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The Pricing and Hedging of Mortgage-Backed Securities: A Multivariate Density Estimation Approach

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Abstract

This chapter presents a non-parametric technique for pricing and hedging mortgage-backed securities (MBS). The particular technique used here is called multivariate density estimation (MDE). We find that MBS prices can be well described as a function of two interest rate factors; the level and slope of the term structure. The interest rate level proxies for the moneyness of the prepayment option, the expected level of prepayments, and the average life of the MBS cash flows, while the term structure slope controls for the average rate at which these cash flows should be discounted. We also illustrate how to hedge the interest rate risk of MBS using our model. The hedge based on our model compares favorably with existing methods.

1 Introduction

The mortgage-backed security (MBS) market plays a special role in the U.S. economy. Originators of mortgages (S&Ls, savings and commercial banks) can spread risk across the economy by packaging these mortgages into investment pools through a variety of agencies, such as the Government National Mortgage Association (GNMA), Federal Home Loan Mortgage Corporation (FHLMC), and Federal National Mortgage Association (FNMA). Purchasers of MBS are given the opportunity to invest in virtually default-free interest-rate contingent claims that offer payoff structures different from U.S. Treasury bonds. Due to the wide range of payoff patterns offered by MBS and their derivatives, the MBS market is one of the largest as well as fastest growing financial markets in the United States. For example, this market grew from approximately $100 million outstanding in 1980 to about in $1.5 trillion in 1993.

Pricing of mortgage-backed securities is a fairly complex task, and investors in this market should clearly understand these complexities to fully take advantage of the tremendous opportunity offered. Pricing MBS may appear fairly simple on the surface. Fixed-rate mortgages offer fixed nominal payments; thus, fixed-rate MBS prices will be governed by pure discount bond prices. The complexity in pricing of MBS is due to the fact that statutorily mortgage holders have the option to prepay their existing mortgages; hence, MBS investors are implicitly writing a call option on a corresponding fixed-rate bond. The timing and magnitude of cash flows from MBS are therefore uncertain. While mortgage prepayments occur largely due to falling mortgage rates other factors such as home owner mobility and home owner inertia play important roles in determining the speed at which mortgages are prepaid. Since these non-interest rate related factors that affect prepayment (and hence MBS prices) are difficult to quantify the task of pricing MBS is quite challenging.

This chapter develops a non-parametric method for pricing MBS. Much of the extant literature (e.g., Schwartz and Torous (1989)) employs parametric methods to price MBS. Parametric pricing techniques require specification and estimation of specific functions or models to describe interest rate movements and prepayments. While parametric models have certain advantages, any model for interest rates and prepayments is bound to be only an approximation of reality. Non-parametric techniques such as the multivariate density estimation (MDE) procedure that we propose, on the other hand, estimates the relation between MBS prices and fundamental interest rate factors directly from the data. MDE is well suited to analyzing MBS because, although financial economists have good intuition for what the MBS pricing fundamentals are, the exact models for the dynamics of these fundamentals is too complex to be determined precisely from a parametric model. For example, while it is standard to assume at least two factors govern interest rate movements, the time series dynamics of these factors and the interactions between them are not well understood. In contrast, MDE has the potential to capture the effects of previously unrecognized or hard to specify interest rate dynamics on MBS prices.

In this chapter, we first describe the MDE approach. We present the intuition behind the methodology and discuss the advantages and drawbacks of non-parametric approaches. We also discuss the applicability of MDE to MBS pricing in general and to our particular application.

We then apply the MDE method to price weekly TBA (to be announced) GNMA securities[1] with coupons ranging from 7.5% to 10.5% over the period 1987-1994. We show that at least two interest rate factors are necessary to fully describe the effects of the prepayment option on prices. The two factors are the interest rate level, which proxies for the moneyness of the prepayment option, the expected level of prepayments, and the average life of the cash flows; and the term structure slope, which controls for the average rate at which these cash flows should be discounted. The analysis also reveals cross-sectional differences among GNMAs with different coupons, especially with regard to their sensitivities to movements in the two interest rate factors. The MDE methodology captures the well-known negative convexity of MBS prices.

Finally, we present the methodology for hedging the interest rate risk of MBS based on the pricing model in this chapter. The sensitivities of the MBS to the two interest rate factors are used to construct hedge portfolios. The hedges constructed with the MDE methodology compare favorably to both a linear hedge and an alternative non-parametric technique. As can be expected, the MDE methodology works especially well in low interest rate environments when the GNMAs behave less like fixed maturity bonds.

2 Mortgage-Backed Security Pricing: Preliminaries

Mortgage-backed securities represent claims on the cash flows from mortgages that are pooled together and packaged as a financial asset. The interest payments and principal repayments made by mortgagees, less a servicing fee, flow through to MBS investors. MBS backed by residential mortgages are typically guaranteed by government agencies such as the GNMA

[1] A TBA contract is just a forward contract, trading over the counter. More details are provided in Section 3.

and FHLMC or private agencies such as FNMA. Because of the reinsurance offered by these agencies MBS investors bear virtually no default risk. Thus, the pricing of an MBS can be reduced to valuing the mortgage pool's cash flows at the appropriate discount rate. MBS pricing then is very much an issue of estimating the magnitude and timing of the pool's cash flows.

However, pricing an MBS is not a straightforward discounted cash flow valuation. This is because the timing and nature of a pool's cash flows depends on the prepayment behavior of the holders of the individual mortgages within the pool. For example, mortgages might be prepaid by individuals who sell their homes and relocate. Such events lead to early repayments of principal to the MBS holders. In addition, MBS contain an embedded interest rate option. Mortgage holders have an option to refinance their property and prepay their existing mortgages. They are more likely to do so as interest rates, and hence refinancing rates, decline below the rate of their current mortgage. This refinancing incentive tends to lower the value of the mortgage to the MBS investor because the mortgages' relatively high expected coupon payments are replaced by an immediate payoff of the principal. The equivalent investment alternative now available to the MBS investor is, of course, at the lower coupon rate. Therefore, the price of an MBS with, for example, a 8% coupon is roughly equivalent to owning a default-free 8% annuity bond and writing a call option on that bond (with an exercise price of par). This option component induces a concave relation between the price of MBS and the price of default-free bonds (the so called "negative convexity").

2.1 MBS Pricing: An MDE Approach

Modeling and pricing MBS involves two layers of complexity: (i) modeling the dynamic behavior of the term structure of interest rates, and (ii) modeling the prepayment behavior of mortgage holders. The standard procedure for valuation of MBS assumes a particular stochastic process for term structure movements and uses specific statistical models of prepayment behavior. The success of this approach depends crucially on the correct parameterization of prepayment behavior and on the correct model for interest rates. We propose here a different approach that directly estimates the relation between MBS prices and various interest rate factors. This approach circumvents the need for parametric specification of interest rate dynamics and prepayment models.

The basic intuition behind the MDE pricing technique we propose is fairly straightforward. Let a set of m variables, denoted by $\mathbf{x_t}$, be the underlying factors that govern interest rate movements and prepayment behavior. The vector $\mathbf{x_t}$ includes interest rate variables (e.g., the level of interest rates) and possible prepayment specific variables (e.g., transaction costs of refinancing). The MBS price at time t, denoted as $P_{mb,t}$, is a function of these factors and can be written as

$$P_{mb,t} \quad = \quad V(\mathbf{x_t}, \theta)$$

where $V(\mathbf{x_t}, \theta)$ is a function of the state variables $\mathbf{x_t}$, and the vector θ is a set of parameters that describe the interest rate dynamics and the relation between the variables $\mathbf{x_t}$ and the prepayment function. The vector θ includes variables such as the speed with which interest rates tend to revert to their long run mean values and the sensitivity of prepayments to changes in interest rates. Parametric methods in the extant literature derive the function V based on equilibrium or no-arbitrage arguments and determine MBS prices using estimates of θ in this function. The MDE procedure, on the other hand, aims to directly estimate the function V from the data and is not concerned with the evolution of interest rates or the specific forms of prepayment functions.

The MDE procedure starts with a similar basic idea as parametric methods, viz. that MBS prices can be expressed as a function of a small number of interest rate factors. MBS prices are expressed as a function of these factors plus a pricing error term. The error term allows for the fact that model prices based on any small number of pricing factors will not be identical to quoted market prices. There are several reason why market prices can be expected to deviate from model prices. First, bid prices may be asynchronous with respect to the interest rate quotes. Furthermore, the bid-ask spreads for the MBS in this paper generally range from $\frac{1}{32}$nd to $\frac{4}{32}$nds, depending on the liquidity of the MBS. Second, the MBS prices used in this paper refer to prices of unspecified mortgage pools in the marketplace (see Section 3.1). To the extent that the universe of pools changes from period to period, and its composition may not be in the agent's information set, this introduces an error into the pricing equation. Finally, there may be pricing factors that are not specified in the model. Therefore, we assume observed prices are given by

$$P_{mb,t} = V(\mathbf{x_t}) + \epsilon_t \tag{1}$$

where ϵ_t represent the aforementioned pricing errors. A well specified model will yield small pricing errors. Examination of ϵ_t based on our model will therefore enable us to evaluate its suitability in this pricing application.

The first task in implementing the MDE procedure is to specify the factors that determine MBS prices. To price MBS we need factors that capture the value of fixed cash flow component of MBS and refinancing incentives. The particular factors we use here are the yield on 10-year Treasury notes and the spread between the 10-year yield and the 3-month T-bill yield. There are good reasons to use these factors for capturing the salient features of MBS. The MBS analyzed in this paper have 30 years to maturity; however, due to potential prepayments and scheduled principal repayments, their expected lives are much shorter. Thus, the 10-year yield should approximate the level of interest rates which is appropriate for discounting the MBS's cash flows. Further, the 10-year yield has a correlation of 0.98 with the mortgage rate (see Table 1B and Figure 1). Since the spread between the mortgage rate and the MBS's coupon determines the refinancing incentive, the 10-year yield should prove useful when valuing the option component.

The second variable, the slope of the term structure (in this case, the spread between the 10-year and 3-month rates) provides information on two factors: the market's expectations about the future path of interest rates, and the variation in the discount rate over short and long horizons. Steep term structure slopes imply lower discount rates for short-term cash flows and higher discount rates for long-term cash flows. Further, steep term structures may imply increases in future mortgage rates, which should decrease the likelihood of mortgage refinancing.

2.2 Multivariate Density Estimation Issues

This subsection explains the details of the multivariate density estimation technique proposed in this chapter. To understand the issues involved, suppose that the error term in equation (1) is uniformly zero and that we have unlimited data on the past history of MBS prices. Now suppose that we are interested in determining the fair price for a MBS with a particular coupon and prepayment history at a particular point in time when, for example, the 10-year yield is 8% and the slope of the term structure is 1%. In this case all we have to do is look back at the historical data and pick out the price of an MBS with similar characteristics at a point in time historically when the 10-year yield was 8% and the slope of the term structure was 1%. While this example illustrates the simplicity of underlying idea behind the MDE procedure, it also highlights the sources of potential problems in estimation. First of all, for reasons discussed in the last subsection, it is unrealistic to assume away the error terms. Secondly, in practice we do not have unlimited historical data, and a particular economic scenario, such as an 8% 10-year yield and a 1% term structure slope, may not have been played out in the past. The estimation technique therefore should be capable of optimally extracting information from the available data.

The MDE procedure characterizes the joint distribution of the variables of interest, in our case the joint distribution of MBS prices and interest rate factors. We implement MDE using a kernel estimation procedure.[2] In our application, the kernel estimator for MBS prices as a function of interest rate factors simplifies to:

$$\hat{P}_{mb,c}(r_l, r_l - r_s) = \frac{\sum_{t=1}^{T} P_{mb,c,t} K\left(\frac{r_l - r_{l,t}}{h_{r_l}}\right) K\left(\frac{[r_l - r_s] - [r_{l,t} - r_{s,t}]}{h_{r_l - r_s}}\right)}{\sum_{t=1}^{T} K\left(\frac{r_l - r_{l,t}}{h_{r_l}}\right) K\left(\frac{[r_l - r_s] - [r_{l,t} - r_{s,t}]}{h_{r_l - r_s}}\right)}, \tag{2}$$

where T is the number of observations, $K(\cdot)$ is a suitable kernel function and h is the window width or smoothing parameter. $\hat{P}_{mb,c}(r_l, r_l - r_s)$ is our model price for a MBS with coupon c when the long rate is r_l and the term structure slope is $r_l - r_s$. $P_{mb,c,t}$ is the market price of the t^{th} observation for the price of a MBS with coupon c. Note that the long rate at the time of observation t are is $r_{l,t}$ and the term structure slope is $r_{l,t} - r_{s,t}$.

The econometrician has at his or her discretion the choice of $K(\cdot)$ and h. It is important to point out, however, that these choices are quite different from those faced by researchers employing parametric methods. Here, the researcher is not trying to choose functional forms or parameters that satisfy some goodness-of-fit criterion (such as minimizing squared errors in regression methods), but is instead characterizing the joint distribution from which the functional form will be determined.

One popular class of kernel functions is the symmetric beta density function, which includes the normal density, the Epanechnikov (1969) "optimal" kernel, and the commonly used biweight kernel as special cases. Results in the kernel estimation literature suggest that any reasonable kernel gives almost optimal results, though in small samples there may be differences (see Epanechnikov (1969)). In this paper, we employ an independent multivariate normal kernel, though it should be pointed out that our results are relatively insensitive to the choice of kernel within the symmetric beta class. The specific functional form for the $K(\cdot)$ that we use is:

$$K(z) = (2\pi)^{-\frac{1}{2}} e^{-\frac{1}{2}z^2},$$

[2] For examples of MDE methods for approximating functional forms in the empirical asset pricing literature, see Pagan and Hong (1991), Harvey (1991) and Ait-Sahalia (1996). An alternative approach to estimating nonlinear functionals in the derivatives market is described by Hutchinson, Lo and Poggio (1994). They employ methods associated with neural networks to estimate the nonlinear relation between option prices and the underlying stock price.

where z is the appropriate argument for this function.

The other parameter, the window width, is chosen based on the dispersion of the observations. For the independent multivariate normal kernel, Scott (1992) suggests the window width

$$\hat{h}_i = k_i \hat{\sigma}_i T^{\frac{-1}{m+4}} ,$$

where $\hat{\sigma}_i$ is the standard deviation of the ith variable (i.e., i may denote either variable r_l or $r_l - r_s$), m is the dimension of the variables, which in our case is 2, and k_i is a scaling constant often chosen via a cross-validation procedure. In our application we need to chose two such scaling constants, one for the long rate r_l and one for the term structure slope $r_l - r_s$. Note that the window width is larger when the variance of the variable under consideration is larger in order to compensate for the fact that observations are, on average, further apart. This window width (with $k_i = 1$) has the appealing property that, for certain joint distributions of the variables, it minimizes the asymptotic mean integrated squared error of the estimated density function. Unfortunately, our data are serially correlated and therefore the necessary distributional properties are not satisfied.

We employ a cross-validation procedure to find the k_i that minimizes the estimation error. To implement cross-validation, the implied MDE price at each data point is estimated using the entire sample, except for the actual data point and its nearest neighbors.[3] We identify the k_i's that minimize the mean-squared error between the observed price and the estimated kernel price. Once the k_i's are chosen based on cross-validation, the actual estimation of the MBS prices and analysis of pricing errors involves the entire sample.

To gain further intuition into the estimation procedure, note that equation (2) takes a special form; the estimate of the MBS price can be interpreted as a weighted average of observed prices:

$$\hat{P}_{mb,c}(r_l^*, r_l^* - r_s^*) = \sum_{t=1}^{T} w_i(t) P_{mb,c,t} \ , \tag{3}$$

where

$$w_r(t) = \frac{K\left(\frac{r_l^* - r_{l,t}}{h_{r_l^*}}\right) K\left(\frac{[r_l^* - r_s^*] - [r_{l,t} - r_{s,t}]}{h_{r_l^* - r_s^*}}\right)}{\sum_{t=1}^{T} K\left(\frac{r_l^* - r_{l,t}}{h_{r_l^*}}\right) K\left(\frac{[r_l^* - r_s^*] - [r_{l,t} - r_{s,t}]}{h_{r_l^* - r_s^*}}\right)} .$$

Note that to determine the MBS price when the interest rate factors are $(r_l^*, r_l^* - r_s^*)$ the kernel estimator assigns to each observation t a weight $w_r(t)$ that is proportional to the

[3]Due to the serial dependence of the data, we performed the cross-validation omitting one year of data, i.e., six months in either direction of the particular data point in question.

"distance" (measured via the kernel function) between the interest rate factors at the time of observation t $(r_{l,t}, r_{l,t} - r_{s,t})$ and the current interest rate factors. The attractive idea behind MDE is that these weights are not estimated in an ad hoc manner, but instead depend on the true underlying distribution (albeit estimated) of the relevant variables. Thus, if the current state of the world, as measured by the state vector $(r_l^*, r_l^* - r_s^*)$, is not close to a particular point in the sample, then this sample price is given little weight in estimating the current price. Note, however, that MDE can give weight (possibly inconsequential) to all observations, so that the price of the MBS with $(r_l^*, r_l^* - r_s^*)$ also takes into account MBS prices at surrounding interest rates. This will help average out the different ϵ errors in equation (1) from period to period. Although our application utilizes only two factors, MDE will average out effects of other factors if they are independent of the two interest rate factors. Thus, for any given long rate r_l^* and a given short rate r_s^*, there is a mapping to the MBS price $P_{mb}(r_l^*, r_l^* - r_s^*)$. These prices can then be used to evaluate how MBS prices move with fundamental interest rate factors.

While the MDE procedure has the advantage that it does not require explicit functional specification of interest rate dynamics and prepayment models, it does have certain drawbacks. The most serious problem with MDE is that it is data intensive. Much data are required in order to estimate the appropriate weights which capture the joint density function of the variables. The quantity of data which is needed increases quickly in the number of conditioning variables used in estimation. How well MDE does at estimating the relation between MBS prices and the interest-rate factors is then an open question, since the noise generated from the estimation error can be substantial.[4]

Another problem with MDE is that the procedure requires covariance stationarity of the variables of interest. For example, when we use only two interest rate factors, the MDE procedure does not account for differences in prices MBS when the underlying pools have different prepayment histories. For this reason the MBS procedure is most suitable for pricing TBA securities which are most commonly used for new originations rather than for seasoned MBS. Accounting for *seasoning* of a mortgage or a mortgage pool's *burnout* will require additional factors that are beyond the scope of this chapter.

A few comments are in order, however, to provide some guidance on how these factors could be accounted for when one is interested in pricing seasoned MBS. First, one could

[4]Boudoukh, Richardson, Stanton and Whitelaw (1997) perform simulation exercises in an economy governed by two factors and some measurement error in reported prices. Within this (albeit simple) environment, the MDE methodology performs quite well.

potentially take account of a mortgage pool's seasoning by nonlinearly filtering out any time dependence. Estimation error aside, this filtering would be effective as long as the seasoning is independent of the other state variables. Second, in order to incorporate path dependence due to a pool's burnout, the only viable way would be to employ a state variable which captures this dependence. For example, Boudoukh, Richardson, Stanton and Whitelaw (1997) and Richard and Roll (1989) describe several variables that might be linked closely with *burnout*. Because the strength of the MDE procedure estimation of nonlinear relations, all that is required is that these variables span the appropriate state space.

3 Data Description

3.1 Data Sources

Mortgage-backed security prices were obtained from Bloomberg Financial Markets covering the period January 1987 to May 1994. Specifically, we collected weekly data on 30-year fixed-rate Government National Mortgage Association (GNMA) MBS, with coupons ranging from 7.5% to 10.5%.[5] The prices represent dealer-quoted bid prices on GNMAs of different coupons traded for delivery on a *to be announced* (TBA) basis.

The TBA market is most commonly employed by mortgage originators who have a given set of mortgages that have not yet been pooled. However, trades can also involve existing pools on an unspecified basis. Rules for the delivery and settlement of TBAs are set by the Public Securities Association (PSA) (see, for example, Bartlett (1989) for more details). For example, an investor might purchase \$1 million worth of 8% GNMAs for forward delivery next month. The dealer is then required to deliver 8% GNMA pools within 2.5% of the contracted amount (i.e., between \$975,000 and \$1,025,000), with specific pool information to be provided on a TBA basis (just prior to settlement). This means that, at the time of the agreed-upon-transaction, the characteristics of the mortgage pool to be delivered (e.g., the age of the pool and its prepayment history) are at the discretion of the dealer. Nevertheless, for a majority of the TBA's, the delivered pools represent newly issued pools.

With respect to the interest rate series, weekly data for the 1987-1994 period were collected on the average rate for 30-year mortgages (collected from Bloomberg Financial

[5]Careful filters were applied to the data to remove data reporting errors using prices reported in the Wall Street Journal. Furthermore, data are either not available or sparse for some of the GNMA coupons during the period. For example, in the 1980's, 6% coupon bonds represent mortgages originated in the 1970's, and not the more recent issues which are the focus of this paper. Thus, data on these MBS were not used.

Markets),[6] and the yields on the 3-month Treasury bill and 10-year Treasury note (provided by the Board of Governors of the Federal Reserve).

3.2 Data Characteristics

Before describing the pricing results and error analysis for MBS using the MDE approach, we briefly describe the environment for interest rates and mortgage rates during the sample period, 1987-1994.

Characteristics of Mortgages (1987-1994)

Since the mortgage rate represents the available rate at which homeowners can refinance, it plays an especially important role with respect to the prepayment incentive. Figure 1 graphs the mortgage rate for 1987 through 1994. From 1987 to 1991, the mortgage rate varied from 9% to 11%. In contrast, from 1991 to 1994, the mortgage rate generally declined from 9.5% to 7%.[7]

For pricing GNMA TBAs, it is most relevant to understand the characteristics of the universe of pools at a particular point in time. That is, the fact that a number of pools have prepaid considerably may be irrelevant if newly originated pools have entered into the MBS market since the MBS from new originations are the one typically delivered in TBA contracts. To get a better idea of the time series behavior of the GNMA TBAs during this period, Figure 2 graphs an artificially constructed index of all the originations of 7.5% to 10.5% GNMA pools from January 1983 to May 1994.[8]

There is a wide range of origination behavior across the coupons. As mortgage rates moved within a 9% to 11% band between 1987 to 1991, Figure 2 shows that GNMA 9s, 9.5s, 10s and 10.5s were all newly originated during this period. Consistent with the decline in mortgage rates in the post 1991 period, GNMA 7.5s, 8s and 8.5s originated while the GNMA 9s–10.5s became seasoned issues. Thus, in terms of the seasoning of the pools most likely to be delivered in the TBA market, there are clearly cross-sectional differences between the coupons.

[6]Bloomberg's source for this rate is "Freddie Mac's Primary Mortgage Market Survey", which reports the average rate on 80% of newly originated 30-year, first mortgages on a weekly basis.

[7]Note that the MBS coupon rate is typically 50 basis points less that the interest rate on the underlying mortgage. The 50 basis point is retained to cover the servicing fee and reinsurance cost.

[8]The dollar amount outstanding for each coupon is normalized to 100 in January 1987. Actual dollar amounts outstanding in that month were \$10,172, \$27,096, \$10,277, \$63,392, \$28,503, \$15,694, and \$5,749 (in millions) for the 7.5% – 10.5% coupons, respectively.

Figure 2 shows that there are several reasons for choosing the TBA market during the post 1986 time period to investigate MBS pricing using the MDE methodology. First, during 1985 and 1986, interest rates dramatically declined, leading to mortgage originations for a wide variety of coupon rates. Thus, the GNMA TBAs in 1987-1994 correspond to mortgage pools with little prepayment history (i.e., no *burnout*) and long maturities. In contrast, prior to this period, the 7.5% to 10.5% GNMAs were backed by mortgages originated in the 1970's and thus represented a different security (in both maturity and prepayment levels). Second, MDE pricing requires joint stationarity between MBS prices and the interest rate variables. This poses a potential problem in estimating the statistical properties of any fixed maturity security, since the maturity changes over time. Recall that the TBA market refers to unspecified mortgage pools available in the marketplace. Thus, to the extent that there are originations of mortgages in the GNMA coupon range, the maturity of the GNMA TBA is less apt to change from week to week. Figure 2 shows that this is the case for the higher coupon GNMAs pre 1991, and for the low coupon GNMAs post 1991. Of course, when no originations occur in the coupon range (e.g., the GNMA 10s in the latter part of the sample), then the maturity of the available pool will decline. In this case, the researcher may need to add variables to capture the maturity effect and possibly any prepayment effects. In our analysis, we choose to limit the dimensionality of the multivariate system, and instead focus on the relation between MBS prices and the two interest rate factors.

Characteristics of MBS Prices and Interest Rates(1987-1994)

Table 1 provides ranges, standard deviations and cross-correlations of GNMA prices (Table 1A), and mortgage and interest rates (Table 1B) during the 1987-1994 period. Absent prepayments, MBS are fixed-rate annuities, and the dollar volatility of an annuity increases with the coupon. In contrast, from Table 1, we find that the lower coupon GNMAs are more volatile than the higher coupon GNMAs. The lower volatility of the higher coupon GNMAs is due to the embedded call option of MBS. The important element of the option component for MBS valuation is the refinancing incentive. For most of the sample (especially 1990 on), the existing mortgage rate lies below 10.5% and the prepayment option is at- or in-the-money.[9] Historically, given the costs associated with refinancing, a spread of approximately 150 basis points between the old mortgage rate and the existing rate is required to induce

[9]Figure 1 also graphs one of the interest rate factors, the 10-year yield. There is a difference in the level between the two series (i.e., on average 1.56%), representing the cost of origination, the option value, and the bank profits, among other factors.

rapid prepayments.[10] The lack of seasoning aside, this would suggest that the higher coupon GNMAs began to prepay in the early 90's.

As mentioned above, Figure 1 graphs the 10-year yield against the mortgage rate. During the 1987 to 1994 period, there are multiple observations of particular interest rates. Since these multiple observations occur at different points of the sample, this will help MDE isolate the potential impact of additional interest rate factors, as well as reduce maturity effects not captured by the MDE pricing (see *Characteristics of Mortgages* above). Similarly, while the spread between the 10-year yield and the 3-month rate is for the most part positive, there is still variation of the spread during the period of an order of magnitude similar to the underlying 10-year rate (see Table 1B). Moreover, the correlation between these variables is only -0.45, indicating that they potentially capture independent information, which may be useful for pricing GNMAs.

4 Empirical Results

This section implements the MDE procedure and investigates how well the model prices match market prices.

4.1 One-Factor Pricing

As a first pass at the MBS data, we describe the functional relation between GNMA prices and the level of interest rates (the 10-year yield). As an illustration, Figure 3 graphs the estimated 9% GNMA price with the actual data points. The smoothing factor, which is chosen by cross-validation, is 0.35 (i.e., $k_i = 0.35$).

Several observations are in order. First, the figure illustrates the well-known negative convexity of MBS. Specifically, the MBS price is convex in interest-rate levels at high interest rates (when it behaves more like a straight bond), yet concave at low interest rates (as the prepayment option becomes in-the-money). Second, the estimated functional relation is not smooth across the entire range of sample interest rates. Specifically, between 10-year yields of 7.1% to 7.8%, there is a *bump* in the estimated relation. While this feature is most

[10] See Bartlett (1989) and Breeden (1991) for some historical evidence of the relation between prepayment rates and the mortgage spread. Note that in the 1990's the threshold spread required to induce refinancing has been somewhat lower – in some cases, 75 to 100 basis points. Some have argued that this is due to the proliferation of new types of mortgage loans (and ensuing marketing efforts by the mortgage companies) (Bartlett (1989)), though it may also be related to aggregate economic factors, such as the implications of a steep term structure.

probably economically spurious, it reflects the fact that the observed prices in this region are high relative to the prices at nearby interest rates. Increasing the degree of smoothing eliminates this bump at the cost of increasing the pricing errors. The source of this variation, which could be missing factors, MDE estimation error, or structural changes in the mortgage market, is investigated further below. Third, there is a wide range of prices at the same level of interest rates. For example, at a 10-year yield of 8%, prices of GNMA 9s vary from 98% to 102% of par. Is this due to the impact of additional factors, measurement error in GNMA prices, MDE estimation error, or some other phenomenon?

Table 2 provides some preliminary answers to this question. Specifically, Table 2A reports summary statistics on the pricing errors (defined as the difference between the MDE estimated price and the observed MBS price) for the 7.5% to 10.5% GNMAs. As seen from a comparison between Table 1A and 2A, most of the volatility of the GNMA price can be explained by a 1-factor kernel using the interest rate level. For example, the volatility of the 9% GNMA is \$5.26, but its residual volatility is only \$0.83. However, while 1-factor pricing does well, it clearly is not sufficient as the pricing errors are highly autocorrelated (from 0.861 to 0.927) for all the GNMA coupons. Though this autocorrelation could be due to measurement error induced by the MDE estimation, it does raise the possibility that there is a missing factor. In addition, the residuals are highly correlated across the 7 different coupon bonds (not shown in the table). Thus, the pricing errors contain substantial common information.

This correlation across different GNMAs implies that an explanation based on idiosyncratic information (such as measurement error in prices) will not be sufficient. Combined with the fact that the magnitude of the bid-ask spreads in these markets lies somewhere between $\frac{1}{32}$nd and $\frac{4}{32}$nds, clearly measurement error in observed prices cannot explain either the magnitude of the pricing errors with 1-factor pricing (e.g., \$2 – \$3 in some cases) or the substantial remaining volatility of the errors (e.g., \$0.70 to \$0.84 across the coupons).

Table 2B looks at the impact of additional interest rate factors. We run a regression of the pricing errors on the level and squared level to check whether any linear or nonlinear effects remain. For the most part, the answer is no. The level has very little explanatory power for the pricing errors, with R^2s ranging from 1.1% to 2.8%. Moreover, tests of the joint significance of the coefficients cannot reject the null hypothesis of no explanatory power at standard significance levels. Motivated by our discussion in Section 2, we also run a regression of the pricing errors for each GNMA on the slope of the term structure (the spread between the 10-year yield and the 3-month yield) and its squared value. The results strongly support the existence of a second factor, with R^2s increasing with the coupon from a low of 2.0% to 40.1%. Furthermore, this second factor comes in nonlinearly as both the linear and nonlinear terms are large and significant.

Most interesting is the fact that the slope of the term structure has its biggest impact on higher coupon GNMAs. This suggests an important relation between the prepayment option and the term structure slope. Due to the relatively lower value of the prepayment option, low coupon GNMAs behave much like straight bonds. Thus, the 10-year yield may provide enough information to price these MBS. In contrast, the call option component of higher coupon GNMAs is substantial enough that the duration of the bond is highly variable. Clearly, the slope of the term structure provides information about the variation in yields across these maturities; hence, its additional explanatory power for higher coupon GNMAs. The negative coefficient on the spread implies that the 1-factor MDE is underpricing when the spread is high. In other words, when spreads are high, and short rates are low for a fixed long rate, high coupon GNMAs are more valuable than would be suggested by a 1-factor model. The positive coefficients on the squared spread suggest that the relation is nonlinear, with a decreasing effect as the spread increases. Note that in addition to information about variation in discount rates across maturities, the spread may also be proxying for variation in expected prepayment rates that is not captured by the long rate.

4.2 Two-Factor Pricing

Motivated by the results in Table 2B, it seems important to consider a second interest rate factor for pricing MBS. Therefore, we describe the functional relation between GNMA prices and two interest-rate factors, the level of interest rates (the 10-year yield) and the slope of the term structure (the spread between the 10-year yield and the 3-month yield). In particular, we estimate the pricing functional given in equation (1) for each of the GNMA coupons. For comparison purposes with Figure 3, Figure 4 graphs the 9% GNMA against the interest rate level and the slope. The smoothing factor for the long rate is fixed at the level used in the 1-factor pricing (i.e., 0.35), and the cross-validation procedure generates a smoothing factor of 1.00 for the spread.

The well-known negative convexity of MBS is very apparent in Figure 4. However, this functional form does not hold in the northwest region of the figure, that is, at low spreads and low interest rates. The explanation is that the MDE approach works well in the regions of the available data, but extrapolates poorly at the tails of the data and beyond. Figure 5 graphs a scatter plot of the interest rate level against the slope. As evident from the figure, there are periods in which large slopes (3%-4%) are matched with both low interest rates (in 1993-1994) and high interest rates (in 1988). However, few observations are available at low spreads joint with low interest rates. Thus, the researcher needs to be cautious when interpreting MBS prices in this range.

Within the sample period, the largest range of 10-year yields occurs around a spread of 2.70%. Therefore, we take a slice of the pricing functional for the 8%, 9% and 10% GNMAs, conditional on this level of the spread. Figure 6 graphs the relation between GNMA prices for each of these coupons against the 10-year yield. Several observations are in order. First, the negative convexity of each MBS is still apparent even in the presence of the second factor. Though the *bump* in the functional form is still visible, it has been substantially reduced. Thus, multiple factors do play a key role in MBS valuation. Second, the price differences between the various GNMA securities narrow as interest rates fall. This just represents the fact that higher coupon GNMAs are expected to prepay at faster rates. As GNMAs prepay at par, their prices fall because they are premium bonds, thus reducing the differential between the various coupons. Third, the GNMA prices change as a function of interest rates at different rates depending on the coupon level, i.e., on the magnitude of the refinancing incentive. Thus, the effective duration of GNMAs varies as the moneyness of the prepayment option changes.

The results of Section 4.1, and Figures 5 and 6, suggest the possible presence of a second factor for pricing MBS. To understand the impact of the term structure slope, Figure 7 graphs the various GNMA prices against interest rate levels, conditional on two different spreads (2.70% and 0.30%).[11] Recall that the slope of the term structure is defined using the yield on a full-coupon note, not a ten-year zero-coupon rate. As a result, positive spreads imply upward sloping full-coupon yield curves and even more steeply sloping zero-coupon yield curves. In contrast, when the spread is close to zero, both the full-coupon and zero-coupon yield curves tend to be flat. Thus, holding the 10-year full-coupon yield constant, short-term (long-term) zero-coupon rates are lower (higher) for high spreads than when the term structure spread is low.

In terms of MBS pricing, note that at high interest rate levels, the option to prepay is out-of-the-money. Consequently, many of the cash flows are expected to occur as scheduled, and GNMAs have long expected lives. The appropriate discount rates for these cash flows

[11]The spreads and interest rate ranges are chosen to coincide with the appropriate ranges of available data, to insure that the MDE approach works well.

are therefore longer-term zero-coupon rates. Consider first the effects on the price of an 8% GNMA. Since this security has its cash flows concentrated at long maturities, its price should be lower for higher spreads, just as we observe in Figure 7. On the other hand, the option component of the 10% GNMA is much closer to being at-the-money, even for the highest interest rates shown in the figure. Hence, at these interest rates, 10% GNMA prices do not follow the same ordering as 8% GNMAs vis-a-vis the level of the spread.

As interest rates fall, prepayments become more likely, and the expected life of the MBS falls for GNMAs of all coupons. As this life declines, the levels of the shorter-term zero-coupon rates become more important for pricing. In this case, high spreads imply lower discount rates at the relevant maturities, for a fixed 10-year full-coupon yield. Consequently, when the GNMAs are priced as shorter-term securities due to high expected prepayments, high spreads imply higher prices for all coupons. This implication is illustrated in Figure 7. While prices always increase for declining long rates, the increase is much larger when spreads are high. For the 8% GNMA, this effect causes the prices to cross at a long rate of approximately 8.3%, while for the 10% GNMA it causes the pricing functionals to diverge further as rates decrease. The effect in Figure 7 is primarily driven by changes in expected cash flow life. The 10-year yield proxies for the moneyness of the option, the expected level of prepayments, and the average life of the cash flows. The addition of the second factor, the term structure slope, also controls for the average rate at which these cash flows should be discounted.

In order to understand the impact of 2-factor pricing more clearly, Table 3 provides an analysis along the lines of Table 2 for 1-factor pricing. Specifically, Table 3A reports some summary statistics on the pricing errors for the 7.5% to 10.5% GNMAs. The addition of a second interest rate factor reduces the pricing error volatility across all the GNMA coupons, i.e., from \$0.70 to \$0.65 for the 7.5s, \$0.83 to \$0.61 for the 9s, and \$0.84 to \$0.52 for the 10.5s. Most interesting, the largest reduction in pricing error volatility occurs with the higher coupon GNMAs, which confirms the close relation between the slope of the term structure and the prepayment option. Table 3B looks at whether there is any remaining level or slope effect on the 2-factor MBS prices. We run nonlinear regressions of the pricing errors on the level and the slope separately. Neither the level nor the slope have any remaining economic explanatory power for the pricing errors, with R^2s ranging from 3.6% to 4.5% for the former and R^2s under 1.0% for the latter. The tests of joint significance of the coefficients exhibit marginal significance for the level, suggesting that reducing the smoothing parameter will generate a small improvement in the magnitude of the pricing errors.

5 Hedging Interest Rate Risk

5.1 Hedging Methodology

This section illustrates how to hedge the interest rate risk of MBS using the pricing model presented here. Since there are two interest rate factors that are important for pricing MBS we need two fixed-income assets to hedge out interest rate risk. The hedging instruments we use are a 3-month T-bill and a 10-year Treasury Note futures contract. Let ω_{t-bill} and $\omega_{futures}$ denote the appropriate positions in T-Bills and T-note futures contracts respectively to hedge the interest rate risk of one unit of a MBS. The hedge position taken in each of the instruments should ensure that:

$$\omega_{t-bill}\frac{\partial P_{t-bill}}{\partial r_l} + \omega_{futures}\frac{\partial P_{futures}}{\partial r_l} = -\frac{\partial P_{mb}}{\partial r_l}$$
$$\omega_{t-bill}\frac{\partial P_{t-bill}}{\partial (r_l - r_s)} + \omega_{futures}\frac{\partial P_{futures}}{\partial (r_l - r_s)} = -\frac{\partial P_{mb}}{\partial (r_l - r_s)},$$

where $\frac{\partial P}{\partial r_l}$ and $\frac{\partial P}{\partial (r_l - r_s)}$ are the sensitivities of these instruments with respect to the long rate r_l and slope of the term structure $r_l - r_s$. The equations above specify that the sensitivity of MBS price to changes in the long rate and the slope of the term structure are exactly offset by the corresponding sensitivities of the hedged positions.

Solving for ω_{t-bill} and $\omega_{futures}$ gives

$$\omega_{t-bill} = \frac{-\frac{\partial P_{mb}}{\partial (r_l - r_s)}\frac{\partial P_{futures}}{\partial r_l} + \frac{\partial P_{mb}}{\partial r_l}\frac{\partial P_{futures}}{\partial (r_l - r_s)}}{\frac{\partial P_{t-bill}}{\partial r_l}\frac{\partial P_{futures}}{\partial (r_l - r_s)} - \frac{\partial P_{t-bill}}{\partial (r_l - r_s)}\frac{\partial P_{futures}}{\partial r_l}}, \tag{4}$$

$$\omega_{futures} = \frac{-\frac{\partial P_{mb}}{\partial r_l}\frac{\partial P_{t-bill}}{\partial (r_l - r_s)} + \frac{\partial P_{mb}}{\partial (r_l - r_s)}\frac{\partial P_{t-bill}}{\partial r_l}}{\frac{\partial P_{t-bill}}{\partial r_l}\frac{\partial P_{futures}}{\partial (r_l - r_s)} - \frac{\partial P_{t-bill}}{\partial (r_l - r_s)}\frac{\partial P_{futures}}{\partial r_l}}. \tag{5}$$

Using equations (4) and (5), these hedged portfolios then can be constructed *ex ante* based on the econometrician's estimate of the partial derivatives of the three fixed-income assets with respect to the two factors. These estimates can be generated from historical data (prior to the forming of the hedge) using kernel estimation. For example, an estimate of $\frac{\partial P_{mb}}{\partial r_l}$ can be calculated from equation (2) using

$$\frac{\partial P_{mb}}{\partial r_l} = \frac{\sum_{t=1}^{T} P_{mb,t} K'\left(\frac{r_l - r_{l,t}}{h_{r_l}}\right) K\left(\frac{[r_l - r_s] - [r_{l,t} - r_{s,t}]}{h_{r_l - r_s}}\right)}{\sum_{t=1}^{T} K\left(\frac{r_l - r_{l,t}}{h_{r_l}}\right) K\left(\frac{[r_l - r_s] - [r_{l,t} - r_{s,t}]}{h_{r_l - r_s}}\right)} -$$

$$\frac{\sum_{t=1}^{T} P_{mb,t} K\left(\frac{r_l - r_{l,t}}{h_{r_l}}\right) K\left(\frac{[r_l - r_s] - [r_{l,t} - r_{s,t}]}{h_{r_l - r_s}}\right) \sum_{t=1}^{T} K'\left(\frac{r_l - r_{l,t}}{h_{r_l}}\right) K\left(\frac{[r_l - r_s] - [r_{l,t} - r_{s,t}]}{h_{r_l - r_s}}\right)}{\left[\sum_{t=1}^{T} K\left(\frac{r_l - r_{l,t}}{h_{r_l}}\right) K\left(\frac{[r_l - r_s] - [r_{l,t} - r_{s,t}]}{h_{r_l - r_s}}\right)\right]^2},$$

where $K'(z) = -(2\pi)^{-\frac{1}{2}} z e^{-\frac{1}{2}z^2}$. Unfortunately, it is difficult to estimate the derivative accurately (see Scott (1992)); therefore, we average the estimated derivative with price sensitivities estimated over a range of long rates or slopes. For example, we calculate the elasticity

$$\frac{\Delta P_{mb}}{\Delta r_l} = \frac{P_{mb}(r_l^a) - P_{mb}(r_l^b)}{r_l^a - r_l^b}$$

for two different pairs of interest rates, (r_l^a, r_l^b), and average these values with the kernel derivative. The points are chosen to straddle the interest rate of interest. Specifically, we use the 10th and 20th nearest neighbors along the interest rate dimension within the sample, if they exist, and the highest or lowest interest rates within the sample if there are not 10 or 20 observations with higher or lower interest rates. The return on the hedged portfolio is then given by

$$\frac{P_{mb,t+1} + \hat{\omega}_{t-bill}(P_{1,t+1} - P_{1,t}) + \hat{\omega}_{futures}(P_{2,t+1} - P_{2,t})}{P_{mb,t}},$$

where it is assumed that the investor starts with one unit of GNMAs at time t. The hedged portfolio can then be followed through time and evaluated based on its volatility and correlation with the fixed-income factors, as well as other factors of interest.[12]

5.2 Hedging Analysis

We conducted an out-of-sample hedging exercise over the period January 1990 to May 1994 to evaluate the hedge performance. Starting in January 1987, approximately three years of data (150 weekly observations) were used on a weekly rolling basis to estimate the MBS prices and interest rate sensitivities as described above. For the T-bill and T-note futures, we assume that they move one-for-one with the short rate and long rate, respectively. This assumption simplifies the analysis and is a good first-order approximation. For each rolling period, several different hedges were formed for comparison purposes:

1. To coincide with existing practice, a linear hedge of the GNMAs against the T-note futures was estimated using rolling regressions. The hedge ratio is given by the sensitivity of the MBS price changes to futures price changes.

[12]The method described here forms an instantaneous hedge, which in theory would require continuous rebalancing. For an alternative hedge based on horizon length, see Boudoukh, Richardson, Stanton and Whitelaw (1995).

2. Breeden (1991) suggests a roll-up/roll-down approach to computing hedge ratios. Specifically, the hedge can be formed for a GNMA by computing the ratio between the T-note futures price elasticity and the GNMA price elasticity. (The GNMA price elasticity of, say, a 8% GNMA is calculated from the difference between the prices of $8\frac{1}{2}\%$ and a $7\frac{1}{2}\%$ GNMA. We investigate hedging of 8%, 9% and 10% GNMAs using GNMAs with 7.5% through 10.5% coupons).

3. We investigate the two-factor MDE hedge described by the portfolio weights given in equations (4) and (5).

4. To the extent that the second factor (the slope) seems to play a small role in pricing it is possible that the slope factor may not be important for hedging. To evaluate this, we employ a one-factor MDE hedge using the T-note futures and GNMA as a function of only the 10-year yield.

Table 4 compares the performance of the four hedges for the 8% (Table 4A), 9% (Table 4B) and 10% GNMAs (Table 4C) over the 1990 to 1994 sample period. Consider first the 10% GNMA. The unhedged GNMA return has a volatility of 0.414% (41.4 basis points) on a weekly basis. The two-factor MDE hedge reduces the volatility of the portfolio to 26.1 basis points weekly. In contrast, the one-factor MDE hedge, the roll-up/roll-down hedge and linear hedge manage only 30.0, 29.4 and 34.9 basis points, respectively. The 10% GNMA is the most in-the-money in terms of the refinancing incentive, and it is comforting to find that, in the GNMA's most nonlinear region, the MDE approach works well.

Figure 8 illustrates how the volatility of the hedged and unhedged returns move through time. While the volatility of the unhedged returns declines over time, this pattern is not matched by the hedged returns. To quantify this evidence Table 4C breaks up the sample into four subperiods: January 1990 – February 1991, March 1991 – April 1992, May 1992 – June 1993, and July 1993 – May 1994. The most telling fact is that the MDE approach does very well in the last subperiod relative to the other hedges (19.2 versus 39.4 basis points for the roll-up/roll-down approach). This is a period in which massive prepayments occurred in the first part of the period. Due to these prepayments, 10% GNMAs are much less volatile than in previous periods. Thus, the linear and roll-up/roll-down approaches tended to overhedge MBS, resulting in large exposures to interest rate risks. This might explain some of the losses suffered by Wall Street during this period.

On the other hand, the MDE approach does not fare as well in the first two subperiods. For example, the one- and two-factor hedges have 38.8 and 29.6 basis points of volatility respectively versus the unhedged GNMA's volatility of 48.1 basis points in the second subperiod. In contrast, the roll-up/roll-down hedge has only 26.4 basis points of volatility. The explanation is that the MDE procedure does not extrapolate well beyond the tails of the data. During the first and second subperiod, the rolling estimation period faces almost uniformly higher interest rate levels than the out-of-sample forecast. Thus, hedge ratios were calculated for sparse regions of the data.

Recall that the MDE two-factor hedge reduces the volatility to 65% of the unhedged GNMA's volatility. Since the hedging was performed on an out-of-sample basis, there is no guarantee that the remaining variation of the GNMA's return is free of interest-rate exposure. Table 4C provides results from a linear regression of the GNMA unhedged and hedged portfolio's return on changes in the interest rate level (i.e., $\Delta r_{l,t}$) and movements in the terms structure slope (i.e., $\Delta(r_{l,t} - r_{s,t})$). It gives the volatility of each portfolio due to interest rate and term structure slope movements. For example, the volatility of the explained portion of the 10% GNMA due to the interest rate level and slope is 28.6 basis points a week; in contrast, the MDE two-factor hedged 10% GNMA's interest rate risk exposure is only 5.4 basis points. Note that the roll-up/roll-down and linear hedges face much more exposure — 11.3 and 16.4 basis points, respectively.[13]

So far, we have described the results for hedging the 10% GNMA. Tables 4A and 4B provides results for the 8% and 9% GNMAs. Essentially, the patterns are very similar to the 10%, except that the MDE approach fares less well relative to the roll-up/roll-down approach. To understand why this is the case, note that the 8% and 9% GNMAs have a lower refinancing incentive. The bonds therefore behave more like a straight bond, and are more volatile (see Table 1). Thus, because the negative convexity of the GNMAs is less prevalent for the 8% and 9% coupons, one explanation for why the MDE approach to hedging GNMAs fares relatively less well with lower coupons is that estimation error is more important. In fact, the roll-up/roll-down method actually produces a lower volatility of the hedged GNMA portfolio than the MDE two-factor approach for both the 8% and 9% GNMAs (27.6 versus 29.4 basis points for the 8%s and 24.6 versus 25.6 basis points for the 9%s).

Multiple factors become less important from a hedging perspective as the GNMA coupon falls (e.g., compare the 8% to 10%). This is to be expected, since we argued that the term

[13]For completeness we also report the volatility of the returns due only to movements in the long rate. These results are very similar to those discussed above, suggesting that most of the volatility on a weekly basis is attributable to variation in the long rate.

structure slope plays a role in pricing as the moneyness of the prepayment option changes through time. The subperiod analysis confirms the intuition based on our findings for the 10% GNMAs. While the relative hedging performance of the various approaches is still related to the subperiods, it is less prevalent for the lower coupon GNMAs. The MDE approach fares relatively best in periods with substantial nonlinearities, e.g., the 10% GNMAs during July 1993 to May 1994. The large prepayments which induced 10% GNMA prices to fall (ceteris paribus) did not occur for the 8% GNMAs. After all, the 8% GNMAs are backed by 8.5% mortgages, and the lowest 30-year fixed-rate mortgage only briefly dropped below 7%.

Of particular interest, both the MDE approach and the roll-up/roll-down hedges substantially reduce the interest rate exposure of their 8% and 9% GNMA hedge portfolios. For example, for the 8% (9%) GNMA, the unhedged GNMA has 59.0 (41.1) basis point of volatility due to the interest rate factors, while the MDE and roll-up/roll-down approaches have only 4.3 (3.9) and 6.8 (1.2) basis points respectively.

6 Conclusion

This chapter presents a non-parametric model for pricing mortgage-backed securities and hedging their interest rate risk exposures. Instead of postulating and estimating parametric models for both interest rate movements and prepayments, as in previous approaches to mortgage-backed security valuation, we directly estimate the functional relation between mortgage-backed security prices and the level of economic fundamentals. This approach can yield consistent estimates without the need to make the strong assumptions about the processes governing interest rates and prepayments required by previous approaches.

We implement the model with GNMA MBS with various coupons. We find that MBS prices can be well described as a function of the level of interest rates and the slope of the term structure. A single interest rate factor, as used in most previous mortgage valuation models, is insufficient. The relation between prices and interest rates displays the usual stylized facts, such as negative convexity in certain regions, and a narrowing of price differentials as interest rates fall. Most interesting, the term structure slope plays an important role in valuing MBS via its relation to the interest rate level and the refinancing incentive associated with a particular MBS. We also find that the interest rate hedge established based on our model compares favorably with existing methods.

On a more general note, the MDE procedure will work well (in a relative sense) under the following three conditions. First, since density estimation is data intensive, the researcher either needs a large data sample or an estimation problem in which there is little disturbance error in the relation between the variables. Second, the problem should be described by a relative low dimensional system, since MDE's properties deteriorate quickly when variables are added to the estimation. Third, and especially relevant for comparison across methods, MDE will work relatively well for highly nonlinear frameworks. As it happens, these features also describe derivative pricing. Hence, while the results we obtain here for GNMAs are encouraging, it is likely that the MDE approach would fare well for more complex derivative securities. Though the TBA market is especially suited for MDE analysis due to its reduction of the maturity effect on bonds, it may be worthwhile investigating the pricing of interest only (IO) and principal only (PO) strips, and collateralized mortgage obligations (CMOs). Since the relation between the prices of these securities and interest rates is more highly nonlinear than that of a GNMA, a multifactor analysis might shed light on the interaction between various interest rate factors and the underlying prices. The advantage of the MDE approach is its ability to capture arbitrary nonlinear relations between variables, making it ideally suited to capturing the extreme convexity exhibited by many derivative mortgage-backed securities.

TABLE 1: SUMMARY STATISTICS

Table 1A – GNMA Prices

	Coupon						
	7.5%	8.0%	8.5%	9.0%	9.5%	10.0%	10.5%
Mean	93.132	95.578	97.876	100.084	102.204	104.347	106.331
Max.	105.156	106.563	107.500	108.281	109.469	110.938	112.719
Min.	78.375	81.625	83.656	86.531	89.531	92.688	95.750
Vol.	6.559	6.287	5.831	5.260	4.722	4.294	3.978

Correlations

	7.5%	8.0%	8.5%	9.0%	9.5%	10.0%	10.5%
7.5%	1.000	0.998	0.993	0.986	0.981	0.983	0.977
8.0%	0.998	1.000	0.997	0.992	0.987	0.987	0.979
8.5%	0.993	0.997	1.000	0.998	0.995	0.993	0.982
9.0%	0.986	0.992	0.998	1.000	0.999	0.995	0.983
9.5%	0.981	0.987	0.995	0.999	1.000	0.997	0.985
10.0%	0.983	0.987	0.993	0.995	0.997	1.000	0.994
10.5%	0.977	0.979	0.982	0.983	0.985	0.994	1.000

Table 1B – Interest Rates

	Long Rate	Spread	Mortgage Rate
Mean	7.779	2.119	9.337
Max.	10.230	3.840	11.580
Min.	5.170	-0.190	6.740
Vol.	1.123	1.101	1.206

Correlations

	Long Rate	Spread	Mortgage Rate
Long Rate	1.000	-0.450	0.980
Spread	-0.450	1.000	-0.518
Mortgage Rate	0.980	-0.518	1.000

Summary statistics for prices of TBA contracts on 7.5% to 10.5% GNMAs, the long rate (10-year), the spread (10-year minus 3-month), and the average mortgage rate. All data are weekly from January 1987 through May 1994. Interest rates are in percent per year.

TABLE 2: 1-FACTOR GNMA PRICING

Table 2A – Pricing Errors

	Coupon						
	7.5%	8.0%	8.5%	9.0%	9.5%	10.0%	10.5%
Mean	0.003	0.006	0.007	0.010	0.010	0.010	0.009
Mean Abs.	0.529	0.605	0.649	0.679	0.660	0.597	0.666
Vol.	0.703	0.747	0.800	0.832	0.824	0.767	0.841
Autocorr.	0.861	0.898	0.918	0.927	0.921	0.917	0.916

Table 2B – Pricing Error Regression Analysis

	Coupon						
	7.5%	8.0%	8.5%	9.0%	9.5%	10.0%	10.5%
Const.	3.226	3.613	3.887	3.857	3.721	3.249	2.755
	(3.190)	(3.413)	(4.133)	(4.419)	(4.402)	(4.028)	(4.320)
Long Rate	-0.963	-1.062	-1.130	-1.117	-1.074	-0.942	-0.805
	(0.887)	(0.941)	(1.135)	(1.216)	(1.210)	(1.119)	(1.216)
(Long Rate)^2	0.069	0.075	0.080	0.078	0.075	0.066	0.057
	(0.059)	(0.063)	(0.075)	(0.081)	(0.080)	(0.075)	(0.082)
R^2	0.028	0.026	0.023	0.020	0.018	0.017	0.011
Joint Test	2.795	2.157	1.709	1.441	1.327	1.228	0.707
p-value	0.247	0.340	0.426	0.487	0.515	0.541	0.702
AC(e)	0.853	0.891	0.913	0.923	0.916	0.912	0.914
Const.	0.491	0.236	0.411	0.607	0.887	1.137	1.494
	(0.148)	(0.194)	(0.212)	(0.211)	(0.194)	(0.165)	(0.135)
Spread	-0.673	-0.373	-0.446	-0.605	-0.948	-1.234	-1.582
	(0.275)	(0.330)	(0.365)	(0.374)	(0.342)	(0.286)	(0.261)
(Spread)^2	0.165	0.098	0.095	0.120	0.199	0.261	0.328
	(0.074)	(0.090)	(0.101)	(0.103)	(0.094)	(0.079)	(0.072)
R^2	0.072	0.020	0.033	0.068	0.145	0.276	0.401
Joint Test	6.480	1.281	2.608	5.916	14.102	34.899	79.195
p-value	0.039	0.527	0.271	0.052	0.001	0.000	0.000
AC(e)	0.848	0.895	0.914	0.920	0.904	0.877	0.847

Summary statistics and regression analysis for the pricing errors from a 1-factor (long rate) MDE GNMA pricing model. The regression analysis involves regressing the pricing errors on linear and squared explanatory variables. Heteroscedasticity and autocorrelation consistent standard errors are reported in parentheses below the corresponding regression coefficient. AC(e) is the autocorrelation of the residuals from the regression.

TABLE 3: 2-FACTOR GNMA PRICING

Table 3A – Pricing Errors

	Coupon						
	7.5%	8.0%	8.5%	9.0%	9.5%	10.0%	10.5%
Mean	0.018	0.020	0.022	0.023	0.025	0.023	0.018
Mean Abs.	0.503	0.489	0.499	0.494	0.483	0.412	0.396
Vol.	0.646	0.616	0.627	0.623	0.613	0.532	0.523
Autocorr.	0.832	0.843	0.859	0.869	0.864	0.840	0.819

Table 3B – Pricing Error Regression Analysis

	Coupon						
	7.5%	8.0%	8.5%	9.0%	9.5%	10.0%	10.5%
Const.	3.470	3.924	4.193	4.188	4.088	3.559	3.002
	(2.751)	(2.784)	(3.112)	(3.064)	(2.924)	(2.228)	(1.875)
Long Rate	-1.036	-1.150	-1.215	-1.207	-1.176	-1.030	-0.878
	(0.763)	(0.754)	(0.830)	(0.814)	(0.780)	(0.605)	(0.526)
$(\text{Long Rate})^2$	0.075	0.082	0.085	0.085	0.082	0.072	0.062
	(0.051)	(0.049)	(0.054)	(0.053)	(0.051)	(0.040)	(0.036)
R^2	0.041	0.045	0.043	0.040	0.039	0.042	0.036
Joint Test	4.775	5.846	5.185	5.059	4.608	5.332	3.792
p-value	0.092	0.054	0.075	0.080	0.100	0.070	0.150
AC(e)	0.825	0.834	0.852	0.865	0.856	0.833	0.814
Const.	0.124	0.096	0.082	0.093	0.117	0.151	0.210
	(0.200)	(0.182)	(0.175)	(0.171)	(0.178)	(0.171)	(0.151)
Spread	-0.203	-0.158	-0.105	-0.105	-0.126	-0.183	-0.308
	(0.279)	(0.265)	(0.265)	(0.255)	(0.243)	(0.210)	(0.177)
$(\text{Spread})^2$	0.057	0.045	0.028	0.027	0.031	0.046	0.081
	(0.072)	(0.068)	(0.069)	(0.065)	(0.061)	(0.052)	(0.043)
R^2	0.009	0.006	0.002	0.002	0.003	0.009	0.027
Joint Test	0.665	0.486	0.172	0.173	0.270	0.777	3.474
p-value	0.717	0.784	0.917	0.917	0.874	0.678	0.176
AC(e)	0.830	0.841	0.857	0.868	0.862	0.836	0.809

Summary statistics and regression analysis for the pricing errors from a 2-factor (long rate, spread) MDE GNMA pricing model. The regression analysis involves regressing the pricing errors on linear and squared explanatory variables. Heteroscedasticity and autocorrelation consistent standard errors are reported in parentheses below the corresponding regression coefficient. AC(e) is the autocorrelation of the residuals from the regression.

TABLE 4: HEDGING RESULTS

Table 4A – 8% GNMA

Period	GNMA	Linear	Roll-Up Roll-Down	MDE 1-fctr	MDE 2-fctr
1/90-5/94	68.3	35.0	27.6	30.0	29.4
1/90-2/91	85.5	26.9	27.6	27.8	30.1
3/91-4/92	72.2	30.5	31.7	34.8	32.1
5/92-6/93	61.3	37.7	25.9	29.3	27.8
7/93-5/94	45.5	43.2	24.8	26.9	27.2
$\sigma_{\Delta r_l, \Delta(r_l - r_s)}$	59.0	15.0	6.8	6.1	4.3
$\sigma_{\Delta r_l}$	59.0	15.0	6.8	6.1	4.2

Table 4B – 9% GNMA

Period	GNMA	Linear	Breeden	MDE 1-fctr	MDE 2-fctr
1/90-5/94	53.0	36.8	24.6	29.3	25.6
1/90-2/91	73.9	24.3	23.5	26.0	27.2
3/91-4/92	55.2	32.3	25.8	38.1	28.2
5/92-6/93	43.8	46.4	25.3	29.3	25.3
7/93-5/94	23.8	39.6	23.7	19.7	20.8
$\sigma_{\Delta r_l, \Delta(r_l - r_s)}$	41.1	18.7	1.2	5.5	3.9
$\sigma_{\Delta r_l}$	41.1	18.6	0.7	5.3	0.1

Table 4C – 10% GNMA

Period	GNMA	Linear	Breeden	MDE 1-fctr	MDE 2-fctr
1/90-5/94	41.4	34.9	29.4	30.0	26.1
1/90-2/91	58.2	24.0	22.3	27.6	27.8
3/91-4/92	48.1	33.8	26.4	38.8	29.6
5/92-6/93	34.8	44.6	29.2	29.5	27.8
7/93-5/94	20.3	32.2	39.4	18.8	19.2
$\sigma_{\Delta r_l, \Delta(r_l - r_s)}$	28.6	16.4	11.3	5.9	5.4
$\sigma_{\Delta r_l}$	28.6	16.4	11.3	5.7	1.4

Results of hedging the 8%, 9% and 10% GNMAs with various methods. Each method's hedge ratios are calculated using the past 150 weeks, for the next week. Hence the hedging period is January 1990 through May 1994. The methods are (i) GNMA – the total volatility of an open position (no hedging), in basis points, (ii) linear – hedging via linear regression on T-note futures returns, (iii) roll-up/roll-down – a method which infers hedge ratios from contemporaneous market prices of near coupon MBS, (iv) MDE – hedge ratios determined via a one factor (long rate only) and two factor (long rate and spread) models, trading in T-note futures and T-bills in the corresponding hedge ratios. The last two rows provide a measure of the quantity of interest rate risk (two factor risk or one factor risk), which remains using each method's hedging results. In all cases the numbers in the tables represent the standard deviation of weekly returns in basis points.

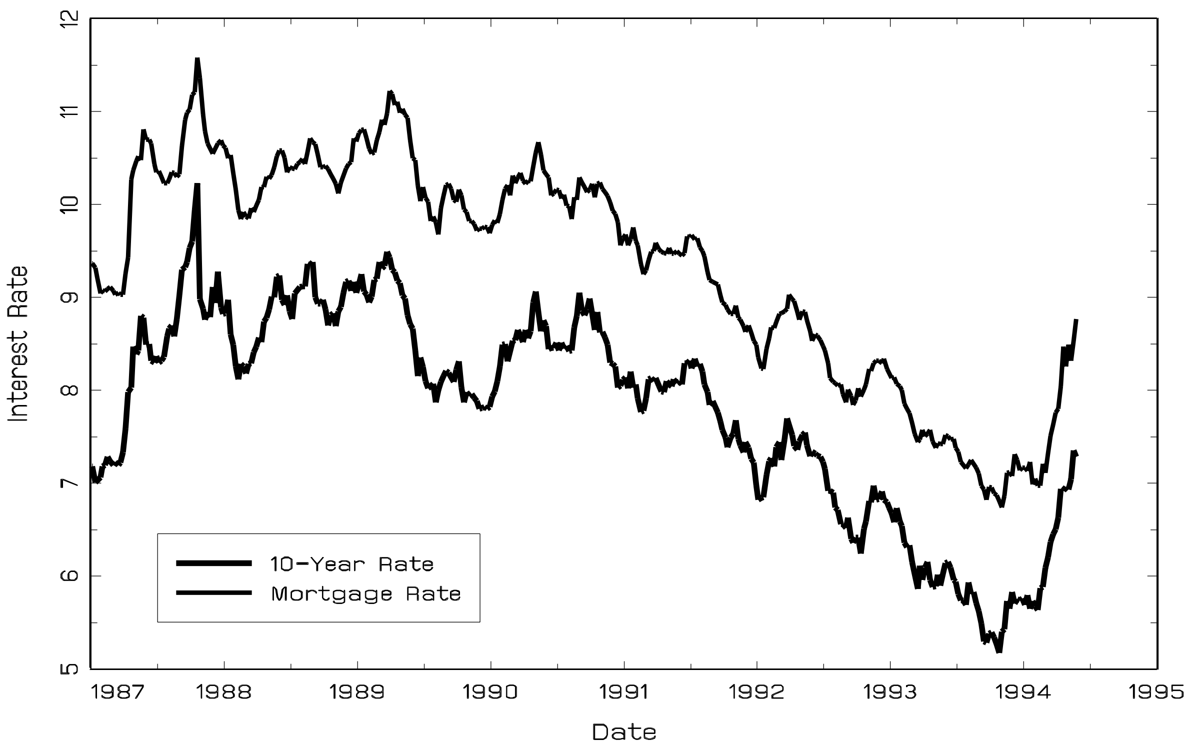

Figure 1: The yield on the "on-the-run" 10-year Treasury note and the average 30-year mortgage rate, from January 1987 to May 1994.

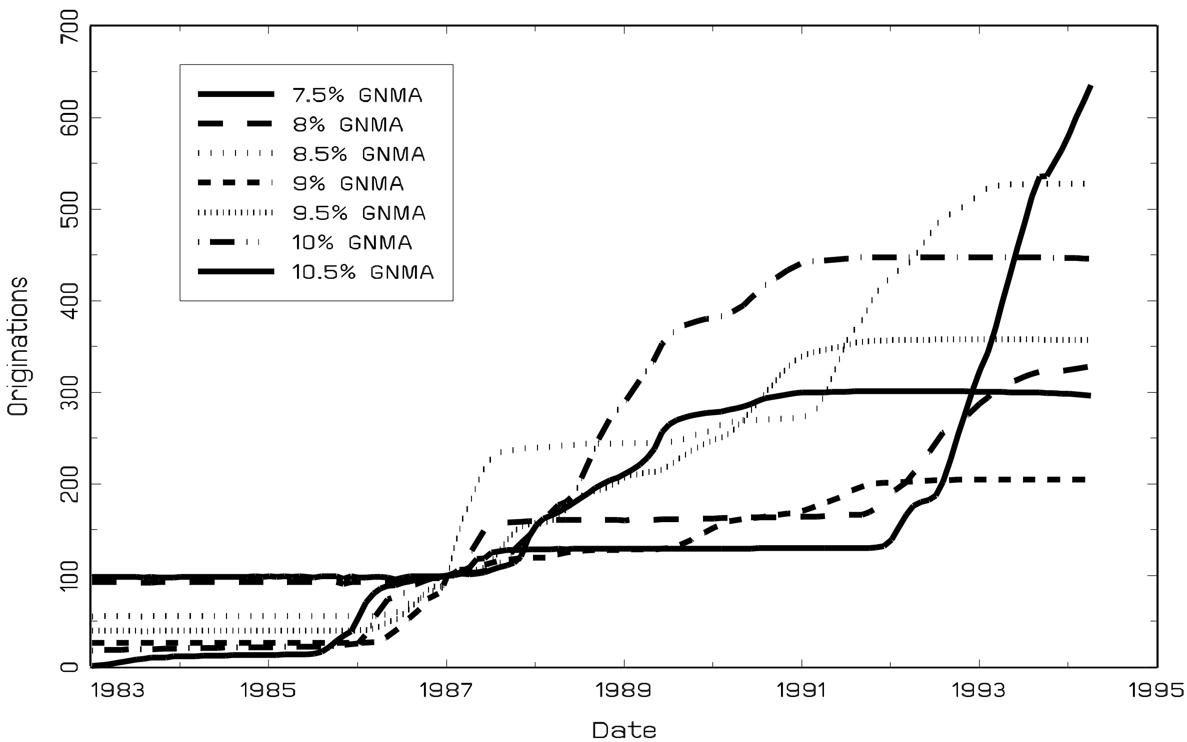

Figure 2: Originations of 7.5%–10.5% GNMAs from January 1983 to April 1994. The dollar amount outstanding is normalized to 100 in January 1987.

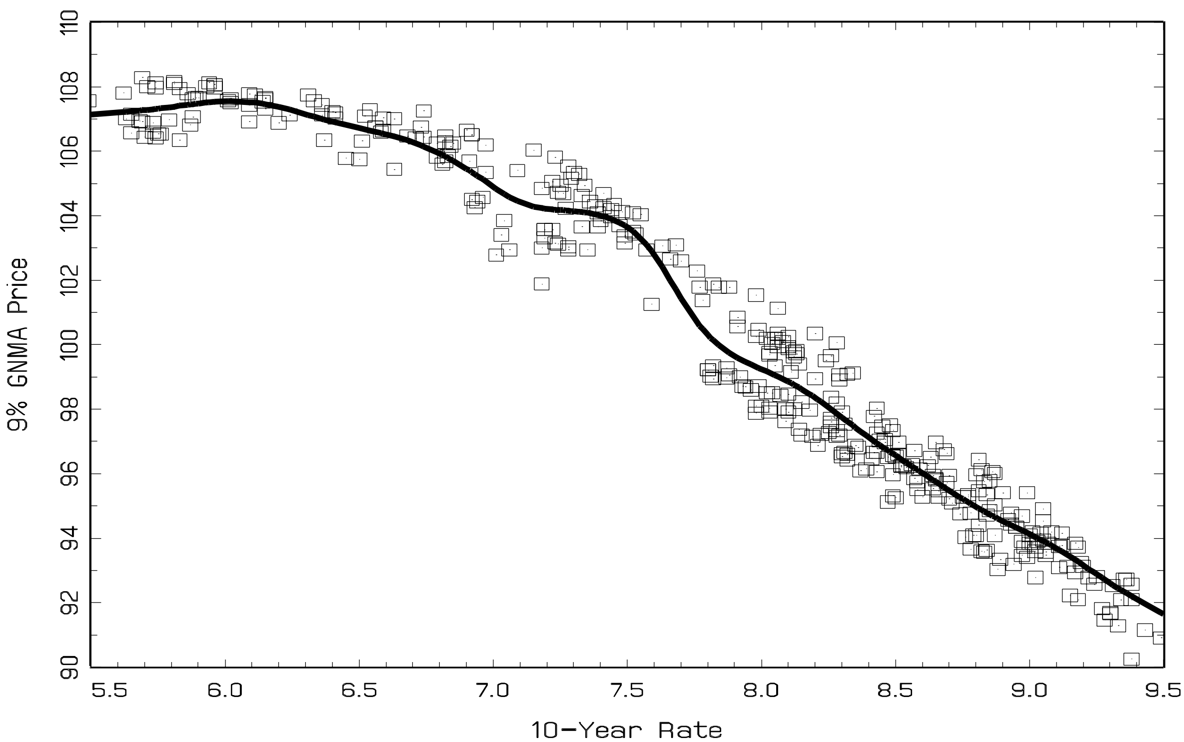

Figure 3: Observed weekly prices and estimated prices from a 1-factor (long rate) MDE model for a 9% GNMA for the period January 1987 to May 1994.

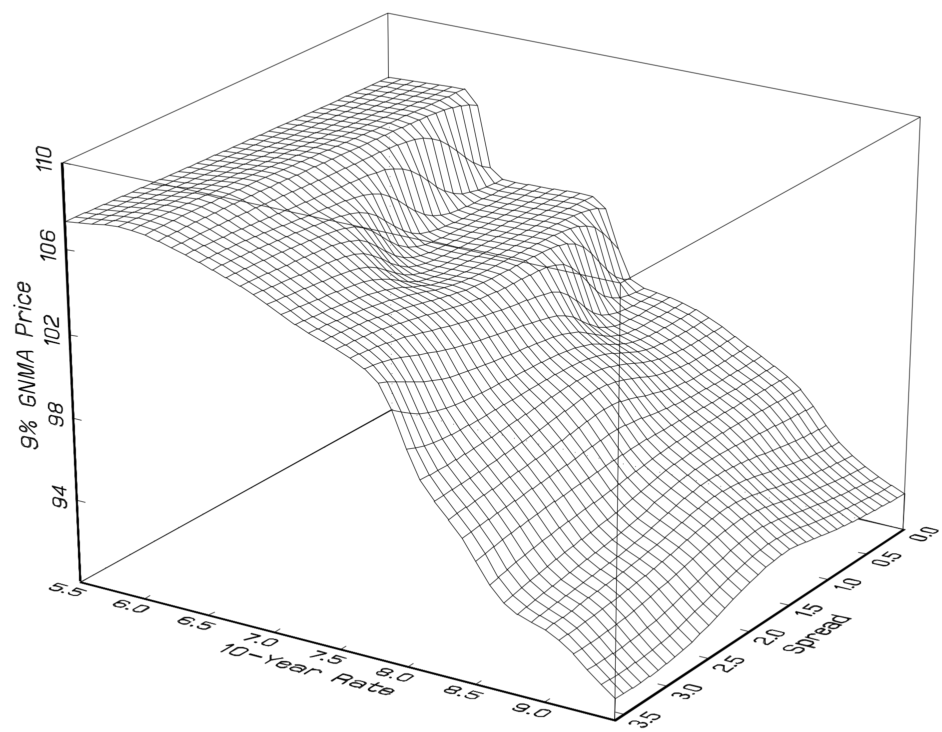

Figure 4: The price of a 9% GNMA as a function of the pricing factors: the long rate and the spread. The pricing functional is estimated using the MDE approach and weekly data from January 1987 to May 1994.

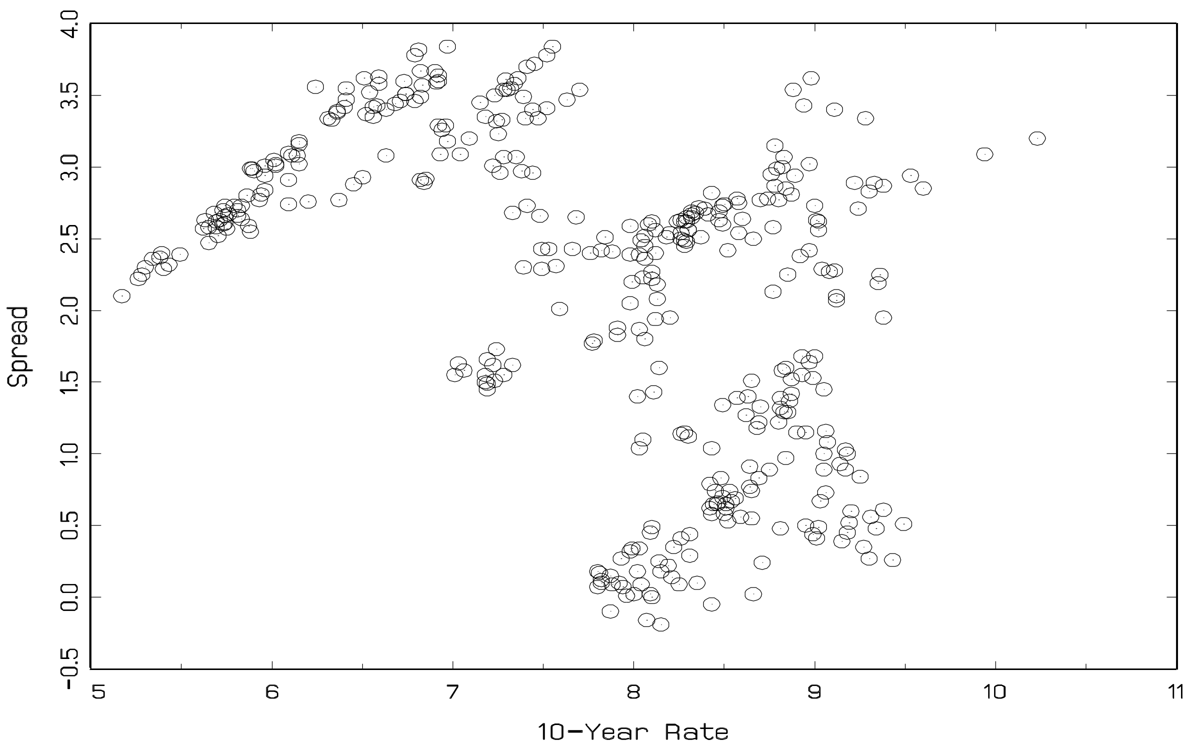

Figure 5: A scatter plot of the pairs of data available for the 10-year rate and the spread between the 10-year rate and the 3-month rate, from January 1987 to May 1994.

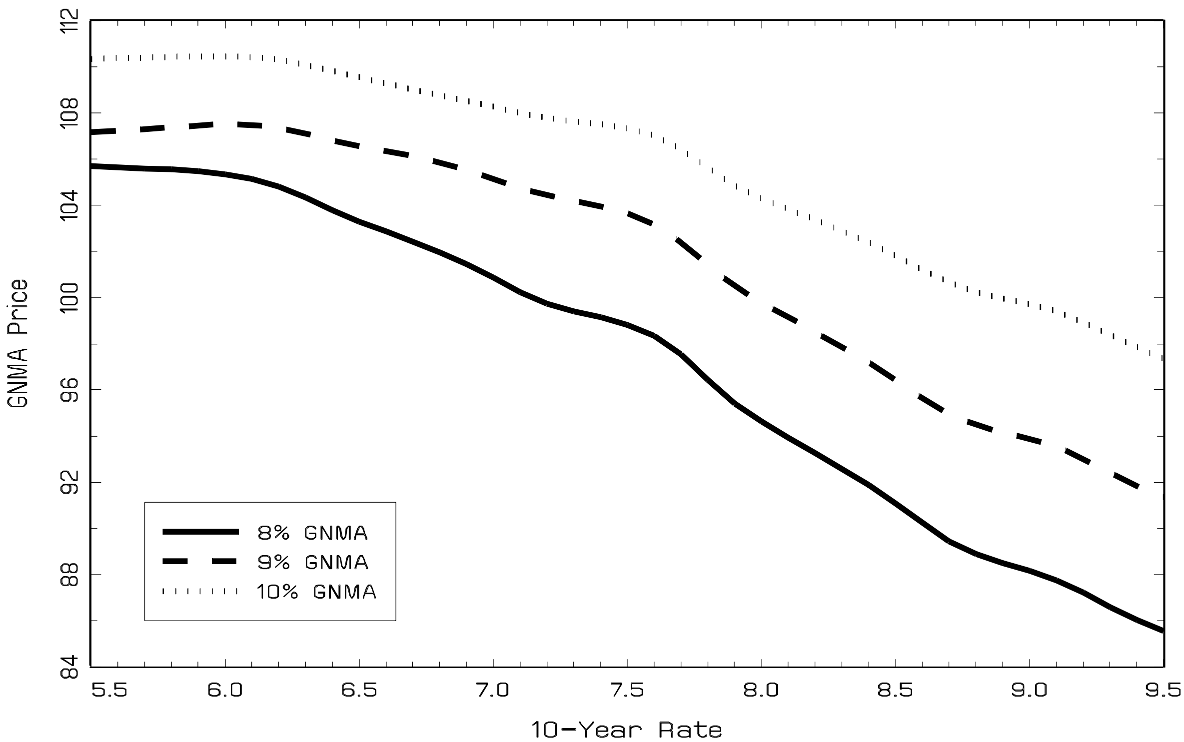

Figure 6: Prices of 8%, 9% and 10% GNMAs for various interest rates, with the spread fixed at 2.70%, as estimated via the MDE approach using weekly data from January 1987 to May 1994.

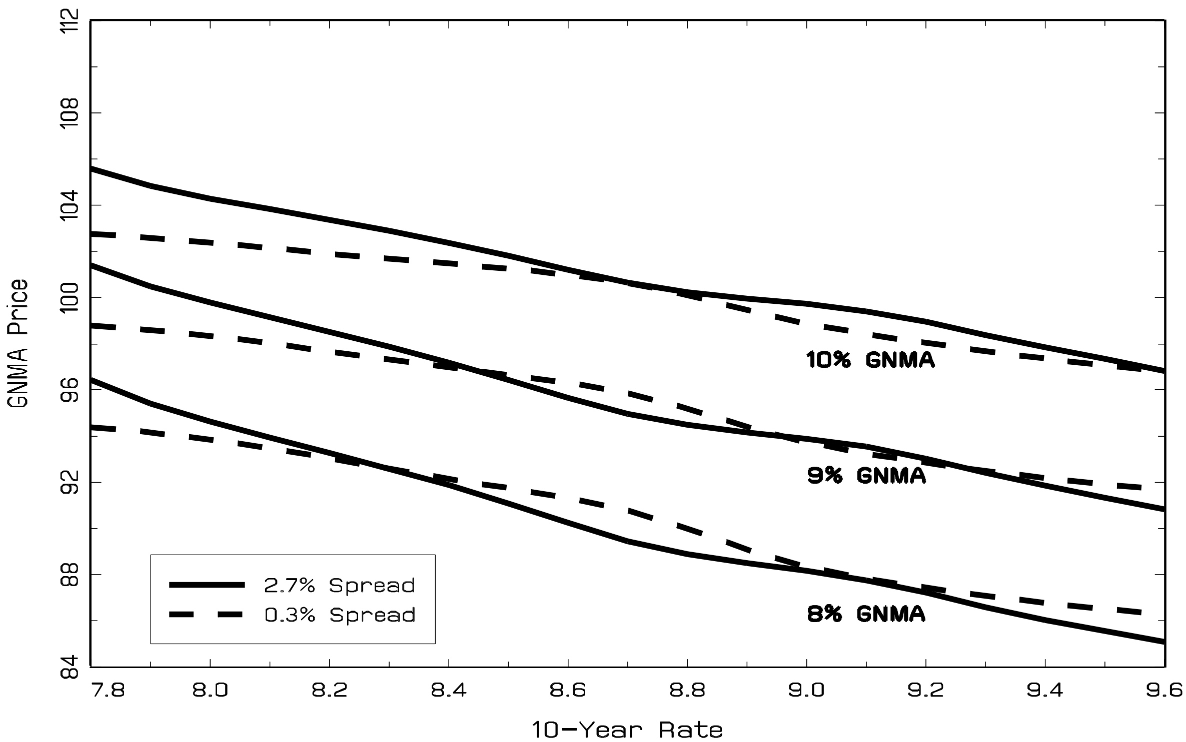

Figure 7: Prices of 8%, 9% and 10% GNMAs for various interest rates, with the spread fixed at 2.70% and 0.30%, as estimated via the MDE approach using weekly data from January 1987 to May 1994.

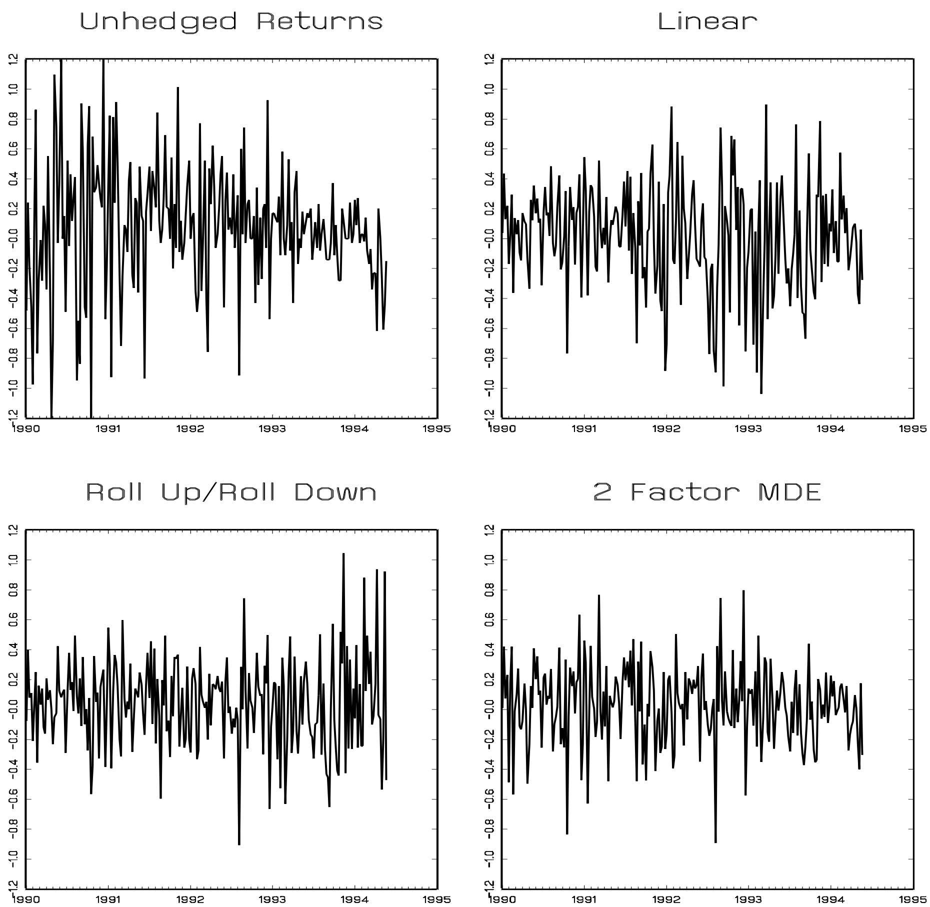

Figure 9: Results from hedging the 10% GNMA using a rolling regression method, where "Linear" is hedging via linear regression of returns on T-note futures, "Roll-Up/Roll-Down" infers hedge ratio from market prices of near coupon MBS, and "2 Factor MDE" uses the two factor MDE approach.

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