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An Empirical Test of a Contingent Claims Lease Valuation Model*

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ABSTRACT

Despite the importance of leases in the US economy, and the existence of several theoretical lease pricing models, there has been little systematic attempt to estimate these models. This paper proposes a simple no-arbitrage based lease pricing model, and estimates it using a large proprietary data set of leases on several property types. We also define a new measure, the Option-Adjusted Lease Spread, or OALS (analogous to an option's implied volatility, or a mortgage-backed security's Option-Adjusted Spread), that allows us to compare leases with different maturities and contract terms on a consistent basis. We find sizeable pricing errors that cannot be explained using interest rates, lease maturity, or information on the options embedded in the contracts. This suggests either that there are significant mispricings in the market for real estate leases, or that lease terms depend heavily on unobservable, property-specific characteristics.

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1 Introduction

Leases are one of the most important financing sources for US corporations.[1] They are in many ways very similar to corporate bonds. Both are contracts in which one party promises to make set payments to another over some period of time. In both cases, the period of the payments may be long or short, the payments may be fixed or adjust over time according to some rule, and the contracts may or may not contain option-like features. In the case of corporate bonds, the most common options are the options to default, to call the bond (i.e. to repurchase it at some fixed price), and to convert it to a fixed share of the firm's equity. In the case of lease contracts, there is again a default option, there may be cancellation options (effectively making the lease callable), and there are often also various equity-like features in which future payments are tied to economic variables such as sales or CPI growth.

Despite the importance of these markets to the US economy, and the recent explosion of both theoretical and empirical research into the valuation of corporate debt (see Duffie and Singleton (2003) for a summary), leases have remained relatively under-studied. There have been many theoretical advances in lease pricing, including Miller and Upton (1976), Brennan and Kraus (1982), McConnell and Schallheim (1983), Schallheim and McConnell (1985), Grenadier (1995), and Grenadier (2005), and there is empirical support for some of these models' predictions. For example, Schwartz and Torous (2004) find that office building construction in 34 metropolitan areas agrees with many of the predictions in the strategic development model of Grenadier (2002). However, there has been little empirical testing of these models' pricing implications.[2] The research that does exist typically regresses the current period's lease payment on various possible explanatory variables to see what factors affect lease rates [see, for example, Glascock, Jahanian and Sirmans (1990), Benjamin, Boyle and Sirmans (1992), Wheaton and Torto (1994), Webb and Fisher (1996), and Mooradian and Yang (2000)]. It is hard, however, to interpret the regression coefficients obtained, since the value of a lease depends not just on its current payment amount, but also on how fast those payments will grow over time, what options there are to renew or cancel the lease, and on the lease's maturity. Gunnelin and Söderberg (2003) and Englund, Gunnelin, Hoesli and

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[1] Although the exact value of the outstanding stock of commercial and manufacturing real estate leases in the U.S. is not known, the economic value of these positions can be approximated through the value of non-residential, non-agricultural structures in the United States. At the end of year 2005, the Federal Reserve, Flow of Funds, B100-B103, reports the value of non residential commercial real estate to be \$9.9 trillion dollars. (Federal Reserve Board, Flow of Funds, L.220). The 2005 Census of Manufacturing reports that building rents account for 14.5% of the total capital expenditures of manufacturing firms (U.S. Census Bureau, Annual Survey of Manufacturers, Table 3, Supplemental Statistics for the United States and States: 2005).

[2] Ambrose et al. (2002), Hendershott and Ward (2003) and Clapham and Gunnelin (2003) develop and simulate valuation models. However, they do not explicitly estimate or test their models using market data.

Söderberg (2004) use regressions to estimate linear term structures of lease rates. However, their data set of Swedish leases does not contain many of the options commonly found in U.S. leases.[3]

There are several reasons why leases have not received as much attention as bonds, despite the close similarities between them. One major reason is the lack of available data. While bond prices are available both on their issue date and subsequently, very little pricing information on leases is available. In general, all we can observe are the terms of the lease, and the fact that both landlord and tenant were willing to sign the contract on a particular date. Not only do we not generally observe the market's assessment of the present value of the future payments due on the lease,[4] but, even if we did, this would not be enough to test a pricing model, since leases differ from bonds in one very fundamental respect. With a bond, the borrower receives a fixed amount of money in exchange for a promise to make the contractual payments on the bond. There is no argument over how much that fixed amount of money is worth. In contrast, in exchange for making lease payments, the lessee obtains use of the underlying asset (e.g. 1,000 square feet of retail space, or a two year old 747 jet) for some specified period of time. We cannot decide whether a lease is correctly priced merely by valuing its payments. We also need to take into account the value of what is obtained in exchange for those payments, something that is often not easily observable.

Another difference between leases and bonds is that leases are substantially more heterogeneous in their terms. Fixed rate bonds differ from each other in maturity, callability, etc., but they have only a few different patterns of cash flows — coupon payments of some fixed size, made at some fixed interval, with some terminal payment at maturity. By contrast, in addition to having a much wider variety of embedded options, the scheduled payments on lease contracts differ widely from one lease to another. For example, Figure 1 shows the monthly contractual rent per square foot for leases in light industrial properties in Phoenix, AR (the figure shows only those contracts that contractually specified adjustments). It can clearly be seen that there is a wide range of payment schedules, even without considering all of the embedded options in these leases.

This paper develops a no-arbitrage based lease valuation model following Brennan and Kraus (1982), and performs a systematic empirical test of the model. In the process, we also define a new lease valuation measure that allows us to compare two leases with different

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[3] There is also some empirical research looking at non-real estate leases. For example, Giaccotto et al. (2007) analyze a large sample of automobile leases to estimate the value of the lease-end purchase option typically embedded in such leases. Empirical analyses of the yields and default behavior on non-real estate leases include Lease et al. (1990) and Schallheim et al. (1987).

[4] There has been increasing capital market interest in directly securitizing leases. Recent examples of this include the securitization of the World Trade Center Lease in New York in 2001, and the securitization of the J.P. Morgan Chase Building in San Francisco, 2002.

terms on a consistent basis. To calculate this measure, we start by explicitly calculating the present value of the service flows obtained in exchange for the promised lease payments. From this we subtract the present value of the lease payments, which gives us the model's estimate of the NPV of the lease. Finally, we annualize this measure, obtaining the lease's "Option-Adjusted Lease Spread" (OALS). This measure, analogous to the Option Adjusted Spread (OAS) of a mortgage-backed security, can be consistently compared across leases with different maturities.

We are able to estimate the values of unobservable parameters (such as the market price of lease risk) by considering multiple leases on the same property or in the same city. Since both parties are willing to sign the lease contract at initiation, on that date its OALS (and NPV) ought to be zero in a competitive market. If the model fits perfectly, and we have observed all relevant information, the OALS we calculate ought therefore to be zero for all leases. If we obtain values that are not all zero,

1. Comparing the model's OALS for each lease with various characteristics of the leases and underlying properties, we should be able to learn something about what characteristics are important, and in turn what this tells us about how the underlying valuation model can be improved.

2. The extent to which the OALS *cannot* be explained by observable characteristics is a measure of either important unobservable characteristics, model misspecifcation or mispricing.[5] We cannot at this point determine which, but we can see how important a question this is for future researchers to investigate.

Besides allowing the model to be calibrated, the OALS is important because it is a single summary statistic that can be consistently compared across different leases, regardless of their maturities, contractual payment amounts, and embedded options.[6] In this light, the OALS measure can be interpreted as a lease counterpart to an option's implied volatility. In the options literature, the first use made of the Black and Scholes (1973) model was as a serious description of option prices, and several authors performed empirical tests of the model [see, for example, Rubinstein (1985)]. However, as the model's shortcomings became clear, and as more realistic, but more complex, alternative models were developed, the Black and Scholes (1973) model has nevertheless retained its usefulness as a simple way of allowing us to compare prices for options with different maturities and strike prices. Our model

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[5] Examples of these unobservable characteristics include both property characteristics (e.g., view, access, location, and tenant mix) and tenant characteristics (such as credit quality).

[6] It is important to note that OALS is independent of the size of a lease, so, just as with IRR, it may not be a good way to compare two leases of very different sizes.

allows us to do the same for leases, summarizing all of the characteristics of a lease and its underlying asset in a single measure, the model's OALS.

In our empirical analysis, we use a proprietary data set of seven hundred and eleven leases from properties located in 11 states. The leases are a subset of a portfolio of leases assembled by the lead underwriter for a \$559,155,971 commercial mortgage pool consisting of 132 fixed-rate, first lien mortgage loans. For each of the 47 properties in our sample we have detailed information about the contract structure of the leases, including the base rent levels, the treatment of expense pass-throughs, the renewal options, the reset periods and level of rent changes, the percentage options, and the maturities on the contracts. We also have detailed information about the local submarket, tenant mix, mortgage contract, and a recent appraisal for each of the properties. There is considerable variability in the leases which provides a unique opportunity to analyze the cross-sectional variation in lease contract structures across locations and properties.

The next section presents the model, outlining our assumptions about service flows, building value, the term structure of lease rates and how these concepts can be combined into a lease valuation model that is empirically tractable. The following section discusses our data set development, and estimates the model. The final section concludes.

2 The Model

We develop a simple contingent-claims model for valuing leases with a wide range of possible terms and embedded options, as a function of the instantaneous spot lease rate. This rate is taken as exogenously specified, as in Brennan and Kraus (1982), McConnell and Schallheim (1983) and Schallheim and McConnell (1985). We use no-arbitrage arguments to derive a partial differential equation for the lease value in terms of the underlying state variable.[7]

Suppose the spot lease rate (equivalently, the instantaneous service flow) from a new building, X_t, follows a geometric Brownian motion process,

$$dX_t/X_t = \mu_x \, dt + \sigma_x \, dZ_t. \tag{1}$$

Write the value of an asset whose payoffs depend on X_t (and possibly time) as $V(x,t)$, where x is the current value of X_t. By Ito's Lemma, we can write

$$\frac{dV(x,t)}{V(x,t)} = m(x,t) \, dt + s(r,t) \, dZ, \tag{2}$$

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[7] Although we derive the model using no-arbitrage arguments, it could also be derived from equilibrium considerations, as in, for example, Goetzmann et al. (2003) (see also Merton (1976) and Ingersoll (2006)).

where

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\begin{aligned}
m(x,t)\,V &= V_t + \mu_x x V_x + \frac{1}{2}\sigma_x^2 x^2 V_{xx}, &(3)\\
s(x,t)\,V &= \sigma_x x V_x. &(4)
\end{aligned}
$$

This equation holds for any asset V. Since everything is driven by a single factor, the instantaneous returns on all assets depending on only X_t and t must be perfectly correlated. As a result, to prevent arbitrage the risk premium on any asset must be proportional to the standard deviation of its return.[8] Substituting for the asset's standard deviation from Equation (4), if the asset pays out dividends at rate d, we can thus write

$$m = r - \frac{d}{V} + q(x,t) x \sigma_x \frac{V_x}{V}, \tag{5}$$

where $q(x,t)$ is the price of risk. Substituting equation (5) into equation (3), assuming that the price of risk is a constant,

$$q(x,t) \equiv \lambda/\sigma_x, \tag{6}$$

yields a partial differential equation that must be satisfied by any contingent claim,

$$\frac{1}{2}\sigma_x^2 x^2 V_{xx} + [\mu_x - \lambda]\, x V_x + V_t - rV + d = 0. \tag{7}$$

Note that in the case where $\lambda = \mu_x - r$, this equation reduces to the familiar Black and Scholes (1973) option pricing equation. Equation (7) can be used to price any contract whose payments depend on the instantaneous spot lease rate, including lease contracts with assorted embedded options, by suitable choice of boundary conditions. For example, consider the building itself. This is an infinitely lived asset which pays out X_t at time t, but also depreciates. If we assume a depreciation rate of δ, the building is paying an effective dividend at rate $x - \delta V$, so its value solves the equation

$$\frac{1}{2}\sigma_x^2 x^2 V_{xx} + [\mu_x - \lambda]\, x V_x + V_t - rV + (x - \delta V) = 0. \tag{8}$$

The homogeneity of this problem implies that the solution must be a multiple of x, and it is simple to verify that the solution is

$$A(x) = \frac{x}{(r + \lambda) - (\mu_x - \delta)}. \tag{9}$$

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[8] Suppose this did not hold for two risky assets. We could then create a riskless portfolio of these two assets with a return strictly greater than r, leading to an arbitrage opportunity (see Ingersoll (1987)).

This is just the standard perpetuity formula, where the expected return on the asset is $r+\lambda$, and the cash flows' (expected) growth rate is $\mu_x - \delta$. The depreciation rate has the same effect as a reduction in the growth rate of the service flows.

2.1 Term Structure of Lease Rates

The partial differential equation above can be used to determine the term structure of fixed lease rates for a given instantaneous lease rate. For a given maturity, T, it must be the case that rolling over a sequence of instantaneous leases has the same present value as taking out a single lease with a constant periodic payment over the same time interval.

Rolling over short term leases

Consider first rolling over a sequence of instantaneous leases. The present value of the remaining payments is the value of an asset which pays out a dividend at rate X_t, and has value 0 at date T. It therefore solves the equation

$$\frac{1}{2}\sigma_x^2 x^2 V_{xx} + \left[\mu_x - \lambda\right] x V_x + V_t - rV + x = 0, \tag{10}$$

subject to the boundary condition that its value at maturity equals zero. It is simple to verify that the solution to this equation is

$$V(x) = \frac{x}{(r+\lambda) - \mu_x}\left[1 - e^{-\{(r+\lambda)-\mu_x\}T}\right], \tag{11}$$

where T is the remaining time to maturity. This is just a version of the familiar annuity formula. Note that as T grows large, this converges to the result in Equation (9), with $\delta = 0$.

Fixed Lease Payments

Now consider a long term lease with constant payment d per period. The present value of the remaining payments is the value of an asset which pays out a dividend at (constant) rate d, and has value 0 at date T. It therefore solves the equation

$$\frac{1}{2}\sigma_x^2 x^2 V_{xx} + \left[\mu_x - \lambda\right] x V_x + V_t - rV + d = 0, \tag{12}$$

subject to the boundary condition that its value at maturity equals zero. It is simple to verify that the solution to this equation is

$$V(x) = \frac{d}{r}\left[1 - e^{-rT}\right], \tag{13}$$

where T is the remaining time to maturity. Since the values of the two leases must be the same, we can immediately solve for the ratio of the long term to the instantaneous lease rate,

$$\frac{d}{x} = \left(\frac{r}{r+\lambda-\mu_x}\right)\left(\frac{1-e^{-\{(r+\lambda)-\mu_x\}T}}{1-e^{-rT}}\right). \tag{14}$$

2.2 Lease NPV and Option-Adjusted Lease Spread (OALS)

The value of a lease depends not only on the payments, but also on the value of the service flow received from the underlying asset. Define the net present value (NPV) of the lease to be the value of the service flows from the underlying asset during the period of the lease, less the value of the lease payments. Write $l(x,t)$ for the contractual lease payment at time t if the underlying spot lease rate is x, the lease's NPV, L, is thus the value of an asset that pays out

$$x - l(x,t)$$

each period. It thus satisfies the partial differential equation

$$\frac{1}{2}\sigma_x^2 x^2 L_{xx} + [\mu_x - \lambda]\, xL_x + L_t - rL + x - l(x,t) = 0. \tag{15}$$

At the lease's final expiration date, T, we have the boundary condition

$$L(x,T) = 0.$$

At any renewal option date, τ, the lease will be renewed only if doing so is preferable to taking out a new lease (which always has NPV equal to zero). Thus, L must also satisfy the boundary condition

$$L(x,\tau) \geq 0$$

at each renewal date. Solving equation (15) subject to these boundary conditions allows us to calculate the NPV for any lease, taking into account any embedded options. Comparing the NPVs of different leases allows us, in principle, to see how well our model matches observed lease prices, but the NPV has the disadvantage of being maturity dependent. Even if our model is wrong, the NPV of a very short lease must be close to zero merely because there is little time for any mispricing to have an effect. This makes it hard to know immediately what to conclude when comparing the NPVs of two leases with different maturities. This is very similar to the problem that arises when comparing the prices of bonds or options with different maturities. Since price automatically varies with maturity, how do you know that any price difference you observe is not due entirely to the maturity difference? To solve this problem, it is customary to quote a transformed version of the price that removes the maturity dependence. For bonds, traders usually talk about yield rather than price. For options, traders often quote implied volatility. In the case of leases, we can define something similar, the "Option-Adjusted Lease Spread", or OALS. Consider the equation

$$\frac{1}{2}\sigma_x^2 x^2 L'_{xx} + [\mu_x - \lambda] x L'_x + L'_t - rL' + x + s - l(x,t) = 0. \tag{16}$$

This equation for $L'(x,t,s)$ is the same as equation (15) for L, except that the payout has been increased by a (constant) amount s. The OALS is the value of s that solves the equation

$$L'(x,t,s) = 0,$$

when equation (16) is solved subject to the boundary conditions[9]

$$\begin{aligned} L'(x,T,s) &= 0, \\ L'(x,\tau,s) &= 0, \qquad \text{if } L(x,\tau) = 0. \end{aligned}$$

This second boundary condition says that the lease's termination behavior is determined by the solution to equation (15). The OALS is an annualized version of the NPV, the (constant) upward shift in the service flow from the underlying asset required for the model to produce an NPV of zero.[10] Focusing on the OALS rather than the NPV removes the maturity dependence of the NPV, but otherwise conveys the same information. In particular, the OALS will be zero/positive/negative whenever the NPV is zero/positive/negative.

3 Empirical Analysis

3.1 Lease Data

Summary statistics for the leases in our data set are reported in Table 1. The mean contract maturity is 5.23 years, about 40% of the leases were written in 1995 and 1996, and the earliest leases were written in 1987. The average appraised value of the properties was about \$5 million and the average monthly base rent in September 1997 was \$11.02 per square foot. Approximately 27.5 percent of the leases had renewal options, and the average number of renewals for those leases that had renewals was one. The maximum number of renewal

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[9] Note that there will always be exactly one solution to this equation, because L' is an increasing function of s, with $L' |_{s=-\infty} = -\infty$, and $L' |_{s=\infty} = \infty$.

[10] This is similar in spirit to the Option Adjusted Spread used in mortgage valuation models (see, for example, Gabaix et al. (2007)).

options was six. The average leased square footage was 3,858 and the range of leased square footage varied from a maximum of 153,480 square feet and a minimum size of 120 square feet for professional office suites. Only 6% of our tenants had percentage rents, and the breaks for these rents were the ratio of the initial stabilized rent to the percentage rate. Nearly all of the leases used expense pass-throughs as the cost sharing mechanism, so this lease feature is not explicitly considered. We had information on the level of tenant improvements for each lease. However, we do not know if it was the tenant or the landlord that paid for these up-front expenses or how they were amortized over time. The mean tenant improvement was $.22 a square foot, but most tenants did not receive tenant improvements.

We identified two types of lease renewal/cancellations in leases. We had leases in which the renewals were exercisable at "market rents". Fifteen percent of the leases had this type of renewal. The second type of renewal option, present in about eleven percent of the sample, was exercisable at an *ex ante* fixed rent. This form of renewal option was usually, though not exclusively, associated with the anchor tenants.

The usual tenant mix in the suburban malls is one large high volume merchandiser, or a movie theater, as the anchor tenant, and small tenants that often include a video store, doctors' offices, nail and beauty salons, book stores, and restaurants. The suburban office were all low-rise buildings, and the tenants appear to be independent professionals, accounting and law firms, travel agencies, insurance companies, restaurants, and some retail. The light industrial properties were one story tilt-up construction, and the tenants are independent professionals and light manufacturing firms such as software companies, tee shirt printers, custom bicycle producers, and back-office financial services uses. The average age of the properties was 16.18 years and the average occupancy rate was 96.6%, ranging from fully occupied buildings to a low of 82% occupancy. The physical condition of the properties was excellent and 18.9% of the older properties had been recently renovated.

The lease documents indicated whether the tenant was an anchor; about 6% of the tenants were anchors. The tenant data also included a marker for "credit" tenants. We separately verified this credit evaluation by checking for a credit rating for the listed tenant. The credit rating scheme we developed is intended to approximate the classifications used by the Urban Land Institute in their publication *Dollars and Cents of Shopping Centers* but with added information about credit worthiness. We classify a tenant as a *National Credit Tenant* if we could find an above investment grade bond rating for the firm. We classified tenants as *Regional Tenants* if the tenant was part of a regional chain of, say, grocery stores or restaurants. We classified tenants as *Local Tenants* otherwise. About 6% of our sample was classified as *National Credit Tenants* and 87.7% were small local tenants.

We allow for a different term structure of spot leases in each of the fourteen metropolitan areas in which the properties are located: Atlanta, Baltimore, Denver, Detroit, Fort Worth, Las Vegas, Los Angeles, Madison, Orange, Philadelphia, Phoenix, Seattle, San Bernardino, and San Jose. We use a metropolitan area specific rental price index from 1987 through 1997, the National Real Estate Index obtained from Ernst and Young, to compute the standard deviation of the percentage rent changes. Those computed values are reported in the sixth column of Table 2.

3.2 Implementing the Model

To solve Equations (15) and (16) for a lease's NPV and OALS, we use a binomial tree similar to Cox, Ross and Rubinstein (1979). The model's parameters are

σ_x	The volatility of the spot lease rate,
μ_x	The expected rate of increase of the spot lease rate,
λ	The market price of lease risk,
r	The riskless interest rate.

Given these parameters, plus an initial value for the spot lease rate, X_0, we construct a binomial tree describing the risk-neutral evolution of the spot lease rate, which can be represented in continuous-time as

$$d\widehat{X}_t/\widehat{X}_t = [\mu_x - \lambda]\ dt + \sigma_x\, dZ_t. \tag{17}$$

Note that, unlike the usual case when building a binomial tree of stock prices, the risk-neutral drift of the lease rate is equal to $(\mu_x - \lambda)$ rather than the riskless rate, r. We match this drift, plus the volatility of the spot lease rate, σ_x, by setting the probability of an upward jump equal to

$$p = \frac{e^{(\mu_x - \lambda)\tau} - d}{u - d},$$

where τ is the time step in the tree, and u and d are the sizes of upward and downward jumps respectively, defined by[11]

$$\begin{aligned} u &= e^{(\mu_x - \lambda)\tau + \sigma\sqrt{\tau}}, \\ d &= e^{(\mu_x - \lambda)\tau - \sigma\sqrt{\tau}}. \end{aligned}$$

Given this tree, we can now value any security whose payoffs depend on the lease rate by discounting its payoffs back through the tree in the usual way. In the process, we can take

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[11] This choice of up and down jumps guarantees that both p and $1 - p$ will be positive for any $\sigma_x > 0$.

into account different promised payment schedules, as well as any embedded options. Let L_t^i be the NPV of a lease contract to the tenant at time t in state i (where i counts the number of up-movements in the tree since time 0). This must be zero at the lease's maturity, since the tenant neither receives any more services from the asset nor makes any further lease payments. Prior to maturity, we value the lease recursively in the usual way. Assuming the lease remains outstanding for another period, the value of the contract today equals its discounted expected value next period, plus the service flow from the underlying asset over the next period, minus the contractual lease payment over the next period:

$$L_t^i = \left[pL_{t+1}^{i+1} + (1-p)L_{t+1}^i\right]/(1+r\,\delta) + X_t^i\delta - C_t^i, \tag{18}$$

where X_t^i is the spot lease rate at time t in state i, and C_t^i is the contractual lease payment. If there is an option to cancel the contract at date τ, the option will be exercised only when it is in the tenant's best interests, i.e.,

$$L_\tau^i = \max\left\{\left[pL_{\tau+1}^{i+1} + (1-p)L_{\tau+1}^i\right]/(1+r\,\delta) + X_\tau^i\delta - C_\tau^i, 0\right\}. \tag{19}$$

Repeated application of this recursion allows us to calculate the initial lease NPV, V_0^0, and the OALS, s is calculated by repeatedly doing the same valuation with X_t^i replaced by $X_t^i + s$ until we find an NPV equal to zero.

3.3 Estimation

For any set of values for the parameters μ_x, λ and σ_x, and an initial spot lease rate, X_0, we can calculate the OALS for each lease, s_i. If our model captures every important feature of the lease contract, and we have the correct values for all of the parameters, the initial NPV and OALS for every lease ought to be zero. However, if factors not captured by the model are important in determining lease contract terms, or if there is (random) mispricing in the marketplace, they will show up as an NPV and OALS different from zero.[12] The important point is that, unlike a single period's contractual payment, the NPV or OALS we have calculated can be compared across different contracts, allowing us to explore the determinants of lease contract terms without worrying about whether all we are seeing is something related to the pattern of contractual cash flows, rather than a true economic relation between lease value and our explanatory variables.

In estimating the model, note that the parameters μ_x and λ appear in the valuation

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[12] One important factor not included in the model as written is a liquidity premium due to the illiquidity of the lease contracts. However, a constant liquidity adjustment will not affect our results at all; it will appear empirically as part of the price of risk, λ.

equation only in the form of their difference, $\mu_x - \lambda$, so we need only to estimate this difference, not the two parameters separately. We also need a value for r, the riskless interest rate. We use the 10 year Treasury rate. σ_x, the volatility of lease rates, is estimated separately for each city and each property type using the metropolitan rent series described above. We also need the current value of the underlying spot lease rate, X_t. The true instantaneous spot lease rate is not really observable, but we do have a rent series for each location, and we assume that the spot lease rate is some constant multiple, α, of the number in the rent series.[13]

We perform the estimation separately on each of fourteen metropolitan areas. Our estimation strategy is similar to that of Quigg (1993), who also used a valuation model and observed market outcomes to back out the implied parameters of the model. Our observables are the base rent levels, the timing and base rent levels on the renewal options, and the maturities on the leases. For each property class and metropolitan area, we estimate the risk-adjusted growth rate of the spot lease rate, $m \equiv \mu_x - \lambda$, and the ratio between the prevailing contractual lease rate R_t and the unobserved underlying spot lease rate, α, by nonlinear least squares, choosing the values that make the OALS's for all leases of that type and in that metropolitan area as close as possible to zero. Formally, let the OALS for lease i be $s_i(\alpha R_t, m; r, \sigma_x)$. Then the estimated values of α and m are given by

$$(\widehat{\alpha}, \widehat{m}) = \operatorname*{argmin}_{\alpha, m} \sum_{i=1}^{N} s_i^2(\alpha R_t, m; r, \sigma_x),$$

where N is the number of leases of the given property type in the given metropolitan area.

Finally, some tenants are so-called “anchor tenants”. These are usually large tenants whose presence in turn attracts both other tenants and customers for those tenants. Since they increase profit for other tenants, they also increase the amount of rent they are willing to pay, and in general are able to negotiate more favorable terms for themselves as a consequence. We assume this benefit is proportional to the current spot lease rate, and value anchor tenants' leases by taking the same state variable as for a non-anchor tenant, then scaling it by an additional (estimated) parameter that reflects this extra benefit.

Estimation results are shown in Table 2. Columns two and three show the fitted parameters for the metropolitan and property class–level multiple for the unobservable spot lease rent. The multiples are positive and quite precisely estimated for most of the cities, except for the smaller sample size cities such as Baltimore, Detroit, Madison, Fort Worth, and San Jose. In contrast, the parameter estimates for the risk adjusted growth rates are highly vari-

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[13] The homogeneity of the spot lease rate process implies that the values of many typical contracts will be some multiple of the current spot lease rate.

able and are quite imprecisely estimated for all of the subsamples as shown in columns four and five of Table 2. The final parameter, representing the anchor tenant's benefit multiple, appears in columns seven and eight when anchor tenants were present in the metropolitan–level property classes. These parameters are again quite precisely estimated for all but the smallest samples, and suggest that anchor tenants command significant discounts for positive externalities.

It is not the model's parameter estimates per se that are interesting, but rather what they imply for lease values. To further consider the implications of our fitted lease valuation models by submarket, we follow the intuition of Grenadier (1995), and plot the implied term structure of spot lease rates by property type and metropolitan area using the parameter estimates and equation 14. Figure 2 shows the implied spot lease rate for light industrial properties in four metropolitan areas: Baltimore, MD; Los Angeles and Orange, CA; and Seattle, WA. Figure 3 shows the implied term structure of spot lease rates for suburban office properties in three metropolitan areas: Phoenix, AZ and Los Angeles and Orange, CA. Finally, Figure 4 shows the results for retail properties in six metropolitan areas: Philadelphia, PA; Los Angeles and Orange, CA; Atlanta, GA; Phoenix, AZ; and Denver, CO. These curves represent the implied structure of forward rents as of September, 1997 in each case.

These plots highlight the variability in the shapes of the term structures across product types and metropolitan real estate markets. Los Angeles has a steeply downward sloping spot rent term structure in all three property types, whereas Orange has an upward sloping light industrial term structure and a flat term structure for retail and suburban office. The severe California recession in the late nineties appears to have been a more significant problem for the Los Angeles market than for the Orange market with an economic base that is focused on the port, its trucking hubs, and U.S. trade with Asia. Phoenix also has a strongly downward sloped term structure for both suburban and retail leases perhaps reflecting the overbuilding in the late nineties. In contrast, the Denver term structure is mildly upward sloping perhaps reflecting the perceived strength of the technology sector that was located there. These plots are again suggestive that our model is able to distinguish across metropolitan real estate markets and that differences in economic fundamentals can be inferred from the parameters of the lease contracting structures.

To further explore the variability in our estimated term structures, we regress the estimated term structure slope by property-type on metro-level economic indicators. Summary statistics for these indicators are reported in Table 3. As shown, the average slope is negative across our metropolitan areas, and there is considerable variability in all of the measures. The numbers of establishments in finance and insurance and in retail are intended as proxy measures for office and retail space demand. Overall, the metro-level CPI growth indicates little inflation in September of 1997, and indicate deflation in Detroit with a CPI growth of $-.1\%$. There is also considerable variability in the unemployment rates across the metropolitan areas, with a high of 6.9% in Detroit and elevated levels in most of the metro areas in the Los Angeles basin. The final measure is an indicator of the average household expenditures on consumption, intended to proxy for differences in average income and purchasing activity across the metropolitan areas.

Table 4 shows the results of regressing the estimated term structure slopes on the metro-level economic indicators. The regression explains little of the overall variance in the slope measures, with an R^2 of .03. In addition, the F test on the null hypothesis that the coefficients are jointly zero is accepted at conventional levels of economic significance. Despite the low explanatory power of this regression (due to the small sample size), the metro-level unemployment rate is positive and statistically significant at the 10% level. This result suggests that the term structures are more steeply sloped in metropolitan areas with higher unemployment rates (the correlation coefficient between unemployment and slope is .32). This weak unemployment rate channel and the important economic downturns caused by base closures in California may explain a part of the steeply negatively sloped term structures we have found in our results.

3.4 Analysis of OALS

Fitting the model gives us a set of parameters that prices the leases correctly on average. It also provides an OALS measure that tells us how far each lease is from the model's predictions. We further evaluate the reasonableness of our model by regressing these OALS values for each lease contract on a variety of lease contract and market indicators. In equilibrium, if our model was correct, and if we had access to all information relevant to pricing the leases, we would expect OALS to be zero for each lease contract. OALS values different from zero may reflect model misspecification, (random) mispricing, or dependence on additional unobservable variables. To investigate this further, Table 5 performs a fixed-effects regression of the OALS, and of the realized rent in September, 1997, on various possible explanatory variables. The fixed effects controls are for properties, since many of the leases are physically located in the same properties.[14] Looking first at the OALS results, we find that few of the individual coefficients are individually significant. However, the significant size coefficients suggest that the lease pricing model does not provide adequate controls for the rental concessions offered to large square footage tenants with long lease maturities. Since these tenants are often, but not always, the anchor tenants, this result indicates that

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[14] We do not report the fixed effects parameters, although a χ^2 test of joint insignificance was rejected at the 1% level

our simple shift parameter for the anchor tenants may not sufficiently capture the rental reductions offered to these tenants.[15] The relation between OALS and maturity may also have something do do with our use of only a single interest rate, the 10 year rate, as a proxy for the entire term structure in our analysis. Overall, however, the regression explains little of the variance in the leases' OALS, so the vast majority of the variability in OALS is due not to misspecification (which would show up as a systematic mispricing, correlated with observable factors) but rather to either unobservable explanatory variables or to (random) mispricings.

As a further visual diagnostic check of possible missing factors in the lease valuation model, we looked at plots of OALS by area and property type against various possible explanatory variables. Figure 5 is one example, showing retail leases in Orange, California, but these plots are representative of the results for other areas and property types. The additional factors we consider in these plots include lease maturity, lease origination date, the short term Treasury rate, and the net square footage of the leased property. As is clear from the plots, there is no discernible relation between these factors and the magnitude of the OALS. The model is doing a good job of picking up almost all of the explainable variation in lease values, though the dispersion of the OALS values suggests that random pricing errors or unobservable factors are also an important feature of lease contracting in these markets.

Finally, Table 5 presents the results of a regression of *ex post* realized rent (the rent on September 9, 1997) on the same set of lease contract and tenant characteristics. This reduced form specification is very common in the empirical lease literature, although in our case it is again a fixed effects model by property.[16] As previously discussed, an important limitation with reduced form regressions of this type is the joint determination of the lease rate and other lease contract terms at the origination of the lease. Since the reduced form regression does not properly account for all the lease contract terms, the reported results cannot be interpreted as offering any causal explanations for observed lease rent rates. Instead, the results are reported as a point of comparison with the structural modelling results.

As shown in columns three and four of Table 5, the length of the lease and lease contracts with renewals at fixed rates all have statistically significant positive effects on the realized rent. Credit worthy tenants, more contractual lease renewals, and the size of the leased space all lead to statistically significant reductions in the realized rents. The finding that the occupancy rate and tenant improvement expenditures appear to have statistically significant

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[15] This result is reminiscent of Schallheim et al. (1987), who find that lease yields are a decreasing function of asset size (among other variables). Crawford et al. (1981) also find some evidence of a relation between lease yields and asset size, though it is less conclusive.

[16] Again, we do not report the fixed effects parameters, although a χ^2 test of joint insignificance was rejected at the 1% level

and negative effects on realized rent levels is somewhat surprising. It is quite possible that these variables proxy for soft leasing markets in which landlords chose to maintain occupancy levels using low rent rates and other inducements such as tenant improvements.

It is interesting to contrast, the reduced form rental results with the results on the same factors in the OALS regression. Since the OALS is obtained from our structural model calibration, it accounts for the true economic value of leases through the combined effects of the drift and volatility of the spot lease rate, the embedded options, and the rent reset structure of leases. Different from the reduced form rental regression, we find that the effects of the options, tenant credit worthiness, occupancy levels, and tenant improvements do not have statistically significant (at the .05 level or better) effects on OALS. These results suggest that the rental regression results for these effects may be spurious due either to problems of endogeneity or to the fact that the rental regressions only account for the initial rental rate of the lease not its full profile of rent rates and options throughout the life of the lease. Another important difference between the two regressions is the realized rent reduced form regression indicates that office and retail rents are statistically significantly higher than warehouse rent, whereas the OALS analysis says that the OALS for these property types are on average higher, indicating a benefit to tenants due to “lower” rents.

4 Conclusions

This paper develops a flexible contingent claims lease valuation model, and estimates the model using a data set giving detailed contract information on 711 leases from three property types in 11 different states. Unlike prior empirical studies, which regress a single period’s lease payment on various explanatory right hand side variables, we analyze the behavior of the NPVs of different leases, estimated from the model. This has the advantage of allowing us to handle, in a consistent framework, leases which differ in their initial payments, how fast the payments grow over time, their maturity, and what options there are to renew or cancel the lease. We find large pricing errors that cannot be explained using interest rates, lease maturity, or information on the options embedded in the contracts, suggesting the presence of significant mispricing or unobservable factors in the market for real estate leases.

In addition to exploring the behavior of our model, we also propose a new measure for comparing different leases, the Option-Adjusted Lease Spread, or OALS. This measure is an annualized version of the model’s estimate of the lease’s NPV, analogous to a bond’s yield, an option’s implied volatility, or a mortgage-backed security’s Option-Adjusted Spread. Like these measures, OALS allows leases with different maturities and contract terms to be compared on a consistent and maturity-independent basis. Since leases are the primary collateral for commercial mortgages, our proposed OALS measure is potentially an important additional underwriting metric for commercial lenders and for investors in the commercial mortgage-backed securities market.

	Variable Name	Mean	Standard Deviation	Minimum	Maximum
Tenant and Property Characteristics	Age of Property	16.1800	8.298	1.000	41.000
	Anchor Lease	0.059	0.236	0.000	1.000
	Appraised Property Value (000)	\$5,056.574	\$3,657.392	\$850.000	\$21,400.000
	National Credit Tenant	0.064	0.244	0.000	1.000
	Regional Tenant	0.059	0.236	0.000	1.000
	Local Tenant	0.877	0.328	0.000	1.000
	Leased Square Footage (000)	3.858	10.636	.120	153.480
	Occupancy Rate	0.966	0.043	0.820	1.00
	Recently Renovated	0.189	0.392	0.000	1.000
	Suburban Office	0.184	0.387	0.000	1.000
	Retail Mall	0.451	0.500	0.000	1.000
	Light Industrial	0.366	0.481	0.000	1.000
Lease Characteristics	Maturity	5.230	5.670	1.000	50.000
	Number of Lease Renewal Options	0.171	0.601	0.000	6.000
	Lease Renewal at Market Rent	0.147	0.354	0.000	1.000
	Lease Renewal at Fixed Rent	0.107	0.310	0.000	1.000
	Percentage Rent Rate	0.002	0.010	0	0.080
	Tenant Improvements psf	0.222	0.151	0	0.50
	Realized Monthly Rent (9/97)	\$11.020	\$5.129	\$2.97	\$30.00
	Ten Year Treasury at Origination	0.061	0.001	0.055	0.081

Table 1: Summary Statistics for Properties and Leases

This table shows the summary statistics for the tenant, property, and lease characteristics of 711 leases originated between 1987 and 1996.

Lease Contract Type	City	N	Spot Lease Rate Multiple	Std. Error	Risk Adj. Growth Rate $\mu - \lambda$	Std. Error	Spot Rent Volatility σ	Anchor Tenant Multiple	Std. Error
Light Industrial	Baltimore	20	0.778	(0.568)	-0.008	(0.176)	0.043	0.657	(0.121)
	Detroit	23	2.018	(1.671)	-0.432	(0.610)	0.031	1.675	(0.637)
	Las Vegas	18	1.481	(0.331)	-0.019	(0.105)	0.028	1.014	(0.629)
	Los Angeles	89	1.132	(0.689)	0.023	(0.288)	0.069		
	Madison	17	1.758	(1.552)	0.157	(0.198)	0.052	0.027	(0.117)
	Orange	64	1.620	(0.384)	0.106	(0.125)	0.064	0.891	(0.341)
	Seattle	22	0.934	(0.337)	0.022	(0.094)	0.032	1.012	(0.034)
Suburban Office	Los Angeles	38	0.504	(0.119)	0.098	(0.028)	0.056		
	Orange	28	0.623	(0.126)	-0.002	(0.057)	0.068		
	Phoenix	64	0.821	(0.167)	0.060	(0.071)	0.065	1.004	(0.501)
Retail Malls	Atlanta	24	1.078	(0.461)	-0.027	(0.131)	0.026		
	Fort Worth	37	0.774	(1.885)	0.009	(0.070)	0.042	0.379	(1.372)
	Denver	26	1.521	(0.627)	-0.061	(0.099)	0.041	0.627	(0.201)
	Detroit	25	1.204	(0.592)	-0.005	(0.130)	0.034	0.938	(0.341)
	Las Vegas	31	0.831	(0.217)	-0.006	(0.007)	0.040		
	Los Angeles	11	1.406	(0.705)	0.004	(0.112)	0.054	0.487	(0.369)
	Orange	34	1.102	(0.417)	-0.004	(0.005)	0.038	0.363	(0.399)
	Philadelphia	27	0.864	(0.286)	0.018	(0.069)	0.059	0.798	(0.283)
	Phoenix	74	1.171	(0.408)	-0.008	(0.007)	0.034	0.974	(0.447)
	San Bernardino	31	1.259	(0.393)	-0.002	(0.005)	0.039	0.675	(0.344)
	San Jose	9	1.311	(1.657)	-0.002	(0.336)	0.055	0.821	(0.769)

Table 2: Coefficient Estimates for the Lease Valuation Model

Parameter estimates by city and property type. Parameters estimated are:

Spot Lease Rate Multiple: Ratio between instantaneous spot lease rate and lease rate in rental series
Risk-adjusted growth rate: Expected rental growth rate, μ, minus price of risk, λ
Spot rent volatility: Annualized volatility of short term lease rate.
Anchor tenant multiple: Anchor tenant lease rate / non-anchor tenant lease rate.

Variable Name	Mean	Standard Deviation	Minimum	Maximum
Slope (15-year lease rate minus 1 year lease rate)	-8.388	9.975	6.959	-40.879
Number of finance and insurance establishments (000)	4.879	2.861	.700	10.500
Number of retail establishments (000)	12.210	7.702	1.800	27.600
CPI Growth (%)	0.600	0.254	-0.1	1.300
Unemployment rate (%)	4.47	0.951	3.400	6.900
Annual Average Household Consumption Expenditures ($ 000)	37.430	2.482	34.400	42.350

Table 3: Summary Statistics for the Metro-level Macroeconomic Indicators

This table shows the summary statistics for the slopes of the spot lease curves in each metropolitan area (measured as the difference between the 15 year lease rate and the 1 year lease rate) and macroeconomic indicators for the respective metropolitan areas. The macroeconomic indicators include the metro-level number of finance and insurance establishments obtained from The 1997 Economic Census, U.S. Census Bureau, the metro-level number of retail establishments obtained from The 1997 Economic Census, U.S. Census Bureau, the metro-level CPI growth in September, 1997 obtained from the Bureau of Labor Statistics, U.S. Department of Labor, the metro-level unemployment rate in May, 1997 (September, 1997 statistics are not available) from the Bureau of Labor Statistics, U.S. Department of Labor, the metro-level annual average household expenditures from the 1997 Consumer Expenditure Survey, Bureau of Labor Statistics, U.S. Department of Labor.

	Coefficient	Std. Error
Intercept	-87.42*	(43.164)
Finance and Insurance Establishments	4.162	(4.189)
Retail Establishments	-1.305	(1.550)
CPI Growth	-6.938	(9.421)
Unemployment Rate	-5.352*	(2.833)
Annual Average Household Consumption Expenditures	1.261	(0.998)
Warehouse	10.119	(7.448)
Retail	8.518	(6.831)
Adjusted R^2	.03	
F Statistic	1.07	
N	21	

** .05 level of statistical significance

* .10 level of statistical significance

Table 4: Determinants of the Slope of the Spot Lease Curve (9/97)

This table reports a regression of the slopes of the spot lease curves in each metropolitan area (measured as the difference between the 15 year lease rate and the 1 year lease rate) on macroeconomic indicators for the respective metropolitan area. The macroeconomic indicators include the metro-level number of finance and insurance establishments in the 1997 Economic Census, U.S. Census Bureau; the metro-level number of retail establishments in the 1997 Economic Census, U.S. Census Bureau; the metro-level CPI growth in September, 1997; the metro-level unemployment rate in May, 1997 (September, 1997 statistics are not available) from the Bureau of Labor Statistics, U.S. Census Bureau; the metro-level annual average household expenditures from the 1997 Consumer Expenditure Survey, U.S. Bureau of Labor Statistics.

	OALS (psf)		Realized Rent (9/97)(psf)	
	Coefficient	Std. Error	Coefficient	Std. Error
Intercept	-521.598*	(233.901)	71.256**	(9.433)
Age of Property	0.229	(1.972)	0.073	(0.080)
Anchor Lease	-2.253	(11.952)	-0.973*	(0.482)
National Credit Tenant	-4.979	(11.649)	0.261	(0.469)
Local Tenant	-5.319	(9.066)	0.060	(0.365)
Leased Square Footage	-2.399**	(0.589)	-0.218**	(0.024)
Sq. Leased Square Footage	-0.011**	(0.004)	0.001**	(0.0002)
Lease Maturity	-2.292**	(.589)	0.144**	(0.030)
Number of Lease Renewal Options	10.591	(8.362)	-1.606**	(0.335)
Lease Renewal at Market Rent	-13.922	(8.013)	0.409	(0.323)
Lease Renewal at Fixed Rent	-15.919	(14.871)	1.917**	(0.600)
Percentage Lease Rate	-22.098	(12.213)	20.812	(11.492)
Property Occupancy Rate	445.221	(247.041)	-56.640**	(8.955)
Property Recently Renovated	28.339	(28.638)	-1.775	(1.155)
Property Suburban Office	62.768	(54.705)	9.820**	(1.091)
Property Retail Mall	30.439	(42.469)	3.715*	(1.712)
Tenant Improvements	392.838	(201.439)	-30.568**	(8.124)
Adjusted R^2	.098		.828	
F Statistic	2.46**		65.53***	
N	711		711	

$**$ .01 level of statistical significance

$*$ .05 level of statistical significance

Table 5: Determinants of OALS and Realized Rental Rate (9/97)

This table shows the results of running a fixed effects regression model on the OALS and on the realized rent observed in September, 1997.

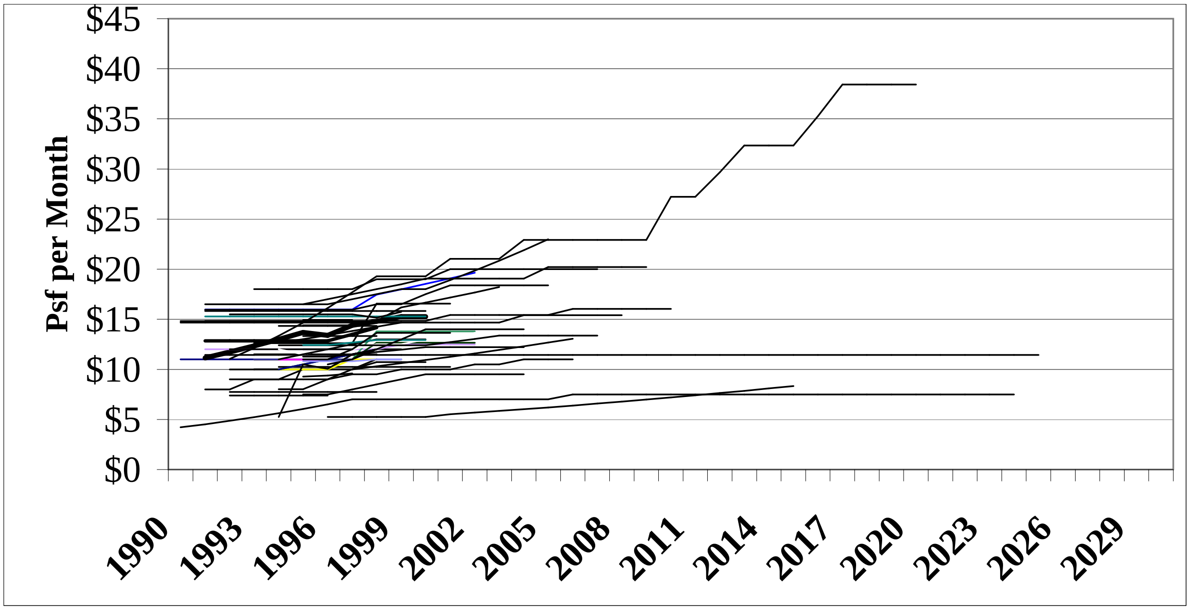


Figure 1: Examples of lease contracts – Phoenix light industrial

Each line corresponds to a single lease on light industrial property in Phoenix, AR, and shows the scheduled monthly lease payment per square foot (psf) for each month after the initiation date of the lease. The varying lease start dates run from 1990 through 1998.

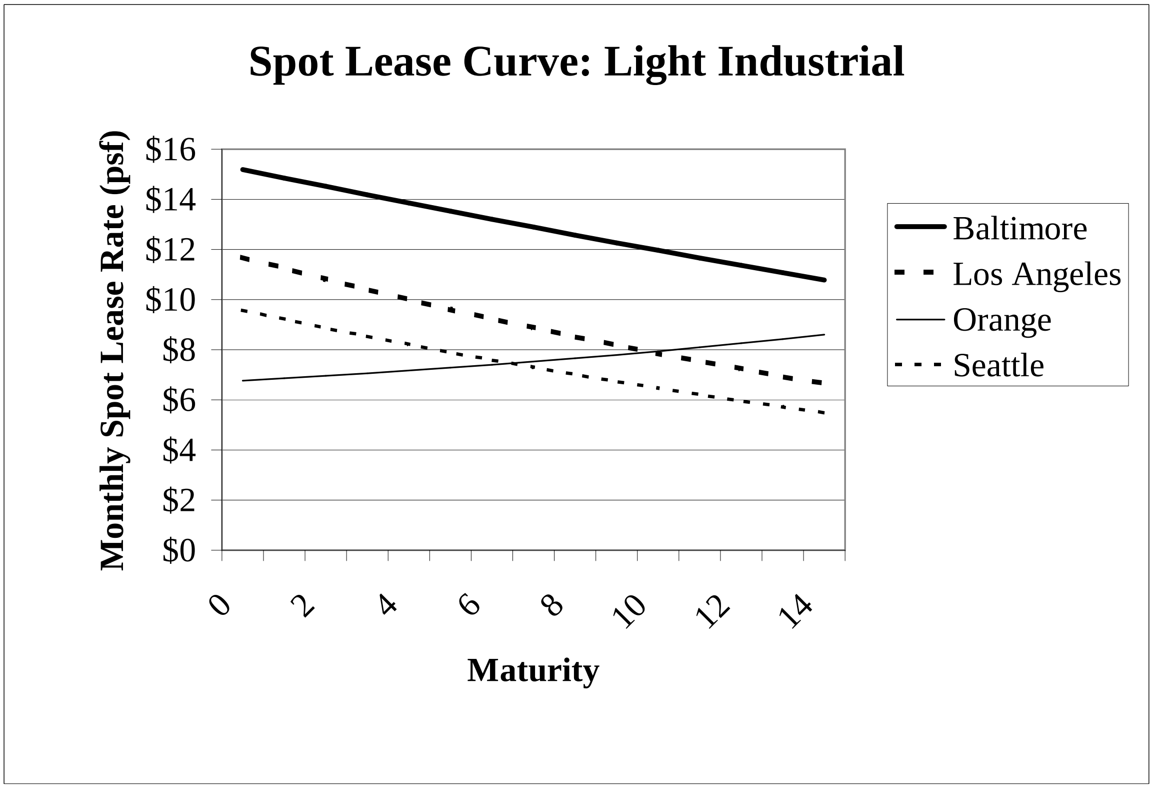


Figure 2: Term Structure of Lease Rates: Industrial

This graph shows the estimated term structure of lease rates for light industrial property in various cities, using the parameter estimates shown in Table 2. The fixed lease payment for a lease with maturity T years (as a multiple of the instantaneous lease rate, x) is calculated from Equation 14,

$$\frac{d}{x} = \left(\frac{r}{r + \lambda - \mu_x}\right)\left(\frac{1 - e^{-\{(r+\lambda)-\mu_x\}T}}{1 - e^{-rT}}\right).$$

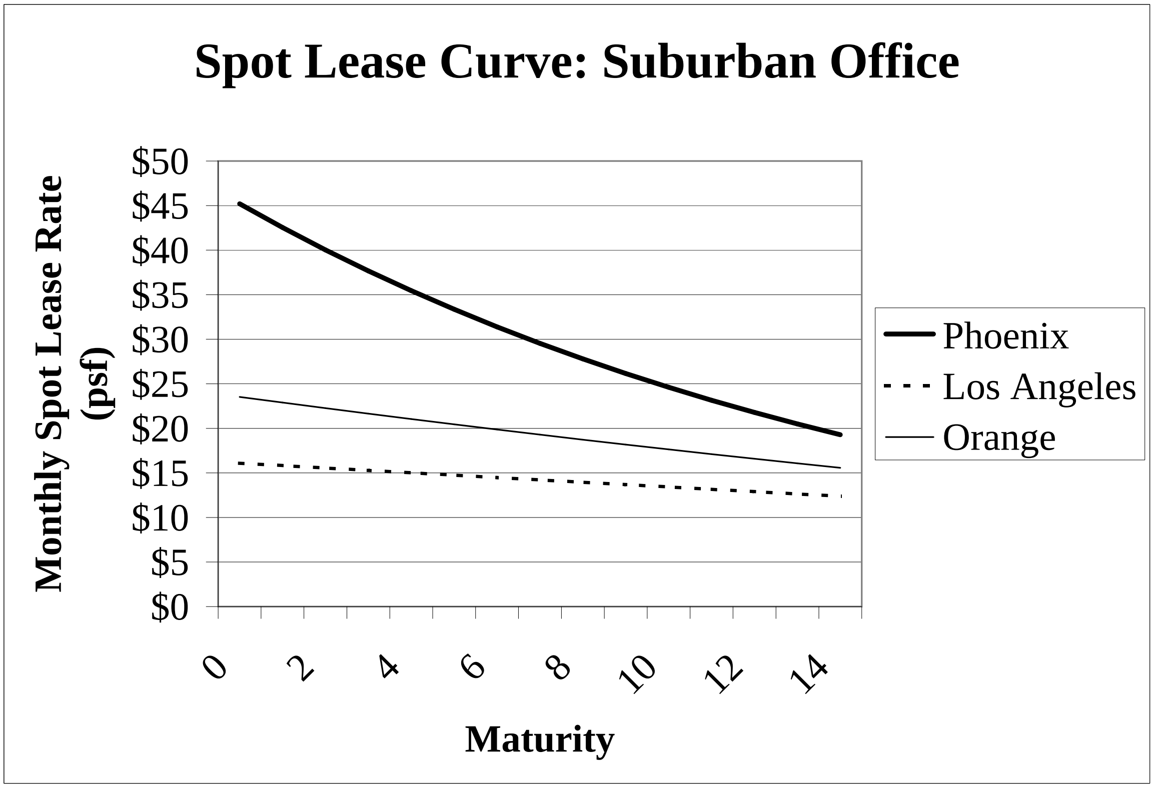


Figure 3: Term Structure of Lease Rates: Office

This graph shows the estimated term structure of lease rates for office property in various cities, using the parameter estimates shown in Table 2. The fixed lease payment for a lease with maturity T years (as a multiple of the instantaneous lease rate, x) is calculated from Equation 14,

$$\frac{d}{x} = \left(\frac{r}{r + \lambda - \mu_x}\right)\left(\frac{1 - e^{-\{(r+\lambda)-\mu_x\}T}}{1 - e^{-rT}}\right).$$

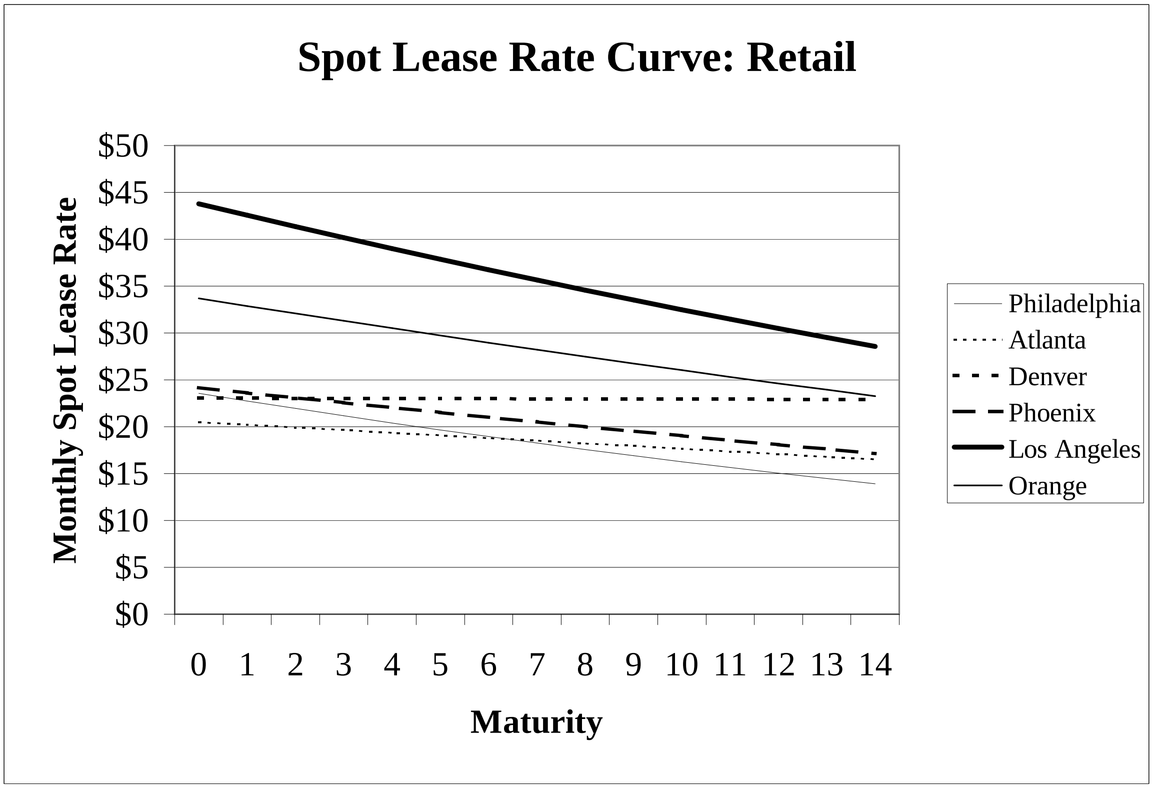
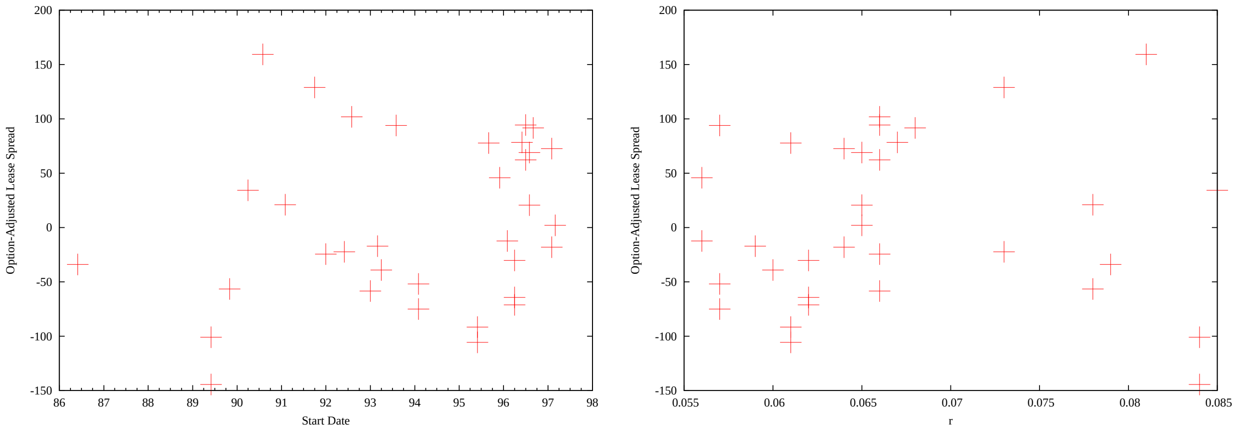


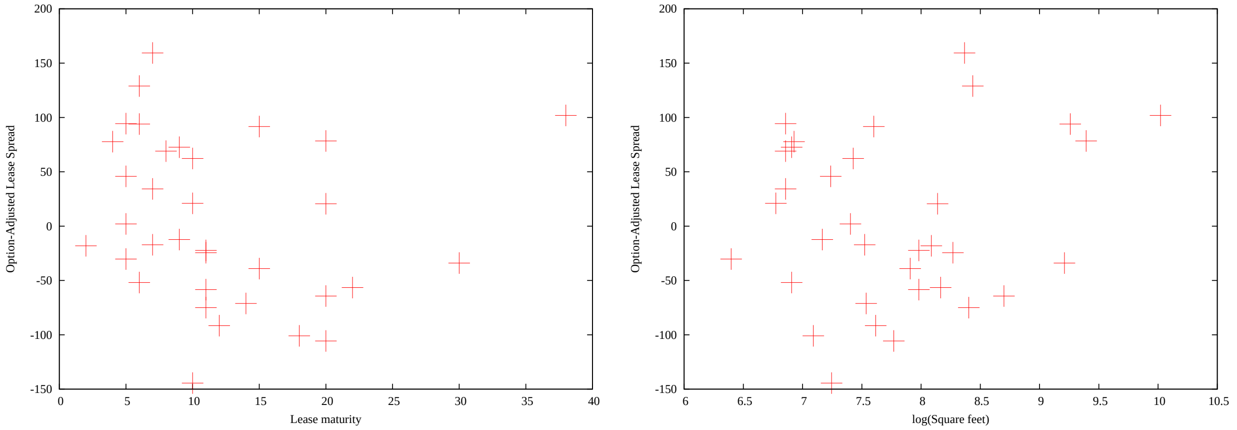
Figure 4: Term Structure of Lease Rates: Retail

This graph shows the estimated term structure of lease rates for retail property in various cities, using the parameter estimates shown in Table 2. The fixed lease payment for a lease with maturity T years (as a multiple of the instantaneous lease rate, x) is calculated from Equation 14,

$$\frac{d}{x} = \left(\frac{r}{r + \lambda - \mu_x} \right) \left(\frac{1 - e^{-\{(r+\lambda)-\mu_x\}T}}{1 - e^{-rT}} \right).$$



(a) Date (b) r



(c) Maturity (d) log(Size)

Figure 5: Option-Adjusted Lease Spread for retail leases in Orange County

This figure plots the OALS for each retail lease in Orange County (calculated using the parameter estimates shown in Table 2) against possible explanatory variables. Panel a. plots the OALS against the start date of the lease, panel b. plots OALS against the interest rate on the lease's start date, panel c. plots OALS against the initial length of the lease contract, and panel d. plots OALS against the log of the area leased.

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