for ARMs based on EDCOFI, due to this lag. We extend a commonly used finite difference valuation algorithm for interest rate contingent claims to value ARMs based on EDCOFI. Our algorithm captures the effects of lags in the index, caps and floors on coupon adjustment, discrete coupon adjustment frequency, and teaser rates. It allows us to use

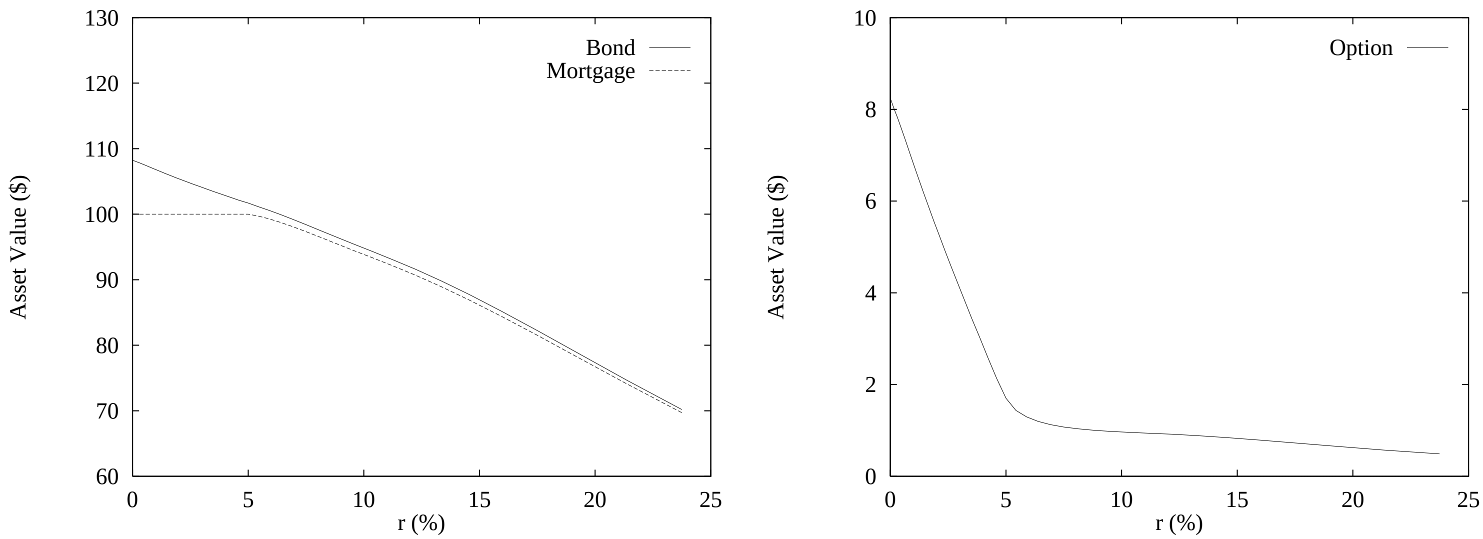

Figure 9: Bond and option values for different values of r. Current EDCOFI is 8.5%. Coupon rate currently equals 9.0%, and resets annually to the prevailing value of EDCOFI plus a margin of 0.5%. Coupon rate has a lifetime cap of 13.5%. Remaining principal is $100.

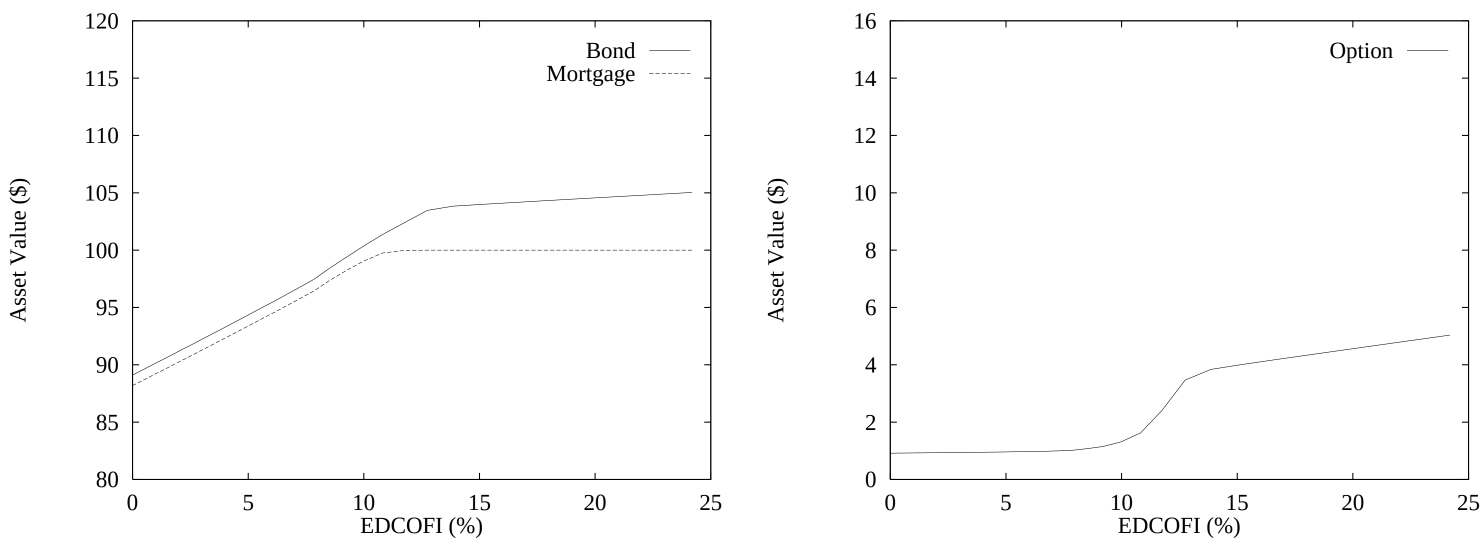

Figure 10: Bond and option values for different values of EDCOFI. Current short term riskless interest rate is 7.5%. Coupon rate equals current value of EDCOFI plus 0.5%, and resets annually to the prevailing value of EDCOFI, plus a margin of 0.5%. Coupon rate has a lifetime cap of 13.5%. Remaining principal is $100.

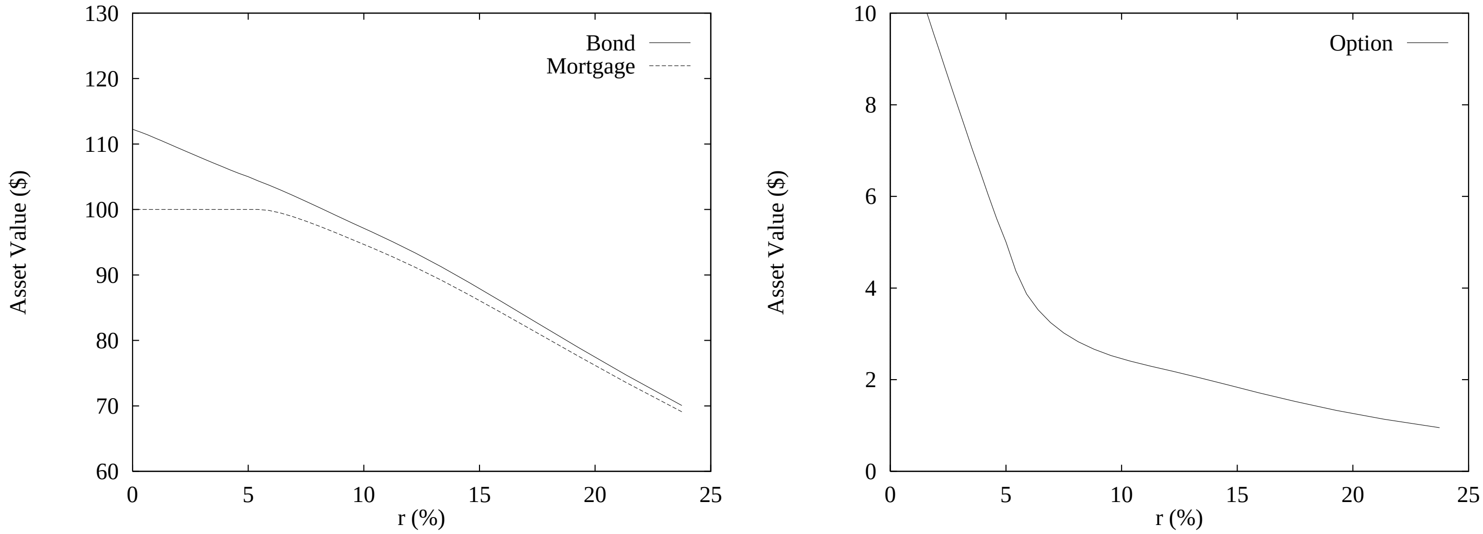

Figure 11: Bond and option values for different values of r. Current EDCOFI is 8.5%. Coupon rate currently equals 9.5%, and resets annually to the prevailing value of EDCOFI, plus a margin of 1%. Remaining principal is $100.

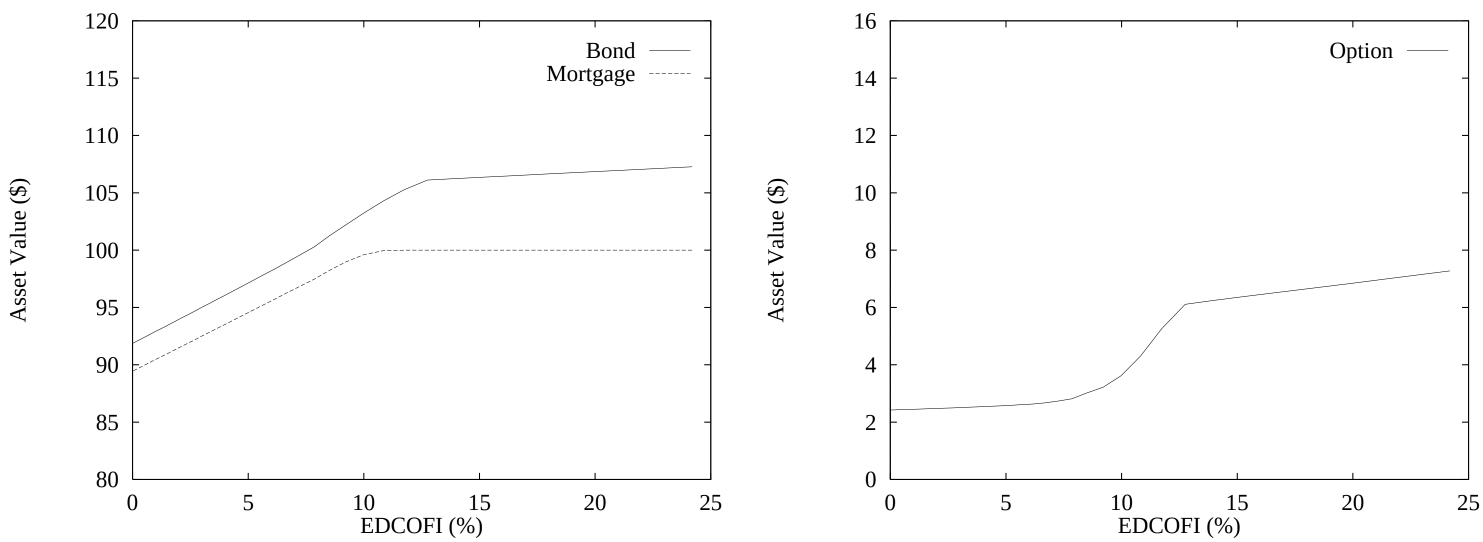

Figure 12: Bond and option values for different values of EDCOFI. Current short term riskless interest rate is 7.5%. Coupon rate equals current value of EDCOFI plus 1%, and resets annually to the prevailing value of EDCOFI, plus a margin of 1%. Coupon rate has a lifetime cap of 13.5%. Remaining principal is $100.

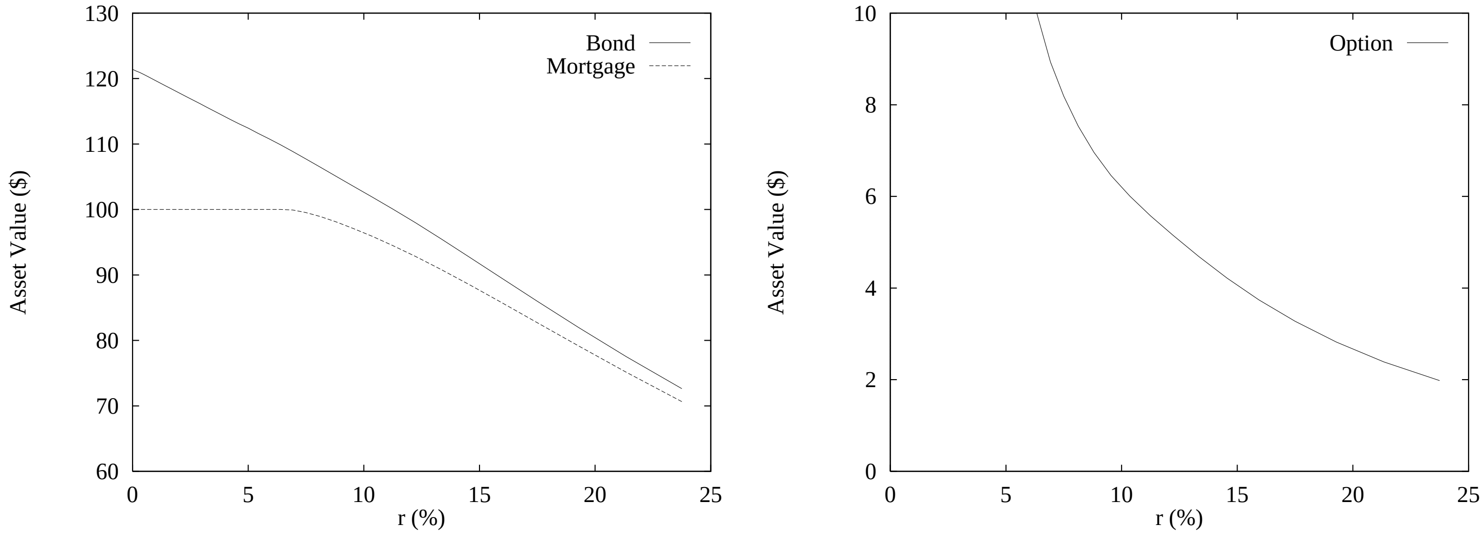

Figure 13: Bond and option values for different values of r. Current EDCOFI is 8.5%. Coupon rate currently equals 10.5%, and resets annually to the prevailing value of EDCOFI, plus a margin of 2%. Remaining principal is $100.

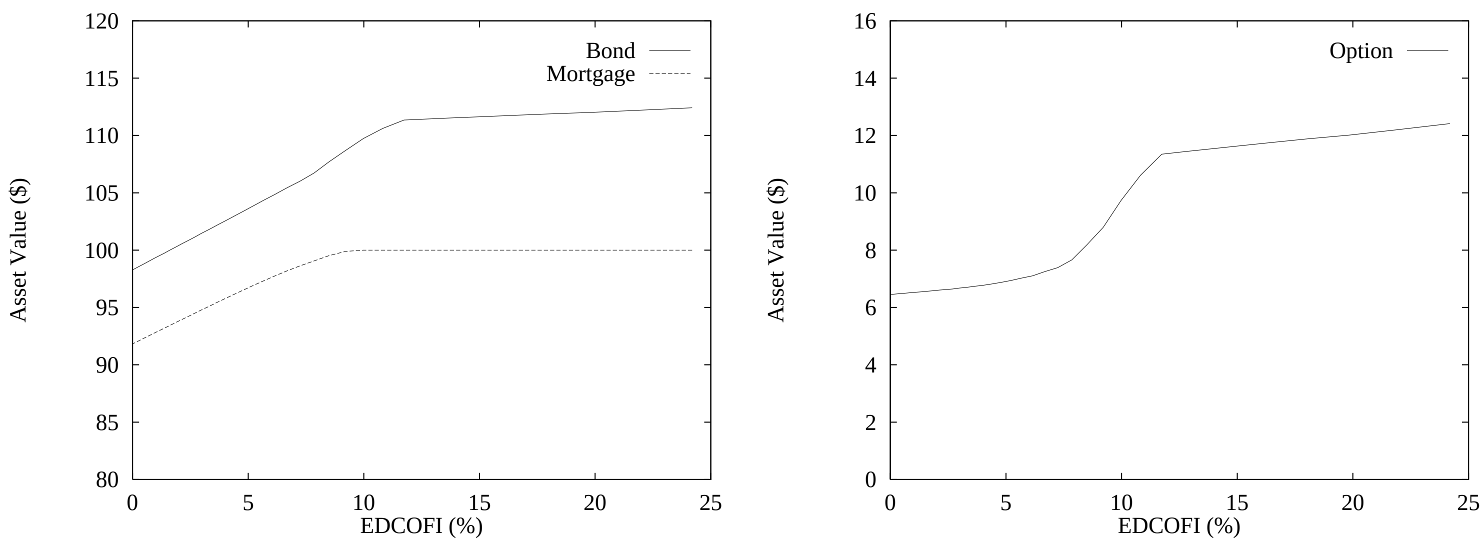

Figure 14: Bond and option values for different values of EDCOFI. Current short term riskless interest rate is 7.5%. Coupon rate equals current value of EDCOFI plus 2%, and resets annually to the prevailing value of EDCOFI, plus a margin of 2%. Coupon rate has a lifetime cap of 13.5%. Remaining principal is $100.

an empirical prepayment function for mortgage holders, or to calculate an optimal prepayment strategy for mortgage holders, leading to the valuation of their prepayment options. We find that the lag in EDCOFI contributes significantly to the value of the mortgage holder's prepayment option under realistic assumptions about contract terms and interest rates.

Appendix - The Crank Nicholson Algorithm

The Crank-Nicholson algorithm is a finite difference approximation used to solve partial differential equations like equation 23.[26] Finite difference methods replace derivatives (such as U_y, U_{yy}) with approximations involving differences between the values of U at neighboring points on a grid of (y,t) values. The partial differential equation is thus replaced with a set of difference equations, which can be solved subject to appropriate boundary conditions. There are several finite difference methods. Each uses a different approximation for the derivatives. Represent the function $U(y,t)$ by its values on the finite set of points,

$$y_j = j\,\Delta y, \tag{37}$$

$$t_k = k\,\Delta t, \tag{38}$$

for $j = 0,1,\ldots,J$, and for $k = 0,1,\ldots,K$. Δy and Δt are the grid spacings in the y and t dimensions respectively. Write

$$U_{j,k} \equiv U(y_j, t_k), \tag{39}$$

for each (j,k) pair. The difference approximations used in the Crank-Nicholson algorithm are

$$U_t(y_j,t_k) \approx (U_{j,k+1} - U_{j,k})\,/\,\Delta t; \tag{40}$$

$$U_y(y_j,t_k) \approx (U_{j+1,k+1} - U_{j-1,k+1} + U_{j+1,k} - U_{j-1,k})\,/4\,\Delta y; \tag{41}$$

$$\begin{aligned} U_{yy}(y_j,t_k) \approx\ & (U_{j+1,k+1} - 2U_{j,k+1} + U_{j-1,k+1} + U_{j+1,k} \\ & - 2U_{j,k} + U_{j-1,k})\,/2\,\Delta y^2. \end{aligned} \tag{42}$$

Substituting these into the original equation (e.g. equation 23), and rearranging, we obtain an equation of the form

$$a_j U_{j-1,k} + b_j U_{j,k} + c_j U_{j+1,k} = d_{j,k}, \tag{43}$$ [27]

for each $k = 0,1,\ldots,K-1$, and for $j = 1,2,\ldots,J-1$. For the extreme values of j, we need to impose additional boundary conditions. For equation 23, $j = 0$ corresponds to $y = 0$, i.e. "$r = \infty$". Any asset with finite cash flows held for a finite length of time therefore has zero value. The

[26] See McCracken and Dorn (1969), or Press, Flannery, Teukolsky and Vetterling (1992) for a discussion.

boundary condition at $j = 0$ is

$$U_{0,k} = C_{0,k}, \tag{44}$$

where $C_{0,k}$ is the cash flow from the asset at time t_k. This can be written in a form corresponding to equation 43 as

$$b_0 U_{0,k} + c_0 U_{1,k} = d_{0,k}, \tag{45}$$

where $b_0 = 1$, $c_0 = 0$, and $d_{0,k} = C_{0,k}$. At the other boundary, $j = J$ corresponds to $y = 1, r = 0$. The boundary condition at $y = 1$ is the equation

$$-\gamma y^2 \kappa \mu U_y + U_t + D = 0. \tag{46}$$

Using the non-central discrete approximation,

$$U_y(J,k) \approx \left(U_{J,k} - U_{J-1,k}\right) / \Delta y, \tag{47}$$

to replace U_y, and equation 40 to replace U_t, allows this equation to be rewritten in the form

$$a_J U_{J-1,k} + b_J U_{J,k} = d_{J,k}. \tag{48}$$

Equations 43, 45 and 48 can be written together in the form

$$M U_k = D_k \tag{49}$$

where M is a tridiagonal matrix with subdiagonal elements a_j, diagonal elements b_j, and super-diagonal elements c_j; D_k is the vector whose jth element is $d_{j,k}$.

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