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Shared-Ride Efficiency of Ride-Hailing Platforms

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Abstract. ***Problem definition:*** Ride-hailing platforms offering shared rides devote effort to reducing the trip-lengthening detours that accommodate fellow customers' divergent transportation needs. By reducing shared-ride delay, improving *shared-ride efficiency* has the twin benefits of making shared rides more attractive to customers and increasing the number of customers a driver can serve per unit time. ***Methodology/results:*** We analytically model a ride-hailing platform that can offer individual rides and shared rides. We establish results that are counter to naive intuition: greater customer sensitivity to shared-ride delay and greater labor cost can reduce the value of improving shared-ride efficiency, and an increase in shared-ride efficiency can prompt a platform to add individual-ride service. We show that when network effects are small, increasing shared-ride efficiency *pushes wages to extremes*: if the current wage is high (low), increasing shared-ride efficiency pushes the wage higher (lower). We provide a sharp characterization of whether shared-ride efficiency and labor supply are complements or substitutes. We provide simple conditions under which increasing shared-ride efficiency reduces (alternatively, increases) labor welfare. We provide evidence that increasing shared-ride efficiency increases consumer surplus. ***Managerial implications:*** Our results inform a platform's decision of whether to invest in improving shared-ride efficiency, as well as how to change its service offering and wage, as shared-ride efficiency improves.

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1. Introduction

Ride-hailing platforms such as Lyft, Uber, and Via offer shared rides to customers seeking transportation (Merced 2016, Omar 2022). In a shared ride, a customer shares the vehicle with another customer for a least a portion of the trip. For a customer, perhaps the most salient feature of a shared ride is the delay experienced as a result of the detours required to accommodate the other customer's transportation needs. The extent of this delay depends on the quality of the ride-hailing platform's algorithms that match shared-ride customers with one another and with drivers, in light of the routing possibilities. To the extent that this matching is done poorly, customers experience lengthy detours. The longer the driving time required to serve a customer, the fewer customers a driver can serve per unit time. Consequently, platforms have twin interests in improving the matching and routing of shared rides so as to minimize trip-extending detours: making shared rides more attractive to customers and increasing the number of customers a driver can serve per unit time.

Ride-hailing platforms have a variety of tools to improve the process by which they match and route shared rides. Platforms can invest in development efforts to improve their algorithms, testing the efficacy of changes by simulation and experiments with drivers and customers (Farronato et al. 2020). To the extent such efforts shrink trip-extending detours experienced by shared-ride customers, they increase *shared-ride efficiency*. This paper addresses five questions faced by ride-hailing platforms that offer shared rides, potentially alongside individual rides. Before stating these questions, we note that shared rides may exhibit *network effects*: as a platform serves more shared-ride customers, it may be easier for the platform to match customers having similar origins and destinations, shrinking trip-extending detours. As we will discuss, the magnitude of network effects may depend on the specific setting.

Under what circumstances should a platform invest in improving shared-ride efficiency? Perhaps the most relevant input to this question from the demand side is the customer sensitivity to shared-ride delay, and from the supply side is the cost of labor. We examine how the sensitivity to delay and the labor cost influence the marginal value of increasing shared-ride efficiency. It might be natural to conjecture that the value of increasing shared-ride efficiency increases the customer sensitivity to shared-ride delay. Intuitively, increasing shared-ride efficiency decreases shared-ride delay, and reducing this delay is more valuable when customers experience greater disutility for this delay. Similarly, it might be natural to conjecture that the value of increasing shared-ride efficiency increases the labor cost. Intuitively, increasing shared-ride efficiency reduces the labor content required to serve a shared-ride customer, and reducing this labor content is more valuable when labor is costly. We demonstrate that these conjectures need not hold—when network effects are small. Then, the marginal value of increasing shared-ride efficiency initially *decreases* in the customer sensitivity to shared-ride delay. Further, the marginal value of increasing shared-ride efficiency *decreases* in the labor cost—if the labor cost and shared-ride efficiency are high. This paper explains the driving forces that reverse the previous conjectures. To do so, it is useful to address the next, more fundamental question.

Are shared-ride efficiency and labor supply complements or substitutes? Shared-ride efficiency and labor supply are complements (substitutes) if increasing shared-ride efficiency increases (decreases) the marginal value of labor. On one hand, increased shared-ride efficiency makes a marginal unit of labor more productive in that it can serve more shared-ride customers, which suggests that shared-ride efficiency and labor supply are complements. On the other hand, increased shared-ride efficiency makes a marginal unit of labor less valuable in that less labor is needed to serve the same number of shared-ride customers, which suggests that shared-ride efficiency and labor supply are substitutes. We provide a sharp characterization of the interplay between shared-ride efficiency and labor supply: for a platform offering both individual rides and shared rides, shared-ride efficiency and labor supply are substitutes. For a platform offering only shared rides, shared-ride efficiency and labor supply are complements if and only if the labor supply is small.

Ride-hailing platforms have a choice of what services they offer to customers. At some points, Lyft and Uber have offered shared rides alongside individual rides, whereas at other times, they have only offered individual rides (Lam 2021, O'Brien 2021). In contrast, Via has only offered shared rides (Solomon 2021).

How should a platform change its service offering as it improves shared-ride efficiency? As shared-ride efficiency increases, the shared-ride service becomes more attractive to the platform relative to the individual-ride service because the former becomes more valuable to consumers and less costly to provide. Hence, one might presume that as shared-ride efficiency improves, it becomes attractive to shift the service offering toward shared rides and away from individual rides. Indeed, when the labor cost is high, the optimal service offering has this expected structure: an increase in shared-ride efficiency prompts a platform that is offering both individual- and shared-ride services to *drop* individual service. Contrary to this intuitive structure, we show that an increase in shared-ride efficiency can prompt a platform to *add* individual service. Whereas the previous questions address longer-term, strategic decisions, the next question addresses a shorter-term, tactical decision.

How should a platform change its wage as it improves shared-ride efficiency? On one hand, increased shared-ride efficiency reduces the labor content required to serve a shared-ride customer, which suggests that the platform may require fewer workers; hence, the platform may reduce the wage. On the other hand, increased shared-ride efficiency reduces shared-ride delay, making the service more attractive to customers, which suggests that the platform may want to expand the size of the shared-ride service, which may entail increasing the wage. We show that when network effects are small, *increasing shared-ride efficiency pushes wages to extremes*: if the current wage is high, increasing shared-ride efficiency pushes the wage higher, and if the current wage is low, increasing shared-ride efficiency pushes the wage lower. We provide evidence that when network effects are large, increasing shared-ride efficiency reduces the wage.

Are driver-workers and consumers hurt or helped by improved shared-ride efficiency? Paralleling the wage findings, we show that increasing shared-ride efficiency reduces labor welfare when network effects are large or when network effects and the labor cost are small. Shared-ride efficiency increases labor welfare when network effects are small and the labor cost is large. We provide evidence that consumer surplus increases in shared-ride efficiency.

These results apply to platforms that are able to offer shared rides and individual rides as well as platforms that are only able to offer shared rides. The caveat is that the conditions for the marginal value of increasing shared-ride efficiency to decrease the labor cost differ when the platform is unable to offer individual rides and network effects are small.

Ride-hailing platforms have received considerable attention in the operations management literature. A large share of this work has focused on pricing, including dynamic pricing (Banerjee et al. 2015, Cachon et al. 2017, Bai et al. 2019, Guda and Subramanian 2019, Hu et al. 2022), spatial pricing (Bimpikis et al. 2019, Besbes et al. 2021), and the impact of driver independence (Taylor 2018, Gurvich et al. 2019). Other dimensions of ride-hailing platforms that have received attention are wage schemes and labor welfare (Nikzad 2018, Hu and Zhou

2019, Benjaafar et al. 2023), regulation (Yu et al. 2020), matching mechanisms (Benjaafar et al. 2019, Benjaafar et al. 2020, Ozkan and Ward 2020, Wu et al. 2020, Bernstein et al. 2021, Daniels and Turcic 2021, Tang et al. 2021, Feng et al. 2023), and competition (Ahmadinejad et al. 2019, Bai and Tang 2022, Cohen and Zhang 2022, Siddiq and Taylor 2022, Chen et al. 2023). Feldman et al. (2022) and Chen and Hu (2023) consider platforms that deliver goods rather than passengers. Chen et al. (2020) identify research opportunities arising from ride-hailing and other platforms.

Within the ride-hailing literature, our work is most closely related to papers that examine platforms that offer shared rides alongside individual rides. Hu et al. (2020) and Jacob and Roet-Green (2021) employ queueing models that capture rich operational details of matching shared-ride customers and congestion-driven delay. A shared-ride customer experiences delay while waiting to match with a second shared-ride customer but incurs the same average transportation time as individual-ride customers. Jacob and Roet-Green (2021) characterize the revenue-maximizing service offering and prices. They show that when the mix of low- and high-type customers is relatively balanced and there are sufficiently many drivers, it is optimal to offer both individual and shared rides. Hu et al. (2020) characterize customers' behavior in choosing between an individual and shared ride. They show that self-interested behavior leads customers to choose shared rides less often than under social welfare maximization. They propose schemes that include elements such as charging a lower price for shared rides and prioritizing shared rides in dispatching drivers, which induce social-welfare maximizing behavior.

Wang and Zhang (2021) and Zhang and Nie (2021) examine social welfare implications of a platform's offering shared rides. Zhang and Nie (2021) employ a spatial driver-customer matching model that captures customer time waiting for pickup and the in-transit detour to pick up a fellow shared-ride customer. In a numerical study based on data from a major city, they find that, in most cases, an unregulated profit-maximizing platform should offer both individual and shared rides, and this results in greater social welfare than other service offerings. Wang and Zhang (2021) find that a platform offering shared rides generally benefits customers but may hurt driver-workers. Wang and Zhang (2021) is the closest paper to ours in that, within its analysis, it explicitly considers shared-ride efficiency. They show that a platform should offer shared rides if and only if shared-ride efficiency exceeds a threshold. Further, as shared-ride efficiency increases, the platform reduces the price of both individual rides and shared rides, increasing the quantity of rides. Our model generalizes that of Wang and Zhang (2021) by allowing for network effects and for customers to experience disutility for shared-ride delay.

In the sequel, Section 2 describes the model, Section 3 presents results, and Section 4 concludes.

2. Model

Normalize the sojourn time, the time between requesting a ride and being dropped off at the destination, of a customer using the individual-ride service to one. For the driver serving an individual-ride customer, the sojourn time consists of the driving time to pick up the customer and the driving transit time. Thus, a driver is able to serve one individual-ride customer per unit time.

A shared-ride customer experiences a longer sojourn time because of the time required to match with a second customer and the detours required to accommodate the second customer's transportation needs. A shared-ride customer's sojourn time is $\tau g(Q_S) > 1$, where Q_S is the demand for shared rides. A driver is able to serve one shared-ride customer in time $\tau g(Q_S)$, and the driver's vehicle is able to accommodate two shared-ride customers. Hence, a driver is able to serve $2/[\tau g(Q_S)]$ shared-ride customers per unit time. We refer to $\psi = 2/\tau$ as *shared-ride efficiency*.

A customer with valuation v receives utility

$$v - p_I$$

from the individual-ride service, where $p_I \geq 0$ denotes the individual-ride price; receives utility

$$v\{\beta - \alpha[\tau g(Q_S) - 1]\} - p_S \tag{1}$$

from the shared-ride service, where $p_S \geq 0$ denotes the shared-ride price; and receives zero utility from no service. Note $\tau g(Q_S) - 1$ is the additional sojourn time experienced by a customer who uses the shared-ride service rather than the individual-ride service; we refer to $\tau g(Q_S) - 1$ as *shared-ride delay.* The parameter $\alpha \in (0, 1)$ represents the customer's disutility per unit time for the additional time associated with the shared-ride service; we refer to α as the customer *sensitivity to shared-ride delay*. The parameter $\beta \in (0, 1)$ represents reduced utility from the shared-ride service due to all aspects not associated with transportation time (e.g., negative psychological aspects of sharing the passenger area of a vehicle with a stranger, including potential unwanted social interaction, reduced personal space and privacy, and safety concerns).

We consider the following form for the function embedded in the shared-ride customer's sojourn time, where $a \geq 0$ and $b \geq 0$,

$$g(Q_S) = a + b/Q_S \tag{2}$$

if $Q_S > 0$; $g(Q_S) = \infty$ if $Q_S = 0$; and $b > 0$ and $g(Q_S) = a$ if $b = 0$. In a stochastic model, Jacob and Roet-Green (2021) model a shared-ride customer's expected sojourn time as having the form in Equation (2). Hu et al. (2020) and Jacob and Roet-Green (2021) model the expected time for a shared-ride customer to match with another customer as having the form b/Q_S. Intuitively, the greater the arrival rate of shared-ride customers, the more quickly a shared-ride customer will match with another customer. More generally, the larger the pool of shared-ride customers, the easier it is to match customers having similar origins and destinations, reducing the detours individual customers incur in accommodating their matched customer's transportation needs. The magnitude of network effects will depend on many factors, such as the distribution of origins and destinations and the distance between origin-destination pairs (e.g., if origins are colocated, destinations are colocated, and distances are lengthy, sojourn times may be dominated by necessary transportation time, making the magnitude of network effects relatively small). To capture this, we assume $a + b = 1$ and refer to b as the *network effects parameter.* Whereas our stylized modeling approach seeks to capture the essence of network effects in a simple, tractable form, we acknowledge that Equation (2) does not capture their full complexity.

Consider a unit of time, and normalize the mass of customers to one. Assume v is distributed uniformly on $[0, 1]$. A customer with utility v selects the option among individual-ride service, shared-ride service, and no service that maximizes utility. Let $Q_I(p_I, p_S)$ denote customer demand for individual rides and $Q_S(p_I, p_S)$ denote demand for shared rides under prices (p_I, p_S). Normalize the mass of drivers to one. Let $f(\cdot)$ denote the density function of the drivers' opportunity cost under labor cost $\theta \in (0, \infty)$ and $F(\cdot)$ denote the corresponding cumulative distribution. The wage $w \geq 0$ denotes the payment received by a driver engaged in serving customers. Then, $K(w, \theta) = F(w)$ denotes the labor supply under wage w and labor cost θ. The labor supply $K(w, \theta)$ increases in w and decreases in θ. All functions described as increasing, decreasing, convex, or concave are strictly so unless stated otherwise. Because a driver is able to serve $2/[\tau g(Q_S)]$ shared-ride customers per unit time, serving $Q_S > 0$ shared-ride customers requires $Q_S \tau g(Q_S)/2 = (aQ_S + b)/\psi$ drivers.

The platform chooses the individual-ride price p_I, shared-ride price p_S, and wage w to maximize profit per unit time

$$p_I Q_I(p_I, p_S) + p_S Q_S(p_I, p_S) - wK(w, \theta),$$

subject to the constraint that the labor supply satisfies the demand $Q_I(p_I, p_S) + \mathbf{1}_{\{Q_S(p_I, p_S)>0\}}\ [aQ_S(p_I, p_S) + b]/\psi \leq K(w, \theta)$, where $\mathbf{1}_{\{\cdot\}}$ is the indicator function. Because the platform satisfies the demand, we refer to (Q_I, Q_S) as sales quantities. Ensuring that a customer's sojourn time is longer for a shared ride than for an individual ride $\tau g(Q_S) > 1$ for the range of shared-rides sales quantities $Q_S \in [0, 1]$ requires that shared-ride efficiency $\psi < 2$.

It is convenient to parameterize the platform's problem in terms of sales and supply quantities. Let $p_I(Q_I, Q_S)$ denote the individual-ride price and $p_S(Q_I, Q_S)$ denote the shared-ride price under sales quantities (Q_I, Q_S). Let $w(K, \theta)$ denote the wage required to induce K drivers to provide service for a unit of time. Assume the wage $w(K, \theta)$ is differentiable and increasing in the labor supply K and labor cost θ. Further, $w(0, \theta) = 0$, $\lim_{\theta \to 0} w(K, \theta) = 0$, $\lim_{\theta \to 0}(\partial/\partial K)w(K, \theta) = 0$, $\lim_{\theta \to \infty} w(K, \theta) = \infty$ for $K > 0$ and $(\partial^2/\partial K \partial\theta)w(K, \theta) \geq 0$. These assumptions are satisfied, for example, if $w(K, \theta) = \theta W(K)$, where $W(\cdot)$ is differentiable and increasing with $W(0) = 0$.

The platform chooses individual-ride sales $Q_I \geq 0$, shared-ride sales $Q_S \geq 0$, and labor supply $K \geq 0$ to maximize profit per unit time

$$\Pi(Q_I, Q_S, K) = p_I(Q_I, Q_S)Q_I + p_S(Q_I, Q_S)Q_S - w(K, \theta)K,$$

subject to the constraint that the labor supply satisfies the demand $Q_I + \mathbf{1}_{\{Q_S>0\}}[aQ_S + b]/\psi \leq K$. We refer to $c(K, \theta) = w(K, \theta)K$ as the supply cost function. Assume $c(K, \theta)$ is weakly convex in K. (Wang and Zhang (2021) show that a sufficient condition for $c(\cdot, \theta)$ to be weakly convex is that the drivers' opportunity cost distribution is log-concave, a restriction satisfied by many commonly used probability distributions, including the normal, uniform, and exponential distributions.)

Let (p_I^*, p_S^*, w^*) denote the optimal prices and wage, (Q_I^*, Q_S^*, K^*) denote the optimal sales and labor supply quantities, and Π^* denote the platform's profit under the optimal decisions. Consumer surplus is the sum of the utility captured by consumers per unit time

$$CS = \int_0^1 \max(v - p_I^*, v\{\beta - \alpha[\tau g(Q_S^*) - 1]\} - p_S^*, 0)dv, \tag{3}$$

and labor welfare is the sum of the utility captured by drivers per unit time

$$LW = \int \max(w^* - y, 0)f(y)dy. \tag{4}$$

One can think of customers as being staggered in their ride request and drop-off times. Consider an example in which a shared-ride customer's sojourn time $\tau g(Q_S) = 1.5$. At time $t + 0.5$, when customer A is being transported, the driver receives a ride request from customer B and proceeds to pick up customer B. At time $t + 1.5$, the driver drops off customer A (who had requested a ride at time t), receives ride request from customer C, and proceeds to pick up customer C. At time $t + 2.0$, the driver drops off customer B, and so on. This example illustrates that a driver serving shared-ride customers is serving customers continuously. This is also true of drivers serving individual customers. Because a driver serving customers receives wage w per unit time, the welfare of a driver with opportunity cost per unit time y is $w - y$, regardless of the type of customer being served, which supports Equation (4).

Our assumption that the platform offers drivers a common wage for individual and shared rides is consistent with the practice of Lyft (Lyft 2023) but differs from that of Uber, which pays a driver transporting a customer a fixed amount for picking up an additional customer (Uber 2023). To sharpen the focus on delay because of trip-extending detours to accommodate the needs of shared-ride customers, we abstract away from delay because of stochasticity in customer arrivals and sojourn times. Section 3.4 shows how our results extend to the setting where the platform is only able to offer shared rides.

3. Results

Section 3.1 characterizes the structure of the platform's optimal service offering. Because much of this structure is intuitive, the section emphasizes the most intriguing element: an increase in shared-ride efficiency can prompt a platform to *add* individual-ride service. Of greater significance, Section 3.2 characterizes how customer sensitivity to delay and labor cost influence the marginal value of increasing shared-ride efficiency. Section 3.3 examines how increasing shared-ride efficiency affects the platform's optimal wage, the shared-ride price, labor welfare, and consumer welfare.

3.1. Impact of Shared-Ride Efficiency on Optimal Service Offering

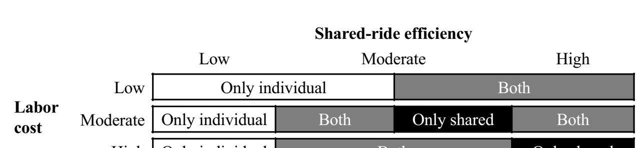
In general, the platform's problem of choosing individual-ride sales Q_I, shared-ride sales Q_S, and labor supply K is not well behaved. More precisely, the presence of network effects (in particular, when the network effects parameter b is moderate) leads the platform's revenue to be neither convex nor concave in the labor supply K. However, we are able to provide analytical results when network effects are small or large. More precisely, in all the formal results that follow, we say that a result holds when *network effects are small* if there exists $\underline{b} > 0$ such that the result holds for $b < \underline{b}$, and a result holds when *network effects are large* if there exists $\overline{b} < 1$ such that the result holds for $b > \overline{b}$. Proposition 1 characterizes the platform's optimal service offering in these two scenarios. We say that a platform offers individual rides if and only if the optimal individual-ride sales quantity $Q_I^* > 0$ and offers shared rides if and only if the optimal shared-ride sales quantity $Q_S^* > 0$. All proofs are in the appendix and Online Supplement except for the results in Section 3.4, which are in the author supplement, which is available directly from the authors.

Proposition 1. (a) *Suppose network effects are small $b < \underline{b}$. There exists $\underline{\psi} > 0$ such that it is optimal to offer only individual rides if and only if shared-ride efficiency is low $\psi \le \underline{\psi}$. There exist $\underline{\theta}$ and $\overline{\theta}$ satisfying $0 < \underline{\theta} \le \overline{\theta}$ such that for each labor cost $\theta \in (0, \underline{\theta}) \cup (\overline{\theta}, \infty)$, there exists $\overline{\psi}(\theta) \in (\underline{\psi}, 2]$ such that it is optimal to offer both individual rides and shared rides if $\psi \in (\underline{\psi}, \overline{\psi}(\theta))$ and to offer only shared rides if $\psi \in (\overline{\psi}(\theta), 2)$, where $\overline{\psi}(\theta) = 2$ if and only if $\theta \in (0, \underline{\theta})$. There exist parameters with $\theta \in (\underline{\theta}, \overline{\theta})$ such that it is optimal to offer both services if $\psi \in (\underline{\psi}, \psi_l) \cup (\psi_h, 2)$ and to offer only shared rides if $\psi \in (\psi_l, \psi_h)$, where $\psi_l < \psi_h$, and optimal individual-ride sales Q_I^* increase and shared-ride sales Q_S^* decrease in ψ on $\psi \in (\psi_h, 2)$.* (b) *Suppose network effects are large $b > \overline{b}$. It is optimal to offer only individual rides for $\psi \in (0, 2)$.*

We begin with part (a), which addresses the case where network effects are small. For this case, Figure 1 conceptually depicts the optimal service offering as a function of shared-ride efficiency and labor cost. How does the optimal service offering change as shared-ride efficiency improves? As shared-ride efficiency increases, the shared-ride service becomes more attractive to the platform relative to the individual-ride service because the former becomes more valuable to consumers and less costly to provide. Hence, one might expect that as shared-ride efficiency improves, it becomes attractive to shift the service offering toward shared rides and away from individual rides. Proposition 1(a) shows that optimal service offering has this intuitively appealing structure—as shared-ride efficiency improves, the optimal service offering shifts from only individual rides to both services and finally to only shared rides—provided that the labor cost is high $\theta > \overline{\theta}$. Thus, in the regime where the platform optimally offers both services, as expected, an increase in shared-ride efficiency prompts the platform to *drop* individual service. Contrary to this expectation, Proposition 1(a) reveals that an increase in shared-ride efficiency can prompt a platform to *add* individual service; this only occurs when the labor cost is moderate $\theta \in (\underline{\theta}, \overline{\theta})$.

To unpack why an increase in shared-ride efficiency, which makes the individual service comparatively less attractive, can prompt a platform to *add* individual service, and to understand why this only occurs when the labor cost is moderate $\theta \in (\underline{\theta}, \overline{\theta})$, it is useful to first explain the structure of the optimal service offering when the labor cost is outside of this range. When the labor cost is high $\theta > \overline{\theta}$, the cost of serving customers is an important concern, and if the cost-effectiveness of

Figure 1. Optimal Service Offering as a Function of Shared-Ride Efficiency and the Labor Cost When Network Effects Are Small



one service offering is notably superior to the other (shared-ride efficiency is low $\psi \le \underline{\psi}$ or high $\psi > \overline{\psi}(\theta)$), it is optimal to only offer the more cost-effective service. Only if the cost effectiveness of the two service offerings is comparable (shared-ride efficiency is moderate $\psi \in (\underline{\psi}, \overline{\psi}(\theta))$) is it optimal to offer both services. When the labor cost is low $\theta < \underline{\theta}$, it is optimal to offer both individual rides and shared rides. Intuitively, when the labor cost is low, revenue concerns dominate. Offering two services allows the platform to price discriminate among customers, extracting more surplus from customers and generating greater revenue (and profit) for the platform.

The interesting result that an increase in shared-ride efficiency can prompt a platform that is offering only shared rides to *add* individual service is driven by the presence of convexity in the supply cost function. As shared-ride efficiency increases, the marginal cost of an individual ride decreases if one keeps the quantity of shared rides fixed because less labor is required to provide that quantity of shared rides. Hence, increased shared-ride efficiency makes it attractive for a platform that is offering shared rides to add individual service. This phenomenon does not occur when the labor cost is low $\theta < \underline{\theta}$ because, as noted earlier, then it is always optimal to offer both services. This phenomenon does not occur when the labor cost is high $\theta > \overline{\theta}$ because then, as shared-ride efficiency increases, the high cost of labor makes it attractive to sharply increase the quantity of shared rides, increasing the marginal cost of an individual ride.

How do the optimal sales quantities change as shared-ride efficiency improves? Because as shared-ride efficiency improves and the shared-ride service becomes more valuable to consumers and less costly to provide, it may be reasonable to expect that optimal shared-ride sales Q_S^* would increase. Indeed, it is straightforward to verify that this occurs when the platform is restricted to only offering shared rides. Proposition 1(a) reveals that the opposite occurs when the platform offers both individual rides and shared rides—for some parameters (again, the necessary condition is that the labor cost is moderate). To understand why this occurs, recall that increasing shared-ride efficiency ψ across the threshold ψ_h prompts the shared-ride-only platform to add individual service. As shared-ride efficiency ψ increases over the range $\psi \in (\psi_h, 2)$, optimal individual-ride sales Q_I^* increase. Because of the convexity in the labor cost function, this increases the marginal cost of labor, prompting the platform to decrease shared-ride sales Q_S^*.

Proposition 1(a) complements Wang and Zhang (2021), who show that in the region where the platform only offers shared rides, optimal shared-ride sales Q_S^* *increase* in shared-ride efficiency by showing how this result can be reversed when the platform offers both individual and shared rides. Proposition 1(a) complements Wang and Zhang (2021), who show that it is optimal to offer *shared rides* if and only if shared-ride efficiency exceeds a threshold, by showing that the optimality of offering *individual rides* depends on shared-ride efficiency in a more complex fashion.

Proposition 1(b) reveals that when network effects are large, a necessary condition for the platform to offer shared rides is that shared-ride efficiency $\psi \ge 2$. We conducted a numerical study replacing the restriction $\psi < 2$ with the looser but essential restriction that under the optimal offering, the customer's sojourn time is longer for a shared ride than for an individual ride $\tau g(Q_S^*) > 1$. Let set A denote $\alpha \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, $\beta \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, $\theta \in \{2^{-6}, 2^{-5.88}, 2^{-5.76}, \ldots, 2^{5.88}, 2^6\}$, and $b \in \{0, 0.00001, 0.05, 0.10, \ldots, 0.95, 0.99999, 1\}$, where $c(K, \theta) = \theta K^2$. Under network effects parameter $b = 1$, we observed that for every combination of $\{\alpha, \beta, \theta\}$ in set A in which there exists a ψ such that it is optimal to offer shared rides, the optimal service offering has a simple structure characterized by a single threshold $\underline{\psi}$: offer only shared rides when shared-ride efficiency is large $\psi > \underline{\psi}$, and offer only individual rides otherwise. Intuitively, when shared-ride efficiency is large, scale economies in shared rides are substantial, making it unattractive to offer individual rides, which would cannibalize sales of shared rides.

Let $\tilde{b} = \inf\{b :$ optimal service offering is either only individual or only shared rides for ψ such that $\tau g(Q_S^*) > 1\}$. For every parameter combination of $\{\alpha, \beta, \theta\}$, we observed that the structure of the optimal offering described earlier that holds at $b = 1$ holds for $b > \tilde{b}$. Finally, for every parameter combination of $\{\alpha, \beta, \theta\}$, we observed that $\tilde{b} = \underline{b} = \overline{b}$, meaning that the structure described in Proposition 1(a) holds for $b < \tilde{b}$. The threshold $\tilde{b}$ weakly decreases in the utility from the shared-ride service β. More precisely, we observed that for each instance of $\{\alpha, \theta\}$, $\tilde{b}$ weakly decreases in β. To see the intuition, note that $\tilde{b}$ decreasing is associated with expanding the parameter space in which the optimal service offering is only shared rides rather than both individual and shared rides. As utility from the shared-ride service β increases, shared rides become more attractive relative to individual rides.

Sections 3.2 and 3.3 provide analytical results when network effects are small $b < \underline{b}$ and when they are large $b > \overline{b}$. At the end of each of these sections, we discuss observations from the numerical study that supplement the analytical results.

In what follows, we restrict attention to the parameter regime in which the platform optimally offers shared rides. This is equivalent to imposing the restriction that shared-ride efficiency is sufficiently large $\psi > \underline{\psi}$.

3.2. Value of Increasing Shared-Ride Efficiency

Perhaps the most critical input to the economic viability and profitability of the shared-ride service is shared-ride efficiency. As a result, an important decision for a platform is how much effort to exert to improve shared-ride efficiency (e.g., by improving matching and routing algorithms) (Farronato et al. 2020). Two of the most fundamental inputs into this investment decision are the customer sensitivity to shared-ride delay α—because increasing shared-ride efficiency decreases shared-ride delay—and the labor cost θ—because increasing shared-ride efficiency reduces the labor content required to serve a shared-ride customer.

This section explores how the attractiveness of investing in improving shared-ride efficiency depends on these two inputs. More precisely, we explore how the sensitivity to delay α and the labor cost θ influence the marginal value of increasing shared-ride efficiency $(\partial/\partial\psi)\Pi^*$. It might be natural to conjecture that the value of increasing shared-ride efficiency increases in the customer sensitivity to shared-ride delay and in the labor cost. The intuition for the former is that increasing shared-ride efficiency decreases shared-ride delay, and reducing this delay is more valuable when customers experience greater disutility for this delay. The intuition for the latter is that increasing shared-ride efficiency reduces the labor content required to serve a shared-ride customer, and reducing this labor content is more valuable when labor is costly. This section shows that these conjectures need not hold. More precisely, this section characterizes parameter regions in which they are violated (and regions in which they hold), and it explains the driving forces that reverse the conjectures.

Proposition 2 characterizes the impact of the customer sensitivity to shared-ride delay α on the marginal value of increasing shared-ride efficiency $(\partial/\partial\psi)\Pi^*$. Counter to the logic outlined earlier, when network effects are small, the marginal value of increasing shared-ride efficiency initially *decreases* in the customer sensitivity to shared-ride delay.

Proposition 2. (a) *Suppose network effects are small $b < \underline{b}$. There exist $\underline{\psi}_\alpha$ and $\overline{\psi}_\alpha$ satisfying $\underline{\psi} < \underline{\psi}_\alpha \le \overline{\psi}_\alpha < 2$ such that the marginal value of increasing shared-ride efficiency $(\partial/\partial\psi)\Pi^*$ decreases in the customer sensitivity to shared-ride delay α for $\psi \in (\underline{\psi}, \underline{\psi}_\alpha)$ and increases in α for $\psi \in (\overline{\psi}_\alpha, 2)$.* (b) *Suppose network effects are large $b > \overline{b}$, and the platform offers only shared rides. Then, $(\partial/\partial\psi)\Pi^*$ increases in α.*

We begin with part (a), which addresses the case where network effects are small. To understand the managerial implications of Proposition 2(a), consider a platform that is contemplating investing in increasing its shared-ride efficiency ψ and is confronted by an increase in customers' sensitivity to shared-ride delay α. Counter to the intuition that this increase in disutility should strengthen the platform's (marginal) payoff from investment in increasing shared-ride efficiency, Proposition 2 uncovers that the opposite occurs—when the platform's shared-ride efficiency is low $\psi \in (\underline{\psi}, \underline{\psi}_\alpha)$. The intuitive result is restored when the platform's shared-ride efficiency is high $\psi \in (\overline{\psi}_\alpha, 2)$.

To identify the driving forces that inform the intuition for Proposition 2(a), it is useful to consider the simpler setting in which the platform only offers shared rides. Let $\pi_S(q_S)$ denote the platform's profit and $P_S(q_S)$ denote the shared-ride price under shared-ride sales quantity q_S and no individual rides; formally, $\pi_S(q_S) = \Pi(0, q_S, (aq_S + b)/\psi)$ and $P_S(q_S) = p_S(0, q_S)$. Let q_S^* denote the optimal (i.e., profit-maximizing) shared-ride sales quantity. Hence, $\pi_S^* = \pi_S(q_S^*)$ denotes the platform's optimal profit, and $P_S(q_S^*)$ denotes the optimal shared-ride price.

The impact of a change in customer sensitivity to shared-ride delay α on the marginal value of increasing shared-ride efficiency $(\partial/\partial\psi)\pi_S^*$ can be written as

$$\frac{\partial^2 \pi_S^*}{\partial\alpha\partial\psi} = \underbrace{\left[\frac{\partial}{\partial\psi}\frac{\partial P_S(q_S^*)}{\partial\alpha}\right] \times q_S^*}_{\substack{>0 \\ \textit{price effect}}} + \underbrace{\frac{\partial P_S(q_S^*)}{\partial\alpha} \times \frac{\partial q_S^*}{\partial\psi}}_{\substack{<0 \\ \textit{quantity effect}}}. \quad (5)$$

Increasing the customer sensitivity to shared-ride delay has two effects on the marginal value of increasing shared-ride efficiency: a positive *price effect* and a negative *quantity effect*.

Within the price effect, the term in square brackets is strictly positive. The logic is that increasing shared-ride efficiency reduces the shared-ride delay, making the shared-ride more attractive to customers; increasing the customer sensitivity to shared-ride delay amplifies this attractiveness. In words, the price effect captures that increasing the customer sensitivity to shared-ride delay contributes to increasing the marginal value of shared-ride efficiency by amplifying the positive impact that shared-ride efficiency has on the price.

We label the second term on the right-hand side of Equation (5) the quantity effect because it reflects the change in the optimal sales quantity. The first term in the quantity effect, the partial derivative of price with respect to customer sensitivity to shared-ride delay, is negative. As the sensitivity to shared-ride delay increases, the shared ride becomes less attractive to customers, pushing down the price. The second term in the quantity effect, the partial derivative of the optimal sales quantity, with respect to shared-ride efficiency, is positive. Increasing shared-ride efficiency reduces the shared-ride delay, making the shared ride more attractive to customers, prompting the platform to increase its sales quantity. Because the two terms in the quantity effect have opposite signs, the quantity effect is negative. In words, the quantity effect captures that increasing the customer sensitivity to shared-ride delay contributes to decreasing the marginal value of shared-ride efficiency by reducing the attractiveness of the shared-ride service.

We now turn to explaining under what circumstance the positive price effect or the negative quantity effect dominates. The optimal shared-ride sales quantity increases in shared-ride efficiency (as noted earlier) and is strictly positive if and only if the shared-ride efficiency exceeds a lower threshold analogous to $\underline{\psi}$. When shared-ride efficiency is close to the lower threshold, the optimal shared-ride sales quantity is very small. Consequently, the price effect is very small, and the (negative) quantity effect dominates. Conversely, when shared-ride efficiency grows large, so does the shared-ride sales quantity q_S^* such that the (positive) price effect dominates. Hence, the marginal value of increasing shared-ride efficiency $(\partial/\partial\psi)\Pi^*$ decreases in the customer sensitivity to shared-ride delay α when shared-ride efficiency is low and increases in α when shared-ride efficiency is high.

The intuition extends to the case where the platform offers individual and shared rides, with the following caveat: as noted in Proposition 1(a), when the platform optimally offers individual rides and shared rides, the optimal shared-ride sales quantity may decrease in shared-ride efficiency ψ for high levels of shared-ride efficiency. To the extent this occurs, it reinforces the intuition that customer sensitivity to shared-ride delay α increases the marginal value of increasing shared-ride efficiency when shared-ride efficiency is high because then, both the price effect and quantity effects contribute positively.

We conclude by summarizing the intuition for Proposition 2(a) in more managerial terms. If shared-ride efficiency is low, the shared-ride service is of limited viability in the sense that customers' willingness to pay for the service is low. As customer sensitivity to shared-ride delay increases, customers' willingness to pay is reduced, further eroding the viability of the shared-ride service. Consequently, increased customer sensitivity to shared-ride delay reduces the value of marginally improving the shared-ride service through better shared-ride efficiency. In contrast, if shared-ride efficiency is high, the shared-ride service is attractive to a large set of customers. For these customers, reducing shared-ride delay (by increasing shared-ride efficiency) becomes more valuable as customers exhibit greater sensitivity to this delay.

Proposition 2(b) reveals that when network effects are large and the platform offers only shared rides, the intuitively appealing result that the value of increasing shared-ride efficiency increases in the customer sensitivity to shared-ride delay is restored. Recall that in the numerical study described at the end of Section 3.1, for every parameter combination of $\{\alpha,\beta,\theta\}$ in set A, when network effects are large $b > \overline{b}$ (where $\overline{b} = \underline{b} = \tilde{b}$) and it is optimal to offer shared rides, it is optimal to offer only shared rides. Hence, when discussing subsequent results that hold when network effects are large and the platform offers only shared rides, we omit mention of the latter restriction.

Proposition 3 characterizes the impact of the labor cost θ on the marginal value of increasing shared-ride efficiency $(\partial/\partial\psi)\Pi^*$. The beginning of this section provides an argument supporting the assertion that the value of increasing shared-ride efficiency increases in the labor cost θ. Contrary to this assertion, Proposition 3(a) shows that the marginal value of increasing shared-ride efficiency can *decrease* in the labor cost. Recall that $\underline{\theta}$, $\overline{\theta}$, and $\overline{\psi}(\theta)$ are defined in Proposition 1(a), and further, $\theta > \overline{\theta}$ implies $\overline{\psi}(\theta) \in (\underline{\psi}, 2)$.

Proposition 3. (a) *Suppose network effects are small $b < \underline{b}$. If the labor cost is low $\theta < \underline{\theta}$, then the marginal value of increasing shared-ride efficiency $(\partial/\partial\psi)\Pi^*$ increases in the labor cost θ. There exists $\theta_v \geq \overline{\theta}$ such that if the labor cost is high $\theta > \theta_v$, then $(\partial/\partial\psi)\Pi^*$ increases in θ for $\psi \in (\underline{\psi}, \overline{\psi}(\theta))$ and decreases in θ for $\psi \in (\overline{\psi}(\theta), 2)$.* (b) *Suppose network effects are large $b > \overline{b}$, and the platform offers only shared rides. Then, $(\partial/\partial\psi)\Pi^*$ increases in θ.*

The intuitively appealing result that the value of increasing shared-ride efficiency increases in the labor cost holds when the network effects are large (part (b)). In what follows, we focus on the scenario where network effects are small (part (a)) because the results and implications are more interesting.

To understand the managerial implications of Proposition 3(a), consider a platform that is contemplating investing in increasing its shared-ride efficiency ψ and is confronted by an increase in the labor cost θ. Because increasing shared-ride efficiency reduces the labor content required to serve a shared-ride customer, it is intuitive that this increase in labor cost should strengthen the platform's (marginal) payoff from investment in increasing shared-ride efficiency. Proposition 3(a) reveals that the opposite is true—if the labor cost and shared-ride efficiency are high. However, if the labor cost is low, the intuitive result is restored.

To lay out the logic behind Proposition 3(a), it is necessary to first formally establish a related result. This related result, Proposition 4, addresses the interplay between *labor supply* and shared-ride efficiency, which is useful in understanding the interplay between *labor cost* and shared-ride efficiency captured in Proposition 3(a).

To understand the interplay between shared-ride efficiency and labor supply, a key question is: are these two elements complements, that is, increasing shared-ride efficiency increases the marginal value of labor $(\partial/\partial K)\Pi$? Or are they substitutes, that is, increasing the former decreases the latter? Proposition 4 characterizes the conditions under which shared-ride efficiency and labor supply are complements or substitutes. Let $(Q_I^*(K), Q_S^*(K))$ denote the revenue-maximizing individual-ride and shared-ride sales under labor supply K. Under labor supply K, in the parameter regime where it is optimal to offer both individual and shared rides (i.e., $Q_I^*(K) > 0$ and $Q_S^*(K) > 0$), platform profit is $\Pi_B = \Pi(Q_I^*(K), Q_S^*(K), K)$. Under labor supply K, in the parameter regime where it is optimal to offer only shared rides (i.e., $Q_I^*(K) = 0$ and $Q_S^*(K) > 0$), platform profit is $\Pi_S = \Pi(0, Q_S^*(K), K)$. Let $\overline{K}(\psi) = (\alpha+\beta)(1+b)/\{4[\psi(\alpha+\beta) - \alpha(1-b)]\}$.

Proposition 4. *For a platform offering both individual rides and shared rides, shared-ride efficiency and labor supply are substitutes* $(\partial^2/\partial\psi\partial K)\Pi_B < 0$. *For a platform offering only shared rides, shared-ride efficiency and labor supply are complements* $(\partial^2/\partial\psi\partial K)\Pi_S > 0$ *if and only if the labor supply is small* $K < \overline{K}(\psi)$.

To understand the interplay between shared-ride efficiency and labor supply, we begin by considering a platform that offers only shared rides. Increasing shared-ride efficiency expands the number of customers served by a unit of labor. We refer to this as the *sales quantity expansion effect*. Marginal revenue decreases in the sales quantity.

When the labor supply is small, the sales quantity is small, and the revenue from a marginal increase in the sales quantity is large. Because the value of expanding the sales quantity is high, the sales quantity expansion effect increases the marginal value of labor. Stated more managerially, if a platform is operating with a small labor supply, an increase in shared-ride efficiency will prompt the profit-maximizing platform to increase its labor supply to take advantage of the fact that each unit of labor is more productive.

When the labor supply is large, the sales quantity is large, and the revenue from a marginal increase in the sales quantity is small. Because the value of expanding the sales quantity is low, the sales quantity expansion effect decreases the marginal value of labor. Stated more managerially, if a platform is operating with a large labor supply, an increase in shared-ride efficiency will prompt the profit-maximizing platform to decrease its labor supply because each unit of labor being more productive means that less labor is needed.

We now turn to the intuition for the platform that offers both individual and shared rides. It is optimal to offer both services only if the labor supply is sufficiently large. Parallel to the case without individual rides, the large labor supply causes the sales quantity expansion effect to decrease the marginal value of labor.

Having established the conditions under which shared-ride efficiency and labor supply are complements or substitutes, we are now in a position to explain the intuition for Proposition 3(a). The marginal value of increasing shared-ride efficiency $(\partial/\partial\psi)\Pi^*$ *increases* in the labor cost θ if and only if shared-ride efficiency and labor supply are *substitutes*; formally, $(\partial^2/\partial\theta\partial\psi)\Pi^*$ has the opposite sign of $(\partial^2/\partial\psi\partial K)\Pi|_{K=K^*}$. The intuition is that the optimal labor supply K^* decreases in the labor cost θ (see Lemma A.8(a) in the appendix). Hence, in terms of the effect on the marginal value of increasing shared-ride efficiency $(\partial/\partial\psi)\Pi^*$, increasing the labor cost has the same directional effect as decreasing the labor supply.

First, consider the following two cases: the labor cost is low (formally, $\theta < \underline{\theta}$), or the labor cost is high, and shared ride efficiency is low (formally, $\theta > \overline{\theta}$ and $\psi < \overline{\psi}(\theta)$). In each case, it is optimal to offer both individual rides and shared rides (by Proposition 1(a)). From Proposition 4, it follows that shared-ride efficiency and labor supply are substitutes. Consequently, the marginal value of increasing shared-ride efficiency *increases* in the labor cost.

Second, consider the case where the labor cost and shared-ride efficiency are high. From Proposition 1(a), under high labor cost $\theta > \overline{\theta}$ and high shared-ride efficiency $\psi > \overline{\psi}(\theta)$, it is optimal to offer only shared rides. Further, if the labor cost is sufficiently high, then the optimal labor supply is sufficiently small $K^* < \overline{K}(\psi)$ that shared-ride efficiency and labor supply are complements. Consequently, the marginal value of increasing shared-ride efficiency *decreases* in the labor cost.

We now summarize our results regarding under what circumstances investing in improving shared-ride efficiency is attractive. Contrary to the naive intuition that high customer sensitivity to shared-ride delay makes such investments attractive, we find that such high sensitivity makes such investments *less* attractive, provided that the platform's current shared-ride efficiency is low, the improvement in shared-ride efficiency is modest, and network effects are small. However, the naive intuition is restored when the platform's current shared-ride efficiency is already high or network effects are large (Proposition 2). Contrary to the naive intuition that high labor cost makes investing in improving shared-ride efficiency attractive, we find that such high cost makes such investments *less* attractive, provided that the platform's current shared-ride efficiency is already high and network effects are small. However, the naive intuition is restored when the platform's current shared-ride efficiency is low and the improvement in shared-ride efficiency is modest or when network effects are large (Proposition 3).

We conclude by discussing observations from the numerical study, where set A and $\tilde{b}$ are defined at the end of Section 3.1. Under every combination of $\{\alpha, \beta, \theta, b\}$ in set A with $b > \tilde{b}$, we observed $(\partial^2/\partial\alpha\partial\psi)\Pi^* > 0$, which is consistent with Proposition 2(b). However, when $b < \tilde{b}$, for some parameters, $(\partial^2/\partial\alpha\partial\psi)\Pi^* > 0$ over the full range of ψ, which departs from Proposition 2(a). The most important insight of Proposition 3 is that when the network effects parameter b is small, the labor cost θ is high, and shared-ride efficiency ψ is high, $(\partial^2/\partial\theta\partial\psi)\Pi^* < 0$. Under every combination of $\{\alpha, \beta\}$ in set A, we observed that there exist θ_v and $\overline{\psi}$ such that if $b < \tilde{b}$, $\theta > \theta_v$, and $\psi > \overline{\psi}$, then $(\partial^2/\partial\theta\partial\psi)\Pi^* < 0$, which is consistent with Proposition 3(a). However, when $b > \tilde{b}$, for some parameters, $(\partial^2/\partial\theta\partial\psi)\Pi^* < 0$, which departs from Proposition 3(b).

3.3. Impact of Shared-Ride Efficiency on Wage, Price, Labor, and Consumers

Whereas the previous section addressed the longer-term, strategic decision of investing to improve shared-ride efficiency, this section addresses the shorter-term, tactical decisions of the platform. Two of platform's primary tactical decisions are the wage it offers to drivers and the price it charges consumers for a shared ride. This section provides insight into how the platform should change its optimal wage and shared-ride price as its shared-ride efficiency improves. This section also addresses the broader questions of whether workers or consumers benefit from increased shared-ride efficiency.

Do workers benefit from increased shared-ride efficiency? On one hand, increased shared-ride efficiency reduces the labor content required to serve a shared-ride customer, which suggests that the platform may require fewer workers; hence, the platform may reduce the wage, to the detriment of workers. On the other hand, increased shared-ride efficiency reduces shared-ride delay, making the service more attractive to customers, which suggests that the platform may want to expand the size of the shared-ride service, which may entail increasing the wage to the benefit of workers. (Note from Equation (4) that labor welfare LW increases in the optimal wage w^* and depends on shared-ride efficiency ψ only through w^*.)

Under what conditions will further increasing shared-ride efficiency prompt the platform to increase the wage or, alternatively, to decrease the wage? Proposition 5 provides a simple sufficient condition for each. We use the notation $LW(\psi)$ to denote the dependence of labor welfare on shared-ride efficiency.

Proposition 5. *Consider any fixed α and β.* (a) *Suppose network effects are small $b < \underline{b}$. For any level of shared-ride efficiency $\psi_0 \in (\underline{\psi}, 2)$, there exist $\underline{w}(\psi_0)$ and $\overline{w}(\psi_0)$ satisfying $0 < \underline{w}(\psi_0) \leq \overline{w}(\psi_0)$ such that if the optimal wage is sufficiently small $w^*(\psi_0) < \underline{w}(\psi_0)$, then the optimal wage $w^*(\psi)$ and labor welfare $LW(\psi)$ decrease in shared-ride efficiency ψ for $\psi \in (\psi_0, 2)$, and if $w^*(\psi_0) > \overline{w}(\psi_0)$, then $w^*(\psi)$ and $LW(\psi)$ increase in ψ for $\psi \in (\psi_0, 2)$.* (b) *Suppose network effects are large $b > \overline{b}$ and the platform offers only shared rides. For any level of shared-ride efficiency $\psi_0 > \underline{\psi}$, $w^*(\psi)$ and $LW(\psi)$ decrease in ψ for $\psi > \psi_0$.*

We begin with part (a), which addresses the case where network effects are small. Simply stated, the message of Proposition 5(a) is that *increasing shared-ride efficiency pushes wages to extremes*: if the current wage is high, increasing shared-ride efficiency will push it higher, and if the current wage is low, increasing shared-ride efficiency will push it lower.

Just as Proposition 5(a) provides a crisp prescription for managers, it provides a parallel insight for labor advocates and others concerned for labor welfare. To see this, observe that because labor welfare increases in the wage, Proposition 5(a) can be stated in terms of labor welfare: $\underline{w}(\psi_0)$ and $\overline{w}(\psi_0)$ are replaced by labor welfare thresholds $\underline{LW}(\psi_0)$ and $\overline{LW}(\psi_0)$, and $w^*(\psi_0)$ is replaced by $LW(\psi_0)$. When labor welfare is low ($LW(\psi_0) < \underline{LW}(\psi_0)$), increasing shared-ride efficiency will push labor welfare lower (a "spiral down" effect), and when labor welfare is high ($LW(\psi_0) > \underline{LW}(\psi_0)$), increasing shared-ride efficiency will push it higher (a "spiral up" effect). Thus, if labor advocates or workers themselves feel that workers are already being "squeezed" by the platform, they should be concerned that workers will be further squeezed by the platform's improving its shared-ride efficiency. To the extent that labor advocates want to ensure that workers are not made worse off by these changes, they could advocate for additional protections for workers or platform-provided benefits (e.g., bonus payments paid to drivers that provide shared rides). To the extent the workers are already doing well, platforms have an additional incentive outside of their narrow self-interest to improve shared-ride efficiency: by doing so, they will benefit their worker-driver "stakeholders."

We now turn to explaining the logic behind Proposition 5(a). Just as in the case of Proposition 3(a), Proposition 5(a) is largely a consequence of Propositions 1(a) and 4. The intuition also relies on some intuitive properties of the labor supply K, the optimal wage w^*, and the shared-ride efficiency threshold $\overline{\psi}(\theta)$: the labor supply K increases in the wage w, the optimal wage w^* increases in the labor cost θ (see Lemma A.9 in the appendix for a formal proof), and if the labor cost is high, then the shared-ride efficiency threshold $\overline{\psi}(\theta)$ decreases in θ. The logic for the last result is that as the labor cost increases, offering individual rides becomes less attractive, and hence, it becomes optimal to offer only shared rides over a larger range of shared-ride efficiency levels.

First, we explain why when the current wage is low, increasing shared-ride efficiency pushes the wage lower. The current wage $w^*(\psi_0)$ being low implies that the labor cost θ is small. From Proposition 1(a), it follows that it is optimal to offer both individual rides and shared rides as shared-ride efficiency increases through $\psi \in (\psi_0, 2)$. From Proposition 4, it follows that shared-ride efficiency and labor supply are substitutes. As a consequence, as shared-ride efficiency increases, the optimal labor supply K^* decreases, which is achieved by decreasing the wage.

Second, we explain why when the current wage is high, increasing shared-ride efficiency pushes the wage higher. The current wage $w^*(\psi_0)$ being high implies that the labor cost is high θ, which, in turn, implies that shared-ride efficiency threshold is small $\overline{\psi}(\theta) \le \psi_0$ and the optimal labor supply is small. From Proposition 1(a), it follows that it is optimal to offer only shared rides as shared-ride efficiency increases through $\psi \in (\psi_0, 2)$. This and the fact that the labor supply is small together imply that shared-ride efficiency and labor supply are complements (by Proposition 4). As a consequence, as shared-ride efficiency increases, the optimal labor supply K^* increases, which is achieved by increasing the wage.

Whereas the sufficient conditions in Proposition 5(a) are stated in terms of the endogenous current wage, the conditions can be stated instead in terms of the exogenous parameters: if the labor cost is low, then the wage and labor welfare decrease in shared-ride efficiency, and if the labor cost is high, then the wage and labor welfare decrease and then increase in shared-ride efficiency (see Lemma A.10 in the appendix).

Over time, as platforms that source labor from independent contractors ("gig workers") have grown and matured, competition for these workers has intensified, increasing the labor costs platforms face (Kelsey 2017, Preetika 2021, Siddiqui 2021). To the extent that network effects are small, past wages were sufficiently low, and current wages are sufficiently high, Proposition 5 suggests that platforms should respond to increased shared-ride efficiency in opposite ways: decreasing wages in the past and increasing them now.

Proposition 5(b) reveals that when network effects are large, the effect of shared-ride efficiency on the wage and labor welfare is unambiguous: increased shared-ride efficiency reduces both. When network effects are large, taking advantage of scale economies necessitates selling a large quantity of shared rides. Hence, the platform's optimal labor supply is large. This implies that shared-ride efficiency and labor supply are substitutes (by Proposition 4). As a consequence, as shared-ride efficiency increases, the optimal labor supply K^* decreases, which is achieved by decreasing the wage, to the detriment of workers.

We now turn from the impact of shared-ride efficiency on the wage and labor welfare to its impact on the shared-ride price and consumer surplus, beginning with the former. To think about impact of shared-ride efficiency on the shared-ride price, it is useful to consider two effects of increasing shared-ride efficiency. First, increasing shared-ride efficiency expands the number of shared-ride customers served by a unit of labor (the aforementioned *sales quantity expansion effect*), which makes decreasing the price attractive. Second, increasing shared-ride efficiency reduces customers' shared-ride delay, increasing their valuation for a shared ride (we label this the *customer value enhancement effect*), which makes increasing the price attractive. The magnitude of the customer value enhancement effect decreases as the customer sensitivity to shared-ride delay α decreases and vanishes as α approaches zero. This suggests that when customer sensitivity to shared-ride delay α is small, the price-dampening sales quantity expansion effect would dominate, so the price would decrease in shared-ride efficiency. Further, the price-dampening sales quantity effect would seem to be most pronounced when the costliness of labor is low so that the labor supply is high. The opposite is true, as demonstrated by the next result. Recall that our setup allows customer sensitivity to shared-ride delay α to be arbitrarily small.

Proposition 6. (a) *Suppose network effects are small $b < \underline{b}$. There exist $\theta_p \in (0, \underline{\theta})$ such that if the labor cost is low $\theta < \theta_p$, then the optimal shared-ride price p_S^* increases in shared-ride efficiency for $\psi \in (\underline{\psi}, 2)$.* (b) *If network effects are large $b > \overline{b}$ and the platform offers only shared rides, then p_S^* increases in shared-ride efficiency ψ for $\psi > \underline{\psi}$.*

We begin with part (a), which addresses the case where network effects are small. Proposition 6(a) shows that when the labor cost is low, the optimal shared-ride price increases in shared-ride efficiency. The intuition relies on a third effect, the labor supply adjustment effect, wherein the platform adjusts its wage, and thereby its labor supply, in response to a change in shared-ride efficiency. In the extreme case where the labor cost goes to zero, the platform sets its wage and prices to induce the labor supply that will support the revenue-maximizing sales quantities. As shared-ride efficiency increases, the platform reduces the supply so as to maintain the revenue-maximizing sales quantities. That is, the labor supply adjustment effect precisely cancels out the sales quantity expansion effect, leaving only the price-boosting customer value enhancement effect. This continues to hold when the labor cost is nonzero but small.

To place Proposition 6(a) in context, it is useful to compare it to the finding in Wang and Zhang (2021), which considers a setting in which: network effects are not present (which corresponds to $b = 0$ in our model), and customers prefer individual rides to shared rides (which corresponds to $\beta < 1$ in our model) but are indifferent to the magnitude of the shared-ride delay they experience (which corresponds to $\alpha = 0$ in our model). In this setting, Wang and Zhang (2021) show that the optimal shared-ride price always decreases in shared-ride efficiency. Proposition 6(a) complements Wang and Zhang (2021) by showing that if customers are sensitive to shared-ride delay, then the shared-ride price increases in shared-ride efficiency if the labor cost is low. (Wang and Zhang (2021) show that the individual-ride price decreases in shared-ride efficiency, and we find no evidence that incorporating customer sensitivity to shared-ride delay reverses this result. Intuitively, increased shared-ride efficiency improves the quality and reduces the cost of the inferior product in the platform's product line; the increased competition from the inferior product drives down the price of the superior product.)

When the labor cost is high $\theta > \theta_p$, we observed numerically that the optimal shared-ride price exhibits relatively little structure: the price can decrease, increase, or be nonmonotonic in shared-ride efficiency. Accordingly, the insight of Proposition 6(a) is narrow: the presence of even a small amount of customer disutility for shared-ride delay reverses the price prescription from the setting where customers are indifferent to shared-ride delay if the labor cost is low. Proposition 5(b) reveals an alternative sufficient condition for this reversal to occur: network effects are large.

When network effects are large, the optimal shared-ride price increases in shared-ride efficiency. As discussed following Proposition 5, when network effects are large, as shared-ride efficiency increases, the optimal labor supply K^* decreases. This price-increasing labor supply adjustment effect dominates the price-decreasing sales quantity expansion effect (formally, the optimal shared-ride sales quantity Q_S^* decreases in shared-ride efficiency). This, coupled with the price-increasing customer value enhancement effect, drives the result that the price increases in shared-ride efficiency.

Taken together, Proposition 5(b), Proposition 6, and the appendix's Lemma A.10 show that the shared-ride price and wage can move in opposite directions as shared-ride efficiency increases. Namely, when network effects are large or when network effects and the labor cost are small, the shared-ride price increases and the wage decreases.

Do consumers benefit from increased shared-ride efficiency? Three effects are relevant. First, increasing shared-ride efficiency reduces the labor content required to serve a shared-ride customer (we label this the *cost reduction effect*), which makes it attractive for the platform to increase its sales of shared rides, to the benefit of consumers. Second, increased shared-ride efficiency reduces customers' shared-ride delay, increasing their utility for a shared ride (the aforementioned *customer value enhancement effect*). On the other hand, from Proposition 6, increased shared-ride efficiency may prompt the platform to increase its shared-ride price (we label this the *increased price effect*), reducing customers' utility for the shared-ride service. The next result shows that surplus-enhancing cost reduction and value enhancement effects dominate the surplus-dampening increased price effect when network effects are small or large.

Proposition 7. *If either* (i) *network effects are small* $b < \underline{b}$, *or* (ii) *the network effects are large* $b > \overline{b}$ *and the platform offers only shared rides, then consumer surplus CS increases in shared-ride efficiency* ψ.

To the extent that consumer surplus increases in shared-ride efficiency, platforms have an additional incentive outside of their narrow self-interest to improve shared-ride efficiency: doing so benefits their customer "stakeholders."

Taken together, Propositions 5 and 7 reveal conditions (namely, that network effects are large or network effects are small and the labor cost is large) under which increasing shared-ride efficiency provides a "win-win-win" in the sense that the platform, workers, and consumers benefit. Of course, these benefits need to be weighed against the cost the platform incurs to increase shared-ride efficiency.

Wang and Zhang (2021) show that from a baseline of only individual rides, the provision of shared rides typically increases consumer surplus but may reduce labor welfare. Similarly, Zhang and Nie (2021) provide evidence that the provision of shared rides tends to increase consumer surplus. Propositions 5 and 7 complement these findings by characterizing the impact of shared-ride efficiency on labor welfare and consumer surplus.

It is useful to note the extent to which our assumption that customers are sensitive to shared-ride delay $\alpha > 0$ drives the results. This assumption is fundamental to Proposition 2 and, as noted earlier, drives Proposition 6. Proposition 7 holds when customers are insensitive to shared-ride delay $\alpha = 0$, with the caveat that if the network effects parameter $b = 1$, then consumer surplus is invariant to shared-ride efficiency. The remaining results continue to hold when the assumption that customers are sensitive to shared-ride delay is eliminated. A limitation of our model is that it does not capture stochasticity in customer arrivals and sojourn times. For a discussion of incorporating this stochasticity, see the author supplement.

We conclude by discussing observations from the numerical study, taking the results in reverse order. Under every combination of $\{\alpha, \beta, \theta, b\}$ in set A, we observed $(\partial/\partial\psi)CS > 0$, which is consistent with Proposition 7. Under every combination of $\{\alpha, \beta, \theta, b\}$ in set A, we observed there exists θ_p such that if $\theta < \theta_p$, then $(\partial/\partial\psi)p_S^* > 0$, which is consistent with Proposition 6(a). However, when $b > \tilde{b}$, for some parameters, $(\partial/\partial\psi)p_S^* < 0$, which departs from Proposition 6(b). Under every combination of $\{\alpha, \beta\}$ in set A, we observed that there exists $\underline{\theta}_w$, $\overline{\theta}_w$ and $\overline{\psi}$ such that if $b < \tilde{b}$, $\psi > \overline{\psi}$, and $\theta < \underline{\theta}_w$, then $(\partial/\partial\psi)w^* < 0$ and $(\partial/\partial\psi)LW < 0$, and if $b < \tilde{b}$, $\psi > \overline{\psi}$, and $\theta > \underline{\theta}_w$, then $(\partial/\partial\psi)w^* > 0$ and $(\partial/\partial\psi)LW > 0$. This is similar in spirit to Proposition 5(a) in that the optimal wage w^* increases in the labor cost θ (per the appendix's Lemma A.9). However, when $b > \tilde{b}$, for some parameters, $(\partial/\partial\psi)w^* > 0$ and $(\partial/\partial\psi)LW > 0$, which departs from Proposition 5(b).

3.4. Platform Only Able to Offer Shared Rides

We have focused on the setting where the platform is able to offer both individual rides and shared rides. This section considers the setting in which the platform is only able to offer shared rides. To the extent that a platform has only developed technology to support shared rides and has only developed relationships with owners of large vehicles (e.g., vans, sport utility vehicles), it may be difficult for the platform to offer individual rides. Proposition 7 is strengthened under this extension: consumer surplus increases in shared-ride efficiency for the full range of network effects parameter $b \in [0,1]$. The portions of Propositions 2, 3, 5, and 6 addressing the case where network effects are large $b > \overline{b}$ directly address the case where the platform is only able to offer shared rides. The portions of Propositions 2, 5, and 6 addressing the case where network effects are small $b < \underline{b}$ extend to the setting where the platform is only able to offer shared rides, with very minor technical adjustments. Specifically, the extension of Proposition 6(a) requires the technical assumption that $\lim_{\theta\to 0}(\partial^2/\partial K^2)c(K,\theta) = 0$; this assumption is satisfied, for example, if $c(K,\theta) = \theta C(K)$, where $C(\cdot)$ is weakly convex. (See the author supplement for formal statements of the results and the proofs.) The managerial implication is that our prescriptions regarding the impact of shared-ride efficiency on the wage and shared-ride price, as well as the impact of customer sensitivity to shared-ride delay on the marginal value of increasing shared-ride efficiency, do not depend on the platform's ability to offer individual rides.

The only other proposition that is relevant to the setting where the platform is only able to offer shared rides is Proposition 3(a). Proposition 3S(a) characterizes the impact of the labor cost on the marginal value of increasing shared-ride efficiency when the platform is unable to offer individual rides and network effects are small.

Proposition 3S(a). *Suppose the platform is only able to offer shared rides, and network effects are small $b < \underline{b}$. The marginal value of increasing shared-ride efficiency $(\partial/\partial\psi)\Pi^*$ decreases in the labor cost θ for $\psi \in (\underline{\psi}, \psi_S)$ and increases in θ for $\psi \in (\psi_S, 2)$, where $\psi_S \in (\underline{\psi}, 2]$. There exists $\theta_S > 0$ such that $\psi_S < 2$ if and only if the labor cost is low $\theta < \theta_S$.*

The central insight from Proposition 3(a), that the marginal value of increasing shared-ride efficiency can *decrease* in the labor cost, continues to hold. However, comparing Propositions 3(a) and 3S(a) reveals that the conditions under which this occurs change when the platform is unable to offer individual rides. When the platform is only able to offer shared rides, the conditions for the marginal value of increasing shared-ride efficiency to decrease in the labor cost are low shared-ride efficiency or high labor cost (Proposition 3S(a)). In contrast, when the platform is able to offer both individual and shared rides, the conditions are high shared-ride efficiency and high labor cost (Proposition 3(a)).

The explanation for why the conditions differ is driven by whether shared-ride efficiency and labor supply are complements or substitutes. As discussed in Section 3.2, the marginal value of increasing shared-ride efficiency decreases in the labor cost if and only if shared-ride efficiency and labor supply are complements. Recall from Proposition 4 that shared-ride efficiency and labor supply are complements if and only if the platform only offers shared rides and the labor supply is small $K < \overline{K}(\psi)$. It remains to explain why the conditions identified cause the optimal labor supply of a platform unable to offer individual rides to be small $K^* < \overline{K}(\psi)$ so that shared-ride efficiency and labor supply are complements. Naturally, if the labor cost is high, the platform's optimal labor supply is small. If shared-ride efficiency is small, shared-ride efficiency and labor supply are complements for a wide range of labor supply (formally, $\overline{K}(\psi)$ decreases in ψ). Hence, for any labor cost, sufficiently low shared-ride efficiency implies that shared-ride efficiency and labor supply are complements.

The managerial implication is that the attractiveness of improving shared-ride efficiency depends on the labor cost in a fundamentally different way when a platform is unable to offer individual rides. When shared-ride efficiency is low and the labor cost is low or high, an increase in the labor cost has opposite effects on the attractiveness of improving shared-ride efficiency: it decreases this attractiveness for a platform that is unable to offer individual rides, and it increases this attractiveness for a platform that is able to offer such rides. In contrast, when shared-ride efficiency is high, the effect of an increase in the labor cost has the same directional effect on the attractiveness of improving shared-ride efficiency, regardless of whether the platform is able to offer individual rides: when the labor cost is high, it reduces the attractiveness of improving shared-ride efficiency, and when the labor cost is low, it increases the attractiveness of improving shared-ride efficiency.

The intuition for the similarity in results when the labor cost and shared-ride efficiency are high is that in this parameter regime, it is optimal for a platform that is able to offer individual rides not to do so. In contrast, in the other parameter regimes, it is optimal for a platform that is able to offer individual rides to do so, which contributes to the divergence in results.

4. Discussion

Ride-hailing platforms offering shared rides devote effort to reducing the trip-lengthening detours associated with accommodating the divergent transportation needs of the customers on a shared ride. By reducing shared-ride delay, improving *shared-ride efficiency* has the twin benefits of making shared rides more attractive to customers and increasing the number of customers a driver can serve per unit time.

The impact of increasing shared-ride efficiency depends on the magnitude of the network effects. We provide evidence that when network effects are large, the platform only offers shared rides. Increasing shared-ride efficiency prompts the platform to increase the shared-ride price and decrease the wage, expanding its margin. Increasing shared-ride efficiency benefits consumers (consumer surplus increases) but hurts workers (labor welfare decreases). When network effects are small, there exist parameters under which all of these results—with the exception of the consumer surplus result—are reversed.

Although our model is static, it is possible to frame our results in terms of a platform's improving shared-ride efficiency over time. For concreteness, for the remainder of this section, consider a platform operating in a setting where network effects are small. As shared-ride efficiency increases from a very low level, it eventually becomes optimal to shift from offering only individual rides to offering both individual and shared rides. When shared rides are newly introduced, customers' expectations for ride-hailing transportation may be conditioned by their experience with individual rides. Consequently, customers may be particularly sensitive and averse to shared-ride delay. It might be reasonable to expect that heightened sensitivity to shared-ride delay would make it more attractive for the platform to invest in reducing shared-ride delay (i.e., increasing shared-ride efficiency). To the contrary, heightened sensitivity to shared-ride delay *decreases* the marginal value of increasing shared-ride efficiency—in this region where efficiency is high enough to make offering shared rides optimal, but still relatively low (Proposition 2(a)). Over time, as customers become acclimated to experiencing shared-ride delay, their sensitivity and aversion to shared-ride delay may decrease. Diminished sensitivity to shared-ride delay *increases* the marginal value of increasing shared-ride efficiency (Proposition 2(a)). The rationale is that in this region of low shared-ride efficiency, the shared-ride service is barely viable in the sense that lengthy shared-ride delay makes the service unattractive to most customers. However, as customers grow more tolerant of shared-ride delay, the set of customers who find the shared-ride service attractive expands. This larger critical mass of shared-ride customers makes it more attractive for the platform to improve the shared-ride service.

Over time, as platforms that source labor from independent contractors (gig workers) have grown and matured, competition for this labor market has grown, increasing the labor costs platforms face (Kelsey 2017, Preetika 2021, Siddiqui 2021). It might be plausible that high labor cost would make it more attractive to invest in increasing the productivity of labor by increasing shared-ride efficiency. However, when shared-ride efficiency is sufficiently high that it is optimal to only offer shared rides, the opposite is true: a high labor cost *decreases* the marginal value of increasing shared-ride efficiency (Proposition 3(a)). The explanation is driven by the fact that for a shared-ride-only platform facing high labor cost and, hence, employing a small labor supply, shared-ride efficiency and labor supply are complements, meaning that increasing shared-ride efficiency increases the marginal value of labor (Proposition 4). Increasing the labor cost reduces the optimal labor supply and, hence, the marginal value of increasing shared-ride efficiency.

Consider a ride-hailing platform whose shared-ride efficiency is sufficiently high that it is optimal to offer only shared rides. (Via is an example of a platform that only offers shared rides.) It might be reasonable to expect that further increases in shared-ride efficiency would prompt the platform to "double-down" on only offering the shared-ride service. In fact, the opposite may be true: an increase in shared-ride efficiency may prompt a shared-ride-only platform to *add* individual rides (Proposition 1(a)). The intuition is that as shared-ride efficiency increases, less labor is required to provide a given quantity of shared rides, which decreases the marginal cost of an individual ride, making it attractive for the platform to add individual service. This phenomenon only occurs when a high level of shared-ride efficiency is coupled with a moderate labor cost.

Finally, increasing shared-ride efficiency pushes wages to extremes: if the current wage is high, increasing shared-ride efficiency pushes the wage higher, and if the current wage is low, increasing shared-ride efficiency pushes the wage lower (Proposition 5(a)).

We have focused on one way that platforms can improve shared rides: by decreasing trip-lengthening detours. However, there are other operational adjustments platforms could make to shared rides that we have not explored. For example, customers requesting Uber's Express Pool service are required to walk to a common pick-up location (this deviates from the traditional shared ride (e.g., Uber Pool), where customers are picked up at the location they specify), which expands the time window for making matches, potentially allowing more customers to be matched with a single driver (Farronato et al. 2020). To the extent that shared-ride delay increases in this time window, this could be captured in our model by simultaneously increasing the shared-ride delay and the number of customers served in a shared ride. In contrast to shared-ride efficiency, which improves shared rides from the perspective of both customers and labor productivity, the expanded time window approach reflects the typical cost-quality trade-off: the cheaper-to-deliver service is less attractive to customers.

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Appendix

The appendix provides analysis and intermediate results that are useful in the proofs of the results in the body of the paper. All proofs are in the Online Supplement. The appendix begins with an analysis that includes Lemmas A.1–A.3 and culminates in Lemmas A.4 and A.5, which are useful in proofs of Propositions 1, 4, and 5. For use in the statements of Lemmas A.1, A.3, and A.4 and proofs of Lemmas A.3, A.4, A.6, and A.7 and Propositions 1, 4, and 6, let $v(\psi) = (\psi - 2a)[\psi(\alpha+\beta) - 2\alpha a] + a^2$. Assume throughout that $\beta - \alpha(\tau - 1) > 0$, or, equivalently, $\psi > \underline{\underline{\psi}}$, where $\underline{\underline{\psi}} = 2\alpha/(\alpha+\beta)$. This assumption is without loss of generality in that if the assumption does not hold, then the shared-ride service has nonpositive utility for all customers.

Lemma A.1. *$v(\psi) > 0$ for $\psi > 0$.*

A necessary condition for a platform to sell $Q_S > 0$ shared rides is

$$\beta - \alpha[\tau g(Q_s) - 1] > 0. \tag{A.1}$$

To see this, note from (1) that if this inequality is reversed, then all customers receive nonpositive utility from the shared-ride service. Note (A.1) holds if and only if $Q_S > \underline{Q}_S$, where $\underline{Q}_S = 2\alpha b/[\psi(\alpha+\beta) - 2\alpha a]$. Further, $Q_I + \mathbf{1}_{\{Q_S^*>0\}}[aQ_S + b]/\psi \le K$ implies $Q_S \le (\psi K - b)/a$. Thus, $Q_S > 0$ implies $K > \underline{K}_S$, where $\underline{K}_S = (\alpha+\beta)b/[\psi(\alpha+\beta) - 2\alpha a]$. We restrict attention to $p_I \le 1$, which is without loss of generality in that $Q_I = 0$ if $p_I \ge 1$. We restrict attention to $p_S \le \beta - \alpha[\tau g(Q_s) - 1]$, which is without loss of generality in that $Q_S = 0$ if $p_S \ge \beta - \alpha[\tau g(Q_s) - 1]$.

Lemma A.2. *Under individual-ride price p_I and shared-ride price p_S, the individual-ride and shared-ride sales quantities $(Q_I, Q_S) = (1 - p_I, 0)$ if $\beta - \alpha[\tau g(Q_s) - 1] \le 0$; otherwise, (Q_I, Q_S)*

$$= \begin{cases} (1 - p_I, 0) & \text{if } p_I < \dfrac{p_S}{\beta - \alpha[\tau g(Q_s) - 1]} \\ \left(0, 1 - \dfrac{p_S}{\beta - \alpha[\tau g(Q_s) - 1]}\right) & \text{if } p_I > p_S + 1 - \beta + \alpha[\tau g(Q_s) - 1] \\ \left(1 - \dfrac{p_I - p_S}{1 - \beta + \alpha[\tau g(Q_s) - 1]}, \right. & \\ \quad \dfrac{p_I - p_S}{1 - \beta + \alpha[\tau g(Q_s) - 1]} & \\ \left. \quad - \dfrac{p_S}{\beta - \alpha[\tau g(Q_s) - 1]}\right) & \text{otherwise.} \end{cases}$$

It follows from Lemma A.2 that if $\beta - \alpha[\tau g(Q_s) - 1] > 0$, then, without loss of generality, we can restrict attention to $p_I \in [p_S/\{\beta - \alpha[\tau g(Q_s) - 1]\}, p_S + 1 - \beta + [\tau g(Q_s) - 1]]$. It follows from Lemma A.2 that under individual-ride sales Q_I and shared-ride sales Q_S, individual-ride and shared-ride prices are

$$\begin{aligned} p_I(Q_I, Q_S) &= 1 - Q_I - Q_S[\psi(\alpha+\beta) - 2\alpha a]/\psi + 2\alpha b/\psi \\ p_S(Q_I, Q_S) &= (1 - Q_I - Q_S)[\psi(\alpha+\beta) - 2\alpha g(Q_S)]/\psi \end{aligned} \tag{A.2}$$

if $\beta - \alpha[\tau g(Q_s) - 1] > 0$; otherwise, $p_I(Q_I, 0) = 1 - Q_I$ and $p_S(Q_I, 0) = 0$.

For use in Statement of Lemmas A.4 and A.7 and proofs of Lemmas A.3–A.9 and Propositions 1, 2, 5, 6, and 7, let $R(Q_I, Q_S) = p_I(Q_I, Q_S)Q_I + p_S(Q_I, Q_S)Q_S$ denote revenue under individual-ride sales Q_I and shared-ride sales Q_S. Let $(Q_I^*(K), Q_S^*(K))$ denote the revenue-maximizing individual-ride and shared-ride sales under supply K. Formally, $(Q_I^*(K), Q_S^*(K))$ is the solution to the optimization problem $\max_{Q_I \ge 0,\, Q_S \ge 0} R(Q_I, Q_S)$ subject to $Q_I + \mathbf{1}_{\{Q_S > 0\}}[aQ_S + b]/\psi \le K$. Let $Q_I^I(K) = \min(K, 1/2)$; under $\psi K \ge b$, let $Q_S^S(K, b) = \min((\psi K - b)/(1 - b), [\psi(\alpha+\beta) + 2\alpha(2b - 1)]/[2\psi(\alpha+\beta) - 4\alpha(1 - b)])$ for $b < 1$ and $Q_S^S(K, 1) = [\psi(\alpha+\beta) + 2\alpha]/[2\psi(\alpha+\beta)]$; and let $Q_I^B(K, b) = \{[2K(\psi - 1 + b) - 1 + b][\psi(\alpha+\beta) - 2\alpha(1 - b)] + (1 - b)^2 + 2[\alpha(1 - b) - (\psi - 1 + b)(\alpha+\beta)]b\}/[2v(\psi)]$; and $Q_S^B(K, b) = \{(1 - 2K)\psi[\psi(\alpha+\beta) - (2\alpha + 1)(1 - b)] + [2\psi(\alpha+\beta) + 2(\psi - 4 + 4b)\alpha - 2(1 - b)]b\}/[2v(\psi)]$. Lemma A.3 is useful in proof of Lemma A.4.

Lemma A.3. *If $Q_I^*(K) > 0$ and $Q_S^*(K) = 0$, then $Q_I^*(K) = Q_I^I(K)$. If $Q_I^*(K) = 0$ and $Q_S^*(K) > 0$, then $Q_S^*(K) = Q_S^S(K, b)$. If $Q_I^*(K) > 0$ and $Q_S^*(K) > 0$, then $Q_j^*(K) = Q_j^B(K, b)$ for $j \in \{I, S\}$.*

Let $\underline{\psi}_S = 2\alpha(\alpha + 2\beta)/(\alpha+\beta)^2$.

Lemma A.4. (a) *Under labor supply $K > 0$, the revenue-maximizing individual-ride and shared-ride sales quantities $(Q_I^*(K), Q_S^*(K))$ are given as follows. If $K \ge 1/2$, then $Q_I^*(K) = Q_I^I(K) > 0$ and $Q_S^*(K) = 0$. Suppose $K < 1/2$ and $b < 1$; if $Q_S^*(K) > 0$, then $Q_S^*(K) = \{\psi[K - Q_I^*(K)] - b\}/(1 - b)$; platform revenue under only shared-rides $R(0, (\psi K - b)/(1 - b))$ is continuous and concave in K and continuous in b.* (b) *If $b = 1$, $Q_I^* = 0$, and $Q_S^* > 0$, then $Q_S^* = Q_I^S(1/\psi, 1)$, $K^* = 1/\psi$, and $\psi > \underline{\psi}_S$.*

In the sequel, we restrict attention to $K \le 1/2$, which is without loss of generality because, by Lemma A.4(a), $(Q_I^*(K), Q_S^*(K))$ is invariant to K for $K \ge 1/2$. Let $\check{\psi} = [2\alpha + \max(a, 2\beta b)]/(\alpha+\beta)$.

Lemma A.5. *If $\psi \le \check{\psi}$, then it is optimal not to offer shared rides $Q_S^* = 0$.*

It is convenient to prove Lemma A.6—of which parts (a) and (c) are useful in proof of Proposition 2 and of which part (b) is useful in proofs of Proposition 5–and Proposition 1(a) together. Let $\underline{\psi}_0 = (2\alpha + 1)/(\alpha+\beta)$.

Lemma A.6. *Suppose network effects are small $b < \underline{b}$.* (a) *For each labor cost $\theta > 0$, there exists $\breve{\psi}(\theta) \in (\underline{\psi}, 2]$ such that it is optimal to offer both individual rides and shared rides if $\psi \in (\underline{\psi}, \breve{\psi}(\theta))$.* (b) *$\overline{\psi}(\theta)$ decreases in θ on $\theta \in (\overline{\theta}, \infty)$, and $\lim_{b\to 0}\lim_{\theta\to\infty}\overline{\psi}(\theta) = \underline{\psi}$.* (c) *$\underline{\psi}$ is continuous in b and $\lim_{b\to 0}\underline{\psi} = \underline{\psi}_0$.*

As stated in the body of the paper following Proposition 1, in the sequel, we restrict attention to the parameter regime in which the platform optimally offers shared-rides $Q_S^* > 0$. This restriction applies to each of the formal results that follow, but after Lemma A.7, for brevity, we omit stating it explicitly. We abuse notation by writing $\Pi(K)$ to denote $\Pi(Q_I^*(K), Q_S^*(K), K)$; in words, $\Pi(K)$ is the platform's profit under labor supply K and the revenue-maximizing sales quantities $(Q_I^*(K), Q_S^*(K))$. The restriction in the (b) part of Propositions 2, 3, 5, 6, and 7 to it being optimal for the platform to offer only shared rides is equivalent to assuming that there exists $\hat{b} < 1$ such that $Q_I^* = 0$ and $Q_S^* > 0$ for $b \in (\hat{b}, 1]$.

Lemma A.7. *Suppose that is it optimal to offer shared rides $Q_S^* > 0$ and that there exists $\hat{b} < 1$ such that $Q_I^* = 0$ for $b \in (\hat{b}, 1]$. There exist $\underline{b}$ and $\overline{b}$, where $0 < \underline{b} \le \overline{b} < 1$ such that for $b \in [0, \underline{b}) \cup (\overline{b}, 1)$, p_I^*, p_S^*, w^*, Q_I^*, Q_S^*, K^*, Π^*, CS, LW, $(\partial/\partial\psi)CS$, $(\partial/\partial\psi)p_S^*$, $(\partial/\partial j)K^*$, and $(\partial/\partial j)\Pi^*$ for $j \in \{\psi, \alpha, \theta\}$ and $(\partial^2/\partial\psi\partial j)\Pi(K)$ for $j \in \{K, \alpha, \theta\}$ are continuous in b, and for $b \in [0, \underline{b}) \cup (\overline{b}, 1)$, $R(Q_I^*(K), Q_S^*(K))$ and $\Pi(K)$ are concave in K.*

Lemma A.8 is useful in proofs of Proposition 3 and Lemma A.10. Suppose $b \in [0, \underline{b}) \cup (\overline{b}, 1)$. Because platform revenue $\Pi(K)$ is concave in K (by Lemma A.7), the optimal labor supply K^* is unique. Note that proof of Lemma A.8 invokes Proposition 4, which is proven subsequently. Note Π_B and Π_S, which are used in proof of Lemma A.8, are defined immediately before Proposition 4.

Lemma A.8. *Suppose there exists $\hat{b} < 1$ such that $Q_I^* = 0$ for $b \in (\hat{b}, 1]$. (a) If $b \in [0, \underline{b}) \cup (\overline{b}, 1)$, then the optimal labor supply K^* decreases in θ. (b) Suppose $b < \underline{b}$. If $\lim_{\psi \to 2}[K^*(\psi) - \overline{K}(\psi)] < 0$, then K^* increases in shared-ride efficiency ψ for $\psi \in (\overline{\psi}(\theta), 2)$. (c) Suppose $b < \underline{b}$. If the labor cost is low $\theta < \underline{\theta}$, then K^* decreases in ψ. There exists $\theta_K \ge \overline{\theta}$ such that if the labor cost is high $\theta > \theta_K$, then K^* decreases in ψ for $\psi \in (\underline{\psi}, \overline{\psi}(\theta))$ and increases in ψ for $\psi \in (\overline{\psi}(\theta), 2)$. (d) If $b > \overline{b}$, then K^* decreases in ψ.*

Lemma A.9 is useful in proofs of Lemma A.10 and Proposition 5. For use in proof of Lemma A.9, let $r(K) = R(Q_I^*(K), Q_S^*(K))$, and with some abuse of notation, let $\pi(K, w) = r(K) - wK$. Note $r(K)$ denotes the platform's revenue under labor supply K and the revenue-maximizing sales quantities $(Q_I^*(K), Q_S^*(K))$. Further, $\pi(K(w,\theta), w)$ denotes the platform's profit under wage w, labor supply $K(w,\theta)$, and revenue-maximizing sales quantities $(Q_I^*(K(w,\theta)), Q_S^*(K(w,\theta))$. Suppose $b \in [0, \underline{b}) \cup (\overline{b}, 1)$; because the profit-maximizing labor supply K^* is unique (by the argument immediately before Lemma A.8) and $w(K,\theta)$ increases in K, the profit-maximizing wage $w^* = w(K^*, \theta)$ is unique.

Lemma A.9. *Suppose there exists $\hat{b} < 1$ such that $Q_I^* = 0$ for $b \in (\hat{b}, 1]$. If $b \in [0, \underline{b}) \cup (\overline{b}, 1)$, then the optimal wage w^* increases in the labor cost θ.*

Lemma A.10 is useful in proof of Proposition 5.

Lemma A.10. *Suppose $b < \underline{b}$. If the labor cost is low $\theta < \underline{\theta}$, then the optimal wage w^* and labor welfare LW decrease in shared-ride efficiency ψ. There exists $\theta_w \ge \overline{\theta}$ such that if the labor cost is high $\theta > \theta_w$, then w^* and LW decrease in ψ for $\psi \in (\underline{\psi}, \overline{\psi}(\theta))$ and increase in ψ for $\psi \in (\overline{\psi}(\theta), 2)$.*

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