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In markets, firms must compete following a set of rules determined by laws, regulations, and social practices or pressures. This article investigates the effect of the degree of competition on the extent to which firms invest in behaving according to the rules of the marketplace. The authors model investments in following these rules as increasing the firm’s marginal costs of production and decreasing its probability of being caught violating the market rules (and thus losing profits). They show that greater competition leads to smaller investments in following the market rules. This leads to (1) the existence of a social optimum degree of competition that is less than perfect competition and (2) more competition in general, thus prompting greater optimal monitoring efforts. Stricter market rules can lead to greater investments in satisfying the market rules and to lower production. The authors also present results on the likelihood of firms having broken the market rules depending on relative market shares, optimal monitoring, and the effect of dynamics on the incentives to satisfy the market rules.

Keywords: corporate social responsibility, rules of the market, competitive strategy, game theory, pricing

Competitive Vices

Behind any great fortune there is a crime.

—Honoré de Balzac

Firms operate in markets within certain rules—the “rules of the game.” As Hutton (2002, p. vii) notes, “The act of incorporation in all capitalist societies was originally conceived as winning a license to trade in return for the acceptance of obligations set out by the government of the day.” The rules of the market are determined by laws, regulations, social practices, or social pressures informed by moral and ethical values. In addition, in several instances, competitors or third parties may argue that a firm has not followed the rules of the market; has behaved unlawfully; has missed some regulations or contract obligations; or has entered in some activity that is unethical, immoral, or socially irresponsible.1

1Bhattacharya and Korschun (2008) discuss the extension of the goals of the firm to include serving the interests of multiple “stakeholders,” not just the shareholders’ interests.

Moreover, the rules of the market are often complex and sometimes poorly defined. For example, firms have to respect a proliferation of laws and regulations regarding their actions. There is also some heterogeneity in the degree to which firms engage in socially responsible behavior (e.g., Bradshaw and Vogel 1981, pp. xii, xxiv; Kaplow 1995; Slemrod 1989). The interpretation of the rules of the market is also often not clear, as evidenced by the sometimes intense use of the judicial system by firms in a market, or the uncertain outcome of public relations endeavors.2

This article considers a model of competing firms investing in (costly) efforts to satisfy the rules of the market under an imperfect monitoring authority (e.g., Becker 1968; Stigler 1970). The more a firm invests in trying to adhere to the rules of the market, the greater its production costs and the lower its probability of getting caught disobeying the rules. If a firm is caught doing so, it may have to pay a penalty, possibly under limited liability.

2In sports, players frequently encounter situations in which the rules of the game are unclear. For example, in a baseball game on September 15, 2010, Derek Jeter, “the Yankee shortstop and generally perceived-to-be boy scout, feigned being hit by a pitch. He pretended that the ball had ricocheted off his hand, though as stop-action replay made evident (and as he readily acknowledged after the game), it actually hit the knob of his bat. Nonetheless, his charade fooled the umpire; he was awarded first base” (Weber 2010). Some viewers reacted with “indignation” at the lack of “rectitude” and asked whether “truth-telling and accountability have no place in sports.” Others argued that this was “testimony to a first-rate athletic instinct” (Weber 2010).

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Throughout business history, there have been many instances of companies or people engaging in inappropriate behavior. For example, while characterizing the unscrupulous “robber baron” industrialists of the nineteenth century, Josephson (1934, p. vii) states, “They were aggressive men, as were the first feudal barons; sometimes they were lawless; in important crises, nearly all of them tended to act without those established moral principles which fixed more or less the conduct of the common people of the community.” Of one of the robber barons, Josephson (p. 72) reports, “He drove himself, men, and things with reckless energy, and with an indifference to established custom and law which stood him in good stead. ‘What do I care about the law? Hain’t I got the power?’” He describes another robber baron as having learned “that it was not enough to conquer the whole legislature; but one must buy the judges as well” (p. 132). After a business encounter with one such robber baron, a person complained, “I said (after settling with him) that it was an almighty robbery; that we had sold ourselves to the devil” (p. 134). During other encounters, market results sometimes depended on “armed conflicts with rivals” (p. 135), meeting “force with force, bribe with bribe and duplicity with duplicity” (p. 139). Other robber barons were accused of “fraud, violence and morally reprehensible practices” (p. 140) or of being “unscrupulous usurpers, who by a sort of legerdemain, seized control of the stockholders’ property, stole . . . money, and so demoralized its service as to bring calamities of unusual horror, damage and death” (p. 140).

In recent years, consumers have heard similar accusations that certain companies have violated safety or antitrust laws, violated consumers’ trust, violated truth in advertising, failed to truthfully report their financial accounts, used child labor, or engaged in actions that hurt the environment, among other misdeeds. Some recent newsworthy cases include, for example, those of Enron (e.g., Fox 2003; in a scandal that also brought down the accounting firm Arthur Andersen) and Archer Daniels Midland (e.g., Eichenwald 2000).

This article investigates the effect of the degree of competition on the extent to which firms adhere to the rules of the market when firms caught not adhering to the rules lose their profits (limited liability). We show that under certain reasonable conditions, more competition leads to firms being less careful in following the rules of the market. The argument is that with competition, firms have less to lose if they are caught not following the rules of the market and, therefore, are more likely to break those rules. We investigate the implications of this effect on social welfare and on the design of optimal rules. We also show that in a stochastic environment, firms that end up with a greater market share may be the ones that are more likely to have broken the rules of the game. We demonstrate this result through both a mixed-strategy equilibrium and an incomplete information game in which firms may have different costs of investing in following market rules. We also investigate the implications for optimal limited monitoring of the market rules in such a setting.

Empirical evidence has provided support for the ideas we present herein. Snyder (2010) considers how liver transplant centers behave with regard to the statement of patients’ health problems and finds that centers in areas with multiple competitors may overstate health problems to gain greater priority on the liver transplant waiting list. Bennett et al. (2013) investigate the behavior of facilities testing for vehicle emissions. They report that with greater intensity of competition, the facilities significantly increase the leniency with which vehicles pass the emissions tests. Hegarty and Sims (1978) provide some evidence that competition leads to unethical behavior in a laboratory setting. Cai and Liu (2009) find a positive correlation between increases in competition and increases in tax avoidance behavior among Chinese manufacturers. Other settings in which to explore these findings include child labor, corruption, and earnings manipulation.

Another notable issue is whether firms with different positions in the market behave differently with respect to following the market rules. We show that firms with more to lose from being caught are more likely to invest more in following the rules. In a dynamic environment, this then leads firms to take greater risks of being caught in earlier periods; furthermore, the firms that are successful in the early periods are more careful in future periods.

A related stream of literature focuses on the question of the development of moral standards and values (e.g., Baron 2010; Dal Bó and Terviö 2008; Kaplow and Shavell 2007; Tabellini 2008). Within this research area, our article could be viewed as investigating the effect of competition on the extent to which economic agents respect the current moral standards or values. Furthermore, our work considers all possible rules of the market more generally as well as potential implications for monitoring and rule making. Another related issue is firms’ practice of investing in social responsibility practices as a response to consumer preferences (e.g., Arora and Henderson 2007; Banerjee and Waterhouse 2010; Gneezy et al. 2010; Sen and Bhattacharya 2001). In relation to this possibility, the punishment for being caught not satisfying the market rules can be viewed as the result of consumer retaliation against a firm that does not invest enough in socially responsible practices. Kopalle and Lehmann (2010) investigate the question of firms overclaiming their quality level. They argue that greater competition causes firms to overstate their quality to a greater extent, a position that is consistent with the results we present herein. The effect we consider can also be viewed as related to the efficiency wage theory in labor economics (e.g., Akerlof 1982; Shapiro and Stiglitz 1984; Yellen 1984), whereby employees exert more effort if they are overpaid because they have more to lose if they are found not exerting effort.

The general problem we address is also related to the questions of how market structure affects an industry’s intensity of innovation and how the possibility of innovation may affect that market structure (see, e.g., Aghion et al. 2005; Dasgupta and Stiglitz 1980; Gilbert and Newbery 1982; Ofek and Sarvary 2003; Reinganum 1983). In relation to that literature, and as we specify next, investments in satisfying the market rules affect profits in a particular way that can differ from how innovation investments affect profits. The former may (1) affect the marginal costs of production, (2) not affect competitors’ profits, (3) affect profits for only one period, and (4) lead to a maximum possible liability if a firm is found in violation of the market rules. Any of these results might not be the case for typical innovation investments.

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3Luo and Bhattacharya (2006) show that firm contributions to corporate social responsibility can negatively affect the market value of firms with lower-quality products.
Furthermore, when considering the topic of satisfying the market rules, there is the issue of monitoring policies and optimal design of market rules, which does not have an obvious parallel in a setting of investment in innovations. Finally, a main message of this research is that more competition may lead to lower investments in satisfying the market rules, whereas the literature on investment in innovations has shown in several cases that more competition leads to more innovation.

In the context of this article, “satisfying the rules of the market” means not getting caught disobeying the rules of the market. This can be done either by actually following the rules of the market or by spending resources to hide any nonsatisfaction of the rules of the market (e.g., lobbying). Under general conditions, if the probability of not being caught is increasing and concave on investments both in the actual satisfaction of the market rules and in deception, market forces that lead to an increase in the probability of not getting caught will lead to greater investments in actually satisfying the market rules as well as greater investments in deception. For example, as we derive subsequently, a greater number of competitors leading to an increased probability of a firm’s being discovered violating the market rules would be associated with lower investments both in satisfying the actual market rules and in hiding that some market rules are being broken. We do not fully explore the effects of several dimensions of working to avoid being caught disobeying the market rules, but the results follow under general conditions.

Another issue we do not explore is that competitors may have information that another firm is failing to satisfy some market rules and may report it to the monitoring authority. This would be a force toward firms satisfying more market rules under more intense competition and would be contrary to the effects we describe herein.

To introduce the main idea parsimoniously, we present in the next section a symmetric quantity competition model showing that more competition leads to firms being less careful in fulfilling the rules of the market. We then discuss implications of this possibility for the design of market rules. To explore ex post asymmetry effects, optimal monitoring, and dynamics, we introduce a price competition model in which equilibria are in mixed strategies and show that the firms that end up with a greater market share are the ones that were less careful in following the market rules. We also study optimal monitoring by the regulator in that setting. Next, we consider the case in which firms have private information and then address the dynamic case in which firms may change the care with which they follow the market rules from period to period. Finally, we offer concluding remarks.

**STATIC SYMMETRIC COMPETITION**

Consider a homogeneous product market with \(N\) symmetric firms competing in quantity. Denote the inverse demand function as \(P(Q)\), where \(Q\) is the total quantity produced in the market, with \(P'(Q) < 0\) and \(2P'(Q) + QP''(Q) < 0^3\). Each firm \(i\) chooses its quantity to produce, which we denote by \(q_i\). We have \(Q = \sum_{i=1}^{N} q_i\).

In addition to choosing the quantity to produce, each firm \(i\) can decide the degree to which the production processes (or suppliers used) satisfy the market rules, with a greater degree of satisfying the market rules associated with higher production costs. For example, the market rules may require several safety and hygiene conditions that firms can invest in satisfying or not satisfying, as it may not be fully clear whether some conditions are absolutely required. Investing in satisfying more safety and hygiene conditions leads to greater production costs but also increases the likelihood that the firm followed a required safety or hygiene condition.

Let \(\gamma_i\) be the extent to which a firm \(i\) decides to adhere to the rules of the market (i.e., how much the firm invests in safety or hygiene conditions). This variable \(\gamma_i\) is the probability of not being caught breaking the market rules. Choosing a higher \(\gamma_i\) means having a higher marginal cost of production, \(c(\gamma_i)^6\). That is, we have \(c'(\gamma_i) > 0\).

Furthermore, let us assume that a greater \(\gamma_i\) has increasing effects on the marginal costs, \(c''(\gamma_i) > 0\). In this base setup, the parameter \(\gamma\) represents both the investment in satisfying the market rules and the probability of being discovered breaking the market rules. More generally, one could consider that the relation between these variables would be moderated by the enforcement and monitoring mechanism. That is, given a certain investment in satisfying the market rules, the probability of being found out for breaking the market rules would depend on the enforcement and monitoring mechanism. We explore this issue in the “Optimal Monitoring” subsection.

If a firm is caught breaking the market rules, let us assume that all its profits are taken away. We consider the case when only its profits can be confiscated by limited liability. The limited liability assumption can be viewed as reasonable in the sense that penalties that can be imposed on firms are limited by the resources firms have (and given existing bankruptcy laws). As we have mentioned, examples of firms losing all their profits after being caught disobeying the market rules include Enron and Arthur Andersen, and these companies went bankrupt. As discussed in Arlen (2012) regarding a particular case of breaking the market rules (corporate crime), more than one-third of convicted organizations do not have sufficient assets to pay the entire criminal fine imposed (and this still does not include civil liabilities). Furthermore, a factor in deciding the level of penalty is the firm’s ability to pay it, and publicly held firms (which can be viewed as more profitable firms) also face larger mean and median fines than those for other firms. Together, these factors suggest that fines increase with firms’ profits. Finally, reputational and market penalties suffered are proportional to the profits

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4 Without the concavity assumption—for example, if there were increasing returns of investments in either satisfying the actual market rules or hiding the nonsatisfaction of market rules—a greater probability of not being caught might lead to either lower investments in satisfying the actual market rules or lower investments in deception.

5 The latter condition is the standard second-order condition for firm pricing with constant marginal costs.

6 In the Technical Appendix, we discuss the case when a higher \(\gamma_i\) has an effect on fixed costs.

7 In the next section, we briefly discuss what might happen if the penalty paid were less than the total profits obtained and if the penalty were contingent on the extent to which the rules were broken (which can be seen as being measured by \(-\gamma_i\)).
obtained. As we discuss subsequently, our results apply immediately to the case in which the penalties for being caught disobeying the market rules are proportional to the profits obtained. Note that we do not know whether penalties that increase with the profits obtained (or penalties that extract the full profits) are optimal in this setting. We merely explore the effects of this type of penalty that we observe in the real world on market interaction. If the penalties for not satisfying the market rules were without bound, they could induce firms to exert the maximum investment in satisfying the market rules. We assume that firms are risk neutral.

The profit of a firm \( i \) can then be represented as 
\[
\pi^i(q_i, Q, \gamma) = \gamma q_i [P(Q) - c(\gamma_i)].
\]
We assume that firms produce for the market even if they are caught not satisfying the market rules. That is, firms produce first for the market, and their profits are expropriated if they are found to be at fault. An alternative model would have firms being caught before producing, which would benefit firms that were not caught.

In such a model, a firm’s profit would also depend on the \( \gamma_i \) chosen by the competitors. We discuss this case further subsequently.

The first-order conditions for a firm \( i \) would then be
\[
\frac{\partial P(Q) - c(\gamma_i) + q_i P^i(Q)}{\partial q_i} = 0, \quad \text{and} \quad \frac{\partial P(Q) - c(\gamma_i) - \gamma_i c'(\gamma_i)}{\partial \gamma_i} = 0.
\]
Under general conditions, the equilibrium is symmetric, with each firm choosing the same \( q \) and \( \gamma \) (whereby we drop the subscript for each firm, given the symmetric equilibrium). Let \( D = \frac{\partial P^i(Q)}{\partial \gamma_i} + \frac{\partial P(Q)}{\partial \gamma} + \frac{\partial q_i P^i(Q)}{\partial \gamma_i} + \frac{\partial q_i P(Q)}{\partial \gamma} + \frac{\partial q_i^2 P(Q)}{\partial \gamma_i^2} + \frac{\partial q_i q_i P^i(Q)}{\partial \gamma_i^2} + \frac{\partial q_i q_i P(Q)}{\partial \gamma_i^2} \), which is negative given the aforementioned conditions.

By totally differentiating the equilibrium \( q \) and \( \gamma \) with respect to \( N \), we obtain
\[
\frac{\partial \gamma_i}{\partial N} = \frac{1}{D} \frac{\partial P(Q)^2}{\partial \gamma_i} < 0.
\]
We state this result in the following proposition.

**P1:** When the intensity of competition increases (i.e., the number of firms increases), each firm invests less in satisfying the market rules.

The intuition is that with more competition, firms have less to lose if they are caught breaking the market rules and therefore are more likely to be less careful about respecting those rules. Because competition leads to lower prices, firms have a greater pressure to decrease their marginal costs, lowering \( \gamma \). In addition, competition leads to lower quantity produced, so firms have less incentive to lower costs (greater \( \gamma \)), but this effect is dominated by the former effect of having less to lose. In terms of the previous example, this would mean that with more competition, firms would invest less in safety or hygiene conditions. This is consistent with Snyder’s (2010) empirical results that, with more competition, liver transplant centers may tend to overstate patients’ health problems to gain priority on the liver waiting list.

For the linear demand example, \( P(Q) = 1 - Q \), the equilibrium \( q \) and \( \gamma \) satisfy \( \gamma c'(\gamma) = q \) and \( c(\gamma) + (1 + N)c'(\gamma) = 1 \). In addition, if we assume the marginal costs to be linear in \( \gamma \), with \( c(\gamma) = c_0 + \alpha \gamma \), where \( \alpha > 0 \) is the effect of being careful about the market rules on the marginal costs, we have \( \gamma = (1 - c_0)/(\alpha(2 + N)) \) and \( q = (1 - c_0)/(2 + N) \). Note that this is the equilibrium at which firms simultaneously optimize on \( q \) and \( \gamma \), not the equilibrium at which they optimize on \( q \) given \( \gamma \), which would be \( q = [1 - c(\gamma)]/(1 + N) \). To observe the effect of competition on how careful firms are about the market rules, note that firms are less careful about the market rules (lower \( \gamma \) the more firms there are in the market (N). Note also that, as we expected, the greater the costs of being respectful of the market rules, \( \alpha \), the less likely firms are to be careful about following the market rules. This effect increases as the number of firms in the market increases.

The previous analysis is done under the assumption that firms are risk neutral. If firms were risk averse, the penalty of ending with a zero payoff would be more costly, and they would invest more in satisfying the market rules.

**Social Welfare**

If one specifies a social welfare function with some costs of breaking the market rules, there is then an optimal (noninfinite) number of firms. The social planner may prefer not to have too much competition. To formalize this notion further, suppose that the social costs of firms not investing enough in following the market rules are proportional to the expected total quantity produced that could be violating the market rules, \( (1 - \gamma)Q \). The social welfare function would then be
\[
S(Q, \gamma) = \int_0^Q P(x)dx - c(\gamma)Q - k(1 - \gamma)Q.
\]
where \( k \) represents the per unit social cost of not following the market rules. A natural threshold to consider for \( k \) is that \( k > c'(1) \), such that given the total quantity \( Q \), a social planner would prefer costs to be higher and market rules to be fully satisfied than to incur the social costs of the market rules not being fully satisfied.\(^8\)

If the social planner could choose \( \gamma \) and \( Q \) and \( k > c'(1) \), it would choose \( \gamma = 1 \) and \( Q \) such that \( P(Q) = c(1) \), the usual condition of price equal to marginal cost. If the social planner could choose \( \gamma \) and the number of firms in the market (even though it cannot regulate the total quantity produced), it could implement the optimum by choosing \( N \to \infty \).

If \( \gamma < 1 \) is fixed, note that the optimal total quantity in the market is no longer determined by \( P(Q) = c(\gamma) \) but rather by \( P(Q) = c(\gamma) + k(1 - \gamma) \). That is, the optimal total quantity produced should be reduced with respect to marginal cost pricing because there is an extra social cost of each unit that is produced without following all market rules. Note also that if \( k > c'(1) \), the optimal total quantity produced is lower than the optimal quantity produced when \( \gamma = 1 \), because \( c(\gamma) + k(1 - \gamma) > c(1) \) for \( \gamma < 1 \). To implement the optimal quantity \( Q' \) for fixed \( \gamma < 1 \), the social planner now must choose a finite \( N \), which is \( N = -[Q'P'(Q')]/k(1 - \gamma) \).

As we have shown, the number of firms in the market also endogenously determines the equilibrium intensity with

\(^8\)Note that if the social planner is not fully aware of the costs incurred in satisfying market rules, it may choose to set rules where \( K < c'(1) \).
which firms try to satisfy the market rules, \( \gamma \). Consider the first-order condition of the optimal number of firms \( N \) for the social planner:

\[
(4) \quad [P(Q) - c(\gamma) - k(1 - \gamma)] \frac{dQ}{dN} + k - c'(\gamma)Q \frac{dP}{dN} = 0.
\]

Proposition 1 shows the effect of the number of firms \( N \) on \( \gamma \) to be negative: a greater number of firms leads firms to be less careful about satisfying the market rules. Therefore, using Equation 4 we observe that the endogenous \( \gamma \) leads the social planner to choose an even lower number of firms if \( k > c'(\gamma) \), which is true for the case of \( k > c'(1) \).

\( P_2 \): When firms can decide how much to invest in satisfying the market rules, and if \( k > c'(1) \), decreasing the intensity of competition (i.e., limiting entry into the market) increases social welfare compared with when the extent of satisfaction of the market rules is exogenous.

The intuition is that by reducing the intensity of competition (having fewer firms in the market), firms invest more in satisfying the market rules, which is beneficial for social welfare. In terms of the previous example, this means that social welfare could benefit from reducing competition because that would lead firms to invest more in safety and hygiene conditions. In terms of Bennett et al.'s (2013) application to facilities testing for vehicle emissions, reducing competition between facilities could lead to greater welfare because the testing facilities may become less lenient on the vehicles passing the emission test.

The aforementioned linear example would result in the following. For the case in which \( \gamma = 1 \), the optimal total quantity would be \( Q = 1 - c_0 - \alpha \). For the case in which we have fixed \( \gamma < 1 \), we would have the total optimal quantity \( Q = 1 - c_0 - \gamma \alpha - (1 - \gamma)k \) and an optimal number of firms \( N = (1 - c_0 - \gamma \alpha - 1 - \gamma k)/[k(1 - \gamma)] \). Finally, for the full endogenous case, in which both the total quantity produced and the investment in satisfying the market rules are endogenous, we have \( Q = 2[(1 - c_0 - \gamma \alpha - (1 - \gamma)k)/(2 + k - \alpha)] \) and \( N = 4(1 - c_0 - \gamma \alpha - (1 - \gamma)k) \).}

Production Only if Satisfying the Rules of the Market

The previous analysis considers the case in which even if a firm is caught not following the rules of the market, the quantity produced by that firm continues to be supplied to the market. In other cases, firms caught disobeying the market rules may be blocked before the market exchange. The monitoring authority could potentially find out before the market exchange which firms are violating the market rules and block them from supplying to the market. For example, a restaurant can be prevented from operating if it is not following hygiene rules, or a manufacturer can be temporarily held from supplying to the market if it is not fully satisfying some safety rules.

Consider the case in which a firm that is caught not following the market rules is then restricted from supplying its intended production to the market, to the benefit of its competitors. The demand and profit of a firm is now stochastic, depending on which firms are caught not satisfying the market rules. Given an inverse demand function \( P(Q) \), the expected profit function for a firm can be written as

\[
\pi'(q_i, q_{i-1}, N; \gamma_i, \gamma_{i-1}) = \gamma_i \left[ \sum_{j=0}^{N-1} \Pr_j P(q_i + q_{j-1}) - c(\gamma_i)q_i \right],
\]

where all other competitors are assumed to each choose quantity \( q_{i-1} \) and intensity of satisfying the market rules \( \gamma_{i-1} \), and \( \Pr_j \) is the probability of \( j \) competitor firms not being caught violating the market rules. Given \( \gamma_{i-1} \), we have \( \Pr_j = |(N - 1)!/[j!(N - 1 - j)!]|^{1/(1 - \gamma_{i-1})} \).

Note that the intensity with which competitors try to satisfy the market rules now (negatively) affects the payoff of a firm. Furthermore, if the competitors are less careful about satisfying the market rules, a firm should be more careful about satisfying the market rules. That is, the \( \gamma_i \) variables are strategic substitutes. The first-order conditions are similar:

\[
(5) \quad \sum_{j=0}^{N-1} \Pr_j P(q_i + q_{j-1}) - c(\gamma_i)q_i + \gamma_i \sum_{j=0}^{N-1} \Pr_j P'(q_i + q_{j-1}) = 0,
\]

but now, to totally differentiate with respect to \( \gamma \) and \( q \), we must take into account that \( \gamma \) and \( q \) are also changing for the competitors at the first-order conditions. For \( P(Q) \) close to linear, we can then observe that in equilibrium (see the Technical Appendix), each firm invests less in satisfying the market rules (lower \( \gamma \)) as the number of competitors increases. That is, as before, more competition leads firms to be less careful about satisfying the rules of the market.

Compared with the previous case—when all firms produced independently of whether they were caught not satisfying the market rules—one can observe that the effect of the number of firms on the equilibrium \( \gamma \) is now smaller in absolute value for high equilibrium \( \gamma \) (which holds with low \( \alpha \)). Now, when the number of competitors increases, we know that only some competitors will actually affect the industry production, which means that the effect on expected profits is lower. Therefore, there is still a lot to lose from not investing sufficiently in satisfying the market rules, and firms do not decrease their investments in satisfying the market rules as much.

Consider the comparison of the equilibrium \( \gamma \) in the two environments. On the one hand, if all firms produce the same amount as when all production stays in the market, and if some output leaves the market, the firms have more to lose if they are caught, and thus they invest more in satisfying the rules of the market. On the other hand, firms realize that the competitor’s output may be reduced and thus increase their quantity produced accordingly, which could lead both to lower profits and to firms being less careful in satisfying the market rules. For demand that is close to linear, the first effect dominates, and with the production of firms caught violating the market rules leaving the market, firms invest more in satisfying the market rules and produce more than in the case in which production always stays in the market.

To observe how we obtain this result, consider the two sets of first-order conditions for Equations 1–2 and 5–6. Note that in both cases, both the quantity produced and the
degree of satisfying the market rules are increasing functions of the firm’s margin (i.e., equilibrium price minus marginal cost). This means that there is a positive relationship through the margin between quantity produced and degree of satisfying the market rules, which is close to the same relationship if the demand curve is close to linear.\(^9\) Because we know that for the same quantity produced, each seller now wants to invest more in satisfying the market rules, we also know that, in equilibrium, firms invest more in satisfying the market rules and intend to produce more. However, the expected quantity supplied in the market ends up being smaller than when no supplied quantity leaves the market. Equation 6 directly demonstrates this phenomenon: as the equilibrium \(\gamma_1\) increases, the expected equilibrium price must increase, which means that the expected quantity in the market decreases. Overall, the effect on social welfare can go up or down because there are now greater equilibrium investments in satisfying the market rules. For the previous linear example, in equilibrium we find that \(\gamma = \sqrt{9\alpha^2 + 4\alpha(N - 1)(1 - c_0) - 3\delta} / [2\alpha(N - 1)]\) and \(q = \alpha \gamma\). We state these results in the following proposition:

**OPTIMAL MARKET RULES**

The previous analysis suggests that a social planner may also be interested in changing the market rules such that firms may find it easier or more difficult to invest in satisfying those rules. That is, the social planner could potentially decide how strict to make the rules of the market.\(^10\) Let \(\lambda\) be an index of how strict the rules are. Stricter rules increase the marginal cost of production—that is, the marginal cost of production can now be written as \(c(\gamma, \lambda)\), with \(\partial c / \partial \lambda > 0\) and \(\partial^2 c / \partial \gamma \partial \lambda \geq 0\).

Market rules that are less strict may decrease the likelihood that a firm will be caught violating the rules. However, social welfare may suffer if firms do not satisfy a potential rule that—though not officially a market rule—would increase social welfare if satisfied. To consider this possibility, we define \(k\) as the total social welfare cost of all potential rules not being satisfied, as previously noted. Suppose that the legislator decides how many rules to set, \(\lambda\), with \(\lambda \in [0, 1]\), whereby the first rules being set are the ones that, if not satisfied, lead to a higher social cost. If there are \(\lambda\) rules, the social cost per unit supplied by a firm disobeying the market rules can be viewed as continuing to be \(k\) per unit supplied. However, there are also social costs from firms that have not been found in violation of the market rules, because those firms may be violating some potential market rules that have not been designated as part of the set of actual market rules. The social cost per unit supplied from the firms that have not been found violating the market rules can be represented by \(G(\lambda)\), where \(G'(\lambda) < 0\) and \(G''(\lambda) > 0\), because less strict rules lead to greater social costs of unsatisfied potential rules. We assume that this effect is greater when the rules are less strict because the legislator first chooses to incorporate market rules that are more important. Furthermore, we assume that \(G(0) = k\) and \(G(1) = 0\): if there are no rules, the social costs are \(k\) per unit for all output, and if all the potential rules are incorporated in the set of market rules, the social costs of firms not caught are zero.

Thus, whereas in the previous section the social costs of not satisfying potential rules were \(k(1 - \gamma)Q\), they are now \(k(1 - \gamma)Q + G(\lambda)Q\). By totally differentiating the equilibrium \(\gamma\) and \(q\) defined by Equations 1 and 2 with respect to \(\lambda\), we obtain

\[
\frac{d\gamma}{d\lambda} = \frac{-1}{D} \left\{ P'(Q) + QP''(Q) \left( \frac{\partial c}{\partial \lambda} + \frac{\partial^2 c}{\partial \gamma \partial \lambda} \right) \right\} + (N + 1)P'(Q) + QP''(Q) \left( \frac{\partial^2 c}{\partial \gamma^2} \right),
\]

and

\[
\frac{dq}{d\lambda} = \frac{1}{D} \left\{ \frac{\partial c}{\partial \lambda} + \frac{\partial^2 c}{\partial \gamma \partial \lambda} \right\} - \gamma \frac{\partial c}{\partial \gamma} + \frac{\partial^2 c}{\partial \gamma^2} \frac{d\gamma}{d\lambda}.
\]

From Equations 7 and 8, we observe that if \(P'(Q) + QP''(Q) < 0\) (the second derivative of the inverse demand function is not too large), then \(d\gamma / d\lambda < 0\); stricter rules cause the firms to choose a lower degree of satisfying the market rules. That is, if the rules of the market are stricter, firms find it too costly to satisfy all the rules of the market and are more likely to be found out as not satisfying the market rules. However, with stricter market rules, firms invest more in satisfying the rules if the second derivatives of the marginal cost function are dominated by the first derivatives—that is, in equilibrium, \(c(\gamma, \lambda)\) is increasing with \(\lambda\), and \(\partial c / \partial \lambda + (\partial c / \partial \gamma)(d\gamma / d\lambda) > 0\).

If the marginal cost function is additively separable in \(\gamma\) and \(\lambda\), we also observe that \(dq / d\lambda < 0\)—that is, stricter market rules lead each firm to produce less. The intuition is that stricter market rules mean that the marginal costs of production are higher, and the firms respond by producing less. We state these results in the following proposition.

**P4:** Suppose that the marginal cost function is additively separable in the investment of satisfying the market rules and the index of how strict the rules are. Then, stricter market rules lead to (1) more firms being caught violating the market rules, (2) greater investment in satisfying the market rules, and (3) lower production.

Next, consider the effects on social welfare of the market rules that are set. Given the previous formulation, we observed that the social welfare function is

\[
S(Q, \gamma, \lambda) = \int_0^Q P(x)dx - c(\gamma, \lambda)Q - k(1 - \gamma)Q - G(\lambda)Q.
\]

By totally differentiating \(S(Q, \gamma, \lambda)\) with respect to \(\lambda\), taking into account that the equilibrium \(Q\) and \(\gamma\) depend on \(\lambda\), we obtain
The first term in Equation 10 represents the social cost of less quantity offered in the market when the market rules are stricter. This term is negative in general. The second term represents the social costs that result from firms being more likely to be caught disobeying the market rules when the market rules are stricter. This term is also negative. The third term represents the increase in production costs when the market rules are stricter, which also negatively affects social welfare. Finally, the fourth term represents the social benefits of having more potential rules satisfied, which is a positive effect of having stricter market rules. The optimal market rules result from the trade-off between these four forces, which depend on the level of competition in the market. Note that if the last potential rules that can be enforced have insignificant value—that is, if \( \lim_{m \to 1} G'(\lambda) = 0 \)—then at the social welfare optimum, the legislator chooses not to include all the potential rules in the rules of the market.

For sharper results, consider the linear example, \( P(Q) = 1 - Q \) and \( c(\gamma, \lambda) = c_0 + \alpha \gamma + \beta \lambda \) (where \( \alpha \) and \( \beta \) are parameters). We can thus calculate the equilibrium \( \gamma = (1 - c_0 - \beta \lambda) / (\alpha (1 + N)) \) and \( Q = N (1 - c_0 - \beta \lambda) / (1 + N) \).

This illustrates that the equilibrium \( \gamma \) and \( Q \) decrease with \( \lambda \). We can also obtain the equilibrium investments in satisfying the market rules, \( c(\gamma, \lambda) = [1 + N (c_0 + \beta \lambda)] / (1 + N) \), which are increasing with \( \lambda \). Finally, for this example, the optimal rules must satisfy the following:

\[
G(\lambda) \left( \frac{1}{\alpha} \right) - G'(\lambda) \left( \frac{1 - c_0 - \beta \lambda}{\alpha \beta} \right) = 2k + N.
\]

Given that the left-hand side of Equation 11 decreases with \( \lambda \), this example shows that more competition (greater \( N \)) and greater costs of satisfying the market rules (greater \( \alpha \)) lead the legislator to choose less strict market rules. Greater competition leads firms to invest less in satisfying the market rules; the legislator alleviates this effect by having less strict market rules. We state this result in the following proposition:

**P5:** For linear demand and cost function, the legislator (i.e., the social planner) chooses less strict market rules when there is greater competition and greater costs of satisfying the market rules.

In terms of the previous safety and hygiene examples, this would mean that if there is greater competition, the social planner may want to have less strict market rules for safety and hygiene for firms to increase their intensity of satisfying the existing market rules (\( \gamma \)) and increase quantity production. If the market rules are too strict, firms invest more in satisfying the market rules and produce less.

Another notable issue we do not consider is what would be the optimal penalty mechanism that a social planner would choose. Although this issue is beyond the scope of this article, we present some discussion. In the previous analysis, we assume the extreme penalty of fully extracting the profit, with limited liability, of the firms found out not to be satisfying the market rules. Consider next what would happen if the penalty were instead a fixed proportion of the profit obtained. For a fixed penalty proportion \( \mu \), the expected profit of a firm would then be \( \pi'(q_i, \gamma) = \gamma_i + (1 - \gamma_i) / (1 - \mu) q_i [P(Q) - c(\gamma_i)] \).

Under this situation, the first-order condition Equation 1 remains the same, and Equation 2 is replaced with \( P(Q) - c(\gamma) - [(\gamma + 1 - \gamma)(1 - \mu)] / \mu c'(\gamma) = 0 \). This new first-order condition means that, for the same \( q \), a firm now chooses a lower \( \gamma \) and has fewer incentives to satisfy the market rules because it makes some profit even if it is caught violating the market rules. The equilibrium then causes each firm to invest less in satisfying the market rules and thus produce more. When the penalty is a fraction of profits, this has a similar effect on how firms behave with varying competition intensity as when the penalty is the profit obtained. In both cases, the firms have more to lose and invest more in satisfying the market rules when there is less competition. If the social costs of firms not satisfying the market rules are sufficiently high, the optimal fixed proportion is indeed 100%, as we assumed previously. This penalty generates the positive result that firms want to invest more in satisfying the market rules.

Another potential penalty for firms not satisfying the market rules could be a fee per unit produced. The social planner could choose a penalty per unit produced, such that firms have greater incentives to invest in satisfying the market rules. It may not be optimal to have a penalty per unit so high that firms fully invest in satisfying the market rules. If there is too high, the social cost of firms not satisfying the market rules is relatively limited; the social planner would prefer to have a lower penalty per unit and give incentives for firms to produce greater quantities. It may also be possible that the optimal penalty per unit produced is zero if, without that penalty, firms are already investing at a sufficiently high level (though not fully) in satisfying the market rules. It would also be worthwhile to investigate what happens when we allow the penalty of not satisfying the market rules to depend nonlinearly on the profit obtained and/or on the quantity produced or be a function of the rules that are not satisfied.

Note that if the penalties are fixed (i.e., independent of profits), and if there are no limited liability issues, firms with greater demand will invest less in satisfying the market rules because of the increased costs. Because greater competition may mean lower demand per firm, we would then find that greater competition could lead to greater investments in satisfying the rules of the market, which counters the previous results because of the reduced demand effect. If there is uncertainty about the size of the penalties, such that limited liability could play a role, and this effect is sufficiently important, we again find that greater competition leads to lower investments in satisfying the market rules.

The analysis on the optimal market rules assumes that all rules not satisfied receive the same maximum penalty, independent of the social cost of the broken rule. This may be perceived as desirable if the regulator wants to satisfy all the rules. However, in the real world, we often observe that more important rules, when broken, receive a greater punishment than do less important rules. The results in P5 can be viewed through this perspective: the more important rules receive a harsh punishment if broken, and the less important rules receive a minimal punishment because their
full enforcement would be too costly in terms of industry output. If the penalty depends on the extent to which a firm violates the market rules, however defined, in some cases competition may not affect the degree of investment in satisfying the market rules. However, if there is some uncertainty as to how the monitoring authority evaluates the extent to which a firm violates the market rules—again, if there is limited liability and sufficient uncertainty—our main messages are still valid.

EX POST MARKET ASYMMETRY AND OPTIMAL MONITORING

Ex Post Market Asymmetry

Next, consider a symmetric competition model in which ex post firms end up with different market shares. This allows us to consider the effects regarding which firms are more likely to have invested less in satisfying the market rules if they have ex post different market shares. To illustrate this in a parsimonious model, we consider price competition in mixed strategies, drawing on Varian (1980) and Narasimhan (1988). This enables us to present price competition simply, wherein asymmetry is generated in the market. The next subsection then investigates optimal differentiated monitoring. The following section (“Firm Private Information”) presents the asymmetric information case (similar to this case), in which firms have pure strategies. This model also enables us to consider dynamics subsequently.

Suppose that N symmetric firms compete in price, with the same assumptions on the marginal cost and effect on profits if found violating the rules of the game as before. A set of consumers of dimension 1 buys the product that has the lowest price if it is below a reservation value r. Per firm, there is a set of consumers of dimension M that buys that firm’s product if the price is below the reservation value r. Firms simultaneously choose the price they charge and the intensity with which they satisfy the market rules.11

The equilibrium is in mixed strategies, with each firm charging a price drawn from a cumulative distribution function F(P). The equilibrium condition for F(P) is that a firm is indifferent for all prices that it charges and γ that it chooses:

\[
\gamma[P - c(\gamma)]\left\{1 - F(P)\right\}^{N-1} + M = K,
\]

for all prices P being charged with positive density, where K is the equilibrium expected profit.

For a certain price P_i charged by firm i, the optimal γ_i is calculated as

\[
P_i - c(\gamma_i) - \gamma_i' c'(\gamma_i) = 0,
\]

the same condition as before, which means that the firm will make a greater effort to adhere to the rules of the market when charging a higher price.

At the highest price charged, r, we calculate the equilibrium γ, γ as \(r - c(\gamma') - \gamma' c'(\gamma') = 0\). The equilibrium expected profit K can be expressed as \(K = \gamma^2 c'(\gamma) M\). The equilibrium mixed strategy is then

\[
F(P) = 1 - \left(\frac{\gamma^2 c'(\gamma') - \gamma' c'(\gamma)}{\gamma^2 c'(\gamma)} M\right)^{\frac{1}{N}},
\]

where γ is the monotonic function of P presented in Equation 12.

Next, consider the effect of the degree of competition on the equilibrium prices and the intensity of satisfying the rules of the market. Note that changing only the number of firms N in this model may lead to higher prices (e.g., Rosenthal 1980), but here a greater N alone may not necessarily be viewed as increased competition, because another firm brings additional M loyal consumers into the market. Increasing N while keeping the total number of loyal consumers in the market fixed (i.e., keeping NM fixed), we find that when the intensity of competition increases, firms charge lower prices and are less careful about satisfying the rules of the market. Alternatively, we can examine the effect of the parameter M alone, which can be viewed as a measure of product differentiation. In this model, given N, the only thing that matters for the intensity of competition is the ratio of locked-in consumers to the consumers that choose the lowest-price product (not M per se). We can then also observe that when the intensity of competition increases (as measured by M), this results in lower prices and less intensity of satisfying the market rules.

This model thus illustrates that the firms that “win” (i.e., have greater market share and profits) are more likely to have invested less in satisfying the rules of the market. We state the formal result in the following proposition.

P_c: In the symmetric model with a mixed-strategy equilibrium, the firm that has the largest profit and market share invested less in satisfying the market rules.

Note that P_c regards the ex post outcomes in the market. Given the symmetric mixed-strategy equilibrium, ex ante, any of the firms is equally likely to have invested less in satisfying the market rules, and, ex ante, all firms have the same expected profits. In the next section, we show how the likelihood of being a firm that invests less in satisfying the market rules can be connected with its private information. In terms of the safety and hygiene conditions’ application, P_c would indicate that the firm that has the largest market share ex post may have invested less in safety or hygiene conditions. This echoes the idea that the market winners may have behaved outside of the market rules. We further explore the implications of this notion for differentiated monitoring levels in the next subsection and more generally in the “Dynamics” section.

For the example in which the marginal cost is linear in γ, we find that Equation 12 leads to \(P = c_0 + 2\alpha\gamma\), which results in \(\gamma' = (r - c_0)/(2\alpha\gamma)\) and an expected profit \(K = [(r - c_0)^2/(4\alpha)]M\). The more costly it is for firms to be careful about the market rules, α, the lower the expected profits. Furthermore, for this case we can express the equilibrium mixed strategy (Equation 13) as

\[
F(P) = 1 - \left[\frac{r - P}{P - c_0}\left(1 + \frac{r - c_0}{P - c_0}\right) M\right]^\frac{1}{N},
\]

which is independent of the cost of being careful about the market rules, α. That is, in this case, the pricing equilibrium is
not affected by changes in \( \alpha \). In other words, for each price draw, an increase in \( \alpha \) is associated with a decrease in using caution regarding the market rules (i.e., a reduction in \( \gamma \)).

**Optimal Monitoring**

In terms of this model, next consider a social planner’s decision regarding the levels of monitoring whether firms are complying with the rules of the market. We restrict attention to the case in which monitoring can be made contingent only on the firm’s market share and not on the price charged by the firm. The idea is that the monitoring authority may not necessarily observe prices (i.e., there is potential for secret price cuts), whereas market shares (or overall profitability) may be observable. This means that the monitoring authority can monitor with different intensities the firm with the largest market share (the firm with the lowest price) and the other firms. We denote the intensity of monitoring the firm with the largest market share as \( m^H \) and the intensity of monitoring the other firms as \( m^L \). A notable case is when \( m^H = m^L \), in which all the firms are monitored equally.

We consider the following timing. First, the monitoring authority commits to \( m^H \) and \( m^L \). Second, the firms choose their prices and the intensity with which they comply with the market rules. Finally, monitoring takes place at the committed levels. The monitoring authority’s ability to commit to the monitoring levels could be due, for example, to reputation building or to the investment in creation of a monitoring structure that is costly to change. The monitoring authority may want to create a reputation of following through with the announced monitoring levels, which may help in generating greater firm investments in satisfying market rules. It may also be that having some monitoring levels may require ex ante investment in a creation of a structure that is able to monitor the firms’ actions, and the firms can observe such a monitoring structure prior to their actions. Subsequently, we briefly discuss the case in which the monitoring authority cannot commit ex ante to the monitoring levels.

To consider the different monitoring rules, we slightly adjust the previous notation regarding the intensity of satisfying the market rules. We consider that firms choose their marginal costs \( c \) (with higher marginal costs meaning greater care in satisfying the market rules), which, together with the monitoring level \( m \), determine the probability that the firm will not be discovered disobeying the rules of the market, \( g(c, m) \). The probability \( g(c, m) \) increases with the marginal costs \( c \) and decreases with the monitoring level \( m \). Denoting the cumulative probability distribution of prices (the equilibrium is in mixed strategies with no mass points) as \( F(P) \), the expected value of profits can be written as

\[
\begin{align*}
\pi = & g(c, m^L)(P - c)M \\
& + (P - c)[1 - F(P)]^{N-1}g(c, m^H)(1 + M) - g(c, m^L)M,
\end{align*}
\]

To obtain sharper results, we concentrate on the case in which \( g(c, m) \) is linear in \( c \) and \( m \), \( g(c, m) = g_0 + \beta c - m \).

Consider the optimal combination of prices and marginal costs for a firm. That is, what is the optimal marginal cost (how much to invest in satisfying the rules of the market) for each price \( P \)? By differentiating Equation 15 with respect to \( c \) and making it equal to zero, we obtain

\[
[\beta(P - c) - g(c, mL)] (M + [1 - F(P)]^{N-1}) + (1 + M)[1 - F(P)]^{N-1}(m^H - m^L) = 0.
\]

This equation enables us to make the following observations. First, if the monitoring level is the same for all firms (\( m^L = m^H \)), this condition reduces to the same as that obtained in the previous subsection (Equation 12). Second, if the price is equal to the reservation price (\( P = r \)), this condition is also the same as obtained previously, as \( F(r) = 1 \). Note that at the price equal to the reservation price, the degree to which a firm invests in satisfying the market rules is expressed by \( \beta(r - c) - g(c, mL) = 0 \), which leads to \( c^* = (\beta r - g_0 + m^L)/\beta \). Third, if the monitoring intensity is higher for the firm that has the greatest market share than for the other firms (\( m^H > m^L \)), then a firm with a price \( P \) chooses to invest more in satisfying the market rules (greater \( c \) than what we found in the “Ex Post Market Asymmetry” subsection). This effect is greater for lower prices, as for lower prices the firm is more likely to be the one with the greatest market share.

Substituting Equation 16 into Equation 15, we have

\[
\pi = \beta(P - c)^2 \frac{M + [1 - F(P)]^{N-1}}{(P - c)^2}.
\]

which is independent of any difference between monitoring levels across firms with different market shares. The pricing strategy of each firm then has to satisfy

\[
F(P) = 1 - \left[ \frac{(r - c^*)^2 - (P - c)^2}{(P - c)^2} \right]^{\frac{1}{m^L}},
\]

which is independent of any difference between monitoring levels across firms with different market shares. That is, the cumulative distribution of the margins \( P - c \) is independent of the difference among monitoring levels, but for each price \( P \), each firm is now “choosing” (investing in satisfying the market rules) a higher marginal cost \( c \). Using Equation 17 in Equation 16, we can then obtain the relationship between the optimal price \( P \) and marginal cost \( c \) as

\[
P = c + \frac{\beta(r - c^*)^2 - 4(1 + M)\Delta(r - c^*)^2}{2(1 + M)\Delta}.
\]

12We assume that at the level of monitoring that is possible, \( g(c, m) \) is bounded such that the probability of being caught violating the market rules does not change too much with the monitoring level \( m \). That is, it is not possible to apply infinite monitoring such that a firm that violates the market rules is inevitably caught.

13There might also be an effect of the interaction between a firm’s investment in complying with the market rules and the monitoring level on the probability of a firm being caught. For example, if firms are very unlikely to be caught not satisfying the market rules, it could be that a greater monitoring level is simply more able to discern between the firms that are investing more and less in satisfying the market rules. However, the interaction may run the other way if most firms are caught not satisfying the market rules.
where $\Delta \equiv m^H - m^L$. Figure 1 illustrates the comparison of the relationship of price and marginal costs for the pairs with positive density for the cases in which $\Delta = 0$ and $\Delta > 0$, showing that, for the same price when $\Delta > 0$, firms invest more in satisfying the market rules and have greater marginal costs. Figure 2 illustrates a comparison of the cumulative distribution function of the marginal costs for the cases of $\Delta = 0$ and $\Delta > 0$, showing that when $\Delta > 0$, firms invest more in satisfying the rules of the market. Figure 3 presents the cumulative distribution function of prices, illustrating that when there is a difference in the monitoring levels, the equilibrium prices are higher.

Next, consider the issue for the monitoring authority of whether to set a difference in the monitoring levels across firms with different market shares—that is, whether to set $\Delta > 0$. To observe this, consider that the monitoring authority is maximizing social welfare, with costs $T(m)$ of monitoring one firm at the level $m$, and that the social costs of firms not investing enough to satisfy the market rules are proportional to the expected quantity produced that could be violating the market rules at an exogenous level of monitoring $\bar{m}$, $k[1 - g(c, \bar{m})]$. We assume $T', T'' > 0$ and focus on the case in which $M$ is small. We also assume $\beta k > 1$, such that the monitoring authority prefers higher costs with more compliance with the rules of the market than lower costs and lower compliance with the market rules. The social welfare function can then be represented by

$$S(m^L, \Delta) = r(1 + NM) - E(c_{\text{min}}) - E(c)NM - k[1 - g_0] - \beta E(c_{\text{min}}) + \bar{m} - kNM[1 - g_0 - \beta E(c) + \bar{m}] - T(m^L + \Delta) - (N - 1)T(m^L),$$

where $E(c_{\text{min}})$ is the expected value of the minimum marginal cost in the market, and $E(c)$ is the expected value of the marginal cost of a given firm.

Considering the effect on social welfare, we can observe that the optimal $\Delta$ is strictly positive, which we state in the following proposition (for the proof, see the Technical Appendix).

**P7:** Under optimal monitoring, it is better to monitor the firm that has the greatest market share with strictly greater intensity than the other firms.

This yields the expected result that the monitoring authority may more aggressively watch firms that do well.
This finding is reasonable because this monitoring policy has a greater effect in incentivizing firms to invest in complying with the rules of the market. In terms of the security and hygiene conditions’ application, this would mean that the firms with greater market share would be monitored more intensely on their security and hygiene conditions.

If the monitoring authority cannot commit to the monitoring levels before evaluating the market competition, the incentives for firm behavior caused by more aggressively monitoring the firms with greater market share will disappear. However, if the monitoring authority itself has the incentive of appropriating the greatest share more aggressively, it should then choose ex post to monitor the firm with the greatest market share more aggressively.

**FIRM PRIVATE INFORMATION**

Consider a variation of the previous model, where firms have private information about their costs of satisfying the market rules. That is, firms are endowed with different costs of satisfying the market rules, and each firm knows about only its own costs. One way to consider this is to have the linear marginal cost example, with α differing across firms, and each firm having private information over its α. A lower α would mean that the firm has lower costs of satisfying the market rules. Suppose that α is distributed with support [α, ±], with cumulative distribution function H(α) with no mass points, and α independently drawn across firms. Then, a firm with private information α will choose a price P(α) and an amount of care of satisfying the market rules γ(α), which, as in the previous analysis, must satisfy

\[ P(\alpha) = c_0 + 2\alpha \gamma(\alpha). \]

Furthermore, we calculate the optimal price P(α) for a firm with private information α by solving the following problem:

\[ \max_{P} \gamma(\alpha)[P - c_0 - \alpha \gamma(\alpha)] \left(1 - F(P)\right)^{N-1} + M. \]

The first-order condition of this problem reduces to

\[ \frac{P - c_0}{1 - F(P)} = \frac{\gamma(\alpha)}{\left(1 - F(P)\right)^{N-1}}. \]

Substituting for \( \gamma(\alpha) = \frac{[P(\alpha) - c_0]}{2\alpha} \), this expression reduces to

\[ [1 - F(P)]^{N-1} + M = \frac{P - c_0}{2\alpha} [N(1 - F(P)) - 2] = 0. \]

Assuming that P(α) is strictly increasing with α (which is shown in the equilibrium we consider subsequently), we find that F[P(α)] = H(α) and, thus, F[P(α)]P(α) = H'(α). Including this notation in Equation 21 results in the following differential equation on P(α):

\[ P'(\alpha) \left(1 - H(\alpha)\right)^{N-1} + M \right) - (N - 1) - \frac{P(\alpha) - c_0}{2\alpha} H'(\alpha) \left(1 - H(\alpha)\right)^{N-2} = 0. \]

which confirms that P'(α) > 0 and has as a solution the equilibrium pricing policy as a function of the private information α,

\[ P(\alpha) = \frac{(r - c_0)\sqrt{M} + c_0}{\sqrt{[1 - H(\alpha)]^{N-1} + M}} + c_0, \]

using the condition that P(±) = r. We can then calculate the equilibrium γ as a function of α as

\[ \gamma(\alpha) = \frac{P(\alpha) - c_0}{2\alpha}. \]

Note that for the firms that have the highest cost of satisfying the market rules, the equilibrium prices are similar to each other, \( \lim_{\alpha \to -\infty} P'(\alpha) = 0 \). From the equilibrium conditions, we can also observe that the firm that has the greatest ex post market share is the one that has the lowest cost of satisfying the market rules (i.e., the firm with the lowest α charges the lowest price). Note, however, that it is not necessarily the firm with the lowest cost of satisfying the market rules that ends up satisfying the market rules to the greatest degree (i.e., choosing a higher γ). That is, γ(α) does not necessarily decrease over the whole range of α. We know from the “Static Symmetric Competition” section that firms with a greater market share have more to lose from not satisfying the market rules and thus invest more in satisfying those rules. When α decreases, then for the same γ, the marginal costs fall, which is a force toward increasing the degree to which firms satisfy the market rules. Furthermore, with a lower α, it also becomes cheaper to satisfy the market rules, which is an additional force in that direction. However, in this setting, a firm with a lower α also has greater ability to charge lower prices, with potentially lower margins, and this is a force for the firm to have lower incentives to invest in satisfying the market rules (lower γ). In this setting, this third market force (which is present in the previous section) can dominate the other two forces in some range, with γ increasing with α in that range.

In particular, observe that

\[ \lim_{\alpha \to -\infty} \gamma'(\alpha) = \frac{(r - c_0)}{2\alpha} \frac{\sqrt{M}}{1 + M} \frac{\sqrt{H'(\alpha)}}{2 - \frac{N - 1}{1 + M}}. \]

which is positive if the support of α is sufficiently small (large H') or the number of competitors is sufficiently large. This means that it is likely that the firm that has the lowest price in a certain market realization is one of the firms that chose the lowest intensity of satisfying the rules of the market (even though it is the one for which the cost of satisfying the market rules is the lowest). Note also that when the support of α converges to zero, the equilibrium converges to the equilibrium in the previous section, which means that P(α) increases from the minimum price of the previous section to the reservation price r, with lower intensities of satisfying the market rules associated with lower prices. We also find that the intensity of satisfying the market rules decreases with α for the high types, \( \lim_{\alpha \to 0} \gamma'(\alpha) < 0 \). We state these results in the following proposition:

P8: If firms have private information about their costs of satisfying the market rules, then a firm’s price increases with its costs of satisfying the market rules. If the support of the costs of satisfying the market rules is sufficiently

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14An alternative model could add a cost of not satisfying the market rules, modeled as η(1 - γ), with private information over η. The main ideas derived here carry through in this alternative model.
small, a firm’s degree of satisfying the market rules increases with the costs of satisfying the market rules in some range, and the firm with the largest market share is likely one that was less careful in satisfying the market rules.

Figure 4 illustrates the equilibrium functions \( P(\alpha) \) and \( \gamma(\alpha) \) for the example \( c_0 = 1, N = 8, M = .5, r = 2.3 \), and \( \alpha \) distributed uniformly on \([1, 1.1]\). Figure 5 illustrates how the equilibrium functions for the same example change for a reduced support of \( \alpha \), \([1, 1.02]\).

In addition to the existence of private information about the firm’s costs of satisfying the market rules, it could also be that firms have private information about their base marginal costs of production, \( c_0 \). In that case, the equilibrium price and investment in satisfying the market rules for each firm would be a function of that firm’s base marginal costs, \( c_0 \), and cost of the investment in satisfying the market rules, \( \alpha \)—that is, \( P(\alpha, c_0) \) and \( \gamma(\alpha, c_0) \). In that case, the equilibrium price for a firm would be increasing with its base marginal cost, and the degree of satisfying the market rules for a firm would decrease with its base marginal cost. That is, firms with lower marginal costs invest more to satisfy the market rules. The intuition is that if a firm has a lower marginal cost, it has more to lose if it is caught violating the market rules. Together with \( P_3 \), this would then mean that firms that are more likely to have greater market share \( \alpha \) post with lower base marginal costs (lower \( c_0 \)) and those that have chosen a lower intensity in satisfying the market rules (lower \( \alpha \), possibly translated into lower \( \gamma \) by \( P_3 \)). The first effect is positive from a social welfare point of view, while the second effect can be negative. Which effect is more important depends on whether there is greater variation in \( c_0 \) or \( \alpha \).

**DYNAMICS**

To investigate the dynamics of firms that invest in satisfying the rules of the market, consider a two-period variation of the model in the “Ex Post Market Asymmetry and Optimal Monitoring” section, where \( L \) consumers who buy at the lowest price become loyal in the second period to the firm from which they bought in the first period. In the second period, the firm that had the lowest price in the first period now has \( M + L \) loyal consumers, and firms compete on price over \( 1 - L \) consumers.

If a firm is caught not satisfying the market rules in one period, the punishment is only any potential profit in that period. That is, if a firm is caught violating the market rules only in the first period, it loses its first-period profits (and only that amount). Likewise, if a firm is caught violating the market rules only in the second period, it will lose only its second-period profits.

We consider the case in which a firm that is caught not satisfying the market rules in the first period can still operate in the market during the second period. We then discuss what happens when a firm that is caught not satisfying the market rules in the first period is not allowed to operate in the market in the second period. Firms discount the second-period profits with the discount factor \( \delta \in (0, 1) \).

**Second Period**

Consider the second period of this industry with \( N \) firms. Suppose that \( N > 2 \), such that if the firm that won the largest market share in the first period were taken out of the market, there would still be competition in the market. In this case, suppose that the firm that won the largest market share in the previous period charged a price equal to the reservation price \( r \). Then, the competitors would behave as described previously, but with \( N - 1 \) firms. The equilibrium strategies for each of those firms would then be characterized by

\[
\gamma = [P_2 - c(\gamma)] \left\{ [1 - F_{2L}(P_2)]^{N-2} (1 - L) + M \right\} = \pi_{2L},
\]

The analysis of the case \( N = 2 \) is slightly different than the one we present next (Narasimhan 1988), but it leads to the same messages as the ones discussed here. With \( N = 2 \), the firm that did not win the largest market share in the first period can benefit from the higher prices charged by its competitor.
where \( P_t \) and \( \gamma_t \) represent the price and investment in satisfying the market rules in period \( t \), \( F_{2L} \) is the cumulative distribution function of the prices in the second period by a firm that did not win the largest market share in the first period, and \( \pi_{2L} \) is the second-period expected profit for a firm that did not win the largest market share in the first period.

Note first that, as we have shown, there is an optimal relationship between the price charged and the firm’s investment in satisfying the rules of the market, \( P - c(\gamma) - \gamma c(\gamma) = 0 \), which leads to \( \gamma|P - c(\gamma)| = \gamma^2 c(\gamma) \). Note also that if a firm without the largest market share in the first period charges a price of \( r \), it will have a demand of only \( M \). We can then determine from Equation 25 that the second-period expected profit for the firms that did not win the largest market share in the first period is \( \pi_{2L} = \gamma^2 c(\gamma^2)M \).

If the firms that did not win the largest market share in the first period followed the strategies described in Equation 25, the best response for the firm with the largest market share would be to charge the reservation price \( r \), as we have assumed, and to invest \( \gamma \) in satisfying the rules of the market. This is because a lower price would lead to lower profits than what this firm can get by charging \( r \), \( \gamma^2 c(\gamma^2)(M + L) \), as \( [1 - F(P)]^{N-1} < [1 - F(P)]^{N-2} \); furthermore, with the proposed equilibrium strategies, the firms that did not win the largest market share in the first period are only able to earn an expected profit of \( \gamma^2 c(\gamma^2)M \).

Therefore, the strategies in Equation 25 for the firms that did not win the largest market share in the first period, and the price equal to the reservation price \( r \) for the firm that did win the largest market share in the first period, constitute a second-period market equilibrium. Indeed, we can show that this is the only equilibrium with the same strategies for all the firms in the same situation (all the firms without extra loyal consumers) because a lower price for the firm with the largest market would lead its competitors to charge even lower prices, which would then lead the firm with the largest market in the first period to charge the reservation price \( r \).

From this, we find that the equilibrium strategies of the firms that did not win the largest market in the first period satisfy the following:

\[
[1 - F_{2L}(P)]^{N-2} = \frac{\gamma^2 c(\gamma^2) - \gamma^2 c(\gamma)}{\gamma^2 c(\gamma)} \times \frac{M}{1 - L}.
\]

The equilibrium expected profits in the second period are \( \pi_{2L} = \gamma^2 c(\gamma^2)M \) for the firms that did not win the largest market share in the first period and \( \pi_{SW} = \gamma^2 c(\gamma^2)(M + L) \) for the firm that did win the largest market share in the first period. Note that the firms that did not win the largest market share in the first period compete more aggressively (and invest less in satisfying the market rules) the greater the \( L \). Furthermore, for an \( L \) close to zero, those firms compete more aggressively (and invest less in satisfying the market rules) than as described in the “Ex Post Market Asymmetry and Optimal Monitoring” section (given the same \( N \)). We can now observe that the firm that won the largest market share in the first period has, as we predicted, a higher expected profit as well as an equilibrium strategy on the investment in satisfying the market rules that stochastically dominates (first-order) the equilibrium strategy of the other firms.

The intuition is that the firm that won the largest market share in the first period has more to lose if it is caught not satisfying the market rules. This could be viewed as an illustration of large companies being more careful in satisfying the market rules because they may have more to lose if caught.\(^{16}\) Indeed, there is anecdotal evidence of larger firms investing more in legal departments, public relations offices, and activities that would make them less of a monitoring target (e.g., environmentally friendly activities, charitable and political contributions). Larger, more successful firms may engage further in charitable contributions to avoid being caught not satisfying the market rules. For example, Bernard Madoff was very generous in charitable and political contributions before he was caught engaging in securities fraud (e.g., Henning 2008). Madoff served on boards of several nonprofit institutions and, through his family foundation, donated to hospitals; theaters; and several educational, cultural, and health charities. In terms of the robber baron examples discussed previously, several of them invested heavily in charitable contributions to education and the arts in later periods, which can be viewed as a way both to reduce the chances of being caught not satisfying the market rules and to gain social acceptance in spite of wrongs committed in the past.

The result that the firms that did not win the largest market share in the first period are more compliant in the second period in satisfying the market rules could then lead the monitoring authority to watch those firms more aggressively in the second period. For example, this is consistent with the Federal Aviation Administration’s being more careful in monitoring the safety practices of airlines that are in weak financial conditions because these airlines may have less to lose and might be less careful in satisfying the appropriate safety practices.\(^{17}\)

**First Period**

Next, consider the first-period decisions. The equilibrium strategies in the first period again involve mixed strategies. The expected present value of profits for a firm can be written as follows:

\[
\gamma|P - c(\gamma)|M + \delta \pi_{2L} + |1 - F_1(P)|^{N-1} [\gamma|P - c(\gamma)| + \delta(\pi_{SW} - \pi_{2L})] = \pi_1 + \delta \pi_2,
\]

where \( F_1(P) \) is the cumulative probability distribution of the equilibrium pricing strategy for each firm, and \( \pi_1 \) is the expected profit in period \( t = 1, 2 \).

From Equation 27, we observe that for a certain price \( P \), each firm chooses an investment in satisfying the rules of the market determined by \( P - c(\gamma) - \gamma c(\gamma) = 0 \). The

\(^{16}\)In the previous example, in the second period the “large” firm ends up with a demand of \( M + L \) while the firm with the lowest price ends with a demand of \( M + 1 - L \). The “large” firm continues to be the firm with the largest market share if \( L > 1/2 \). Alternatively, models could be constructed in which these effects go through and the “large” firm continues to be the one with the largest market share in the second period for all parameter values.

\(^{17}\)As stated in the U.S. General Accounting Office (1996, p. 17) Report to Congressional Requesters on Aviation Safety from October 1996: “Over the years, FAA has targeted specific airlines and areas of commercial airline operations for increased surveillance on the basis of a variety of factors. For example, FAA has used an increased frequency of noncompliance with federal aviation regulations, an increased frequency of incidents by individual airlines, the deteriorating financial conditions of individual airlines [emphasis added], and non-airline-specific attributes (such as aging aircraft) to target its surveillance activities.” We thank the Associate Editor for suggesting this example.
equilibrium expected value of profits in the first period is then \( \pi_1 + \delta \pi_2 = \gamma' c' (\gamma') M + \delta \pi_{2L} \), and the equilibrium pricing strategy in the first period is determined by

\[
F_1(P) = 1 - \frac{\gamma' c' (\gamma') - \gamma' c' (\gamma' \gamma M)}{\gamma' c' (\gamma') + \delta \gamma' c' (\gamma' \gamma M)}.
\]

This then yields the finding that, in the first period, firms price more aggressively and invest less in satisfying the market rules than in the case described in the “Ex Post Market Asymmetry and Optimal Monitoring” section. This is because firms try to gain the second-period prize of having been the firm with the largest market share in the first period. That is, in markets in which there are complementary dynamic effects (i.e., a greater market share today leads to competitive advantages in the future), firms will be less careful in satisfying the rules of the market in early periods. We summarize these results in the following proposition.

**P:** In a dynamic two-period environment in which early advantages carry on into the future, firms are more aggressive in not satisfying the market rules in the first period. In the second period, the largest firm from the first period is more careful to satisfy the market rules.

This proposition can also be viewed as consistent with the anecdotes about the robber barons. They may have chosen not to invest too much in satisfying the market rules in the early days, but after their success was guaranteed, they may have become more cautious in satisfying the market rules. This proposition can also be viewed as consistent with the anecdotes about the robber barons. They may have chosen not to invest too much in satisfying the market rules in the early days, but after their success was guaranteed, they may have become more cautious in satisfying the market rules.

**CONCLUDING REMARKS**

In this article, we have investigated the question of how firms decide to invest in satisfying the rules of the market depending on its effects on costs and the monitoring technology. We demonstrate that greater competition intensity may lead firms to invest less in complying with the market rules. This could be viewed as a force for a social planner not to allow too much competition in a market. We also show that in a dynamic environment, firms may invest less in satisfying the market rules in the early periods, but in the later periods, firms that did well in the past may become more careful in satisfying the rules of the market. Finally, we illustrate how a monitoring authority may benefit from choosing different monitoring levels for different firms depending on their market shares. This analysis focused on the case in which a firm found to be violating the market rules forfeits all its profit. The main messages of this model are also apparent if the penalty of not satisfying the market rules is proportional to the profit obtained. In that case, a firm also chooses to invest less in satisfying the market rules with more competition because it also has less to lose if it is caught not satisfying the market rules.

A straightforward application of this model is in a firm’s production process decision. As we have noted, a firm can decide how much effort to put in its production process (or suppliers) to satisfy the market rules. Another worthwhile application of these ideas would be in the context of deceptive advertising. Suppose that the more a firm uses deceptive advertising, the more profits it makes, but it is also more likely to be caught using deceptive advertising. This then generates the same payoff structure as in the previous model, leading to similar results. If there is more intense competition, firms have less to lose if caught using deceptive advertising and therefore engage more in deceptive advertising (see also Kopalle and Lehmann 2010).

Herein, we have explored the case in which the rules of the market consist of societal “moral standards” or “values”
(the definition of which is not fully clear), whereby a firm can have greater costs of satisfying these market rules. There is some likelihood that a firm will be caught violating one of these market rules, leading to a loss that is proportional to the firm’s profit. The point we have illustrated could also be relevant in settings in which a principal offers an incentive scheme to agents. Steeper incentive schemes could lead agents to invest less in satisfying the rules of the market, which could be interpreted as a cost to the principal. It would be worthwhile for further research to investigate these effects in greater detail.

**TECHNICAL APPENDIX**

**Fixed Costs of Satisfying the Rules of the Market**

Consider a variation of the model in the “Static Symmetric Competition” section, in which investing in satisfying the market rules is a fixed cost and does not affect the marginal costs. Suppose also that the fixed costs of satisfying the market rules are compensated if the firm is found not to satisfy the market rules. The profit function for a firm i is then defined as \( \pi^i(q_i, Q, \gamma_i) = \gamma_i \left( q_i \left( |P(Q) - c| - F(\gamma_i) \right) \right) \), where \( F(\gamma_i) \) are the fixed costs of choosing to satisfy the market rules at the level \( \gamma_i \), with \( F', F'' > 0 \). With any given number of firms in the market, N, we can compute the equilibrium \( q[P(Q) - c] \), which decreases with N and is independent of \( \gamma_i \), because the quantity produced by each firm is dependent only on the marginal costs, which are independent of \( \gamma_i \).

If we write the equilibrium variable profits \( q[P(Q) - c] \) as \( \pi^m(N) \), the first-order condition for \( \gamma_i = F(\gamma_i) + \gamma_i F'(\gamma_i) = \pi^m(N) \). Because the equilibrium firm profits, \( \pi^m(N) \), decrease with the number of firms N, we can immediately observe that a greater number of firms in the market leads to each firm investing less in satisfying the market rules (lower \( \gamma_i \)). As before, a greater number of firms in the market causes firms to be less careful about satisfying the market rules because each firm has less to lose if caught. The social welfare results stated previously would also follow in this case.

**Proof of P2**

For a \( P(Q) \) that is close to linear, we have that \( P' \) is close to a constant and that \( \sum_{j=0}^{N-1} P_j |q_j + q_{j+1} - q| \) is close to \( P(q_i + (N - 1)q_{i+1}) \). Then, totally differentiating Equations 5 and 6 under this approximation with respect to \( \gamma, q, \) and \( N \) at the symmetric equilibrium yields

\[
\frac{d\gamma}{dN} = \frac{\gamma qP^2}{D},
\]

where \( D \) is the Jacobian determinant of a firm’s first-order conditions under the market equilibrium,

\[
D = (c' + c'' \gamma) \left[ 2 + \gamma(N - 1) \right] P' + c'q(1 + \gamma(N - 1))^2 P'' - c'q[N + [(c' + c'' \gamma) \left[ 2 + \gamma(N - 1) \right] P' + c'q(1 + \gamma(N - 1))^2 P' - c'].
\]

For a \( P' \) that is sufficiently close to zero, \( D < 0 \), and we have \( d\gamma/dN < 0 \).

Next, consider the comparison with the case in which firms that are caught violating the market rules can still supply to the market. For a \( P(Q) \) that is linear, note that the two sets of first-order conditions, Equations 1–2 and Equations 5–6, are the same except that \( Q = q \) when all the supply remains in the market and \( Q = q + \gamma q(N - 1) \) when the supply of firms caught not satisfying the market rules is removed from the market (under the symmetric equilibrium). From both sets of conditions, we find that \( q = c(\gamma) + \gamma c'(\gamma) \), which is a positive relationship between \( q \) and \( \gamma \). Next, note that both Equations 2 and 6 represent a negative relationship between \( q \) and \( \gamma \). For the same \( \gamma \), Equation 6 represents a higher \( q \) than does Equation 2; thus, the equilibrium in Equations 5–6 has a greater \( q \) and \( \gamma \) than the equilibrium in Equations 1–2. Finally, note that because the equilibrium \( \gamma \) is greater when the supply of the firms caught not satisfying the market is removed from the market, we observe through Equations 2 and 6 that \( P(Q) \) is greater in equilibrium in this case, which means that the expected quantity is now lower.

**Proof of P7**

Consider the effect on social welfare of increasing \( \Delta \) when \( \Delta = 0 \), under the condition that the firm is choosing the optimal level of \( m \). This latter condition is

\[
\frac{dS}{dm\lambda = 0} = (b_k - 1) \left[ \frac{\partial E(c^\text{min})}{\partial dm} + \frac{NM \partial E(c)}{\partial dm} \right] - NT^{(m^k)} = 0.
\]

Taking the derivative of \( S(m^k, \Delta) \) with respect to \( \Delta \) evaluated at \( \Delta = 0 \), using Equation A1, we calculate the following:

\[
\frac{dS}{d\Delta_{\lambda = 0}} = \left( \frac{\partial E(c^\text{min})}{\partial \Delta} - \frac{1}{N} \frac{\partial E(c^\text{min})}{\partial dm} \right) + \left( \frac{\partial E(c)}{\partial \Delta} - \frac{1}{N} \frac{\partial E(c)}{\partial dm} \right).
\]

Define the margin at the reservation price \( u' = r - c \) (which, as we have noted, is independent of \( \Delta \)) and the general margin as \( u \equiv P - c \). Then we can obtain the cumulative distribution of the margin as \( F(u) = 1 - \left\{ [(u'' - u^2)/u']^2 \right\}^{1/(N - 1)} \), and as before, \( F(u) \), both independent of \( \Delta \).

Next, from Equation 16, we can express \( c \) as a function of \( u \) as

\[
c(u) = u - \frac{g_0}{\beta} + \frac{m^k}{\beta} + \left( \frac{1}{\beta} - \frac{u^2}{w^2} \right) \left( 1 + \frac{M}{\beta} \right) \Delta.
\]

Next, consider the value of \( E(c^\text{min}) \) and \( E(c) \) for an \( M \) that is close to zero. For an \( M \) close to zero, the cumulative distribution \( F(u) \) converges to 1—that is, firms are offering \( u \) close to the lowest value of \( u \) as \( u \rightarrow P - c \). Therefore, we then observe that \( \lim_{M \rightarrow 0} E(c^\text{min}) = \lim_{M \rightarrow 0} E(c) = c(0) \).

Therefore, \( \lim_{M \rightarrow 0} E(c^\text{min})/E(c) = \lim_{M \rightarrow 0} E(c)/E(c) = 1/\beta \).

This then yields this equation in Equation (A2), we find that \( \lim_{M \rightarrow 0} (dS/d\Delta_{\lambda = 0}) = 0 \)—that is, the optimal \( \Delta \) is strictly greater than zero.

**REFERENCES**


