A Stock Return Decomposition Using Observables

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Abstract

We propose a new decomposition approach for stock returns that is based on the sensitivity of the stock price with respect to expected returns and dividends at various horizons. The decomposition does not rely on log-linearization or VAR estimation, and can be implemented at a daily frequency using observable data on the term structure of real rates, the Martin (2017) lower bound of equity risk premia, and dividend futures. We apply our approach to shed light on the evolution of the return on US stocks during the COVID crisis in 2020. The equity risk premium increased sharply in the near term as the crisis intensified in March, contributing 14 percent of the 26 percent market decline up to March 18. The market recovery was heavily influenced by declining real rates even at long maturities, with lower real rates contributing a positive 18 percent to the realized stock return for the year. News about dividends out to 10 years had a modest effect with a larger role for a decline and subsequent recovery of expectations for more distant dividends.

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I. Introduction

A central theme in asset pricing is what news drives fluctuations in asset prices. The standard approach to assessing this is to exploit the Campbell and Shiller (1988) decomposition of unexpected returns into cash flow news and discount rate news. This decomposition is commonly implemented by estimating a vector-autoregressive (VAR) model that includes realized equity returns and predictors of equity returns. A problem with this approach is that results tend to be sensitive to which predictors are included, as shown by Chen and Zhao (2009). Any misspecification of the process for expected returns results in imprecise estimates of not only discount rate news but also cash flow news since the latter is calculated as a residual.

To overcome these issues, we argue that one can get a long way towards a decomposition of unexpected returns into cash flow news and discount rate news without making assumptions about return predictors and without estimating a VAR. The stock price is the present value of expected dividends discounted using the expected return on stocks which in turn equals the real riskless rate plus the equity risk premium. Therefore, in order to decompose unexpected returns into riskless rate news, risk premium news and cash flow news, one needs data on the evolution of the term structures of the real riskless rate, the equity risk premium, and expected dividends.

A lot of information is available about each of these inputs. The term structure of the real riskless rate can be measured out to around 30 years from data on either nominal Treasuries and inflation swaps, or data on inflation-indexed Treasuries (TIPS). The term structure of the equity risk premium is not directly observable but Martin (2017) provides a lower bound on the equity risk premium based on S&P500 index options. He argues that this lower bound is approximately tight and thus is close to the actual equity risk premium.\footnote{We supplement Martin’s analysis with theoretical analysis of how the change in the Martin lower bound relates to the true change in the equity risk premium. In particular, we show that for the CRRA log-normal case, the same parameters that ensure that the lower bound is in fact a lower bound (Martin’s negative correlation condition) also ensure that the change in the lower bound is smaller than the change in the true risk premium. This suggests that our approach will understate the role of risk premium changes for realized returns to the extent that Martin’s lower bound is not tight.} While Martin studies the equity risk premium out to 1 year, this can be extended out to around 2 years in recent years, based on available S&P500 options. If fluctuations in the equity risk premium are concentrated at the short end of the term structure, we can estimate most of the risk premium news based on S&P500 options (with less transitory fluctuations, one can combine this with assumptions about the speed of mean-reversion in risk-premia past year 2). Finally, some information about expected dividends
is available from dividend futures, available out to 10 years. The residual unexpected return not explained by any of the measured components will then capture news about dividends past year 10, as well as any news about the real riskless rate or equity risk premium past the horizons stated.

To implement this idea, we derive a new decomposition of returns that maps more directly to available data than the Campbell-Shiller decomposition (and avoids log-linearization). Result 1 shows that the effect of an instantaneous change to the expected return for year \( t + k \) on today’s stock price can be expressed as a function of one minus the fraction of the stock price paid for dividends out to year \( k \).\(^2\) Result 2 shows that effect of an instantaneous change to expected dividend for year \( t + k \) on today’s stock price. Result 3 combines the above to decompose realized returns into its expected component and the three unexpected components: real risk-free rate news, risk premia news and dividend news.

We use our approach to understand the evolution of the stock market over the COVID crisis in 2020. We provide a decomposition of daily returns and document the cumulative series for each of the return components over the year. The evolution of the US stock market during the COVID crisis in 2020 has been dramatic. Figure 1 graphs the cumulative return on the S&P500 index over the year 2020. The market fell 31 percent from January 1 to March 23, before rebounding sharply. It had full recovered by June 8 and ended the year with a 16 percent annual realized return. Figure 1 also graphs the cumulative return of the contributors of stock market return as set out in Result 3. While the financial press has covered an apparent disconnect between the recovery of the stock market against the continued struggles of the real economy in 2020, our decomposition can go a long way to explaining the realised stock market returns.

The decomposition reveals three key facts. First, the equity risk premium increases sharply until March 18 and had a substantial role in the market crash. We estimate that from the start of the year up to March 18, the equity risk premium for the one-year horizon increased from 2.6% to 15.6%, with further increases in the year-2 risk premium. Together, the increase in the risk premium for the first two years contributed a minus 14.3 percent effect on the stock return up to March 18. A downward sloping equity risk premium During the recovery period, these risk premia decline quickly. An “A-shaped” pattern for the equity risk premium thus helps explain the V-shaped pattern of the stock price. Equity premia remain somewhat higher at the end of 2020.

\(^2\)We map the fraction of the stock price paid for dividends out to year \( k \) to dividend futures. Past the 10-year point, we assume a Gordon growth model and constant expected growth of dividends to estimate the fraction of the stock price paid for dividends out to year \( 10 + k \).
than at the start of the year.

Second, with the exception of an upward spike in long rates from March 9-18, real riskless rates drop dramatically across all maturities and do not recover by the end of the year. The 10-year real riskless rate declines over 100 bps over the year and real forward rates fall even out to the 30-year horizon. The forward real rate from year 21 to 30 drops over 50 bps. For the year 2020, the decline in the term structure of real rates out to year 30 contributes a 18.3 percent increase in the stock market. Evidence from 50-year UK inflation-linked bonds suggests that real rates fell even beyond year 30.

Third, changes to expected dividends out to year 10 have a modest effect on the market, contributing minus 2.5 percentage to the stock return over the year and never more than minus 4.5 percent during the year. This is unsurprising given that the first decade’s dividends generally contribute only about 1/5 of the value of the stock market. More interestingly, we can get a sense of how important changes to expected dividends past year 10 were as these will drive the residual component in our return decomposition after accounting for the expected return, the riskfree rate news component, the equity risk premium news component and the effects of news about dividends out to year 10. We estimate that the more distant dividends contributed about 20 percentage point of the stock market crash but that this effect fully reverted by the end of the year. About 7 percentage points of the reversion occurred in early November following the presidential election and the news about the BioNTech/Pfizer vaccine.

Aside from its link to the long literature on stock-return decomposition, our paper is related to an evolving literature on the stock market during the COVID crash and recovery. Several papers have constructed measures of the cash flow impact and argued that it is difficult to explain the sharp decline in the market in March. Landier and Thesmar (2020) analyze analyst earnings forecasts (up to May 2020). They document that downward revisions occurred smoothly and affected mainly earnings estimates for 2020-2022, with longer-term forecasts remaining stable. Gradual and modest reductions of earnings expectations are inconsistent with the sharp market decline and recovery. Cox et al. (2020) studies the COVID crisis using the estimated structural asset pricing model of Greenwald et al. (2019). They conclude that it is difficult to explain the V-shaped trajectory of the stock market over the COVID crisis with plausible fluctuations in economic activity, corporate profit shares, or short-term interest rates. A central input to their estimation is that, based on data from the Survey of Professional Forecasters, the COVID shock
was expected to be quite transitory based on GDP growth forecasts for 2020:Q2 and 2020:Q3. Gormsen and Koijen (2020) study dividend futures. They show that to explain the decline in the stock market from February 12 to March 12, the value of dividends past year 10 must have declined substantially. Furthermore, during the recovery period up to July 20, they show that the value of near-term (up to year 10) dividends do not recover, implying that price recovery must be due to recovery in the value of distant (past year 10) dividends. Our contributions compared to this literature is to provide a simple return decomposition framework that allows for quantification of each of the components of realized returns using observable data. Compared to Landier and Thesmar (2020) we take the complementary approach of focusing on measuring discount rate news rather than cash flow news. Relative to Cox et al (2020) we avoid the need for a structural model by arguing that many of the inputs to a return decomposition can be estimated directly from available data. Our focus on measuring discount rate news supplements Gormsen and Koijen (2020) in that discount rate news drives the changes in the value of dividends they document.

II. A new stock return decomposition

We derive a new result for how changes in expected stock returns affect the stock price. This result enables a simple decomposition of returns into riskfree rate news, risk premium news and cash flow news components. We compare the new decomposition to the Campbell-Shiller decomposition and argue that the former maps directly to available data and may be more accurate since it does rely on log-linearization around the historical average of the log dividend-price ratio.

A. The effect of expected return and expected dividend changes on the stock price

Start from the present value formula, valuing the stock market as the present value of expected dividends

\[ P_t = \sum_{n=1}^{\infty} \frac{E_t[D_{t+n}]}{1 + R_t^{(n)}} \]

where

\[ 1 + R_t^{(n)} = E_t \prod_{k=1}^{n} (1 + R_{t+k}) \]

and time is in years. \( R_{t+k} \) is the 1-year discount rate for year \( t+k \), i.e., from date \( t+k-1 \) to date \( t+k \). Unless otherwise noted, all dividends and returns are in real terms. We then characterize
the effect of expected return changes on stock returns as follows.

**Result 1 (expected return news and stock returns).**

If \( E_t \prod_{k=1}^n [1 + R_{t+k}] = \prod_{k=1}^n E_t [1 + R_{t+k}] \), then the effect of an instantaneous change to the expected return for year \( t+k \) on the stock return can be expressed as:

\[
\frac{\partial P_t}{P_t} \simeq - \frac{1}{E_t [1 + R_{t+k}]} \left( 1 - \sum_{n=0}^{k-1} w_{n,t} \right)
\]

where

\[
w_{n,t} = \frac{E_t [D_{t+n}] / (1 + R_t^{(n)})}{P_t} = \frac{F_{n,t}}{P_t} / (1 + y_{t,n}^{\text{nom}})^n
\]

for all \( n \)

with \( F_{n,t} \) denoting the date \( t \) price of a dividend future paying the nominal dividends for year \( t+n \) at \( t+n \) and \( y_{t,n}^{\text{nom}} \) is the (annualized) riskless nominal yield at date \( t \) for a \( n \)-year investment.

**Proof:** See appendix.

Result 1 is related to the standard bond pricing formula that relates bond price changes to duration and yield changes. However, in Result 1, expected returns are allowed to differ across future years and we derive the effect of a change to the expected return for one future year. To see the intuition, consider a change in the expected return for year \( t+k \), \( E_t R_{t+k} \), of one percentage point. With the higher discount rate for year \( t+k \), all dividends to be paid at \( t+k \) or later will now be discounted by one percentage point more when we discount back from \( t+k \) to \( t+k-1 \). Therefore, if there were no dividends before date \( t+k \), then \( \frac{\partial P_t}{P_t} / \frac{\partial E_t R_{t+k}}{} \) would simply be -1 (ignoring the term \( \frac{1}{E_t [1 + R_{t+k}]} \)). However, if there are dividends before date \( t+k \), their present value is unaffected by the change in the expected return for year \( t+k \), resulting in a smaller effect of \( E_t R_{t+k} \) on \( P_t \). The factor \( (1 - w_{1,t} - ... - w_{k-1,t}) \) captures the fraction of today’s price \( P_t \) that is due to dividends at date \( t+k \) and later.

The numerator \( E_t D_{t+n} / (1 + R_t^{(n)}) \) in the weight \( w_{n,t} \) is the price at \( t \) of a dividend strip paying \( D_{t+n} \) at \( t+n \). It is well known that dividend strips (which are not traded) can be valued from dividend futures (e.g. van Binsbergen et al. (2013)). Since dividend futures pay off at maturity \((t+n)\), dividend strips and dividend futures prices are related by \( E_t [D_{t+n}] / (1 + R_t^{(n)}) = F_{n,t} / (1 + y_{t,n}^{\text{nom}})^n \). In this expression, \( F_{n,t} \) is nominal (since actual dividend futures contracts pay the nominal dividend) and therefore discounted using the nominal yield.
The assumption used to derive Result 1, \( E_t \prod_{k=1}^{n} [1 + R_{t+k}] = \prod_{k=1}^{n} E_t [1 + R_{t+k}] \), states that realized returns are independent, conditional on information known at date \( t \). Importantly, this does not rule out time-variation in expected returns and expected returns for different maturities can update in a correlated fashion. What needs to hold is that realized returns in one year are not informative for realized returns in another year, conditional on what is known at \( t \). For example, 

\[
E_t [(1 + R_{t+1})(1 + R_{t+2})] = E_t [1 + R_{t+1}] E_t [1 + R_{t+2}] + \text{cov}_t (R_{t+1}, R_{t+2})^{3}
\]

Thus, the assumption holds for horizon \( n=2 \) if \( \text{cov}_t (R_{t+1}, R_{t+2}) = 0 \), i.e., if the distance of \( R_{t+1} \) from its conditional mean is uninformative for the distance of \( R_{t+2} \) from its conditional mean.

We next characterize the effect of expected dividend changes on stock returns as follows.

**Result 2 (dividend news and stock returns).**

The effect of an instantaneous change to the expected dividend for year \( t + k \) on the stock return can be expressed as:

\[
\frac{\partial P_t}{P_t} = \frac{1}{1 + R_t^{(k)}} \frac{\partial E_t D_{t+k}}{
\}
\]

A unit change in expected dividend \( t + k \) has less impact on the aggregate stock market return as the maturity \( k \) of the dividend increases. This is because later cash flows are discounted more, and thus contribute less weight to the overall value of the stock market.

**B. The stock return decomposition**

The expected stock return for year \( t + k \) can be expressed as

\[
E_t R_{t+k} = f_{t+k} + e p_{t+k}
\]

where \( f_{t+k} \) denotes the forward rate for a riskless 1-year investment in year \( t + k \) and \( e p_{t+k} \) denotes the equity risk premium for year \( t + k \). Result 1 holds whether changes to \( E_t R_{t+k} \) are due to

\[
E_t [(1 + R_{t+1})(1 + R_{t+2})] = 1 + E_t [R_{t+1}] + E_t [R_{t+2}] + E_t [R_{t+1} R_{t+2}] \\
= 1 + E_t [R_{t+1}] + E_t [R_{t+2}] + E_t [R_{t+1}] E_t [R_{t+2}] + \text{cov}_t [R_{t+1}, R_{t+2}]
\]
changes in $f_{t+k}$ or $ep_{t+k}$. Therefore, we can use Results 1 and 2 to decompose the capital gain over a one-day period (assumed short enough that the first-order approximation is accurate) as follows

\[
\text{Realized capital gain}_d - \text{Expected capital gain}_d = \sum_{k=1}^{\infty} \frac{\partial P_t}{\partial E_t R_{t+k}} \partial f_{t+k} + \sum_{k=1}^{\infty} \frac{\partial P_t}{\partial E_t R_{t+k}} \partial ep_{t+k} + \sum_{k=1}^{\infty} \frac{\partial P_t}{\partial E_t D_{t+k}} \partial E_t D_{t+k}
\]

The realized return on day $d$ is then

\[
\text{Realized return}_d = \text{Realized capital gain}_d + \text{Realized dividend yield}_d
\]

We then have the following decomposition.

**Result 3 (Realised return decomposition).**

\[
\text{Realized return}_d = \text{Expected return}_d + \sum_{k=1}^{\infty} \frac{\partial P_t}{\partial E_t R_{t+k}} \partial f_{t+k} + \sum_{k=1}^{\infty} \frac{\partial P_t}{\partial E_t R_{t+k}} \partial ep_{t+k} + \sum_{k=1}^{\infty} \frac{\partial P_t}{\partial E_t D_{t+k}} \partial E_t D_{t+k}
\]

In our application to 2020, we implement the return decomposition for each day of 2020 and then aggregate each component across days.

**C. Comparison to the Campbell-Shiller decomposition**

Let $r_t = \ln(1 + R_t)$, $p_t = \ln P_t$, $d_t = \ln D_t$

\[
\begin{align*}
r_{t+1} &= \ln(P_{t+1} + D_{t+1}) - \ln P_t \\
&= \ln \left( P_{t+1} \left[ 1 + \frac{D_{t+1}}{P_{t+1}} \right] \right) - \ln P_t \\
&= p_{t+1} - p_t + \ln (1 + \exp (d_{t+1} - p_{t+1}))
\end{align*}
\]

The Campbell-Shiller decomposition is based on a first-order Taylor approximation of
\[ \ln (1 + \exp (d_{t+1} - p_{t+1})) \] around the historical average value of \( d - p \) (denote this by \( \overline{d - p} \))

\[
\begin{align*}
\ln (1 + \exp (d_{t+1} - p_{t+1})) &= \ln (1 + \exp (d - p)) + \frac{\exp (d - p)}{1 + \exp (d - p)} \left[ d_{t+1} - p_{t+1} - (d - p) \right] \\
&= k + (1 - \rho) [d_{t+1} - p_{t+1}]
\end{align*}
\]

with

\[
\rho = \frac{1}{1 + \exp (d - p)}
\]

\[
k = -\ln \rho - (1 - \rho) \ln \left( \frac{1}{\rho} - 1 \right)
\]

This implies,

\[
r_{t+1} = p_{t+1} - p_t + k + (1 - \rho) [d_{t+1} - p_{t+1}] \implies p_t = \rho p_{t+1} + k + (1 - \rho) d_{t+1} - r_{t+1}
\]

Iterating forward

\[
p_t = \frac{k}{1 - \rho} + \sum_{j=0}^{\infty} \rho^j (1 - \rho) d_{t+1+j} - \sum_{j=0}^{\infty} \rho^j r_{t+1+j}
\]

Therefore,

\[
\frac{\partial p_t}{\partial E_{tR_{t+k}}} = -\rho^{k-1}
\]

(7) is close to our Result 1 since

\[
\frac{\partial P_t}{\partial E_{tR_{t+k}}} = \frac{\partial P_t/P_t}{\partial \ln (1 + E_t R_{t+k})} \frac{\partial \ln (1 + E_t R_{t+k})}{\partial E_t R_{t+k}} = \frac{\partial P_t/P_t}{\partial \ln (1 + E_t R_{t+k})} \frac{1}{1 + E_t R_{t+k}}.
\]

Result 1 therefore implies \( \frac{\partial P_t/P_t}{\partial \ln (1 + E_t R_{t+k})} \simeq -\left( 1 - \sum_{n=0}^{k-1} w_{n,t} \right) \). Like the \( w_t \) weights in Result 1, the \( \rho < 1 \) in the Campbell and Shiller (1988) (and Campbell (1991)) approach captures the fact that the price effect of changes in expected returns in a later period are smaller the more dividends are received before that period. Our Result 1 makes this more transparent than the Campbell-Shiller approach. Furthermore, the \( w_t \) weights map directly to dividend futures at \( t \) as we have laid out, whereas \( \rho \) in the Campbell-Shiller approach is a historical average. Gao and Martin (2020) argue that the Campbell-Shiller log-linearization can be inaccurate when the log price-dividend ratio is
far from its historical average. This issue may be particularly relevant for the year 2020 and the years to come given the COVID recession. One could consider a version of the Campbell-Shiller approach in which the \( \ln (1 + \exp (d_{t+k} - p_{t+k})) \) was log-linearized around \( E_t (d_{t+k} - p_{t+k}) \). Then

\[
\frac{dp_t}{dE_t r_{t+k}} = -\rho_{t+1}\rho_{t+2}\ldots\rho_{t+k-1}
\]

with

\[
\rho_{t+1} = \frac{1}{1 + \exp (E_t (d_{t+1} - p_{t+1}))}, \quad \rho_{t+2} = \frac{1}{1 + \exp (E_t (d_{t+2} - p_{t+2}))}, \ldots
\]

This would be more accurate than the standard Campbell-Shiller approach but because \( d_{t+1} - p_{t+1} \) in \( \rho_{t+1} \) is in logs, \( d_{t+1} \) does not map directly to dividend futures. Furthermore, \( p_{t+1} \) is a future price so implementing \( E_t (d_{t+1} - p_{t+1}) \) would require assumptions about price expectations (similarly for \( \rho_{t+2} \) etc.).

III. Implementation

A. Data

We implement the stock return decomposition in Result 3 for the S&P500 for year 2020. We use forward rates \( f_{t+k} \) calculated from zero-coupon Treasury yields obtained from the Federal Reserve and inflation swaps from Bloomberg. For \( ep_{t+k} \) we use the methodology of Martin (2017) who calculates a lower bound on the risk premium using prices of stock market index options and argues that this lower bound is approximately equal to the true risk premium. Martin’s data covers the period 1996-2012. We extend his series to 2020 using data from OptionMetrics for 2013-2019 and from the CBOE for 2020. We are able to almost exactly replicate Martin’s series over his sample period. Appendix B details our data construction.

For the cash flow news component of stock returns, expected dividends are extracted from dividend futures

\[
E_t D_{t+k} = \frac{1 + R_t^{(k)}}{(1 + y_{n,t}^{\text{nom}})} F_{t+k}
\]  

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4 Gao and Martin (2019) propose a different log-approach and use it to understand the expected growth rate investors must have to be happy hold the market at a given point in time.

5 One would also need a lower bound for expected log returns, as opposed to expected returns, but that is possible based on Gao and Martin (2020).

6 https://www.federalreserve.gov/data/nominal-yield-curve.htm
where discount rates $R_t^{(k)}$ are implied from the risk-free and risk premium data described above. Dividend futures are obtained from Bloomberg and exist out to the 10 year maturity. The effect of changes in expectations of long-dated dividends are therefore not captured by observables and the cash flow news past year 10 is instead calculated as the residual in the stock return decomposition.

In our baseline estimation we assume that riskless forward rates do not change past year 30 and that the equity risk premium does not change past year 5. This is motivated by data availability (more on this below), but we will argue empirically that these horizons will allow us to capture the majority or discount rate news given actual mean-reversion of riskless rates and equity risk premia. Any changes to real rates past year 30 or risk premia past year 5 will also enter the residual in the decomposition.

**B. Dividend Weights**

To calculate the effect of changing discount rates using Result 1, we need daily estimates of $E_t [1 + R_{t+k}]$, and dividend weights, $w_{k,t}$, out to $k = 30$ years. For the discount rates, we use the data for riskless rates and the equity premium as stated, assuming that $e_{p_{t+k}}$ has mean-reverted to equal its pre-crisis average past $k = 5$. For the weights, we can calculate $w_{1,t},...w_{10,t}$ from (2) using available data on dividend futures, zero-coupon Treasury yields and the stock price. To estimate dividend weights past 10 years, we assume a Gordon growth model for dividends past 10 years. The value of long-term dividends is then

$$
L_t = \sum_{k=11}^{\infty} \frac{E_t D_{t+k}}{1 + R_t^{(k)}}
$$

$$
= \frac{E_t D_{t+10}}{1 + R_t^{(10)}} \left( \frac{1 + g_{11}}{1 + R_{t+11}} + \frac{(1 + g_{11})(1 + g_{12})}{(1 + R_{t+11})(1 + R_{t+12})} + \ldots \right)
$$

$$
= \frac{E_t D_{t+10}}{1 + R_t^{(10)}} \left( \frac{1 + g}{R - g} \right)
$$

where $g$ is a constant dividend growth rate and $R$ is a constant forward discount rate. Rearranging this equation, the growth rate of dividends past year 10 is

$$
g = \frac{R - x}{1 + x}
$$
where

\[ x = \frac{F_{n,t}/(1 + y_{n,t}^{\text{nom}})^n}{L_t} \]

is the ratio of the 10 year dividend strip value to the sum of the long-term dividend values. To estimate \( g \), we set the constant forward discount rate equal to the observed year 11 (real) forward discount rate, \( R = R_{t+11} \), and compute the value of long-term dividends as the difference between aggregated stock price and the sum of dividend prices up to 10 years

\[ L_t = P_t - \sum_{k=1}^{10} \frac{F_{t+k}}{1 + y_{k,t}^{\text{nom}}} \cdot \]

Given the above estimated growth rate in dividends past year 10, dividend weights are

\[ w_k = w_{10} \left( \frac{1 + g}{1 + R} \right)^{k-10} \quad \text{for } k > 10. \]

Figure 2 plots the cumulative weight of dividends (averaged over the year) at each dividend maturity. Dividends up to 10 (30) years have averaged a combined weight of 17% (44%) of the total stock market value. These weights are very similar to those extracted by van Binsbergen (2020). We update the weight schedule daily when implementing the stock return decomposition.

C. Estimating changes to equity risk premia

Martin (2017) starts from the fact that the time \( t \) price of a claim to a cash flow \( X_T \) at time \( T \) can either be expressed using the stochastic discount factor \( M_T \) as

\[ \text{Price}_t = E_t(M_T X_T) \]

or using risk-neutral notation as

\[ \text{Price}_t = \frac{1}{R_{f,t}} E_t^*(X_T) \]

where the expectation \( E_t^* \) is defined by

\[ E_t^*(X_T) = E_t(R_{f,t} M_T X_T). \]
The return on an investment can similarly be written in terms of the SDF or using risk-neutral notation

\[
1 = E_t (M_T R_T) \\
= \frac{1}{R_{f,t}} E_t (R_{f,t} M_T R_T) \\
= \frac{1}{R_{f,t}} E_t^* (R_T).
\]

The conditional risk-neutral variance can be expressed as

\[
\text{var}_t^* R_T = E_t^* R_T^2 - (E_t^* R_T)^2 = R_{f,t} E_t (M_T R_T^2) - R_{f,t}^2
\]

The risk premium expressed as a function of the risk-neutral variance is

\[
E_t R_T - R_{f,t} = \left[ E_t \left( M_T R_T^2 \right) - R_{f,t} \right] - \left[ E_t \left( M_T R_T^2 \right) - E_t R_T \right] \\
= \frac{1}{R_{f,t}} \text{var}_t^* R_T - \text{cov}_t (M_T R_T, R_T) \\
\geq \frac{1}{R_{f,t}} \text{var}_t^* R_T \text{ if } \text{cov}_t (M_T R_T, R_T) \leq 0
\]

Thus \( \frac{1}{R_{f,t}} \text{var}_t^* R_T \) provides a lower bound on \( E_t R_T - R_{f,t} \) if \( \text{cov}_t (M_T R_T, R_T) \leq 0 \), denoted the "negative correlation condition" (NCC).

Martin (2017) shows that the lower bound \( \frac{1}{R_{f,t}} \text{var}_t^* R_T \) can be calculated from put and call option as follows

\[
\frac{1}{R_{f,t}} \text{var}_t^* R_T = \frac{2}{S_t^2} \left[ \int_0^{F_{t,T}} \text{put}_{t,T} (K) dK + \int_{F_{t,T}}^{\infty} \text{call}_{t,T} (K) dK \right]
\]

where \( S_t \) is the stock price at \( t \), \( F_{t,T} = E_t^* (S_T) \) is the forward stock price, and \( K \) denotes the strike price. On any date, it is therefore possible to extract a lower bound estimate for each available maturity of expiring options. Consistent with Martin (2017), we use linear interpolation to calculate constant maturity lower bounds, which post 2006 allows estimates out to two years and 6 months.

To account for changes in (forward) equity risk premia past year two, we first run factor analysis on the constant maturity 1, 2, 3, 6, 12, 18, 24 and 30 month equity risk premia, extracting the
first two factors and also the corresponding factor loadings. We then fit the factor loadings as a function of maturity. Guided by the data, for the first factor (on which loadings are all positive) we use a Box–Cox regression, transforming the factor loading $y_i$ and regressing it on maturity $\tau_i$ as follows

$$\frac{y_i^\lambda - 1}{\lambda} = \alpha + \beta \tau_i + \epsilon_i$$

with $\lambda$, $\alpha$ and $\beta$ estimated by maximum likelihood. For the second factor, we estimate the following relation by nonlinear least squares

$$y_i = \alpha + \beta \left( \frac{1 - e^{-\lambda \tau_i}}{\lambda \tau_i} \right) + \epsilon_i$$

The functional form used for the second factor is the same as typically used for the slope factor in the literature on the term structure of riskless rates. With the estimated functional forms of loadings against maturity, we can predict the loadings of longer-dated unobserved risk premia, and finally estimate longer-dated risk premium themselves.

Figure 3 summarises the results of the above factor analysis. Row one presents the time series of the two factors. The factor analysis uses standardized inputs (mean zero, unit standard deviation risk premia). Row two presents loadings on the factors across the observed risk premium maturities (up to 2.5 years). Note that the factor loadings in row two are reminiscent of the loadings on the well known level and slope factors in the interest rate term-structure literature. All maturities load similarly (close to one) on the first level factor, while short (long) maturities loading positively (negatively) on the second slope factor. Factor loadings on the first (most persistent) factor start to fall for the highest observed maturities.

To model non-standardized risk premia, we multiply the factor loadings for a given maturity by the standard deviation of the risk premium for that maturity. These rescaled factor loadings are shown in row 3 and the above-described modeling of factor loadings is done using these as inputs. The figures in row 3 include (solid lines) the predicted values from our factor modeling. For both factors, the estimated functional forms provide a close fit. Row four presents the extrapolated factor loadings up to 10 year maturity. To avoid extrapolating far past the range of available maturities, we only use extrapolated risk premia out to year 5 in the return decomposition.
IV. Relating the true change in the equity premium to the change in the Martin lower bound

A central element of the implementation of our return decomposition is that we use the Martin (2017) methodology to calculate equity risk premium estimates. This is an alternative to estimate a VAR to decompose discount rate news and cash flow news, an approach that is often sensitive to which variables are included and relies on the strong assumption that relations are stable over time. Since our return decomposition relies on changes in the risk premium, it is essential to know how the change in the bound relates to the change in the true risk premium. We therefore state the general condition for when the change in the lower bound is smaller than the true change in the equity risk premium and test this condition in data for 1996-2020 finding supportive evidence.

We supplement this empirical evidence with theoretical analysis for the log-normal case and the CRRA log-normal case. In particular, we show that for the CRRA log-normal case, the same parameters that ensure that the lower bound is in fact a lower bound (Martin’s negative correlation condition) also ensure that the change in the lower bound is smaller than the change in the true risk premium. To the extent the lower bound is not right, our return decomposition will thus tend to understate the role of risk premium changes.

A. The tightness of the Martin lower bound

Martin documents an average lower bound over the 1996-2012 period of about 5%, close to the equity premium estimates obtained by Fama and French (2002) using average realized dividend (or earnings) growth rates as an estimate of ex-ante expected capital gains. Martin also tests whether the lower bound is a good predictor of the realized excess return. He estimates the relation

\[ \frac{1}{T-t} (R_T - R_{f,t}) = \alpha + \beta \times \frac{1}{T-t} \text{var}_t^* R_T \]  

(9)

and cannot reject the null of \( \beta = 1, \alpha = 0 \) for any horizon from 1 month to 1 year.\(^7\)

\(^7\)Martin’s defines a variable \( SVIX^2_{t\to T} = \frac{1}{T-t} \text{var}_t^* \left( \frac{R_T}{R_{f,t}} \right) \) and his regressor is thus expressed as \( R_{f,t} SVIX^2_{t\to T} \).
B. The change in the lower bound

Suppose an underlying state variable $s_t$ changes and that $s_t$ is signed such that $\frac{\partial \left[ \frac{1}{R_{f,t} \text{var}_t^* R_T} \right]}{\partial s_t} > 0$. Then

$$\frac{\partial [E_t R_T - R_{f,t}]}{\partial s_t} = \frac{\partial \left[ \frac{1}{R_{f,t} \text{var}_t^* R_T} \right]}{\partial s_t} - \frac{\partial \text{cov}_t (M_T R_T, R_T)}{\partial s_t} \geq \frac{\partial \left[ \frac{1}{R_{f,t} \text{var}_t^* R_T} \right]}{\partial s_t} \text{ iff } \frac{\partial \text{cov}_t (M_T R_T, R_T)}{\partial s_t} \leq 0$$

It follows that the change in the lower bound is, on average, equal to the true change in the risk premium if the regression coefficient $\beta$ in (9) equals one. If instead $\beta > 1$ that would suggest that the regressor is positively correlated with the omitted variable $-\text{cov}_t (M_T R_T, R_T)$ implying that $\frac{\partial \text{cov}_t (M_T R_T, R_T)}{\partial s_t} < 0$ and thus that the true change in the risk premium is larger than the change in the lower bound. Martin’s regression coefficients are below one for the shortest horizon (1 month) but above one for the 6 month and 1-year horizons, but standard errors are large. We expand Martin’s the sample from 1996-2012 to 1996-2020.

Over the longer sample, we find that $\beta$ is higher than one for most horizons, though not significantly so (see Table 1). We cannot reject that changes in the lower bound are unbiased estimates of changes in the true risk premium. The $\beta$ estimates above one imply that the true risk premium change exceeds that of the change in the lower bound. It is possible, however, that realized excess returns exceeded expected returns over this particular time period, more so in times of stress (high values of the risk-neutral variance). Fama and French (2002) argue that realized returns exceeded expected returns even over a sample as long as 1951-2000. Cieslak, Morse and Vissing-Jorgensen (2019) argue that over the post-1994 period, unexpectedly accommodating monetary policy has contributed to much of the realized excess return on the stock market. If the unexpected positive component of realized returns is sufficiently correlated with risk-neutral variance, then an estimated $\beta$ above one may not imply that changes in the lower bound are conservative. Given the lack of conclusive empirical evidence on this issue, we next consider what theory says about whether it is likely that $\frac{\partial \text{cov}_t (M_T R_T, R_T)}{\partial s_t} \leq 0$. 

15
C. The log-normal case

Assume conditional log-normality as follows:

\[
M_T = e^{-r_{f,t} + \sigma_{M,t} Z_{M,T} - \frac{1}{2} \sigma_{M,t}^2} \\
R_T = e^{\mu_{R,t} + \sigma_{R,t} Z_{R,T} - \frac{1}{2} \sigma_{R,t}^2}
\]

where \( Z_{M,t} \) and \( R_{M,t} \) are (potentially correlated) standard normal random variables.

\( E_t(M_T R_T) = 1 \) implies that

\[
cov_t(M_T R_T, R_T) = E_t(M_T R_T^2) - E(R_T)
\]

Consider each term on the right hand side separately.

\[
\ln E_t(M_T R_T^2) = E_t(\ln M_T + 2 \ln R_T) + \frac{1}{2} V_t(\ln M_T + 2 \ln R_T)
\]

\[
= -r_{f,t} - \frac{1}{2} \sigma_{M,t}^2 + 2 \left( \mu_{R,t} - \frac{1}{2} \sigma_{R,t}^2 \right) + \frac{1}{2} \left( \sigma_{M,t}^2 + 4 \sigma_{R,t}^2 - 4 \left( \mu_{R,t} - r_{f,t} \right) \right)
\]

\[
= r_{f,t} + \sigma_{R,t}^2
\]

\[
\ln E_t(R_T) = E_t(\ln R_T) + \frac{1}{2} V_t(\ln R_T)
\]

\[
= \mu_{R,t}
\]

Combining these two expressions

\[
cov_t(M_T R_T, R_T) = e^{r_{f,t} + \sigma_{R,t}^2} - e^{\mu_{R,t}}
\]

Therefore, the NCC holds iff the conditional Sharpe ratio exceeds the conditional standard deviation:

\[
cov_t(M_T R_T, R_T) \leq 0 \text{ iff } e^{r_{f,t} + \sigma_{R,t}^2} \leq e^{\mu_{R,t}} \iff \sigma_{R,t} \leq \frac{\mu_{R,t} - r_{f,t}}{\sigma_{R,t}}
\]
\( E_t (M_T R_T) = 1 \) furthermore implies that

\[
\ln (E_t (M_T R_T)) = E_t (\ln M_T + \ln R_T) + \frac{1}{2} V_t (\ln M_T + \ln R_T)
\]

\[
= \left( \mu_{R,t} - r_{f,t} - \frac{1}{2} \sigma_{M,t}^2 - \frac{1}{2} \sigma_{R,t}^2 \right) + \frac{1}{2} \left( \sigma_{M,t}^2 + \sigma_{R,t}^2 + 2 \text{cov}_t (\ln R_T, \ln M_T) \right)
\]

\[
= \mu_{R,t} - r_{f,t} + \text{cov}_t (\ln R_T, \ln M_T) = 0
\]

and thus

\[
\mu_{R,t} - r_{f,t} = -\text{cov}_t (\ln R_T, \ln M_T)
\] (10)

The above results for the log-normal case are known from Martin (2017). The following result adds conditions that relate the true change in the risk premium to the change in the lower bound.

**Result 4:** The true change in the risk premium is larger than the change in the lower bound iff

\[
\frac{\partial \text{cov}_t (M_T R_T, R_T)}{\partial s_t} \leq 0.
\]

Under log-normality, it is sufficient for \( \frac{\partial \text{cov}_t (M_T R_T, R_T)}{\partial s_t} \leq 0 \) that

1. The NCC holds: \( \text{cov}_t (M_T R_T, R_T) \leq 0 \iff \frac{\mu_{R,t} - r_{f,t}}{\sigma_{R,t}} \geq 0 \), and
2. \[
\frac{\partial}{\partial s_t} \left[ \frac{\mu_{R,t} - r_{f,t}}{\sigma_{R,t}} \right] \geq \frac{\partial \sigma_{R,t}}{\partial s_t}.
\]

**Proof:** See appendix.

**D. The CRRA log-normal case**

In addition to log-normality, assume CRRA utility,

\[
M_T = \beta \left( \frac{C_T}{C_t} \right)^{-\gamma}
\]

\[
= e^{\ln \beta - \gamma \ln (C_T/C_t)}
\]

with \( \ln (C_T/C_t) \) is normal \( \mu_{c,t}, \sigma_{c,t}^2 \) conditional on information known at \( t \). We can map this assumption to the above log-normal framework

\[
M_T = e^{-r_{f,t} + \sigma_{M,t} Z_{M,T} - \frac{1}{2} \sigma_{M,t}^2}
\]

\[
R_T = e^{\mu_{R,t} + \sigma_{R,t} Z_{R,T} - \frac{1}{2} \sigma_{R,t}^2}
\]
where $Z_{M,t}$ and $R_{M,t}$ are (potentially correlated) standard normal random variables. To link the two assumptions for $M_T$, equate the two expressions:

$$-r_{f,t} + \sigma_{M,t} Z_{M,T} - \frac{1}{2} \sigma^2_{M,t} = \ln \beta - \gamma \ln (C_T/C_t) \tag{11}$$

Calculating the variance of each side of (11) we get

$$\sigma^2_{M,t} = \gamma^2 \sigma^2_{e,t}.$$ 

Taking expectations in (11) implies

$$r_{f,t} = -\ln \beta - \frac{1}{2} \sigma^2_{M,t} + \gamma E_t \ln (C_T/C_t)$$
$$= -\ln \beta - \frac{1}{2} \gamma^2 \sigma^2_{e,t} + \gamma E_t \ln (C_T/C_t).$$

Thus

$$Z_{M,T} = \frac{1}{\sigma_{M,t}} \left[ \ln \beta - \gamma \ln (C_T/C_t) + r_{f,t} + \frac{1}{2} \sigma^2_{M,t} \right]$$
$$= \frac{1}{\gamma \sigma_{e,t}} \left[ \gamma E_t \ln (C_T/C_t) - \gamma \ln (C_T/C_t) \right]$$
$$= \frac{1}{\sigma_{e,t}} \left[ E_t \ln (C_T/C_t) - \ln (C_T/C_t) \right]$$

We can thus exploit (10) to get

$$\mu_{R,t} - r_{f,t} \sigma_{R,t} = -\text{cov}_t (\ln R_T, \ln M_T)$$
$$= \gamma \text{cov}_t (\ln R_T, \ln (C_T/C_t)).$$

This implies,

$$\frac{\mu_{R,t} - r_{f,t}}{\sigma_{R,t}} = \gamma \frac{\text{cov}_t (\ln R_T, \ln (C_T/C_t))}{\sigma^2_{R,t}}$$
$$= \gamma \beta^C_t \sigma_{R,t}$$

where $\beta^C_t$ is the (potentially time-varying) beta of $\ln (C_T/C_t)$ with respect to $\ln R_T$. Thus, $\frac{\mu_{R,t} - r_{f,t}}{\sigma_{R,t}} \geq$
\( \sigma_{R,t} \) (the NCC holds) iff \( \gamma \beta^C_t \geq 1 \). Furthermore,

\[
\frac{\partial}{\partial s_t} \left[ \frac{\mu_{R,t} - r_{f,t}}{\sigma_{R,t}} \right] = \gamma \beta_t \frac{\partial \sigma_{R,t}}{\partial s_t}
\]

This implies that

\[
\frac{\partial}{\partial s_t} \left[ \frac{\mu_{R,t} - r_{f,t}}{\sigma_{R,t}} \right] \geq \frac{\partial \sigma_{R,t}}{\partial s_t} \text{ iff } \gamma \beta_t \geq 1.
\]

Therefore, the same condition that ensures the NCC holds, \( \gamma \beta^C_t \geq 1 \), also ensures that the true change in the risk premium is larger than the change in the lower bound. In the CRRA log-normal case, the NCC is thus sufficient to ensure that the true change in the risk premium is larger than the change in the lower bound. Martin (2017) argues that the NCC is very likely to hold in the log-normal case since the Sharpe ratio based on realized returns has substantially exceeded the realized standard deviation.

V. Empirical Results

A. The risk premium

Figure 4 shows our estimated risk premia (the lower bounds) over 2020, by maturity. All risk premia shown in the figure are annualized. The top graph shows that 5-day risk premia peaked above 100 percent in March. For a one-year horizon, the risk premium increases from around 3 percent at the start of the year to about 15 percent on March 18. Annualized risk premia for longer horizons rise less.

As a supplementary way to describe the term structure of risk premia, Figure 5 graphs the cumulative equity risk premium by maturity for the beginning and end of the year as well as for March 18, the day risk premia peak. Higher annualized risk premia at shorter horizons translate into a concave cumulative equity risk premium. At the peak of the crisis, investors required a risk premium of 4.1 percent to invest for a 30-day period and a risk premium of 15.7 percent to invest over the next year.

Figure 6 illustrates the time series for the (annualized) risk premia for various 6-month periods. The red line in the left graph shows the dramatic increase and subsequent reversal of the risk premium for the month 1-6 horizon. By contrast, the forward equity premium for the subsequent 6-month period increases much less during the initial months of the year, about 2 percentage
points, and stays largely flat after that. The figure to the right compares the series for months 7-12 to that for months 19-24. The latter increases more gradually but also remains higher at the end of the year than at the beginning. We infer from these facts that near-term risk premia increased sharply during the COVID crisis, as they did during the financial crisis as documented in Martin (2017) but that investors standing in March expected a lot of the uncertainty generated by COVID to be resolved over the first six-month period.

Given that there is some increase in the risk premium even for months 19-24 it is likely that risk premia increase to some extent even past this horizon. As described in Section III.C, we therefore use factor analysis to estimate the perceived persistence of risk premium changes from the maturity structure at a given point in time in order to account for changes to the risk premium past year 2. Figure 7 shows (demeaned) estimated forward risk premia out to 5 years maturity. Although longer-dated forward risk premia move less than shorter-date forward risk premium, the five year forward still increased by about 100 basis points over 2020. To avoid issues with over-extrapolation, we assume there is no change in forward risk premia at maturities past five years.

B. The real rate

Figure 8, top graph, shows the evolution of the 10-year and 30-year real rates estimated from nominal Treasuries and inflation swaps. The bottom graph in Figure 8 illustrates the nominal yields and inflation swaps. Real yields fall dramatically over the year, with a 119 bps decline in the 10-year real rate and an 85 bps decline in the 30-year real rate. The decline is interrupted by a sharp spike in real long yields from March 9 to March 18. Vissing-Jørgensen (2020) studies this spike which led to Federal Reserve purchases of over $1T of Treasuries in 2020Q1 in order to stabilize Treasury markets. The spike is associated with heavy selling by bond mutual funds, foreign central banks and hedge funds and reverses as the Federal Reserve increases its daily purchases sharply starting on March 19. It is possible that the March spike in real Treasury yields is disconnected from the stock market in the sense that selling pressure affected Treasury yields without changing investors’ view of the fundamental value of stocks. If so, our riskfree rate news component will overestimate the negative return effect of the spike on realized stock returns and will assign too small a role to declines in dividends past year 10 in explaining the market crash. This issue will not affect our decomposition for the full year, nor our assessment of the role of the
risk premium for the crash, nor our estimate of the role of the real riskless rate outside of this short period of Treasury market dislocations.

Figure 9 seeks to determine whether our assumption of no changes to real rates past year 30 is realistic. We graph the real (annualized) 10-year forward rates for each of the next 3 decades. based on real rates from nominal Treasuries and inflation swaps (top left) or inflation-indexed bonds (top right). The real forward rate for the 3rd decade from now falls over the year, but less than the real forward rates for the first two decades. In the top left graph we illustrate the real forward rate for year 30 separately and even that appears to decline a bit (about 30 bps). It is thus possible that real rates change somewhat even past year 30. In the UK, inflation-indexed bonds are traded with 50-year maturity and as shown in the bottom graph, the real forward rate for years 31-50 falls about 40 bps for the year.

As a robustness check, we have therefore also calculated our main results assuming the change in the year-30 forward real risk-free rate is also the change in all longer-dated forward rates. However, despite the year-30 forward real risk-free rate falling by 33 bps over the year, the effect on the stock return over the year is approximately zero. This counter-intuitive result is due to the denominator in the right hand side of Result 1 not being a constant. If the right hand side tends to be higher on days with positive changes in long real rates than on days with negative changes of long real rates, then the net effect can be small even if long real rates decline overall for the year.

C. Dividends

Figure 10 shows the constant maturity expected dividend 2, 5 and 8 years ahead over the course of 2020. The left hand side figure shows nominal expected dividends and the right hand side shows real expected dividends. The year-2 expected real dividend fell by 36% from January 2nd to its lowest point on 03 April. It ended the year down 8% relative to the start of the year expectation. The moves in longer term dividends follow a similar, but less dramatic pattern. As the first 10 years of dividends make up less than 20% of total stock value, even these large moves in expected dividends have a small impact on the aggregate stock return.
D. Return decomposition results

Figure 1 reports the main result of our return decomposition based on the above-described inputs. The upward spike in risk premia in March generates a negative realized return effect which accounts for minus 14.3 percentage point of the realized return of minus 26 percent up to March 18. Although the risk premia news effect recovers somewhat from the height on crisis, it still ends the year negative. Our baseline specification only uses observed risk premia (to 2 years maturity), and shows that the increases in risk premia over 2020 generated a negative 4 percentage point return. By extrapolating forward risk premia to 5 years, we increase the estimated effect, with risk premia changes over 2020 generating a negative 7 percentage point return.

The fall in real riskless rates up to March 9 contributes a positive effect on realized stock returns as does the fall in the real riskless rates for the year as a whole. We estimate that the decline in the real riskless rates out to year 30 generate a plus 18.3 percent return component for the stock market for the year 2020. Changes to expected dividends out to year 10 play a minor role, consistent with most of the stock market value coming from later dividends. The expected return component contributes about 6 percent to the realized return for the year. We estimate this component from the 1-year real rate and the 1-year risk premium (compounded on a daily basis).

The top plot of Figure 11 presents the implied return from all of our observables combined. The residual (or unexplained) component of stock market return is then presented in the bottom plot. The residual captures the effect of dividends past year 10 and any changes to risk premia past year 2 and real riskless rates past year 30. We therefore call it long-term news. The long-term news component is large, contributing about 20 percentage points to the crash and a roughly equal amount to the recovery.

VI. Conclusion

The paper contributes to answering a core question in asset pricing: what is the role of discount rate news versus cash flow news. We develop a simple return decomposition starting from the observation that a lot can be observed about how the real riskless rate and the risk premium evolves over time, with some additional observable information dividends out to year 10. We apply the decomposition to understand the evolution of the US stock market during the COVID
crisis. Our findings highlight the role of discount rate variation in driving stock market variation. In particular, volatility in short-term (1 year to 2 year) equity risk premia had a substantial role in the market crash and rebound in March and April, while the fall in long-term (1 year to 30 year) real risk-free rates over the year was a key driver of the stock market ending the year with strong positive returns.
References


VII. Figures

Figure 1. Decomposition of the S&P500 index return, 2020.
This figure shows the cumulative return of the S&P500 index in 2020, along with the return contribution from real rate news, risk premium news, dividend news and also the realisation of the expected return. All components are extracted from observables. The effect of real rate news is estimated from changes in the real rates on index-linked government bonds (available up to a maturity of 30 years). The effect of risk premium news is estimated from changes in the Martin (2017) lower bound of equity risk premia (available up to a maturity of 2 years). The effect of dividend news is estimated from changes in the price of dividend futures (available up to a maturity of 10 years).
Figure 2. Cumulative sum of dividend weights by dividend maturity.

This figure shows the cumulative weight of dividend prices relative to the overall stock market price. The cumulative sum weight at each maturity is the average weight across all days in 2020.
Figure 3. Estimating the equity risk premium term structure.

This figure summarises the factor analysis on the term structure of the equity risk premium. Row one presents the time series of the two factors, row 2 presents loadings on the factors across the observed risk premium maturities (up to 2.5 years), row 3 presents estimated loadings across maturities, and row 4 presents the extrapolated factor loadings up to 10 year maturity.
Figure 4. Equity risk premia by maturity (annualized).

This figure shows annualized Martin (2017) equity risk premium estimates over 2020. The estimates are plotted for various observed maturities of the risk premium.
Figure 5. Cumulative equity risk premium.
This figure shows the unannualised Martin (2017) equity risk premium estimates (day 0 to this day). We shot the cumulative risk premium for three separate days within the sample.
Figure 6. Observed forward equity risk premia (annualized).

This figure shows the 6 month risk premium as well as the 6 months forward and 18 months forward 6 month risk premium.
Figure 7. Estimated (demeaned) forward equity risk premia (annualized).
This figure shows the implied 1 year to 5 year forward 1 year risk premium, as estimated by the factor analysis. The left hand side shows the forward rates across the full sample (from 2006) and the right hand side focuses in on 2020.
Figure 8. Real risk-free rate and the underlying components.

The top figure shows the 10 year and 30 year real yield over 2020. The bottom figures show the nominal yield and inflation swap rate (10 years on the left hand side and 30 years on the right hand side).
Figure 9. Real risk-free rate forwards

This figure shows the real rate forwards
Figure 10. Constant maturity expected dividends

These figure shows the constant maturity expected dividends 2, 5 and 8 years from each date. The left and right figures show nominal and real expected dividends respectively.
Figure 11. Stock market return from observables and long-term news.

The top figure shows the cumulative return of the stock market return along with the implied return from our observables: news about real risk-free rates (1y-30yr), equity risk premium (1yr-5yr), expected dividends (1yr-10yr), and the realisation of the expected return. The bottom figure shows the residual stock return that is not explained by our observed variables. We call it long-term news.
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Table I. Risk premium estimate as a predictor variable.
This table reports the parameter estimate from the following time series regression:

$\frac{1}{T-t} (R_T - R_{f,t}) = \alpha + \beta \times R_{f,t} \cdot SVIX_{t \rightarrow T}^2 + \epsilon_T$ together with Newey-West standard errors with lag selection based on the number of overlapping observations. Columns refer to separate estimations with $T-t = 1, 2, 3, 6$ and 12 months respectively. The sample period is 1996-2020.
IX. Appendix

A. Proofs

A.1. Proof of Result 1

Write out the first few terms of the expression for $P_t$

$$P_t = \frac{E_t [D_{t+1}]}{E_t [1 + R_{t+1}]} + \frac{E_t [D_{t+2}]}{E_t [1 + R_{t+1}]} \frac{E_t [1 + R_{t+2}]}{E_t [1 + R_{t+1}]} + \frac{E_t [D_{t+3}]}{E_t [1 + R_{t+1}]} \frac{E_t [1 + R_{t+2}]}{E_t [1 + R_{t+1}]} \frac{E_t [1 + R_{t+3}]}{E_t [1 + R_{t+2}]} + ...$$

The effects of changes in the first three periods’ expected returns are as follows

$$\frac{\partial P_t}{\partial E_t R_{t+1}} = -\frac{1}{E_t [1 + R_{t+1}]}$$

$$\frac{\partial P_t}{\partial E_t R_{t+2}} = -\frac{1}{E_t [1 + R_{t+2}]} \left[ 1 - \frac{E_t D_{t+1}}{E_t [1 + R_{t+1}]} \right] = -\frac{1}{E_t [1 + R_{t+2}]} \left[ 1 - w_{t,1} \right]$$

$$\frac{\partial P_t}{\partial E_t R_{t+3}} = -\frac{1}{E_t [1 + R_{t+3}]} \left[ 1 - \frac{E_t D_{t+1}}{E_t [1 + R_{t+1}]} - \frac{E_t D_{t+2}}{E_t [1 + R_{t+1}]} \frac{E_t [1 + R_{t+2}]}{E_t [1 + R_{t+1}]} \right]$$

where we exploited that

$$P_t = \frac{E_t [D_{t+1} + P_{t+1}]}{E_t [1 + R_{t+1}]} \Leftrightarrow \frac{E_t P_{t+1} / E_t [1 + R_{t+1}]}{P_t} = 1 - \frac{E_t D_{t+1} / E_t [1 + R_{t+1}]}{P_t}$$

and

$$P_t = \frac{E_t [D_{t+1}]}{E_t [1 + R_{t+1}]} + \frac{E_t [D_{t+2} + P_{t+2}]}{E_t [1 + R_{t+1}]} \frac{E_t [1 + R_{t+2}]}{E_t [1 + R_{t+1}]}$$

$$\frac{E_t P_{t+2} / \{E_t [1 + R_{t+1}][1 + R_{t+2}]\}}{P_t} = 1 - \frac{E_t D_{t+1} / E_t [1 + R_{t+1}]}{P_t} - \frac{E_t [D_{t+2} / \{E_t [1 + R_{t+1}][1 + R_{t+2}]\}]}{P_t}.$$

The effect of changes to later expected returns follows from similar steps.

A.2. Proof of Result 4

From (A.1),

$$\text{cov}_t (M_T R_T, R_T) = e^{r_{f,t} + \sigma^2_{R,t}} - e^{\mu_{R,t}}$$

(A.1)

The derivatives with respect to a state variable $s_t$ is

$$\frac{\partial \text{cov}_t (M_T R_T, R_T)}{\partial s_t} = e^{r_{f,t} + \sigma^2_{R,t}} \left[ \frac{\partial r_{f,t}}{\partial s_t} + 2 \sigma_{R,T} \frac{\partial \sigma_{R,t}}{\partial s_t} \right] - e^{\mu_{R,t}} \left[ \frac{\partial \mu_{R,t}}{\partial s_t} \right].$$
If the NCC holds, $\text{cov}_t(M_T R_T, R_T) \leq 0$ and thus $e^{r_{f,t}^*+\sigma_{R,t}^2} \leq e^{\mu_{R,t}}$. Therefore, it is sufficient for $\frac{\partial \text{cov}_t(M_T R_{T,T})}{\partial s_t} \leq 0$ that $\frac{\partial r_{f,t}}{\partial s_t} + 2\sigma_{R,t} \frac{\partial \sigma_{R,t}}{\partial s_t} \leq \frac{\partial \mu_{R,t}}{\partial s_t}$. Rewrite this sufficient condition as follows

$$
\left( \frac{\partial \mu_{R,t}}{\partial s_t} - \frac{\partial r_{f,t}}{\partial s_t} \right) \frac{1}{\sigma_{R,t}} - \frac{\partial \sigma_{R,t}}{\partial s_t} \geq \frac{\partial \sigma_{R,t}}{\partial s_t}
$$

Consider now the change in the conditional Sharpe ratio (for log returns):

$$
\frac{\partial}{\partial s_t} \left[ \frac{\mu_{R,t} - r_{f,t}}{\sigma_{R,t}} \right] = \frac{1}{\sigma_{R,t}^2} \left[ \left( \frac{\partial \mu_{R,t}}{\partial s_t} - \frac{\partial r_{f,t}}{\partial s_t} \right) \sigma_{R,t} - (\mu_{R,t} - r_{f,t}) \frac{\partial \sigma_{R,t}}{\partial s_t} \right]
$$

$$
= \left( \frac{\partial \mu_{R,t}}{\partial s_t} - \frac{\partial r_{f,t}}{\partial s_t} \right) \frac{1}{\sigma_{R,t}} - \frac{(\mu_{R,t} - r_{f,t}) \partial \sigma_{R,t}}{\sigma_{R,t}^2} \frac{\partial \sigma_{R,t}}{\partial s_t}
$$

$$
\geq \left( \frac{\partial \mu_{R,t}}{\partial s_t} - \frac{\partial r_{f,t}}{\partial s_t} \right) \frac{1}{\sigma_{R,t}} - \frac{\partial \sigma_{R,t}}{\partial s_t}
$$

where the last line follows from the fact that $\frac{(\mu_{R,t} - r_{f,t})}{\sigma_{R,t}^2} \geq 1$ under the NCC and we are considering a state variable that increases risk $(\frac{\partial \sigma_{R,t}}{\partial s_t} \geq 0)$. Thus, it is sufficient for $\frac{\partial \text{cov}_t(M_T R_{T,T})}{\partial s_t} \leq 0$ that the change in the conditional Sharpe ratio is at least as large as the change in the conditional standard deviation

$$
\frac{\partial}{\partial s_t} \left[ \frac{\mu_{R,t} - r_{f,t}}{\sigma_{R,t}} \right] \geq \frac{\partial \sigma_{R,t}}{\partial s_t}
$$
B. Estimating the Martin lower bound

For 2020, we use option price data from CBOE to construct a time series of the Martin (2017) lower bound of the equity risk premium. The data includes the trading prices for every traded SP500 Index Option on each day (with intra-day data available), as well as each option’s best bid and ask price, strike price, expiry date. The data also reports the underlying SP500 index price at the time of trade. We clean the initial data in several ways. First we delete all trades with a highest bid or ask of zero. Second, we delete trades where the trade price is greater (lower) than the highest ask (bid). Third, we delete all trades where the underlying index price is missing. Finally, from this selection of cleaned trades, we select the latest traded option for each date, expiry date, strike, option type (call/put) combination.

For each date, expiry date, strike combination in the dataset we then estimate the equity risk premium by discretizing the right-hand side of Martin (2017)’s lower bound given by

\[
\frac{1}{R_{f,t}} \text{var}^* R_T = \frac{2}{S_t^2} \left[ \int_0^{F_{t,T}} \text{put}_{t,T}(K) \, dK + \int_{F_{t,T}}^{\infty} \text{call}_{t,T}(K) \, dK \right].
\]

The forward price \( F_{t,T} \) is the unique solution \( K \) of the equation \( \text{call}_{t,T}(K) = \text{put}_{t,T}(K) \). We estimate the forward price by first interpolating trade prices across strikes for both calls and puts at each date and expiry date combination, and second identifying the intersection of these two interpolated series. We do not use the interpolated prices in discretization of the above equation.

Once equity risk premium estimates have been estimated for each date expiry date combination, we clean the data again. First, we only keep equity risk premium estimates where the number of strikes used in the estimation is greater than 15. Second, we delete estimates when the minimum call or put price is over 40% of the maximum trade price for that date and expiry date combination. These cleaning procedures delete estimates where the discretization is too coarse and where a large tail of options are missing, both of which cause biased estimates.

Finally, we generate constant maturity equity risk premiums by interpolating between those estimated at available expiry dates on any given date.\(^8\) We also extrapolate to extend maturity. However, to avoid over extrapolation, we limit this extrapolation to a maximum of 150 days greater than the longest maturity option available at that date.

\(^8\)As an alternative to interpolation, we have also estimated the equity risk premium yield curve at each date using cubic spline methods. The results are very similar.