Trading, Profits, and Volatility in a Dynamic Information Network Model*

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Abstract

We introduce a dynamic noisy rational expectations model in which information diffuses through a general network of agents. In equilibrium, agents who are more closely connected have more similar period-by-period trades, and an agent's profitability is determined by a centrality measure that is related to eigenvector centrality. Volatility after an information shock is more persistent in less central networks, and volatility and trading volume are also influenced by the network's degree of asymmetry. Using account level data of all portfolio holdings and trades on the Helsinki Stock Exchange between 1997 and 2003, we find support for the aggregate predictions, altogether suggesting that the market's network structure is important for these dynamics.

Keywords: *Information networks, heterogeneous investors, portfolio choice, asset pricing*

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1 Introduction

There is extensive evidence that heterogeneous and decentralized information diffusion influences investors' trading behavior. Shiller and Pound (1989) survey institutional investors in the NYSE, and find that a majority attribute their most recent trades to discussions with peers. Ivković and Weisbenner (2007) find similar evidence for households. Hong, Kubik, and Stein (2004) find that fund managers' portfolio choices are influenced by word-of-mouth communication. Heimer and Simon (2012) find similar influence from on-line communication between retail foreign exchange traders.

Such information diffusion may help explain several fundamental stylized facts of stock markets. First, investors are known to hold vastly different portfolios, in contrast to the prediction of classical models that everyone should hold the market portfolio. The standard explanations for such diverse portfolio holdings are hedging motives and heterogeneous preferences, but several studies indicate that there are limitations to how well such motives can explain observed heterogeneous investor behavior.[1] With decentralized information diffusion, however, it is unsurprising if significantly heterogeneous behavior of investors is observed in the market. Second, stock markets are known to experience large movements that are unrelated to public news, as documented in Cutler, Poterba, and Summers (1989), and Fair (2002). These studies find that over two thirds of major stock market movements cannot be attributed to public news events, suggesting that there are other channels through which information is incorporated into asset prices. Third, the dynamics of trading volume and asset prices are known to be very rich. Returns and trading volume in many markets are heavy-tailed, time varying, have "long memory" in that shocks are very persistent, and are related to each other in a complex way (see Gabaix, Gopikrishnan, Plerou, and Stanley 2003; Karpoff 1987; Gallant, Rossi, and Tauchen 1992; Bollerslev and Jubinski 1999; Lobato and Velasco 2000). Lumpy information diffusion provides a potential explanation for such behavior. In periods when more information diffuses, volatility is higher, as is trading volume (see, Clark 1973; Epps and Epps 1976; Andersen 1996). To generate rich trading volume dynamics, however, such information diffusion must necessarily be heterogeneous.

In this paper, we follow a recent strand of literature that uses information networks to model information diffusion in the market (see Colla and Mele 2010, Ozsoylev and Walden 2011, Han and Yang 2013, and Ozsoylev, Walden, Yavuz, and Bildik 2014). Agents who are directly linked in a network share information, which consequently diffuses among the population over time in a well-specified manner. This literature has made important observations about the effects of information networks, but several key questions remain open. How does the network structure in a market determine the dynamic trading behavior of its agents, and their performance? How does the network structure influence aggregate returns and trading volume? What does heterogeneous information diffusion "add" compared with, e.g., what can can be generated by heterogeneous preferences alone? The answers to

[1]For example, Massa and Simonov (2006) find that hedging motives for human capital risk—a fundamental source of individual investor risk—does not explain heterogeneous investment behavior among individual investors well. Similarly, Calvet, Campbell, and Sodini (2007, 2009) find that diversification and portfolio rebalancing motives do not explain investors' portfolio holdings well.

these questions, which we analyze in this paper, are obviously fundamental for our understanding of the impact of information networks on financial markets.

We introduce a dynamic noisy rational expectations model in which agents in a network share information with their neighbors. Agents receive private noisy signals about the unknown value of an asset in stochastic supply, and trade in a market over multiple time periods. In each period, they share all the information they have received up until that point with their direct neighbors, leading to gradual diffusion of the private signals. The structure of the network is completely general.

As a first contribution, we prove the existence of a noisy rational expectations equilibrium, and present closed-form expressions for all variables of interest. Theorem 1 provides the main existence and characterization result for a Walrasian equilibrium in a large network economy. To define the large economy equilibrium, we use the concept of replica networks, assuming that that there exists a 'local' network structure and that there are many similar such local network structures in the economy. The locality may be interpreted literally; each network could, e.g., represent a municipality in a country. It may also represent some other way of dividing the agents in an economy into groups, e.g., into a large number of nationwide sports communities (recreational sports leagues), interest groups, movie clubs, etc. This replica economy approach allows for a clean characterization of equilibrium, as well as justifies the assumption that agents act as price takers and are willing to share information. We know of no other network model of information diffusion in a centralized financial market (i.e., exchange) that allows for a complete characterization of equilibrium, that is completely general with respect to network structure, and that is based on first principles of financial economics. We believe that the introduction of such a work-horse model is valuable in itself, by allowing for further study of the general relation between information networks and asset pricing.

The structure of the network is crucial in determining asset pricing dynamics. We show in a simple example that price informativeness and volatility at any given point in time does not only depend on the specific information agents in the network have obtained at that point, but also on how the information has diffused through the network. The equilibrium outcome thus depends on complex properties of the network, beyond the mere precision of agents' signals at any specific point in time.

We next study how the network structure determines the trading behavior and profitability of agents. We show that within the model's setting, the correlation of period-by-period agent trades is positively related to agent proximity, justifying using short-horizon similarity of trades for network identification. This result provides our second contribution.

As a third contribution, we study what determines who makes profits in the network. It is argued in Ozsoylev and Walden (2011) and elsewhere in the literature that some type of centrality measure should determine agent profitability.[2] Centrality—a fundamental concept in network theory—captures

[2] In an empirical study, Ozsoylev, Walden, Yavuz, and Bildik (2014) study the trades of all investors on the Istanbul Stock Exchange in 2005, and find a positive relationship between investors' so-called eigenvector centrality and profitability, but this choice of centrality measure is not theoretically justified. Several other finance papers discuss and use various centrality measures (several different centrality measures exist) without a complete theoretical justification, see e.g., Das and Sisk (2005), Adamic, Brunetti, Harris, and Kirilenko (2010), Li and Schurhoff (2012) and Buraschi and Porchia (2012).

the concept that it is not only who your direct neighbors are that matters, but also who your neighbors' neighbors are, who your neighbors' neighbors' neighbors are, etc. The argument is that agents who are centrally placed tend to receive information signals early, and therefore perform better in the market than peripheral agents, who tend to receive information later. It is a priori unclear, however, what is the appropriate definition of centrality in this context.

We show that profits are determined by a profitability centrality measure that is related to eigenvector centrality, but that also has important differences. To the best of our knowledge, this is the first complete characterization of the relationship between agent centrality and performance in a general information network model of financial markets. A major difference compared with eigenvector centrality, for which the relative importance of connections at different distances is determined mechanically by an eigenvalue, is that for the profitability centrality measure the relative importance is determined by the speed at which information is incorporated into asset prices over time, which in turn is an equilibrium property of the model. Our main results that characterize trading behavior, profitability, and welfare of agents are Theorems 2 and 3.

Our fourth contribution is to derive and analyze several aggregate results regarding the dynamic behavior of price volatility and trading volume in the model. Specifically, in line with the Mixture of Distributions Hypothesis, the information diffusion process generated by the network determines volatility and trading volume in the time series. Volatility after an information shock is more persistent in less central networks, whereas variation of volatility is higher in more asymmetric networks. Similar results hold for trading volume.

We test these aggregate predictions empirically. Using account level data of all portfolio holdings and trades on the Helsinki Stock Exchange between 1998-2003, we construct monthly ownership networks for 74 stocks over 72 months, using geographical closeness as a proxy for network proximity. We find support for the prediction that network centrality matters for persistence of volatility shocks, both statistically and economically. We also find some support for that network asymmetry is related to variation in volatility. This provides our fifth contribution. Altogether, our results provide initial support for that network structure matters for aggregate dynamics.

The rest of the paper is organized as follows. In the next section, we discuss related literature. In Section 3, we introduce the model and characterize equilibrium. In Section 4, we analyze trading behavior and profitability of individual agents. In Section 5, we study the implications of network structure for aggregate volatility and trading volume, and in Section 6, we test the predicted relationships for network centrality and asymmetry, with persistence and variation of volatility. Finally, Section 7 concludes. All proofs are delegated to the appendix.

2 Related Literature

Our paper is most closely related to the recent strand of literature that studies the effects of information diffusion on trading and asset prices. Colla and Mele (2010) show that the correlation of trades among agents in a network varies with distance, so that close agents naturally have positively correlated trades, whereas the correlation may be negative between agents who are far apart. Their model is dynamic, and assumes a very specific symmetric network structure, namely a circle, where each agent has exactly two neighbors. This restricts the type of dynamics that can arise in their model. Ozsoylev and Walden (2011) introduce a static rational expectations model that allows for general network structures and study, among other things, how price volatility varies with network structure. Their model is not appropriate for studying dynamic information diffusion, however, and is therefore not well-suited for several of the questions analyzed in this paper, e.g., the relationship between agent profitability and centrality, and the short-term correlation between agents' trade. Manela (2014) analyzes how the speed of information diffusion affects the welfare of agents, showing that the value is hump-shaped in the diffusion speed. Again, the diffusion process is quite specific.

Han and Yang (2013) study the effects of information diffusion on information acquisition. They show that in equilibrium, information diffusion may reduce the amount of aggregate information acquisition, and therefore also the informational efficiency and liquidity in the market. Their model is also static, and does thereby not allow for dynamic effects. In an empirical study, Ozsoylev, Walden, Yavuz, and Bildik (2014) test the relationship between centrality—constructed from the realized trades of all investors in the market—and profitability. They find that more central agents, as measured by eigenvector centrality, are more profitable. However, they do not justify this choice of centrality measure theoretically. Pareek (2012) studies how information networks—proxied by the commonality in stock holdings–among mutual funds is related to return momentum.

A different strand of literature studies information diffusion through so-called information percolation (Duffie and Manso 2007, Duffie, Malamud, and Manso 2009). In the original setting, a large number of agents meet randomly in a bilateral decentralized (OTC) market and share information, and the distribution of beliefs over time can then be strongly characterized. Recently, the model has been adapted to centralized markets, with exchange traded assets and observable prices—a setting more closely related to ours. Andrei (2012), shows that persistent price volatility can arise in such a model, and Andrei and Cujean (2014) analyze momentum and reversal in a similar setting. In contrast to our model, in which some agents may be better positioned than others, these models are ex ante symmetric in that all agents have the same chance of meeting and sharing information.

Babus and Kondor (2016) also introduce a model of information diffusion in a bilateral OTC market. As in our paper, their network can be perfectly general. In contrast to our model, there is no centralized information aggregation mechanism in their setting, in that there is no price observable by all agents (except for in the case of a complete network in which all agents are connected). Moreover, agents have private values in their model and do not observe private signals, and their model is static.

This paper is also related to the literature on information diffusion and trading volume (Clark 1973). Lumpy information diffusion was suggested to explain heavy-tailed unconditional volatility of asset prices, as an alternative to the stable Paretian hypothesis. Under the Mixture of Distributions Hypothesis (MDH), lumpiness in the arrival of information leads to variation in return volatility and trading volume, as well as to a positive relation between the two (see Epps and Epps 1976 and Andersen 1996). Foster and Viswanathan (1995) build upon this intuition to develop a model with endogenous information acquisition, leading to a positive autocorrelation of trading volume over time. Similar results arise in He and Wang (1995), in a model where an infinite number of ex ante identical agents receive noisy signals about an asset's fundamental value. Admati and Pfleiderer (1988) explain U-shaped intra-daily trading volume in a model with endogenous information acquisition.

Our paper further explores the richness of the dynamics of volatility and volume that arises when agents share their signals, allowing for completely general asymmetry in how some agents are better positioned than others. This extension may potentially shed further light on the very rich dynamics of volatility and volume, and the relationship between the two (see Karpoff 1987, Gallant, Rossi, and Tauchen 1992, Bollerslev and Jubinski 1999, Lobato and Velasco 2000, and references therein). A related strand of literature explores the role of trading volume in providing further information to investors about the market, see Blume, Easley, and O'Hara (1994), Schneider (2009), and Breon-Drish (2010). Our model does not explore this potential informational role of trading volume.

Our study is related to the large literature on games on networks, see the survey of Jackson and Zenou (2012). The games in these models are typically not directly adaptable to a finance setting. Specifically, the mapping between risk averse agents' joint actions in the market for risky assets and their equilibrium utility is significantly more complex in this setting than what is generally assumed in that literature. Our existence result and the characterization of equilibrium in a model based on first principles of financial economics are therefore of interest. Since the welfare of agents in equilibrium can be simply characterized, our model could potentially also be used to study endogenous network formation, see Jackson (2005) for a survey of this literature.

Finally, our paper is related to the (vast) general literature on asset pricing with heterogeneous information (see, e.g., the seminal papers by Grossman 1976, Hellwig 1980, Kyle 1985, and Glosten and Milgrom 1985). Technically, we build upon the model in Vives (1995), who introduces a multi-period noisy rational expectations model in a similar spirit as the static model in Hellwig (1980). Like Vives, we assume the presence of a risk-neutral competitive market maker, to facilitate the analysis in a dynamic setting. This simplifies the characterization of equilibrium considerably. Unlike Vives, we allow for information diffusion among agents, through general network structures. Specifically, in contrast to Vives (1995) the number of signals received varies with agent and over time, depending on the network's structure.

3 Model

There are N agents, enumerated by $a \in \mathcal{N} = \{1, \dots, N\}$, in a $T+1$-period economy, $t = 0, \dots, T+1$, where $T \geq 2$. We define $\mathcal{T} = \{1, \dots, T\}$. Each agent, a, maximizes expected utility of terminal wealth, and has constant absolute risk aversion (CARA) preferences with risk aversion coefficient γ_a,

$a = 1, \ldots, N$,

$$U_a = E[-e^{-\gamma_a W_{a,T+1}}].$$

We summarize agents' risk aversion coefficients in the N-vector $\Gamma = (\gamma_1, \ldots, \gamma_N)$.

There is one asset with terminal value $v = \bar{v} + \eta$, where $\eta \sim N(0, \sigma_v^2)$, i.e., the value is normally distributed with mean $\bar{v}$ and variance σ_v^2. Here, $\bar{v}$ is known by all agents, whereas η is unobservable.

Agents are connected in a network, represented by a graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$. The relation $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ describes which agents (vertices, nodes) are connected in the network. Specifically, $(a, a') \in \mathcal{E}$, if and only if there is a connection (edge, link) between agent a and a'. We will subsequently assume that there are many identical "replica" copies of this network in the economy, each copy representing a "local" network structure. This will make the economy "large" and justify price taking behavior of agents, as well as simplify the characterization of equilibrium. For the time being, we focus on one representative copy of this large network.

We use the convention that each agent is connected to himself, $(a, a) \in \mathcal{E}$ for all $a \in \mathcal{N}$, i.e., $\mathcal{E}$ is reflexive. We also assume that connections are bidirectional, i.e., that $\mathcal{E}$ is symmetric. A convenient representation of the network is by the *adjacency matrix* $E \in \{0, 1\}^{N \times N}$, with $(E)_{aa'} = 1$ if $(a, a') \in \mathcal{E}$ and $(E)_{aa'} = 0$ otherwise.

The distance function $D(a, a')$ defines the number of edges in the shortest path between agents a and a'. We use the conventions that $D(a, a) = 0$, and that $D(a, a') = \infty$ whenever there is no path between a and a'. The set of direct neighbors to agent a is $S_{a,1} = \{a' : (a, a') \in \mathcal{E}\}$. Moreover, the set of agents at distance $m > 1$ from agent a is $S_{a,m} = \{a' : D(a, a') = m\}$, and the set of agents at distance not further away than m is $R_{a,m} = \cup_{j=1}^{m} S_{a,m}$. The number of agents at a distance not further away than m from agent a is $V_{a,m} = |R_{a,m}|$. Here, $V_{a,1}$ is the *degree* of agent a, which we also refer to as agent a's *connectedness*, whereas $V_{a.m}$ is agent a's *mth order degree*. We use the convention that $V_{a,0} = 0$ for all a. We also define $\Delta V_{a,m} = |S_{a,m}|$.

We define N-vectors V^m, $m = 1, \ldots, N$, where the ath element of V^m is $V_{a,m}$ Equivalently,

Definition 1 *The mth order degree vector, $V^m \in \mathbb{R}_+^N$, $m = 1, 2, \ldots,$ is defined as*

$$V^m = \chi(E^m)\mathbf{1}. \tag{1}$$

Here E^m is the mth power of the adjacency matrix, and $\chi : \mathbb{R}^{N \times N} \to \{0, 1\}^{N \times N}$ is a matrix indicator function, such that $(\chi(A))_{i,j} = 0$ if $A_{i,j} = 0$ and $(\chi(A))_{i,j} = 1$ otherwise. Moreover, $\mathbf{1}$ is an N-vector of ones.

First order degree is commonly referred to as *degree centrality.*

Finally, the number of agents within a distance of m from both agents a and a' is $V_{a,a',m} = |R_{a,m} \cap R_{a',m}|$, and the number of neighbors at distance exactly m from both agents is $\Delta V_{a,a',m} = |S_{a,m} \cap S_{a',m}|$

3.1 Information diffusion

At $t = 0$, each agent receives a noisy signal about the asset's value, $s_a = v + \sigma\xi_a$, where $\xi_a \sim N(0,1)$ are jointly independent across agents, and independent of v. At $T+1$, the true value of the asset, v, is revealed. It will be convenient to use the precisions $\tau_v = \sigma_v^{-2}$ and $\tau = \sigma^{-2}$.

The graph, $\mathcal{G}$, determines how agents share information with each other. Specifically, at $t+1$, agent a shares all signals he has received up until t with all his neighbors. We let $\mathcal{I}_{a,t}$ denote the information set that agent a has received up until t, either directly or via his network.

It is natural to ask why agents would voluntarily reveal valuable information to their neighbors. Of course, in a large economy with an infinite number of agents, sharing signals with ones' (finite number of) neighbors has no cost, since the actions of a finite number of agents will not influence prices. In such a market, agents are *informationally negligible.* Even in an economy in which agents are informationally nonnegligible, in that the information an individual shares may impact prices, as long as signals can be verified ex post, truthful information revelation may be optimal in a repeated game setting, since an agent who provides misinformation can be punished by his neighbors, e.g., by being excluded from the network in the future. Even if signals are not ex post verifiable, an agent may still draw inferences about the truthfulness of another agent's signal, by comparing it with other received signals. Again, the threat of future exclusion from the network could be used to enforce truthful information sharing.

In a market with informationally nonnegligible agents, information sharing will be more difficult to sustain in equilibrium. Indeed, an agent will trade off the short-term gains he makes by sharing misleading information with the costs of not receiving future information from his connections when punished by exclusion from the network. Such a framework will lead to potentially interesting restrictions on the networks that are feasible in equilibrium. Moreover, the framework is potentially more tractable than a fully strategic model in which agents also impact market outcomes through their direct trades. In this paper, we take the truthful information sharing behavior of agents as given, but we view the extension of the model to include markets in which agents are informationally nonnegligible as a fruitful area for future research.

As in Ozsoylev, Walden, Yavuz, and Bildik (2014), we formalize the information sharing role of the network by defining

Definition 2 *The graph $\mathcal{G}$ represents an* information network *over the signal structure $\{s_a\}_a$, if for all agents $a \in \mathcal{N}$, $a' \in \mathcal{N}$ and times $t = 1, \ldots, T$, $s_{a'} \in \mathcal{I}_{a,t}$ if and only if $D(a, a') \leq t$.*

The information about the asset's value that an agent has received through the network up until time t can be summarized (as we shall see) by the sufficient statistic

$$z_{a,t} \stackrel{\text{def}}{=} \frac{1}{V_{a,t}} \sum_{j \in R_{a,t}} s_j = v + \zeta_{a,t},$$

where $\zeta_{a,t} = \frac{\sigma}{\sqrt{V_{a,t}}}\xi_{a,t}$, and $\xi_{a,t} \sim N(0,1)$.[3] The number of signals agent a receives at t is $\Delta V_{a,t}$, and we therefore expect $\{\Delta V_{a,t}\}_{a\in\mathcal{N},t\in\mathcal{T}}$ to be important for the dynamics of the economy.

3.2 Market

The market is open between $t = 1$ and $T + 1$. Agents in the information network submit limit orders, and a risk-neutral competitive market maker sets the price such that at each point in time it reflects all publicly available information, $p_t = E_t[v|\mathcal{I}_t^p]$, where $\mathcal{I}_t^p$ is the time-t publicly available information set. In line with earlier literature, as expanded upon in the appendix, the publicly available information at time t consists of aggregate order flows up until t, which in equilibrium will be equivalent to observing prices up until t. At $T + 1$, the asset's value is revealed so $p_{T+1} = v$. Before trading begins, the price is set as the asset's ex ante expected value, $p_0 = \bar{v}$.

To avoid fully revealing prices, we make the standard assumption of stochastic supply of the asset. Specifically, in period t, noise traders submit market orders of u_t per trader in the network, where $u_t \sim N(0, \sigma_u^2)$. These shocks are jointly independently distributed, and are independent of agent signals and the asset's value. In other words, the noise trader demand is defined relative to the size of the population in the information network. As argued elsewhere in the literature, the noise trader assumption needs not be taken literally, but is rather a reduced-form representation of unmodeled supply shocks. It could, e.g., represent hedging demand among investors due to unobservable wealth shocks, or other unexpected liquidity shocks. We do not further elaborate on the sources of these shocks. We will use the precision $\tau_u = \sigma_u^{-2}$.

Agents in the network are price takers. At each point in time they submit limit orders to optimize their expected utility of terminal wealth. They thus condition their demand on contemporaneous public information, as well as on their private information. An agent's total demand for the asset at time t is

$$x_{a,t} = \arg\max_x E\left[e^{-\gamma_a W_{a,T+1}}|\bar{\mathcal{I}}_{a,t}\right], \tag{2}$$

subject to the budget constraint

$$W_{a,t+1} = W_{a,t} + x_{a,t}(p_{t+1} - p_t), \qquad t = 1, \ldots, T,$$

and his net time-t demand is $\Delta x_{a,t} = x_{a,t} - x_{a,t-1}$, with the convention that $x_{a,0} = 0$ for all agents. Here, $\bar{\mathcal{I}}_{a,t}$ contains all public and private information available to agent a at time t.

In the linear equilibrium we study, $z_{a,t}$ and p_t are jointly sufficient statistics for an agent's information set, $\bar{\mathcal{I}}_{a,t} = \{z_{a,t}, p_t\}$, leading to the functional form $x_{a,t} = x_{a,t}(z_{a,t}, p_t)$. Of course, an agent's optimal time-t strategy in (2) depends on the (optimal) future strategy. The dynamic problem can therefore be solved by backward induction. The primitives of the economy are summarized by the

[3] A variation is to let agents receive new private signals in each time period. The analysis in this case is qualitatively similar, but not as clean because of the increased number of signals.

tuple $\mathcal{M} = (\mathcal{G}, \Gamma, \tau, \tau_u, \tau_v, \bar{v}, T)$.

We note that the assumption that the asset's value is revealed at $T+1$ means that any residual uncertainty at T of the asset's value is completely mitigated at $T+1$. We think of this as public information, which becomes available to all agents at $T+1$. Alternatively, we could have assumed that residual uncertainty is gradually incorporated into the market between T and T' for some $T' > T+1$, keeping the assumption that the information diffusion between agents in the network only occurs until T.[4]

The graph, $\mathcal{G}$, determines how information diffuses in the network over time, whereas Γ captures agent preferences. We wish to separate dynamics that can be generated solely by heterogeneity in preferences from those that require heterogeneity in network structure. To this end, we define an economy to be *preference symmetric* if $\gamma_a = \gamma$ for all agents, and some constant $\gamma > 0$. There are several symmetry concepts for graphs. The notion we use is so-called distance transitivity.[5] Informally, symmetry captures the idea that any two vertices can be switched without the network changing its structure. To formalize the concept, we define an automorphism on a graph to be a bijection on the vertices of the graph, $f : \mathcal{N} \leftrightarrow \mathcal{N}$, such that $(f(a), f(a')) \in \mathcal{E}$ if and only if $(a, a') \in \mathcal{E}$. A graph is distance-transitive if for every quadruple of vertices, a, a', b, and b', such that $D(a, b) = D(a', b')$, there is an automorphism, f, such that $f(a) = a'$ and $f(b) = b'$. An economy is said to be *network symmetric* if its graph is distance-transitive. Preference symmetric economies and network symmetric economies provide useful benchmarks to which the general class of economies can be compared. Especially, models with symmetric information structures typically fall into the class of network symmetric economies (e.g., the model in Vives (1995)). The observed heterogeneous behavior and performance of investors in the market, as discussed in the introduction, suggests that asymmetry is important in practice.

We point out that network symmetry does *not* imply that the same amount of information is diffused among agents at each point in time. It does, however, still impose severe restrictions on how information may spread in the economy, as shown by the following lemmas:[6]

Lemma 1 *In a network symmetric economy, $\Delta V_{a,t}$ is the same for all agents at each point in time. That is, for each t, for each a, $\Delta V_{a,t} = \Delta V_t$ for some common ΔV_t.*

Thus, in a network symmetric economy, all agents have an equal precision of information at any point in time, although their signal realizations differ. This follows from the linear filtering properties of normal distributions.

[4] Specifically, it would be straightforward to extend the model to allow for gradual revelation of the asset's value between $T+1$ and $T+K$, for some $K > 1$. In the case when all private information has already diffused at T, the analysis would become especially tractable since no private trading motive exist beyond T in that case.

[5] Other notions include vertex transitivity, distance regularity, arc-transitivity, t-transitivity, and strong regularity, see Briggs (1993). Distance transitivity is a stronger concept than vertex transitivity, arc-transitivity, and distance regularity, respectively, but neither stronger, nor weaker, than t-transitivity and strong regularity.

[6] The first result follows immediately from the fact that automorphisms preserve distances between nodes, see Briggs (1993), page 118. The second result follows from Taylor and Levingston (1978), where the larger class of distance regular graphs i analyzed (see also Brouwer, Cohen, and Neumaier 1989, page 167).

Lemma 2 *In a network symmetric economy the sequence $\Delta V_1, \Delta V_2, \ldots, \Delta V_T$, is unimodal. Specifically, there is a time $1 \leq s \leq T$, such that $\Delta V_{t+1} \geq \Delta V_t$ for all $t \leq s$, and $\Delta V_{t+1} \leq \Delta V_t$ for all $t > s$.*

In other words, the typical behavior of the information diffusion process in a network symmetric economy is "hump-shaped," initially increasing, until it reaches a maximum and then decreasing. This is similar to the S-shaped learning curve obtained in information percolation models with random matching, see, e.g., Amador and Weill (2006) and Fogli and Veldkamp (2011). In the network symmetric setting, agents typically initially receive signals from a limited number of other agents in their immediate neighborhood, but over time this number increases drastically as they receive signals from a larger number of agents farther away, until eventually the whole network is exhausted and information diffusion comes to a halt. The process is therefore hump-shaped. As we shall see, for asymmetric networks the information diffusion process can be quite different.

3.3 Replica network

To justify the assumption that agents are price takers and to avoid issues with strategic information sharing, the number of agents needs to be large. Moreover, as analyzed in Ozsoylev and Walden (2011), restrictions on the distribution of number of connections agents have are needed, to ensure existence of equilibrium. Ozsoylev and Walden (2011) carry out a fairly general analysis of the restrictions needed for the existence of equilibrium to be guaranteed. They show that a sufficient condition is that the distribution of number of connections is not too fat-tailed. Compared with their static model, our model has the additional property of being dynamic. Therefore, not only would restrictions on first-order connections be needed to ensure the existence of equilibrium, but also on connections of all higher orders. In the dynamic economy, signals spread over longer distances, thereby "fattening" the tail of the distribution of signals among agents over time. We therefore believe that a general analysis would be technically challenging, while adding limited additional economic insight, which is why we choose the simplified approach.

We build on the concept of replica economies, originally introduced by Edgeworth (1881) to study the game theoretic core of an economy (see also Debreu and Scarf 1964). We assume that the full economy consists of a large number, M, of disjoint identical replicas of the network previously introduced, and that agents' random signals are independent across these replicas. We then let M tend to infinity. A replica network approach provides the economic and technical advantages of a large economy in a mathematically rigorous way, namely that price taking behavior is rationalized and that the law of large numbers makes most idiosyncratic signals cancel out in aggregate, while avoiding the issues of signals spreading too quickly among some agents, causing equilibrium to break down. An intuitively equivalent approach would be to assume a continuum of finite, identical, local networks, again justifying the price taking behavior of individual agents. We restrict our attention to linear equilibria in which agents in the same position in different replica networks are (distributionally) identical. Such equilibria are thus characterized by the behavior of agents $a = 1, \ldots, N$, who are "representative." Further details are provided in Appendix A

3.4 Equilibrium

Our main existence result is the following theorem, that shows existence of a linear equilibrium in the large economy under general conditions and, furthermore, characterizes this equilibrium.

Theorem 1 *Consider an economy characterized by $\mathcal{M}$. For $t = 1, \ldots, T$, define*

$$\begin{aligned}
A_t &= \frac{\tau}{N}\sum_{a=1}^{N}\frac{V_{a,t}}{\gamma_a}, \\
y_t &= \tau_u(A_t - A_{t-1})^2, \\
Y_t &= \sum_{s=1}^{t} y_s, \\
C_{a,t} &= \left(\frac{\tau_v + \tau V_{a,t+1} + Y_{t+1}}{\tau_v + \tau V_{a,t} + Y_t}\right)\left(\frac{\tau_v + Y_t}{\tau_v + Y_{t+1}}\right)\left(1 + \tau V_{a,t}\left(\frac{1}{\tau_v + Y_t} - \frac{1}{\tau_v + Y_{t+1}}\right)\right), \\
D_{a,t} &= \prod_{s=t+1}^{T} C_{a,s}^{-1/2},
\end{aligned}$$

with the convention that $A_0 = 0$, and $Y_{T+1} = \infty$. There is a unique linear equilibrium, in which prices at time t are given by

$$p_t = \frac{\tau_v}{\tau_v + Y_t}\bar{v} + \frac{Y_t}{\tau_v + Y_t}v + \frac{\tau_u}{\tau_v + Y_t}\sum_{s=1}^{t}(A_s - A_{s-1})u_s. \tag{3}$$

In equilibrium, agent a's time-t demand and expected utility, given wealth $W_{a,t}$ and the realization of signals summarized by $z_{a,t}$, take the form

$$x_{a,t} = \frac{\tau V_{a,t}}{\gamma_a}(z_{a,t} - p_t), \tag{4}$$

$$U_{a,t} = -D_{a,t}e^{-\gamma_a W_{a,t} - \frac{1}{2}\frac{\tau^2 V_{a,t}^2}{\tau_v + Y_t + \tau V_{a,t}}(z_{a,t} - p_t)^2}. \tag{5}$$

Several observations are in place. First, note that the price function (3) has a fairly standard structure. It is determined by the fundamental value (v) and the aggregate supply shocks (u_s, $s = 1, \ldots, t$). The weights on these different components are determined by how signals spread through the network. Especially, A_t summarizes how aggressive—and thereby informative—the trades of agents are at time t, consisting of a weighted average of t-degree connectivity of all agents. The variable Y_t corresponds to a cumulative average of squared innovations in A up until time t, and determines how much of the fundamental value is revealed in the price. The main generalization compared with Vives (1995) is that $V_{a,t}$ varies with agent and over time, depending on the network structure. Moreover, preferences are allowed to vary across agents, through γ_a. This allows us to compare the equilibrium dynamics that may arise because of heterogeneous preferences with the dynamics that may arise because of heterogeneous information diffusion.

It is notable that Y_t does not only depend on the total amount of information that has been diffused through the network at time t, but also on *how* this information has diffused over time. In other words the price at a specific point in time is information path dependent. For example, consider two economies with 4 agents, all with unit risk aversion ($\gamma_a = 1$), and with parameters $\tau = \tau_v = \tau_u = 1$. The first network, shown in panel A of Figure 1, is tight-knit (it is even complete) with every agent being directly connected to every other agent. It is straightforward to calculate $V_{1,a} = V_{2,a} = 4$, $A_1 = A_2 = 4$, $Y_1 = Y_2 = 16$, via (3) leading to $p_2 - v \sim N\left(0, \frac{1}{17}\right)$. The second network, shown in Panel B of Figure 1, is not as tightly knit, and agents have to wait until $t = 2$ before they have received all signals. In the latter case, $V_{1,a} = 3$, $V_{2,a} = 4$, $A_1 = 3$, $A_2 = 4$, $Y_1 = 9$, $Y_2 = 10$, via (3) leading to $p_2 - v \sim N\left(0, \frac{1}{11}\right)$. Thus, the price at $t = 2$ is less revealing in the second case, even though all agents in the network have received the same information at $t = 2$ in both economies. The reason is that in the tight-knit economy, the information revelation is more lumpy, whereas it is more gradual in the less tight-knit economy. Lumpy information diffusion leads to more revealing prices, since it generates more aggressive trading behavior in some periods, in turn making it easier for the market maker to separate informed trading from supply shocks. This is our first example of how the network structure impacts asset price dynamics.

We define

$$h_t = \frac{1}{N}\sum_{a=1}^{N}\left(\frac{\bar{\gamma}}{\gamma_a}\right)\Delta V_{a,t}, \qquad t = 1, \ldots, T, \tag{6}$$

which measures the (weighted) average number of agents at distance t from any agent in the network, or simply time-t “agent distance.” Here, $\bar{\gamma}$ is the (harmonic) average risk aversion coefficient of agents in the economy, $\bar{\gamma} = (\sum_a \gamma_a^{-1})^{-1}$, and weights are chosen such that agents with lower risk aversion get higher weights, which is natural since they will be more active in the market. With this definition, we can write

$$y_t = \frac{\tau_u \tau^2}{\bar{\gamma}^2} h_t^2, \tag{7}$$

showing that y_t is closely related to this average.

4 Trading, profits, and centrality of individual agents

We study how the trades and performance of agents are determined by their positions in the network.

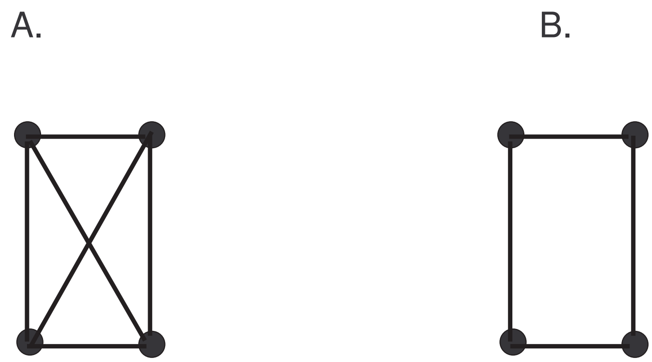

Figure 1: Impact of network structure. *The figure shows two networks with four agents: In Panel A, a tight-knit network is shown, in which every agent is connected with every other agent. In panel B, a less tight-knit network is shown. At $t = 2$, prices are more revealing in the tight-knit network, since the aggregate information arrival has been more lumpy, in turn leading to more revealing trading behavior of informed agents.*

4.1 Correlation of trades

The following theorem characterizes covariances of trades for an individual agent over time, and between agents at a specific point in time.

Theorem 2 *The covariance between an agent's trade at t and $t+1$ is*

$$Cov(\Delta x_{a,t+1}, \Delta x_{a,t}) = \frac{\tau^2}{\gamma_a^2}\Delta V_{a,t}\left(\frac{V_{a,t+1}}{\tau_v + Y_{t+1}} - \frac{V_{a,t}}{\tau_v + Y_t}\right). \tag{8}$$

The covariance of agent a and b's trades at time t is

$$Cov(\Delta x_{a,t}, \Delta x_{b,t}) = \frac{\tau^2}{\gamma_a \gamma_b}\left(\frac{y_t}{(\tau_v + Y_{t-1})(\tau_v + Y_t)}V_{a,t-1}V_{b,t-1} + \frac{\Delta V_{a,t}\Delta V_{b,t}}{\tau_v + Y_t} + \Delta V_{a,b,t}\right). \tag{9}$$

Here, $\Delta V_{a,b,t}$ represents the number of agents who are at the distance t from both agent a and b.

Equation (9), shows how the time-t trades of two agents are related. The first two terms in the expression represent covariance induced by the fact that two informed agents will tend to trade in the same direction because, both being informed, they will take a similar stand on whether the asset is over-priced or under-priced. This part of the expression increases in the total amount of information the agents have received at $t-1$ (through $V_{a,t-1}V_{b,t-1}$), as well as in how much additional information they expect to receive between $t-1$ and t (through $\Delta V_{a,t}\Delta V_{b,t}$). Offsetting these effects is the aggregate informativeness of the market, through the terms $\frac{y_t}{(\tau_v+Y_{t-1})(\tau_v+Y_t)}$ and $\frac{1}{\tau_v+Y_t}$. The third term in the expression provides an additional positive boost to the covariance, and is increasing in the number of common agents at distance t of both agent a and b. This term is zero if the agents are further apart than a distance of $2t$, but will otherwise typically be positive. The term captures the natural intuition that agents who receive identical information signals have more similar trades than agents who receive signals with independent error terms.

An implication of (9) is that the period-by-period covariance between agents' trades is always strictly positive, consistent with previous literature.[7] Ozsoylev, Walden, Yavuz, and Bildik (2014), used the positive relationship between trades over short horizons to reverse engineer a proxy of the information network in the Istanbul Stock Exchange from individual investor trades. Loosely speaking, agents who repeatedly traded in the same stock, in the same direction, at similar points in time, were assumed to be linked in the market's information network.

The fact that trades between close neighbors are always positively correlated is nontrivial in the dynamic setting. Specifically, one may a priori expect a positive relationship between network proximity and portfolio holdings, since agents who are close in the network have many overlapping signals and thereby similar information, leading to similar portfolio holdings. However, for period-by-period trading behavior, the timing of information arrival is important in the dynamic model, and this timing is different even for agents who are close. It is easy to envision a situation where one agent gets a positive signal with a delay from a neighboring agent, and ramps up his investment when the other agent who traded earlier ramps down to realize profits, suggesting negatively correlated trades in that period.

To understand why negatively correlated trades do not arise, we use (4) to rewrite agent a's time-t demand as

$$\Delta x_{a,t} = \frac{\tau}{\gamma_a}\left(\Delta V_{a,t}\left(\frac{\sum_{j\in\Delta S_{a,t}} s_j}{\Delta V_{a,t}} - p_t\right) - V_{a,t}(p_t - p_{t-1})\right).$$

The first term in this expression represents the agent's demand because of additional information received between $t-1$ and t. We note that $\sum_{j\in\Delta S_{a,t}} s_j/\Delta V_{a,t} = v + \zeta^a$, where the error term $\zeta^a \sim N(0, \sigma^2/\Delta V_{a,t})$ is independent of prices. The second term represents the agent's downward sloping demand curve, causing him to rebalance portfolio holdings when the price catches up, by selling (buying) stocks when the price increases (decreases) between $t-1$ and t. For an agent who has an information advantage at time $t-1$, but receives no new information between $t-1$ and t, this second term is the only one present (since $\Delta V_{a,t} = 0$). Now, agent b's demand function has the same form as agent a's and, assuming that agent b receives a lot of new information between $t-1$ and t, the first term dominates. Negative correlation would then arise if agent b tends to ramp up when agent a ramps down, which is the case if $Cov\left(v + \zeta^b - p_t, -(p_t - p_{t-1})\right) < 0$. However, since the market is semi-strong form efficient, $v - p_t$ is independent of $p_t - p_{t-1}$. Furthermore, since ζ^b is independent of aggregate variables, $Cov\left(\zeta^b, -(p_t - p_{t-1})\right) = 0$. In other words, since agent a's rebalancing demand between $t-1$ and t is publicly known at t, it must be independent of agent b's time-t demand which in turn is due to

[7] Feng and Seasholes (2004) studied retail investors in the People's Republic of China, and found that the geographical position of investors was related to the correlation of their trades: geographically close investors had more positively correlated trades than investors who were farther apart. Colla and Mele (2010) showed that information networks can give rise to such patterns of trade correlations, under the assumption that geographically close agents are also close in the information network. Their analysis was restricted to a cyclical network, but the effect was also shown to arise in general networks in the static model of Ozsoylev and Walden (2011).

informational advantage at time t.

Our result is model dependent. It depends on the linear structure of agents' demand functions. However, to a first order approximation we expect the result to hold in more general settings in semi-strong form efficient markets, given that trading for rebalancing purposes mainly depends on price changes, trading for informational purposes depends on the difference between the true value and market price, and the two terms are uncorrelated in a weak-form efficient market.

We stress that this result on the positive period-by-period correlation between agent trades is relevant because it suggests that trade correlation can be used to draw inferences about network proximity in a dynamic setting, not because it allows the network model to be distinguished from other models of trade behavior. Indeed, we would expect other models to lead to similar predictions.

4.2 Profits and centrality

Who makes profits in an information network? Our starting point is the following theorem:

Theorem 3 *Define*

$$\begin{aligned} \pi_{a,t} &= \tau V_{a,t}\left(\frac{1}{\tau_v + Y_t} - \frac{1}{\tau_v + Y_{t+1}}\right), \qquad t = 1, \ldots, T-1,\ a = 1, \ldots, N, \\ \pi_{a,T} &= \tau \frac{V_{a,T}}{\tau_v + Y_T}, \qquad a = 1, \ldots, N. \end{aligned}$$

The ex ante certainty equivalent of agent a is

$$U_a = \frac{1}{2\gamma_a}\log(C), \qquad \textit{where } C = \prod_{t=1}^{T}(1 + \pi_{a,t}). \tag{10}$$

The expected profit of agent a is

$$\frac{\tau}{\gamma_a}\Pi_a, \tag{11}$$

where

$$\Pi_a = \sum_{t=1}^{T}(\tau_v + Y_t)^{-1}V_{a,t} \tag{12}$$

is the profitability *of agent a.*

Our focus in this study is on profitability, but we have also stated the formula for welfare, to allow for future research on endogenous network formation. Equation (11) determines (ex ante) expected profits of an agent. It shows that expected profits depends on three components. First, profits are inversely proportional to an agent's risk-aversion, γ_a, because more risk-averse agents take on smaller positions—all else equal. This follows immediately, since an agent's equilibrium trading position is proportional to γ_a, so it corresponds to pure scaling. Therefore, we do not include it in our measure of profitability, as defined by Equation (12). Neither do we include the signal precision, τ, which is constant across agents. Second, expected profits depend on an agent's position in the network through $\{V_{a,t}\}_t$, $t \in \mathcal{T}$: the higher any given $V_{a,t}$ is, the higher the agent's expected profits. Third, expected profits depend inversely on the amount of aggregate information available in the market, in that at any given point in time, the higher the total amount of aggregate information, the lower the expected profits of any given agent. The third part represents a negative externality of information. Equation (12) thus provides a direct relationship between the properties of a network, local as well as aggregate, and individual agents' profitability.

Equation (12) shows that an agent's profitability is determined by his *centrality*, appropriately defined. Recall that $V_{a,t}$ denotes the number of agents that are within distance t from agent a. So, $V_{a,1}$ is simply the degree of agent a. For $t > 1$, higher order connections are also important in determining profitability. For example, $V_{a,2}$ does not only depend on how connected agent a is, but also on how connected his neighbors are. We use (1) to rewrite (12) on vector form as

$$\Pi = \sum_{t=1}^{\infty} \beta_t \chi(E^t)\mathbf{1}, \tag{13}$$

where $\beta_t = (\tau_v + Y_t)^{-1}$, for $t \in \mathcal{T}$, and $\beta_t = 0$ for $t > T$, is a measure of the degree of price discovery that has occurred in the market up until t. Here, Π is an N-vector where the ath element is the profitability of agent a. The functional form of (13) is close to the standard Katz and eigenvector centrality measures. Recall that the *Katz centrality* vector with parameter α is the vector $K \in \mathbb{R}_+^N$, defined as

$$K = K^\alpha = \sum_{t=1}^{\infty} \alpha^t E^t \mathbf{1}. \tag{14}$$

Here, E^t denotes the t:th power of the adjacency matrix, E, and $\mathbf{1} \in \mathbb{R}^N$ is an N-vector of ones. Moreover, the *eigenvector centrality* vector is the eigenvector corresponding to the largest eigenvalue of E, i.e., the vector C that solves the equation $C = \lambda E C$, for the largest possible eigenvalue, λ, where we normalize C such that $\sum_{a\in\mathcal{N}} C_a = 1$. It is a standard result that eigenvector centrality can be viewed as a special case of Katz centrality, since $C \propto K^{\lambda^{-1}}$ in the sense that $C = \lim_{\alpha \nearrow \lambda^{-1}} \frac{K^\alpha}{\sum_a K_a^\alpha}$.[8]

The structures of the profitability measure (13) and equation (14) are similar. They are both made up by a weighted sum of powers of the adjacency matrix, multiplied with the vector of ones. The main difference is that the weighting is a power of α for Katz and eigenvector centrality but varies more generally with t for profitability. Importantly, both expressions highlight that an agent's centrality depends on direct as well as higher-order connections, in contrast to the degree measure which only measures direct connections. But whereas the variation over time only depends on powers of α for

[8]Uniqueness of the eigenvector centrality measure is not guaranteed, but is almost never an issue in practice.

Katz centrality, it depends on the equilibrium variable β_t for profitability. Specifically, the contribution to an agent's profitability is high in a period if the agent has received many signals while aggregate price discovery has been limited (Y_t is low, implying that β_t is high). Centrality measures with fixed α will not capture this equilibrium driven component of profitability, but we have verified in (unreported) numerical simulations that Katz and eigenvector centrality usually serve as good proxies for profitability. An example of an economy in which the measures would *not* be good proxies for profitability is one in which there is an "information bottleneck" so that there is very little price discovery for $k > 1$ periods, but then massive discovery in period $k+1$. In such an economy, $\beta_1, \ldots, \beta_k$ are large whereas $\beta_{k+1}, \beta_{k+2}, \ldots$ are small, a pattern that is not possible to replicate well with a constant α. We use (13) as the definition of *profitability centrality*—or just centrality—within our model.

There are several straightforward variations and extensions of the model's information diffusion mechanism. First, we assume that the information shock arrives at $t = 0$, but an arbitrary, even random, arrival time would lead to similar dynamics, as long as when the shock arrives, agents know it. Allowing for multiple information shocks to arrive over time is also possible, and especially straightforward if these shocks are sufficiently far apart in time, say by a time period $T' > T$, so that a shock diffuses completely before the next arrives. In this case, the dynamics are repeated every time a new shock arrives, and the profitability expression (13) can be written as $\Pi = \sum_{t=1}^{T} \hat{\beta}_t \chi(E^t)\mathbf{1}$, where the information shocks arrive at $t_1, t_2, \ldots$, and the new coefficients, $\hat{\beta}_t$ "collapse" the total effect of all information shocks, $\hat{\beta}_1 = \sum_k \beta_{t_k}, \hat{\beta}_2 = \sum_k \beta_{t_{k+1}}$ etc. It is also straightforward to allow the network to change over time, for example by having agents adding and severing links in-between—and even during—the diffusion of different information events. In this case, E^t needs to be adjusted to account for this dynamic network evolution. The interpretation is especially straightforward when the network changes between the arrival of information events, but not during the subsequent information diffusion period. In this case network centrality can be defined event-wise and the profitability for an agent during an event will depend on his centrality when the shock arrives.

5 Aggregate volatility and trading volume

After expected returns, return volatility is the second most studied property of asset prices. According to the Mixture of Distributions Hypothesis (MDH), return volatility varies over time, which leads to heavy-tailed unconditional return distributions. A common explanation for such time varying volatility—as well as trading volume—is lumpy diffusion of information into the market (Clark 1973; Epps and Epps 1976; Andersen 1996).

Rich dynamics of volatility and trading volume have indeed been documented in the literature. Especially, return volatility of individual stocks, and markets are positively autocorrelated over extended periods. In other words, volatility shocks are persistent, see Bollerslev and Jubinski (1999) and Lobato and Velasco (2000). Similar results also holds for shocks to trading volume. Also, volume and volatility are related (Karpoff 1987; Crouch 1975, Rogalski 1978), in line with the Wall Street wisdom that it takes volume to move markets. Contemporaneously, trading volume and absolute price change are highly positively correlated. The two series also have positive lagged cross-correlations. Using a semi-parametric approach to study returns and trading volume on the NYSE, Gallant, Rossi, and Tauchen (1992) show that large price movements predict large trading volume. In other markets, there is evidence for a reverse casualty, i.e., that large trading volume leads large price movements. For example, Saatcioglu and Starks (1998) find such evidence in several Latin American equity markets.

In our model, agents' preferences (Γ) and the network structure ($\mathcal{E}$) determine volatility and volume dynamics in the market, which we next explore.

5.1 Volatility

In a general network economy, we would expect price volatility to vary substantially over time. For example, information diffusion may initially be quite limited, with low price volatility as an effect, but eventually reach a hub in the network at which point substantial information revelation occurs with associated high price volatility. The following result characterizes the price volatility over time, and moreover shows that any volatility structure can be supported in a general economy.

Theorem 4 *For $t = 1, \cdots, T$, the variance of prices between $t-1$ and t, is*

$$\sigma_{p,t}^2 = \frac{y_t}{(\tau_v + Y_t)(\tau_v + Y_{t-1})}, \tag{15}$$

where we use the convention that $Y_0 = 0$, and between T and $T+1$, it is

$$\sigma_{p,T+1}^2 = \frac{1}{\tau_v + Y_T}. \tag{16}$$

Moreover, given coefficients, $k_1, \ldots, k_{T+1}$, such that $k_t > 0$ and $\sum_{t=1}^{T+1} k_t = 1$, and an arbitrarily small $\epsilon > 0$, there is a preference symmetric economy, such that

$$\left| \sigma_{p,t}^2 - \frac{k_t}{\tau_v} \right| \leq \epsilon, \qquad t = 1, \ldots, T+1. \tag{17}$$

From the first part of the theorem, we see that the volatility (the square root of variance) has a general decreasing trend over time because of the increasing denominator in (15), but that it can still have spikes in some time periods because of large values in the numerator. In fact, (17) shows that any structure of time-varying volatility after an information shock can be generated. The proof of Theorem 4 is constructive, but the network constructed in the proof bears little resemblance with the networks that we expect to see in practice, suggesting that volatility dynamics alone would not be sufficient to allow the identification of a market's network.

Note that the total cumulative variance up until time $t \leq T$ is

$$\sigma_{P,t}^2 = \frac{1}{\tau_v} \frac{Y_t}{\tau_v + Y_t} = \frac{1}{\tau_v} \frac{1}{1 + \frac{\tau_v}{Y_t}}. \tag{18}$$

This part of the variance that is incorporated until T represents the "information diffusion" component of asset dynamics, whereas the part between T and $T+1$, i.e., (16), represents the component due to public information sources, in line with the discussion in Section 3.2.

It is clear from (18) that volatility after an information shock will be more persistent in economies with lower Y_t for all times $t = 1, \ldots, T$, in that at each t there is more residual volatility left to be incorporated into prices, the lower is Y_t. Specifically, the remaining variance at time t is $\sum_{s=t+1}^{T+1} \sigma_{p,t}^2 = \sigma_v^2 - \sigma_{P,t}^2$.[9] If the remaining variance after shock A is (weakly) higher than after shock B at each point in time t, i.e., if $\sigma_{P,t}^A \leq \sigma_{P,t}^B$ for all t, then shock A is said to be more persistent than shock B.

We use this fact to link the persistence of volatility to the centrality of the network. Specifically, recall that h_t describes the (weighted) average number of nodes at distance t from any node in the network as shown in (6). It is reasonable to call a network with sequence h_t more central that one with sequence h'_t, $t = 1, \ldots, T$, if $h_t > h'_t$ for all t, since the average number of nodes at distance t is higher in the former network than in the latter, for *all* $t \leq T$.[10]

From (7) and (18), and the fact that if the network with sequence h_t is more central than that with sequence h'_t, then $Y_t > Y'_t$ for all t, it immediately follows that

Corollary 1 *Volatility shocks are more persistent in less central networks than in more central networks, all else equal.*

Corollary 1 thus provides a direct link between the persistence of volatility and network structure, a link that we will explore in Section 6.

If we restrict our attention to network symmetric economies, the possible dynamic behavior of volatility is quite restricted. This is not surprising, given the restrictions on information diffusion dynamics described in Lemmas 1 and 2. From (18), it follows that y_t is proportional to $\Delta\eta_t = \eta_t - \eta_{t-1}$, where $\eta_t = \frac{1}{\sigma_v^2 - \sigma_{P,t}^2}$ is the inverse remaining variance. Since y_t is proportional to h_t^2, and therefore to ΔV_t^2 which is the same for all agents in a network symmetric economy in this case, since ΔV_t is unimodal, and since the square of a nonnegative unimodal function is also unimodal, it follows that the sequence $\Delta\eta$ is unimodal.

[9] As discussed in Ozsoylev, Walden, Yavuz, and Bildik (2014), such information shocks could, for example, consist of substantiated rumors that a company executive will step down from his/her position, that a product launch will be delayed because of technological challenges, a company's executives' meetings with officials of another company over a multi-day visit during which a sequence of statements about deepening ties are made that to a varying degree are reported in the local and national press, etc. The important thing is that the information spreads over time across the population, in contrast to an announcement that reaches all agents at the same time and thereby leads to a very short volatility spike. The information event itself may be anticipated (like an earnings announcement), or unanticipated.

[10] Note that this concept, which provides a measure of the average centrality of agents in the network, is distinct from network *centralization*, which measures the difference between the most and least central nodes in a network.

Moreover, consider the case when the public information component is large compared with the information received from diffusion, i.e., $\tau << \tau_v$, implying that $Y_T << \tau_v$. It then follows from (7) and (15) that

$$\sigma_{p,t}^2 \approx \frac{\tau_u \tau^2}{\tau_v^2 \bar{\gamma}} h_t^2, \tag{19}$$

so in this case with a network symmetric economy, $\sigma_{p,t}$ is also unimodal. More generally, (19) provides a quantifiable measure of how network asymmetry affects volatility, in that if we define network asymmetry by the network's variation of h_t over time, i.e., by $Var(h_t)$, it follows that when the public information component is large, a more asymmetric network implies a higher volatility of volatility (vol-vol), all else equal. We summarize these results in

Corollary 2

1. *In a network symmetric economy, the change in inverse remaining variance, $\Delta\eta_t$, is unimodal.*
2. *In a network symmetric economy with a high public information component, volatility, $\sigma_{p,t}$, is unimodal.*
3. *In economies with high public information components, more asymmetric networks have higher volatility of $\sigma_{p,t}$.*

5.2 Volume

Just like with volatility, rich dynamics of trading volume can arise within the network model. Aggregate trading volume will be made up by the heterogeneous trades of many different agents. This contrasts to the uniform behavior in models with a representative informed agent (e.g., Kyle 1985), as well as to the ex ante symmetric behavior in economies with symmetric information structures (e.g., Vives 1995; He and Wang 1995).

We focus on the aggregate period-by-period trading volume of agents in the network, since the stochastic supply is (quite trivially) normally distributed. To this end, we define:

Definition 3 *The time-t aggregate (realized) trading volume is $W_t = \frac{1}{N}\sum_a |\Delta x_{a,t}|$, and the (ex ante) expected trading volume is $X_t = E[W_t]$.*

The following theorem characterizes the expected trading volume, and mirrors our volatility results by showing that any pattern of expected trading volume can be supported in the model.

Theorem 5 *The time-t expected trading volume is*

$$X_t \quad = \quad \frac{\tau}{N}\sum_{a=1}^{N}\frac{1}{\gamma_a}\sqrt{\frac{2}{\pi}\left(\frac{V_{a,t-1}^2}{\tau_v+Y_{t-1}}-\frac{V_{a,t}^2}{\tau_v+Y_t}+2\frac{\Delta V_{a,t}V_{a,t}}{\tau_v+Y_t}+\frac{\Delta V_{a,t}}{\tau}\right)}. \tag{20}$$

Given positive coefficients, $c_1, c_2, \ldots, c_{T+1}$, and any $\epsilon > 0$, there is an economy such that

$$|X_t - c_t| \leq \epsilon, \qquad t = 1, \ldots, T+1.$$

It is clear from (20) that, as is the case for volatility, heterogeneous preferences alone cannot generate such complete generality of trading volume dynamics. In a network symmetric economy, all terms under the square root are identical across agents, and (20) collapses to

$$X_t \quad = \quad \sqrt{\frac{2\tau^2}{\pi\bar{\gamma}^2}\left(\frac{V_{t-1}^2}{\tau_v+Y_{t-1}}-\frac{V_t^2}{\tau_v+Y_t}+2\frac{\Delta V_t V_t}{\tau_v+Y_t}+\frac{\Delta V_t}{\tau}\right)}. \tag{21}$$

Again, differences in preferences in this case are only important through the effect they have on the average risk aversion coefficient, $\bar{\gamma}$. Moreover, the restrictions on ΔV_t imposed by network symmetry carry over to trading volume. It is easily seen that if the public information component is large compared with the diffusion component, the fourth term under the square root in (21) dominates and $X_t \approx \sqrt{\frac{2\tau}{\pi\bar{\gamma}^2}\Delta V_t}$. In this case X_t is therefore unimodal.

If the information network is not completely symmetric, but only moderately asymmetric in that $\Delta V_{a,t} \approx C(1+\omega_{a,t})$ for some constant C, where $|\omega_{a,t}| << 1$, which in turn implies that h_t has little variation, then, given that the public information component is high ($\tau << \tau_v$), it follows from (6) and (20), and a Taylor expansion of the square root function that

$$X_t \approx \sqrt{\frac{2\tau}{\pi}} \times \frac{1}{N}\sum_{a=1}^{N}\frac{1}{\gamma_a}\sqrt{\Delta V_{a,t}} \approx \sqrt{\frac{2\tau C}{\pi\bar{\gamma}^2}} \times \frac{1}{N}\sum_{a=1}^{N}\frac{\bar{\gamma}}{\gamma_a}\left(\frac{1}{2}+\frac{1}{2}(1+\omega_{a,t})\right) \stackrel{\text{def}}{=} \alpha + \beta h_t, \tag{22}$$

where $\alpha, \beta > 0$. For such moderately asymmetric networks, there is thus an approximately linear positive relationship between X_t and h_t.

We summarize these results in

Corollary 3 *In economies in which the public information component is high:*

- *If the economy is network symmetric, expected trading volume, X_t, is unimodal,*
- *If the economy is moderately asymmetric, there is an approximately linear positive relationship between time-t agent distance, h_t, and expected trading volume, X_t, $X_t \approx \alpha + \beta h_t$, $\alpha, \beta > 0$, and more asymmetric networks therefore have higher volatility of X_t.*

To summarize, the degree of nonmonotonicity of volatility and trading volume after an information shock depend on the degree of asymmetry of the network. In the network symmetric case, after an information shock trading volume and price volatility typically increase, reach a peak, and then decrease. In the asymmetric case, the dynamics of volume and volatility after an information shock may be more severely nonmonotone, with several peaks.

The difference between the dynamics in symmetric and asymmetric networks is exemplified by the two networks in Figure 2. The left panel shows a symmetric cyclical network with 11 agents, whereas the right panel shows an asymmetric network, in which two additional links have been added to the cyclical network. The agent distance function, h_t, see (6), is shown in the left panel of Figure 3. In line with Lemma 2, for the symmetric network h_t is unimodal, whereas it is not for the asymmetric network. This carries over to the same behavior of volatility and trading volume after an information shock, as shown in the middle and right panels of Figure 3, in line with our previous arguments.

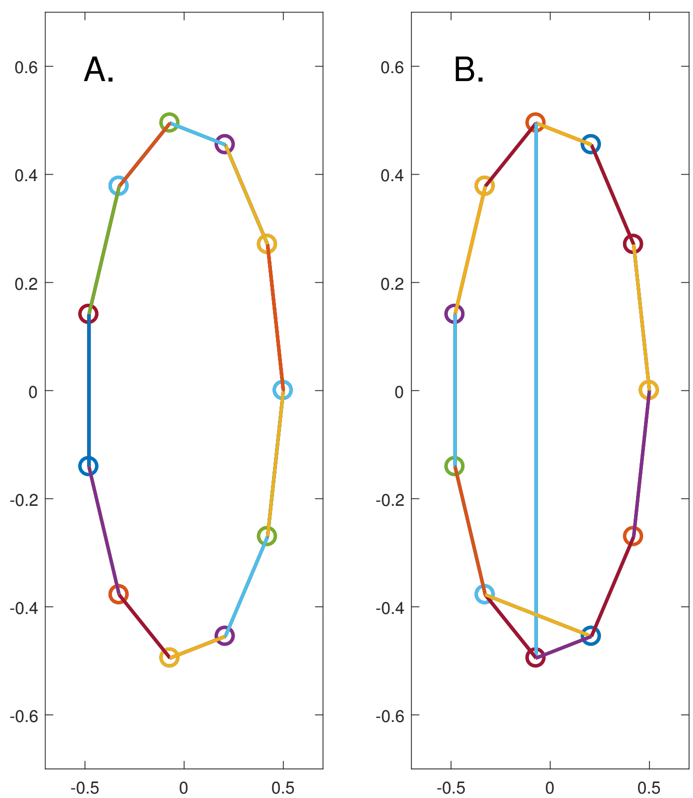

Figure 2: Symmetric and asymmetric network. *The left panel (A.) shows the symmetric cyclical network, with 11 agents. The right panel (B.) shows an asymmetric network in which there are two additional links compared with the network in the left panel.*

In the completely general case, with heterogeneity over both preferences (γ_a) and network structure

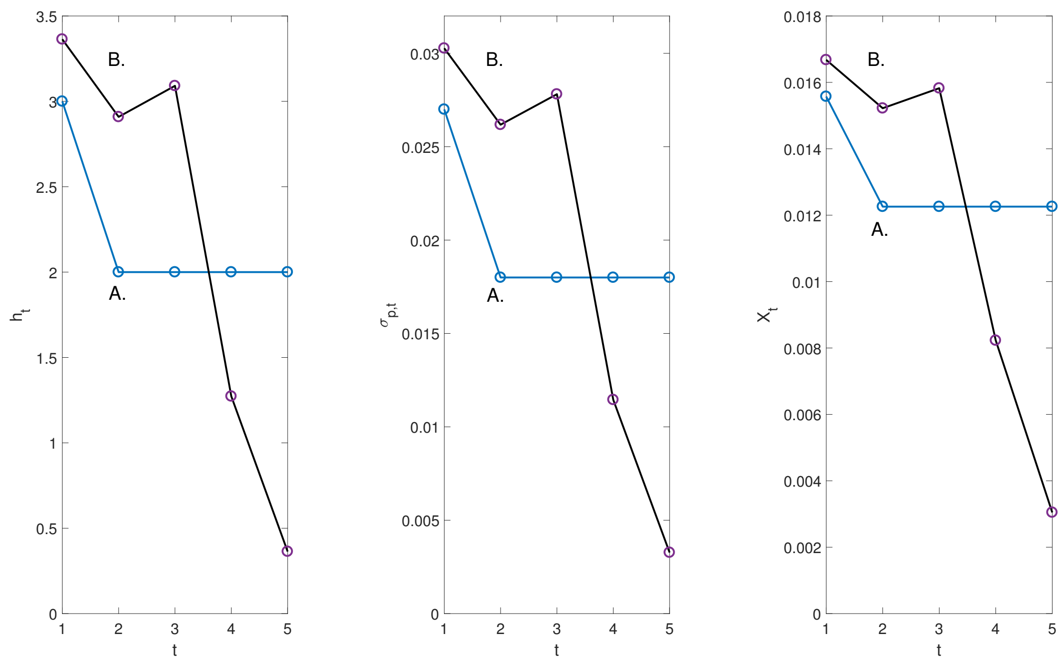

Figure 3: Dynamics of different networks. *The figure shows the dynamics of average number of connections (left panel), volatility (middle panel) and expected trading volume (right panel) of the two networks in Figure 2. The dynamics of the symmetric network (A.) is unimodal in contrast to the dynamics of the asymmetric network (B.), in all panels. Parameters:* $\sigma_v = 3$, $\sigma_u = 1$, $\sigma = 10$, $\gamma = 10$ *for all agents.*

($V_{a,t}$), we expect the interplay between the two to give rise to quite arbitrary trading volume and volatility dynamics. For example, at a large time, t, almost all information may have diffused among the bulk of agents, leading to a small and decreasing $\Delta V_{a,t}$, and thereby low volatility. A peripheral agent with very low risk aversion, who receives many signals very late, may still generate large trading volume at such a late point in time, despite the low volatility. This argument captures the important distinction between trading volume driven by high aggregate information diffusion, and by demand from agents with low risk aversion, a distinction that does not arise in either the preference symmetric or the network symmetric benchmark cases.

6 Empirical results

Our model suggests that network structure influences investor behavior and performance, as well as the aggregate dynamics of asset prices and trading volume. Especially, the centrality of agents is important for the equilibrium outcome. The following two main predictions for individual investors follow from our previous analysis:

Prediction 1 (Individual investors)

a) The closer two agents are in the network, the more similar are their trades.

b) The more central an agent is in the network, the better is his/her performance.

As discussed, Prediction 2a) is relevant, not as a test of the model—other models will likely also predict a positive relation between proximity and trades—but rather because it justifies identifying networks from trades over short time horizons, given that a network approach is taken. Prediction 2b) was explored in Ozsoylev, Walden, Yavuz, and Bildik (2014), who found support for a positive relation between performance and centrality, using a network identification methodology based on trades.[11]

In contrast to the partial equilibrium model in Ozsoylev, Walden, Yavuz, and Bildik (2014), our model also leads to predictions about aggregate dynamics, following Corollaries 1-3:

Prediction 2 (Aggregate dynamics)

a) The less central the network, the more persistent are shocks to volatility.

b) The less symmetric the network, the more volatile is volatility.

Note that the second prediction depends on the assumption, which we take as given, that the public information component is large in the network.

To test these aggregate predictions we use a dataset that, in addition to trades and portfolio holdings, contains information about the individual investors.

6.1 Data

Our main dataset was obtained from Euroclear.[12] It contains portfolio holdings and trades (over 62 Million) of all investors on the Helsinki Stock Exchange between 1997-2003, as well as information about age, gender, and most importantly postal code at a disaggregated level (containing over 3000 postal codes for the approximately 1.2 million accounts that were active during this time period, almost all of which representing individual investors). Summary statistics are provided in Table 1.

We use geographical proximity as a proxy for network proximity, based on the assumption that investors receive their information from people in their geographical vicinity. We argue that this is a reasonable assumption for the time period and market that the data covers. Although it may be argued that geographical distance is not as important a hurdle in the present, with information spreading widely and rapidly on social media platforms like Facebook, Twitter, and Instagram, these platforms did not exist before 2004. In fact, only about one third of the Finnish population used the Internet in 2000.[13]

[11] Pareek (2012), in a somewhat similar approach, uses correlations between mutual fund managers' portfolio holdings to identify information networks.

[12] Euroclear acquired the Finnish Central Securities Depositary in 2008. The dataset has also been used in Grinblatt and Keloharju (2000, 2001).

[13] The fraction of the population was using the Internet in 2000 was 37% in 2000. Source: International Telecommunication Union, ITU.

The country, furthermore, has a low population density. In 2000, the density was 17 people per square kilometer,[14] and over half of its population resides in rural areas, making it one of the most rural countries in the European Union.[15] It is therefore plausible that there would be a significant link between geographical and network proximity within this context.

We associate each stock with a separate network. Investors who own a stock and who live in the same or neighboring postal code areas are assumed to be connected in that stock's network,[16] leading to an *ownership network* for each stock at each point in time. The weight of each investor at time t is $\frac{1}{N_t}$, where N_t is the number of investors in the market at time t, so that each investor is equally weighted and investors' total weight equals one at each point in time. The general network structure shown in Figure 4 is thus the same for each stock, but the number of investors associated with each node (postal code) varies, leading to different network structures across stocks and over time.

Based on our previous discussion, we calculate investors' Katz centrality, and then a network's centrality as the average centrality of the investors in the network. For the free parameter in the Katz centrality measure (α in equation (14)), we choose $\frac{\lambda^{-1}}{2}$, thus in the middle between zero and the value that corresponds to eigenvector centrality. We have verified that our results are robust to varying α. As a measure of network asymmetry, we use the variation of h_t. We calculate the ownership network and associated network centrality and asymmetry month-by-month, for all stocks.

As discussed, one interpretation of the random supply shock is that it actually represents unpredictable changes in total supply of the asset. Within our fairly short time horizons, such shocks are likely quite rare, however, and a more natural interpretation is that some traders trade for exogenous reasons, and not because they belong to the information network. Our empirical measure does not try to distinguish these noise traders from the traders in the information network. This empirical approach is justified in Ozsoylev, Walden, Yavuz, and Bildik (2014), who show that "perturbing" the information network with the noise traders only marginally affects outcomes. In Internet Appendix C.2, we verify that similar results hold within our setting. Specifically, we simulate a large number of networks, each of which is extended to include a significant portion of noise traders, and show that individual centrality, network centrality, and asymmetry, are similar in the actual and extended networks (the correlation between actual and extended measures are in the range of 0.7-0.95, even when the proportion of noise traders is three times that of informed investors). This justifies our empirical focus on the complete ownership network in this section.

We use data from Nasdaq OMX Nordic to calculate daily stock returns for the 74 stocks for which data was available for at least 30 months during the time period. We use a Markov Switching GARCH (MSGARCH) model specification to estimate persistence of return volatility shocks (see Haas, Mittnik,

[14] Source: World Bank, World Development Indicators.

[15] 53.6% of the Finnish population live in predominantly rural regions, compared with a 27.1% average within the European Union, placing Finland third among the EU countries behind Sweden and Slovenia. Source: European Commission, Eurostat rural development database.

[16] Islands are treated as being disconnected from mainland Finland.

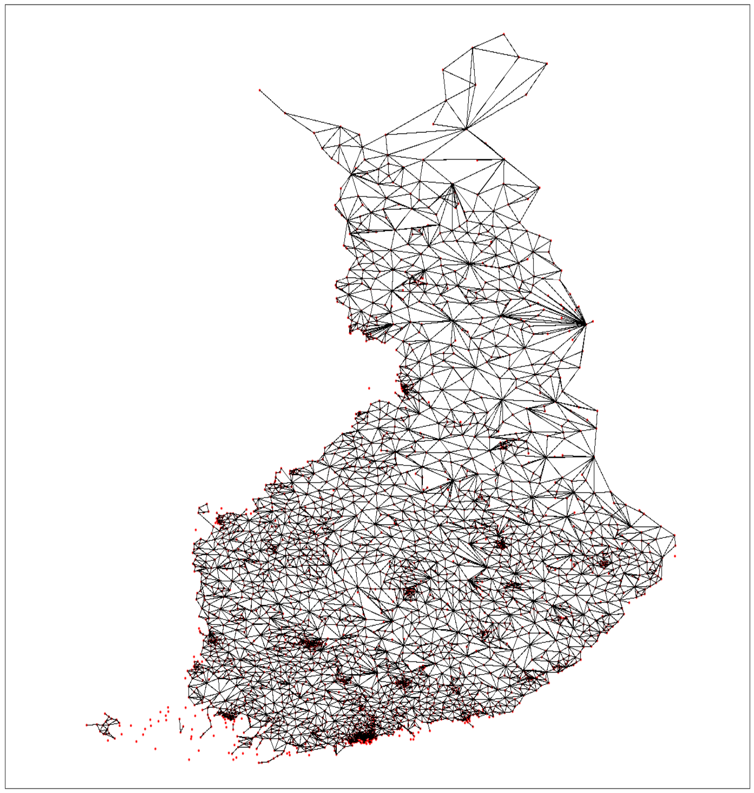

Figure 4: Network representation of Finland. *The figure shows a network representation of Finland, based on neighboring postal codes. Each shareholder is assumed to be connected to all agents within his own postal code, as well as in neighboring postal codes.*

and Paolella 2004). Specifically, volatility of stock i evolves according to

$$\sigma_{i,t}^2 = \alpha_{i,s_t}^0 + \alpha_i^1 \epsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2,$$

where $s_t \in \{1,2\}$ evolves according to a two state Markov process, and without loss of generality we assume that $\alpha_{i,1}^0 \leq \alpha_{i,2}^0$. The arrival of an information shock is represented by a switch from the low-volatility state 1 to the high volatility state 2. The β_i coefficient captures volatility persistence, and should according to prediction 2a) be negatively related to network centrality. The specification thus allows for arrival of information shocks over time, in line with the discussion in Section 4.2. We use maximum likelihood estimation with daily returns over a 240 day time window (approximately one year) to estimate monthly persistence coefficients for each stock. We also estimate each stock's volatility of volatility, using a non-parametric approach. We use a window length of 15 days to calculate sample volatility, and 16 such consecutive windows to calculate the sample volatility of volatility (vol-vol), thus again using a total window length of 240 days for each calculation, which we repeat month-by-month.

The results are robust to varying these window lengths. This gives us 72 monthly observations for each stock. The volatility of volatility should according to prediction 2b) be positively related to network asymmetry.

Additional data on firm characteristics, i.e., market beta, price-to-book ratio, size, and trading volume, were obtained from Thomson ONE, Reuters. Here, size is measured as the logarithm of one plus market capitalization. We exclude firm-months for which data was missing.[17] Such missing observations were mainly due to stocks not being traded during some periods. Altogether, we are left with 2,633 firm-month observations.

6.2 Results

We estimate the dependence of persistence and volatility of volatility on network centrality and asymmetry, using unbalanced panel regressions with both stock and time fixed effects. The standard errors we report are clustered at the firm level.

Columns 2 and 3 of Panel A in Table 2 show the estimated coefficient of dependence for persistence on average centrality, as well as standard errors. The negative sign of the coefficient is in line with prediction 2a), that more central networks are associated with less persistent shocks to volatility. The coefficient is statistically significant at the 1% level, with a p-value of 0.0003. When firm characteristics are included, in Columns 4 and 5 of Panel A, the statistical significance level decreases slightly, with a p-value of 0.0011, but still remains highly significant. The result is also economically quite relevant: a one standard deviation increase in average centrality is associated with an decrease in the persistence coefficient (which lies between 0 and 1), of 0.048. Since the average persistence coefficient is approximately 0.25, this corresponds to a 19% decrease. Of the firm characteristics coefficients, only size seems to be relevant—although not statistically significant—with larger firms being associated with lower persistence of volatility shocks. This is in line with the natural intuition that information diffusion is slower in smaller stocks. The effect is thus above and beyond what can be explained by network effects.

A potential concern is whether home bias may drive these effects. Specifically, if agents are prone to purchase stock in firms that they live nearby, this will give rise to heterogeneous ownership networks across stocks, and home bias may also potentially be related to persistence of volatility shocks, affecting the results. Such an omitted variable effect is likely not an issue for our results, however. Since firm head quarters are fixed for each firm within our sample during the relevant time period, the fixed stock effects that we include in our regressions would also capture home bias effects. Instead, our estimates capture changes in average centrality versus changes in persistence, whether home bias is present or not.

Another potential alternative explanation for the relation between centrality and persistence is liquidity. Specifically, if centrality is positively related to a stock's liquidity, and illiquid stocks have higher persistence to volatility shocks, then our regression may potentially capture this relation. To address

[17] Observations for which over 10% of returns were missing during the estimation period, or for which MSGARCH failed to converge were excluded from the sample.

these concerns, we include Amihud illiquidity (measured as absolute return over volume) in columns 6 and 7 of Panel A. The estimated coefficient on Amihud illiquidity is insignificant, and including the variable only marginally affects the centrality coefficient, which is still statistically significant at the 1% level. Thus, the negative relation between average centrality and persistence remains when illiquidity is included.

To ensure that the results are not driven by a few outliers, we Winsorize the data at the 95th percentile. The results, shown in columns 8 and 9 of Panel A, are very similar to the previous ones. We also show the average persistence coefficient of firms sorted on average centrality, in the left panel of Figure 5. Indeed, the relationship between persistence and average centrality is negative, except for an up-tick for bin 4 from the left. This provides further support for that the predicted relationship is present in practice.

Finally, we find some support for a positive relationship between network asymmetry and volatility of volatility, in line with Prediction 2b), as shown in Panel B of Table 2. The results are significant at the 1% level in all tests. We also show volatility of volatility for groups of firms sorted on asymmetry, in the right panel of Figure 5. The data is right-skewed and fat-tailed, and we choose cutoff points so that the bins are of the same size as if it were normally distributed. The relationship is monotonically increasing, as predicted.

6.3 Robustness

The Amihud measure may not completely capture liquidity. We therefore cannot rule out that liquidity may influence our results, but we can further explore whether the *spatial* dimension of ownership in the network drives our results, as opposed to non-spatial *concentration* of ownership, which may in turn be related to liquidity. We therefore include a measure of ownership concentration across postal codes. Specifically, we calculate a stock's Herfindahl index, based on fraction of ownership in different postal codes, again calculated month-by-month. The new measure thus uses the same ownership network that we have used for centrality and asymmetry calculations, but it does not use any geographic information for the postal codes.

The results when we include the Herfindahl index are shown in columns 10 and 11 of Table 2. We see that the coefficient estimates and standard errors for persistence barely change (Panel A). In unreported additional tests, we used standard deviation as the measure of variation across postal codes, with similar results. The negative relationship between centrality and persistence thus seems to depend on the spatial dimension of the ownership network, as opposed to a liquidity based explanation. The results are quite different for the estimated relation between asymmetry and volatility of volatility, as seen in Panel B of Table 2. When the Herfindahl index is included, the results are no longer statistically significant, although the coefficient sign remains positive. Further tests are therefore needed to rule out liquidity as a driver behind the positive relationship between asymmetry and volatility of volatility.

A potential concern of studying the Helsinki Stock Exchange during the late 1990s and early 2000s is that Nokia was very dominant during this time period, making up about half of the stock market's value. Since our regressions are not value-weighted and Nokia accounted for only a small fraction of the total observations, we would not expect this to be a serious concern. For the sake of robustness, however, we run all the previous tests while excluding Nokia from the sample. The results (not reported) are almost identical.

One may be concerned that large systematic events during the crisis—the Asian financial crisis starting in July 1997, the LTCM bailout triggered by the Russian financial crisis in August 1998, and the NASDAQ crash beginning in March 2000—may influence the results. We perform the same regressions, but exclude a four-month period around each of these events (the month before until three months after each event). The results (not reported) are very similar and remain statistically significant at the same significance levels. To further ensure that correlation across stocks is not an issue, we run the regression with double clustered errors (both firm and month). The coefficient estimate for centrality remains highly significant (coefficient estimate -0.0247, with a standard error of 0.0068, corresponding to a t-statistic of -3.61). To ensure that the statistical significance is not driven by persistent errors in the estimated β coefficients, we also run the regression using Newey-West robust standard errors with 12 lags. The coefficient estimate for centrality remains highly significant (coefficient estimate -0.0246, with a standard error of 0.0077, corresponding to a t-statistic of -3.18).

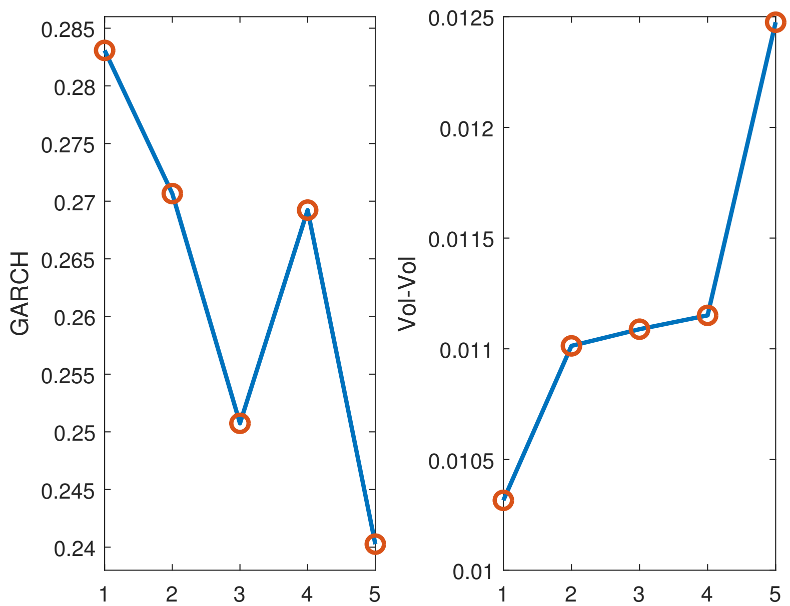

Figure 5: Sorted bins. *The left figure shows average persistence coefficient sorted over five bins of average centrality: Bin 1 represents lowest average centrality (< -1.5 standard deviations below average), and Bin 5 represents highest average centrality (> 1.5 standard deviations above average). The right figure, similarly shows average volatility of volatility sorted over five bins of asymmetry.*

7 Concluding remarks

We have introduced a general network model of a financial market with decentralized information diffusion, allowing us to study the effects of heterogeneous preferences and asymmetric diffusion of information among investors, and the interplay between the two. At the individual investor level, our results show that the trading behavior of investors is closely related to their positions in the network: Closer agents have more positively correlated trades even over short time periods, and more central agents make higher profits. At the aggregate level, network structure affects the dynamics of a market's volatility and trading volume. We find support for the predicted relations between volatility and network structure, using account level data for all traders on the Helsinki Stock Exchange between 1998 and 2003. These results suggest that network structure may be important for the dynamics of asset prices and trading volume in capital markets.

Several extensions of the model would be fairly straightforward. In its current version, signals are assumed to be perfectly communicated from agent to agent. A more realistic extension may be to assume that some "miscommunication" noise is introduced each time an agent shares information with another agent. Agents who are farther apart would then not only receive each others' signals later, but also with more noise than agents who are close. The most straightforward way to incorporate such an effect would be to make the signals an agent receives increasingly noisy over time, i.e., to make the signal precision in Theorem 1 decreasing over time, $\tau_1 > \tau_2 > \cdots > \tau_T$. The main effect of this extension would be to offset the increased information agents gain over time, by decreasing the signal informativeness. For example, in the expression for agents' expected profits, (11,12), the signal precision for each agent at higher t would be lower than in the economy with perfect information sharing, acting as a force to lower profits. On the other hand, the aggregate informativeness of the market, represented by Y_t, would also be lower, increasing the total opportunity for informational rents. It is a priori unclear whether agents would be better or worse off in total with more noise, similar to the hump-shaped behavior of equilibrium welfare in signal precision discussed in Ozsoylev and Walden (2011) and Manela (2014).

A perhaps more interesting extension would take a step toward endogenous network formation, by introducing a cost of sustaining a link between any two agents and studying which networks can be sustained with such a cost present. As we discussed before, in a large network with price-taking agents, there are only advantages for an agent of being linked to another agent. With a cost of sustaining links there is a trade-off. In the simplest version of this extension, there is a one-time cost—the same for any link—imposed at time $t = 0$ on both agents. Only links valuable enough for both agents, as measured in welfare terms in (10), are then sustainable in equilibrium, suggesting that (i) Agents are less likely to sever links to other agents who are central, and to agents whose information is not also obtained through other links (redundant information); (ii) There may be a threshold centrality such that agents below the threshold cannot sustain links with any other agents, since the value of their information does not outweigh the cost of linking to them. Such agents thus end up isolated; (iii) There may be

Number of months	72	
Number of stocks	74	
Number of postal codes	3,036	
Number of accounts	1,266,850	
Number of trades	62,946,475	
Average number of months per stock	46	
	Mean	**Standard deviation**
Average centrality	3.176	2.170
Asymmetry	0.0029	0.018
Persistence coefficient, β	0.2501	0.1790
Volatility of volatility	0.01097	0.005918
Market capitalization	3.327 Billion FIM	13.87 Billion FIM
Ownership Herfindahl index	0.3206	0.2056

Table 1: Summary statistics. The table shows summary statistics for ownership network in Helsinki Stock Exchange, 1998-2003. Sources: Euroclear, Thomson ONE, Nasdaq OMX Nordic.

a threshold centrality, above which it is not worthwhile for an agent to sustain any further link, since the marginal benefit of additional information is decreasing, but the marginal cost of sustaining each link is the same; (iv) All else equal, an agent with higher risk aversion will be less likely to sustain a link than an agent with low risk aversion, since risk averse agents trade less aggressively on private information. Together, (i) and (iii) describe two forces pulling in opposite directions: On the one hand, it is very valuable for an agent—especially one with low centrality—to be connected to a central agent. On the other hand, the value for the central agent of sustaining a link to an agent with low centrality is low. Altogether, we expect such considerations to have interesting implications for which information networks are sustainable in equilibrium, complementing other studies, e.g., Han and Yang (2013), who focus on endogenously determined information acquisition.

Further extensions along these lines are possible. A more general form of the cost function would introduce a period-by-period cost of sustaining links. In this case we may expect the price discovery process to decrease over time—and possibly even come to a halt—as profit opportunities are mitigated when the price becomes more informative, and agents therefore do not find it worthwhile to sustain links. Heterogeneous cost functions across agents may also be assumed, where some agents are more "talented" than others in sustaining links. By assuming increasing marginal costs of sustaining a link in the number of links an agent already has, an interesting trade-off between the costs of having a high degree (many direct links) and the benefits of having a high centrality (many higher order links) would be introduced. We leave these interesting extensions for future research.

A.	Coefficient Estimate	Standard Error	Coefficient Estimate	Standard Error	Coefficient Estimate	Standard Error	Coefficient Estimate	Standard Error	Coefficient Estimate	Standard Error
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Average Centrality	-0.0253 ***	0.0072	-0.0220 ***	0.0072	-0.0191 ***	0.0072	-0.0199 **	0.0085	-0.0191 ***	0.0072
Stock beta			0.0130	0.0401	0.0053	0.0405	0.0053	0.0405	0.0052	0.0406
Price-to-book ratio			-0.0083	0.0319	-0.0291	0.0446	-0.0291	0.0446	-0.0302	0.0445
Firm size			-0.0354	0.0277	-0.0356	0.0331	-0.0356	0.0331	-0.0364	0.0329
Amihud illiquidity					-0.8796	1.5285	-0.8796	1.5285	-0.8483	1.5360
Ownership Herfindahl index									-0.0407	0.0824
Firm fixed effects	Yes		Yes		Yes		Yes		Yes	
Month fixed effects	Yes		Yes		Yes		Yes		Yes	
N =	2,663		2,397		2,092		2,092		2,092	

B.	Coefficient Estimate	Standard Error	Coefficient Estimate	Standard Error	Coefficient Estimate	Standard Error	Coefficient Estimate	Standard Error	Coefficient Estimate	Standard Error
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Asymmetry	0.0122 ***	0.0042	0.0126 ***	0.0036	0.0133 ***	0.0036	1.0361 ***	0.2677	0.0049	0.0067
Stock beta			0.0007	0.0019	0.0012	0.0019	0.0007	0.0012	0.0013	0.0018
Price-to-book ratio			-0.0004	0.0006	-0.0013	0.0020	-0.0001	0.0012	-0.0014	0.0020
Firm size			0.0018 **	0.0009	0.0026 *	0.0015	0.0011	0.0008	0.0028 *	0.0015
Amihud illiquidity					-0.0069	0.0047	-0.0336	0.0397	-0.0062	0.0050
Ownership Herfindahl index									0.0041	0.0031
Firm fixed effects	Yes		Yes		Yes		Yes		Yes	
Month fixed effects	Yes		Yes		Yes		Yes		Yes	
N =	2,693		2,423		2,117		2,1117		2,117	

*** (≤ 0.01)
** (≤ 0.05)
* (≤ 0.1)

Table 2: Panel A. Panel regression of persistence coefficient on average centrality in stock network. **Panel B.** Panel regression of volatility of volatility on variation in centrality. Columns (2-3) show coefficient estimate and standard error for regression with firm and month fixed effects. Columns (4-5) include firm characteristics (beta, price-to-book, size), and columns (6-7) include Amihud illiquidity. Columns (8-9) use 95% Winsorized data, and columns (10-11) include ownership Herfindahl index. All standard errors are clustered at the firm level.

A Replica Network

The total number of agents in the economy is $\bar{N} = N \times M$. Formally, we define the set of agents in an M-replica economy as $\mathcal{A}_m = \mathcal{N} \times \{1, \ldots, M\}$, where $a = (i, j) \in \mathcal{A}_m$ represents the ith agent in the jth replica network, in an economy with M replica networks. There is still one asset, one market, and one competitive market maker in the market with $\bar{N}$ agents. We use the enumeration $a = 1, \ldots, MN$, of agents, where agent (i, j) maps to $a = (j-1)N + i$.

Agent (i, j) and (i, j') are thus ex ante identical in their network positions and in their signal distributions, although their signal realizations (typically) differ. We let M increase in a sequence of replica economies, with the natural embedding $\mathcal{A}_1 \subset \mathcal{A}_2 \cdots \subset \mathcal{A}_m \subset \cdots$, and take the limit $\mathcal{A} = \lim_{M\to\infty} \mathcal{A}_M$, letting $\mathcal{A}$ define our large economy, in a similar manner as in Hellwig (1980). The network $\mathcal{G}$ is thus a representative network in the large economy, $\mathcal{A}$. Our interpretation is that the network, $\mathcal{G}$, represents a fairly localized structure, perhaps at the level of a town or municipality in an economy, whereas $\mathcal{A}$ represents the whole economy.

At time t, the market maker observes the average order flow per agent in the network[18]

$$w_t = u_t + \frac{1}{\bar{N}} \sum_{a=1}^{\bar{N}} \Delta x_{a,t}. \tag{23}$$

B Proofs

Proof of Theorem 1:

We prove the result using a slightly more general formulation, where the volatility of noise trade demand is allowed to vary over time, so instead of τ_u, we have $\tau_{u_1}, \ldots, \tau_{u_T}$. We first state three (standard) lemmas.

Lemma 3 (Projection Theorem) *Assume a multivariate signal* $[\tilde{\mu}_x; \tilde{\mu}_y] \sim N([\mu_x; \mu_y], [\Sigma_{xx}, \Sigma_{xy}; \Sigma_{yx}, \Sigma_{yy}])$. *Then the conditional distribution is*

$$\tilde{\mu}_x | \tilde{\mu}_y \sim N\left(\mu_x + \Sigma_{xy}\Sigma_{yy}^{-1}(\tilde{\mu}_y - \mu_y), \Sigma_{xx} - \Sigma_{xy}\Sigma_{yy}^{-1}\Sigma_{yx}\right).$$

Lemma 4 (Special case of projection theorem) *Assume an K-dimensional multivariate signal* $\mathbf{v} = [v; \mathbf{s}] \sim N(\bar{v}\mathbf{1}, \sigma_v^2 \mathbf{1}\mathbf{1}' + \Lambda^2)$, *where* $\Lambda = diag(0, \sigma_1, \ldots, \sigma_{M-1})$. *This is to say that* $v \sim N(\bar{v}, \sigma_v^2)$, $\mathbf{s}_i = v + \xi_i$, *where* $\xi_i \sim N(0, \sigma_i^2)$*'s are independent of each other and of* v, $i = 1, \ldots, K-1$. *Then the conditional distribution is*

$$v|\mathbf{s} \sim N\left(\frac{\tau_v}{\tau_v + \tau}\bar{v} + \frac{1}{\tau_v + \tau}\boldsymbol{\tau}'\mathbf{s}, \frac{1}{\tau_v + \tau}\right).$$

Here, $\boldsymbol{\tau} = (\tau_1, \ldots, \tau_{K-1})^T$, $\tau_i = \sigma_i^{-2}$, $\tau = \sum_{i=1}^{M-1} \tau_i$, *and* $\tau_v = \sigma_v^{-2}$.

Lemma 5 (Expectation of exponential quadratic form) *Assume* $x \sim N(\mu, \Sigma)$, *and that* $\mathbf{B}$ *is a symmetric positive semidefinite matrix. Then*

$$E\left[e^{-\frac{1}{2}(2\mathbf{a}'\mathbf{x} + \mathbf{x}'\mathbf{B}\mathbf{x})}\right] = \frac{1}{|I + \Sigma\mathbf{B}|^{1/2}} e^{-\frac{1}{2}(\mu'\Sigma^{-1}\mu - (\Sigma^{-1}\mu - \mathbf{a})(\Sigma^{-1} + \mathbf{B})^{-1}(\Sigma^{-1}\mu - \mathbf{a}))}.$$

The structure of the proof is now quite straightforward, the extension compared with previous literature being the heterogeneous information diffusion. We first assume that agents' demand takes a linear form at each point in time, and calculate the market maker's pricing function given observed aggregate demand in (23). This turns out to be linear in a way such that the market maker's information is completely revealed in prices. Thus, p_t and w_t convey the same information. We then close the loop by verifying that given the market maker's pricing function in each time period, each agent when solving their backward induction problem will derive demand and utility according to (4,5), verifying that agents' demand functions are indeed linear.

It will be convenient to use the variables $Q_{a,t} = \tau V_{a,t}$. We enumerate the agents from one-dimensionally from 1 to $\bar{N}$, so that agent $1, \ldots, N$ represents the agents in the first replica network, agents $N+1, \ldots, 2N$, the agents in the second

[18] Technically, the market maker observes $u_t + \lim_{M\to\infty} \frac{1}{MN}\sum_{a=1}^{MN} \Delta x_{a,t}$. We avoid such limit notation when this can be done without confusion.

replica network, etc. Assume that agent a's time-t demand function is

$$x_{a,t}(z_{a,t}, p_t) = A_{a,t} z_{a,t} + \eta_{a,t}(p_t).$$

Then the total average agent demand is

$$x_t(v, p_t) = \frac{1}{\bar{N}} \sum_{a=1}^{\bar{N}} A_{a,t} z_{a,t} + \eta_{a,t}(p_t) = A_t v + \eta_t(p_t),$$

where $A_t = \frac{1}{N}\sum_{a=1}^{N} A_{a,t}$, and $\eta_t = \frac{1}{N}\sum_{a=1}^{N} \eta_{a,t}(p_t)$, with the convention, $A_0 = 0$, $\eta_0 \equiv 0$. Here, we are using the fact that in our large network $\lim_{M\to\infty} \frac{1}{M}\sum_{r=0}^{M-1} z_{a+rM,t} = v$ for all a and t (almost surely). This allows us to use the L.L.N. for each node, and collapse the sum from $\bar{N}$ to N. The net demand at time t is then the difference between time t and $t-1$ demands,

$$\Delta x_t = x_t(v, p_t) - x_{t-1}(v, p_{t-1}) = (A_t - A_{t-1})v + \eta(p_t) - \eta(p_{t-1}).$$

Now, the market maker observes total time t net demands,

$$w_t = \Delta x_t + u_t,$$

and since the functions η_t and η_{t-1} are known, the market maker can back out

$$R_t = (A_t - A_{t-1})v + u_t. \tag{24}$$

This leads to the following pricing formula, which immediately follows from Lemma 2.

Lemma 6 *Given the above assumptions, the time-t price is given by*

$$p_t = \frac{\tau_v}{\tau_v + \hat{\tau}_u^t}\bar{v} + \frac{\hat{\tau}_u^t}{\tau_v + \hat{\tau}_u^t} v + \frac{1}{\tau_v + \hat{\tau}_u^t}\sum_{s=1}^{t}(A_s - A_{s-1})\tau_{u_s} u_s, \tag{25}$$

where $\hat{\tau}_u^t = \sum_{s=1}^{t}(A_s - A_{s-1})^2 \tau_{u_s}$, $\tau_{u_s} = \sigma_{u_s}^{-2}$.

Equivalently,

$$p_t = \lambda_t R_t + (1 - \lambda_t(A_t - A_{t-1}))p_{t-1}, \tag{26}$$

where $\lambda_t = \frac{\tau_{u_t}(A_t - A_{t-1})}{\tau_v + \hat{\tau}_u^t}$, *and* $p_0 = \bar{v}$.

Proof of Lemma 6: At time t, the market maker has observed $R_1, \ldots, R_t$. We define the vector $\mathbf{s} = (R_1/(A_1 - A_0), R_2/(A_2 - A_1), \ldots, R_t/(A_t - A_{t-1}))'$, and it is clear that $\mathbf{s}_i \sim N(\bar{v}, \sigma_v^2 + \sigma_{u_i}^2/(A_i - A_{i-1})^2)$. It then follows immediately from Lemma 2 that

$$v|\mathbf{s} \sim N\left(\frac{\tau_v}{\tau_v + \hat{\tau}_u^t}\bar{v} + \frac{1}{\tau_v + \hat{\tau}_u^t}\sum_{i=1}^{t}(A_i - A_{i-1})^2 \tau_{u_i}\frac{R_i}{A_i - A_{i-1}}, \frac{1}{\tau_v + \hat{\tau}_u^t}\right),$$

i.e.,

$$v = V_t + \sigma_{V_t}\xi_t^V, \tag{27}$$

where $V_t = \frac{\tau_v}{\tau_v + \hat{\tau}_u^t}\bar{v} + \frac{1}{\tau_v + \hat{\tau}_u^t}\sum_{i=1}^{t}(A_i - A_{i-1})\tau_{u_i} R_i$, $\sigma_{V_t}^2 = \frac{1}{\tau_{V_t}}$, where $\tau_{V_t} = \tau_v + \hat{\tau}_u^t$.

So, $p_t = V_t = E[v|\mathbf{s}]$ takes the given form in the first expression of the lemma. A standard induction argument, assuming that the second expression is valid up until $t-1$, shows that the expression then is also valid for t.

Note that (27) is the posterior distribution of v given the information $R_1, \ldots, R_t$, so $v \sim N(V_t, \sigma_{V_t}^2)$. We have shown Lemma 6.

Thus, linear demand functions by agents imply linear pricing functions in the market, showing the first part of the proof. We next move to the demand functions and expected utilities of agent a, given the pricing function of the market maker. We have (using lemma 1 for the posterior distribution) at time t, the distribution of value given $\{z_{a,t}, p_t\}$ is

$$v|\{z_{a,t}, p_t\} \sim N\left(\frac{\tau_{V_t}}{\tau_{V_t} + \tau_{a,t}} V_t + \frac{\tau_{a,t}}{\tau_{V_t} + \tau_{a,t}} z_{a,t}, \frac{1}{\tau_{V_t} + \tau_{a,t}}\right).$$

The time-T demand of an agent can now be calculated. Since individual agents condition on prices, they also observe $\check{R}$, and agent a's information set is therefore $\{z_a, \check{R}\}$, which via Lemma 1 (with $\Lambda = diag(0, \sigma_i^2, \sigma_u^2/\bar{A}^2)$) leads to

$$v|\{z_a, \check{R}\} \sim N\left(\underbrace{\frac{\tau_v}{\tau_v + \tau_a + \hat{\tau}_u}\bar{v} + \frac{\tau_a}{\tau_v + \tau_a + \hat{\tau}_u} z_a + \frac{\hat{\tau}_u}{\tau_v + \tau_a + \hat{\tau}_u}\check{R}}_{\mu_a}, \underbrace{\frac{1}{\tau_v + \tau_a + \hat{\tau}_u}}_{\hat{\sigma}_a^2}\right).$$

At time T, given the behavior of the market maker, the asset's value, given agent a's information set is therefore conditionally normally distributed so given agent a's CARA utility, the demand for the asset (2) takes the form:

$$x_{a,T}(z_a, p) = \frac{E[v|\mathcal{I}_{a,T}] - p_T}{\gamma_a \sigma^2[v|\mathcal{I}_{a,T}]}, \qquad a = 1, \ldots, \bar{N}. \tag{28}$$

The demand of agent a is therefore

$$\begin{aligned}
x_{a,T}(z_{a,T}, p_T) &= \frac{\mu_a - p_T}{\gamma_a \hat{\sigma}_a^2} \\
&= \frac{1}{\gamma_a}\left(\tau_v \bar{v} + \tau_a z_a + \hat{\tau}_u \check{R} - (\tau_v + \tau_a + \hat{\tau}_u) p\right) \\
&= \frac{1}{\gamma_a}\left(\tau_v \bar{v} + \tau_a z_a + \hat{\tau}_u \check{R} - \left(1 + \frac{\tau_a}{\tau_v + \hat{\tau}_u}\right)(\tau_v \bar{v} + \hat{\tau}_u \check{R})\right) \\
&= \frac{1}{\gamma_a}\left(\tau_a z_a - \tau_a p\right) \\
&= \frac{\tau_{a,T}}{\gamma_a}\left(z_{a,T} - p_T\right).
\end{aligned}$$

It follows that $A_T = \frac{1}{N}\sum_{a=1}^{N} \frac{N_{a,T}}{\sigma^2 \gamma_a}$.

Since $v - p_T \sim N(0, \sigma_{V_t}^2)$, and $z_{a,T} - p_T = \zeta_{a,T} + (v - p_T)$, where $\zeta_{a,T}$ is independent of $v - p_T$, it follows that $\zeta_{a,T}|(z_{a,T} - p_T) \sim N(\frac{\tau_{V_T}}{\tau_{V_T} + \tau_{a,T}}(z_{a,T} - p_T), \frac{1}{\tau_{V_T} + \tau_{a,T}})$. The expected utility of the agent at time T (with time-T wealth of zero), given $z_{a,T} - p_T$, is then

$$\begin{aligned}
U_{a,T} &= -E\left[e^{-\gamma_a x_{a,T}(v - p_T)}\Big| z_{a,T} - p_T\right] \\
&= -E\left[e^{-\tau_{a,T}(z_{a,T} - p_T)(z_{a,T} - p_T - \zeta_{a,T})}\Big| z_{a,T} - p_T\right] \\
&= -e^{-\tau_{a,T}(z_{a,T} - p_T)^2} E\left[e^{-\tau_{a,T}(z_{a,T} - p_T)(-\zeta_{a,T})}\Big| z_{a,T} - p_T\right] \\
&= -e^{-\tau_{a,T}(z_{a,T} - p_T)^2} e^{\tau_{a,T}(z_{a,T} - p_T)\frac{\tau_{V_T}}{\tau_{V_T} + \tau_{a,T}}(z_{a,T} - p_T) - \frac{1}{2}\tau_{a,T}^2 (z_{a,T} - p_T)^2 \frac{1}{\tau_{V_T} + \tau_{a,T}}} \\
&= -e^{-\frac{\tau_{a,T}}{\tau_{V_t} + \tau_{a,T}}(z_{a,T} - p_T)^2\left((\tau_{V_t} + \tau_{a,T} - \tau_{V_t}) - \frac{1}{2}\tau_{a,T}\right)} \\
&= -e^{-\frac{1}{2}\frac{\tau_{a,T}^2}{\tau_{V_t} + \tau_{a,T}}(z_{a,T} - p_T)^2}.
\end{aligned}$$

This shows the result at T.

It is easy to check that the unconditional expected utility is $-E_0[e^{-\gamma_a x_{a,T}(v - p_T)}] = -\sqrt{\frac{\tau_{V_T}}{\tau_{V_T} + \tau_{a,T}}}$, using lemma 3. We define $Y_t = \hat{\tau}_u^t = \sum_{i=1}^{t} y_i$, where $y_i = (A_i - A_{i-1})^2 \tau_{u_i}$, and recall that $Q_{a,t} = \tau_{a,t} = \frac{V_{a,t}}{\sigma^2} = \sum_{i=1}^{t} q_{a,i}$, where

$q_{a,i} = \frac{V_{a,i}-V_{a,i-1}}{\sigma^2} = \frac{\Delta V_{a,i}}{\sigma^2}$. With this notation we have

$$
\begin{aligned}
U_{a,T} &= -e^{-\frac{1}{2}\frac{Q_{a,T}^2}{\tau_v+Y_T+Q_T}(z_{a,t}-p_t)^2}, \\
x_{a,T} &= \frac{Q_{a,T}}{\gamma_a}(z_{a,T}-p_T),
\end{aligned}
$$

and $-E_0[e^{-\gamma_a x_{a,T}(v-p_T)}] = -\sqrt{\frac{\tau_v+Y_T}{\tau_v+Y_T+\tau V_{a,T}}} = -\frac{1}{\sqrt{C_{a,T}}} = -D_{a,T}$.

We proceed with an induction argument: We show that given that (4,5) is satisfied at time t, then it is satisfied at time $t-1$. As already shown, p_{t-1}, and $z_{a,t-1}$ sufficiently summarizes agent a's information at time $t-1$ (given the linear pricing function). From the law of motion, $W_{a,t} = W_{a,t-1} + x_{a,t-1}(p_t - p_{t-1})$, an agent's optimization at time $t-1$ is then

$$
\begin{aligned}
U_{a,t} &= \arg\max_{x_{a,t-1}} -E_{a,t-1}\left[e^{-\gamma_a W_{a,t-1}-\gamma_a x_{a,t-1}(p_t-p_{t-1})} D_{a,t} e^{-\frac{1}{2}\frac{Q_t^2}{\tau_v+Y_t+Q_t}(z_{a,t}-p_t)^2}\Big| z_{a,t-1}, p_{t-1}\right] \\
&= \arg\max_{x_{a,t-1}} -D_{a,t}e^{-\gamma_a W_{a,t-1}} E_{a,t-1}\left[e^{-\gamma_a x_{a,t-1}(p_t-p_{t-1})-\frac{1}{2}\frac{Q_t^2}{\tau_v+Y_t+Q_t}(z_{a,t}-p_t)^2}\Big| z_{a,t-1}, p_{t-1}\right] \\
&\overset{\text{def}}{=} \arg\max_{b} -D_{a,t}e^{-\gamma_a W_{a,t-1}} E_{a,t-1}\left[e^{-b(p_t-p_{t-1})-\frac{1}{2}\frac{Q_t^2}{\tau_v+Y_t+Q_t}(z_{a,t}-p_t)^2}\Big| z_{a,t-1}, p_{t-1}\right]. \qquad (29)
\end{aligned}
$$

Thus, we need to calculate the distributions of $p_t - p_{t-1}$ and $z_{a,t} - p_t$ given $z_{a,t-1}$ and p_{t-1}. From the signal structure, we have the following relationship

$$
\begin{aligned}
z_{a,t-1} &= v + \xi_{t-1}, \qquad \xi_{t-1} \sim N\left(0, \frac{1}{Q_{t-1}}\right), && (30)\\
z_{a,t} &= v + \xi_t = v + \frac{Q_{t-1}}{Q_t}\xi_{t-1} + \frac{q_t}{Q_t}e_t, \qquad e_t \sim N\left(0, \frac{1}{q_t}\right), && (31)
\end{aligned}
$$

where e_t and ξ_{t-1} jointly independent and independent of all other variables. In the new notation, from (3), we have

$$
\begin{aligned}
p_t &= \frac{\tau_v}{\tau_v+Y_t}\bar{v} + \frac{Y_t}{\tau_v+Y_t}v + \frac{1}{\tau_v+Y_t}\sum_{s=1}^{t}(A_s - A_{s-1})\tau_{u_s}u_s, && (32)\\
p_{t-1} &= \frac{\tau_v}{\tau_v+Y_{t-1}}\bar{v} + \underbrace{\frac{Y_{t-1}}{\tau_v+Y_{t-1}}}_{A_4}v + \frac{1}{\tau_v+Y_{t-1}}\sum_{s=1}^{t-1}(A_s - A_{s-1})\tau_{u_s}u_s, && (33)
\end{aligned}
$$

so

$$
\begin{aligned}
p_t - p_{t-1} &= \left(\frac{\tau_v}{\tau_v+Y_t} - \frac{\tau_v}{\tau_v+Y_{t-1}}\right)\bar{v} + \underbrace{\left(\frac{Y_t}{\tau_v+Y_t} - \frac{Y_{t-1}}{\tau_v+Y_{t-1}}\right)}_{A_1}v + \frac{1}{\tau_v+Y_t}\underbrace{(A_t - A_{t-1})\tau_{u_t}}_{\sqrt{y_t\tau_{u_t}}}u_t \\
&+ \underbrace{\left(\frac{1}{\tau_v+Y_t} - \frac{1}{\tau_v+Y_{t-1}}\right)}_{B_1}\sum_{s=1}^{t-1}(A_s - A_{s-1})\tau_{u_s}u_s, \qquad (34)
\end{aligned}
$$

and also

$$
\begin{aligned}
z_{a,t} - p_t &= \frac{Q_{t-1}}{Q_t}\xi_{t-1} + \frac{q_t}{Q_t}e_t - \frac{\tau_v}{\tau_v + Y_t}\bar{v} + \underbrace{\frac{\tau_v}{\tau_v + Y_t}}_{A_2} v \\
&- \frac{1}{\tau_v + Y_t}\underbrace{(A_t - A_{t-1})\tau_{u_t}}_{\sqrt{y_t \tau_{u_t}}} u_t - \frac{1}{\tau_v + Y_t}\sum_{s=1}^{t-1}(A_s - A_{s-1})\tau_{u_s} u_s. \qquad (35)
\end{aligned}
$$

This leads to the unconditional distribution:

$$
\begin{bmatrix} p_t - p_{t-1} \\ s_t - p_t \\ s_{t-1} \\ p_{t-1} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ \bar{v} \\ \bar{v} \end{bmatrix}, \begin{bmatrix} \Sigma_{XX} & \Sigma_{XY} \\ \Sigma'_{XY} & \Sigma_{YY} \end{bmatrix} \right), \qquad (36)
$$

Here,

$$
\begin{aligned}
\Sigma_{XX} &= \begin{bmatrix} \frac{A_1^2}{\tau_v} + \frac{y_t}{(\tau_v+Y_t)^2} + B_1^2 Y_{t-1} & \frac{A_1 A_2}{\tau_v} - \frac{y_t}{(\tau_v+Y_t)^2} - \frac{B_1 Y_{t-1}}{\tau_v+Y_t} \\ \frac{A_1 A_2}{\tau_v} - \frac{y_t}{(\tau_v+Y_t)^2} - \frac{B_1 Y_{t-1}}{\tau_v+Y_t} & \frac{1}{Q_t} + \frac{A_2^2}{\tau_v} + \frac{y_t}{(\tau_v+Y_t)^2} + \frac{Y_{t-1}}{(\tau_v+Y_t)^2} \end{bmatrix} \\
&= \begin{bmatrix} \frac{y_t}{(\tau_v+Y_t)(\tau_v+Y_{t-1})} & 0 \\ 0 & \frac{1}{Q_t} + \frac{1}{\tau_v+Y_t} \end{bmatrix},
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_{YY} &= \begin{bmatrix} \frac{1}{\tau_v} + \frac{1}{Q_{t-1}} & \frac{A_4}{\tau_v} \\ \frac{A_4}{\tau_v} & \frac{A_4^2}{\tau_v} + \frac{Y_{t-1}}{(\tau_v+Y_{t-1})^2} \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{\tau_v} + \frac{1}{Q_{t-1}} & \frac{Y_{t-1}}{\tau_v(\tau_v+Y_{t-1})} \\ \frac{Y_{t-1}}{\tau_v(\tau_v+Y_{t-1})} & \frac{Y_{t-1}}{\tau_v(\tau_v+Y_{t-1})} \end{bmatrix},
\end{aligned}
$$

$$
\begin{aligned}
\Sigma_{XY} &= \begin{bmatrix} \frac{A_1}{\tau_v} & \frac{A_4 A_1}{\tau_v} + \frac{B_1}{\tau_v+Y_{t-1}} Y_{t-1} \\ \frac{1}{Q_t} + \frac{A_2}{\tau_v} & \frac{A_2 A_4}{\tau_v} - \frac{1}{\tau_v+Y_{t-1}} \frac{1}{\tau_v+Y_t} Y_{t-1} \end{bmatrix} \\
&= \begin{bmatrix} \frac{y_t}{(\tau_v+Y_t)(\tau_v+Y_{t-1})} & 0 \\ \frac{1}{Q_t} + \frac{1}{\tau_v+Y_t} & 0 \end{bmatrix}.
\end{aligned}
$$

We use the projection theorem to write $[p_t - p_{t-1}; z_{a,t} - p_t] \sim N(\mu, \hat{\Sigma})$, where $\mu = \Sigma_{XY}\Sigma_{YY}^{-1}[z_{a,t-1} - \bar{v}; p_{t-1} - \bar{v}]$, and $\hat{\Sigma} = \Sigma_{XX} - \Sigma_{XY}\Sigma_{YY}^{-1}\Sigma'_{XY}$. It follows that

$$
\mu = \begin{bmatrix} \frac{Q_{t-1} y_t}{(\tau_v+Y_t)(\tau_v+Q_{t-1}+Y_{t-1})} \\ \frac{Q_{t-1}(\tau_v+Y_{t-1})(\tau_v+Q_t+Y_t)}{Q_t(\tau_v+Q_{t-1}+Y_{t-1})(\tau_v+Y_t)} \end{bmatrix} (z_{a,t-1} - p_{t-1}).
$$

We rewrite (29) as

$$
U_{a,t} = \arg\max_q -D_{a,t} e^{-\gamma_a W_{a,t-1}} E\left[e^{-\mathbf{a}\mathbf{x}_1 - \frac{1}{2}\mathbf{x}'\mathbf{B}\mathbf{x}} \right],
$$

where $\mathbf{x} = [p_t - p_{t-1}; z_t - p_t]$, $\mathbf{B} = \left[0, 0; 0, \frac{Q_t^2}{\tau_v+Y_t+Q_t}\right]$, $\mathbf{a} = [q; 0]$, and $q = \gamma_a x_{a,t-1}$. From Lemma 3, it follows directly that this maximization problem is equivalent to

$$U_{a,t} = \arg\max_q \frac{-D_{a,t}e^{-\gamma_a W_{a,t-1}}}{|I + \hat{\Sigma}\mathbf{B}|^{1/2}} e^{-\frac{1}{2}(\mu'\hat{\Sigma}^{-1}\mu - (\hat{\Sigma}^{-1}\mu - \mathbf{a})\mathbf{Z}(\hat{\Sigma}^{-1}\mu - \mathbf{a}))},$$

where $\mathbf{Z} = (\hat{\Sigma}^{-1} + \mathbf{B})^{-1}$. Clearly, the optimal solution is given by

$$\arg\max_q \; q(\mathbf{Z}\hat{\Sigma}^{-1}\mu)_1 - \frac{1}{2}\mathbf{Z}_{11}q^2,$$

leading to $q^* = \frac{(\mathbf{Z}\hat{\Sigma}^{-1}\mu)_1}{\mathbf{Z}_{11}}$. It is easy to verify that $\hat{\Sigma}^{-1}\mu = \frac{Q_t Q_{t-1}}{Q_t - Q_{t-1}}[1;1](z_{a,t-1} - p_{t-1})$, and some further algebraic manipulations shows that indeed $q^* = Q_{t-1}(z_{a,t-1} - p_{t-1})$, leading to the stated demand function at $t-1$, (4).

Given the form of q, it then follows that

$$\mu'\hat{\Sigma}^{-1}\mu - (\hat{\Sigma}^{-1}\mu - \mathbf{a})\mathbf{Z}(\hat{\Sigma}^{-1}\mu - \mathbf{a}) = \frac{Q_{t-1}^2}{\tau_v + Q_{t-1} + +Y_{t-1}}(z_{a,t-1} - p_{t-1})^2, \tag{37}$$

leading to the form of the utility stated in the theorem (5), with $C_{a,t-1} = |I + \hat{\Sigma}\mathbf{B}|^{1/2}$. It is easy to check that $C_{a,t-1}$ takes the prescribed form, as does then $D_{a,t-1} = C_{a,t-1}^{-1/2}D_{a,t}$.

Thus, given a linear pricing function, agents' demand take a linear form and, moreover, the coefficients take the functional forms shown in the Theorem, as do agents' expected utility. We are done. ∎

C Internet Appendix

C.1 Proofs

Proof of Theorem 2: The proof follows immediately from (4), (30-31), and (35).

∎

Proof of Theorem 3: The certainty equivalent satisfies $-e^{-\gamma_a CE} = E_0[-e^{-W_{T+1}}] = -E_0\left[\prod_{t=2}^{T} C_{t-1}^{-1} e^{-\frac{1}{2}\frac{Q_1^2}{\tau_v+Q_1+Y_1}(z_{a,1}-p_1)^2}\right]$. It is easy to see that

$$\prod_{t=2}^{T} C_{t-1}^{-1} = \left(\frac{\tau_v + Q_T + Y_T}{\tau_v + Q_1 + Y_1})\right)\left(\frac{\tau_v + Y_1}{\tau_v + Y_T}\right)\prod_{t=2}^{T}\left(1 + \frac{Q_{t-1}(Y_t - Y_{t-1})}{(\tau_v + Y_{t-1})(\tau_v + Y_t)}\right).$$

Moreover, since at $t = 0$, $z_{a,1} - p_1 \sim N(0, \frac{1}{Q_1} + \frac{1}{\tau_v+Y_1})$, it follows that

$$E_0\left[e^{-\frac{1}{2}\frac{Q_1^2}{\tau_v+Q_1+Y_1}(z_{a,1}-p_1)^2}\right] = \frac{\tau_v + Q_1 + Y_1}{\tau_v + Y_1},$$

and the result for the ex ante certainty equivalent follows.

The ex ante expected profits between t and $t+1$ are $E_0[(z_{a,t} - p_t)(p_{t+1} - p_t)]$. Plugging in the form (34,35) yields the result. Using a similar approach for expected profits, we get that the expected total, time T trading profit of agent a's trade in time t is

$$\frac{1}{\gamma_a}\frac{Q_{a,t}}{\tau_v + Y_t},$$

and the total expected trading profit over time therefore is

$$\frac{1}{\gamma_a}\sum_{t=1}^{T}\frac{Q_{a,t}}{\tau_v + Y_t}, \tag{38}$$

We are done.

∎

Proof of Theorem 4: For the first part, we note that since $(p_t - p_{t-1})$ is independent of p_{t-1} (given publicly available information), it follows that the price volatility between $t-1$ and t is equal to $(\Sigma_{XX})_{11}$, $\sigma^2_{p,t} = (\Sigma_{XX})_{11}|_{t-1} = \frac{y_t}{(\tau_v+Y_t)(\tau_v+Y_{t-1})}$. with the convention $Y_0 = 0$. Also, the final period volatility of $v - p_T$ is $\sigma^2_{p,T+1} = \frac{1}{\tau_v+Y_T}$. This proves the first part of the theorem.

For the second part, we first note that from the definition of y_1, it follows that $y_1 = \tau_v \frac{k_1}{1-k_1}$ and—defining $K_t = \sum_{i=1}^{t} k_i$—a simple induction argument further shows $y_t = \tau_v \frac{k_t}{(1-K_{t-1})(1-K_t)}$, $t = 1, \ldots, T-1$, and $y_T = \tau_v \frac{1-K_T}{k_T(1-K_{T-1})}$. We note that all the $y_1, \ldots, y_T$ are all well defined.

Next, we back out the connectedness that is needed to be consistent with the y's. We have $A_1 = \sqrt{\frac{y_1}{\tau_u}}$, $A_t = A_{t-1} + \sqrt{\frac{y_t}{\tau_u}}$, leading to $\bar{V}_1 \stackrel{\text{def}}{=} \frac{\sum_a V_{a,1}}{N} = \frac{\gamma}{\tau}\sqrt{\frac{y_1}{\tau_u}}$, $\Delta\bar{V}_t \stackrel{\text{def}}{=} \bar{V}_t - \bar{V}_{t-1} = \frac{\gamma}{\tau}\sqrt{\frac{y_t}{\tau_u}}$. Thus, if we can replicate, arbitrarily closely, any sequence of diffusions, through which the average number of signals, $\bar{V}_t$, increases over time, then we can generate any y_t, and thereby any volatility structures. We note that γ is a free parameter that allows us to scale the network to arbitrary sizes. The result now follows from the following lemma:

Lemma 7 *For any T, there are networks of size N, such that $\bar{V}_{T'} = (1 + o(1))\bar{V}_T$ for $T' > T$, and $\bar{V}_{T'} = \frac{1}{N(1+o(1))}\bar{V}_T$ for $T' < T$.*

This lemma thus states that we can always find a (possibly large) network such that very little happens before and after time T, with respect to information diffusion. The result follows immediately: For $T=1$, a tightly-knit network would have these properties. For $T=2$, a large star network. For $T=3$, a star-like network with N^2+N nodes, in which there are N tightly-nit nodes in the center, each connected to N peripheral agents. For even $T\geq 4$, adding longer distance to the $T=2$ (star) network, and for odd $T\geq 5$, adding longer distances to the $T=3$ network will generate these properties. Let's call such a network a T-network.

Finally, any sequence of $\frac{\bar{V}_{t+1}}{\bar{V}_t}$, $t=1,\dots,T$ can be generated by choosing a network with many disjoint $1-,2-,\dots,T-$networks in such a way so that the relative sizes of the networks match the fractions.

We are done. ∎

Proof of Theorem 5: The proof is based on the following standard lemma:

Lemma 8 *Assume a normally distributed random variable, $y\sim N(\mu,\sigma^2)$. Then $E[|y|]=\sigma\sqrt{\frac{2}{\pi}}e^{-\frac{\mu^2}{2\sigma^2}}+\mu(1-2\Phi(-\mu/\sigma))$, where Φ is the cumulative normal distribution of a standard normal variable.*

We note that from (4) and given that $v=\bar{v}+\eta$, it follows that agent a's net time-t demand is

$$
\begin{aligned}
\gamma_a\Delta x_{a,t} &= Q_{a,t}(z_{a,t}-p_t)-Q_{a,t}(z_{a,t-1}-p_{t-1})\\
&= Q_{a,t}\left(\bar{v}+\eta+\frac{Q_{t-1}}{Q_{a,t}}\xi_{a,t-1}+\frac{q_t}{Q_{a,t}}e_{a,t}-\left(\frac{\tau_v}{\tau_v+Y_t}\bar{v}+\frac{Y_t}{\tau_v+Y_t}(\bar{v}+\eta)+\frac{1}{\tau_v+Y_t}\sum_{i=1}^{t}(A_i-A_{i-1})\tau_{u_i}u_i\right)\right)\\
&- Q_{a,t-1}\left(\bar{v}+\eta+\xi_{a,t-1}+-\left(\frac{\tau_v}{\tau_v+Y_{t-1}}\bar{v}+\frac{Y_{t-1}}{\tau_v+Y_{t-1}}(\bar{v}+\eta)+\frac{1}{\tau_v+Y_{t-1}}\sum_{i=1}^{t-1}(A_i-A_{i-1})\tau_{u_i}u_i\right)\right)\\
&= \underbrace{q_{a,t}e_{a,t}}_{\sim N(0,q_{a,t})}\underbrace{-\left(\frac{Q_{a,t}}{\tau_v+Y_t}-\frac{Q_{a,t-1}}{\tau_v+Y_{t-1}}\right)\left(\tau_v\eta-\sum_{i=1}^{t-1}(A_i-A_{i-1})\tau_{u_i}u_i\right)-\frac{Q_{a,t}}{\tau_v+Y_t}(A_t-A_{t-1})\tau_{u_i}u_t}_{\sim N(0,\hat{r}^2_{a,t})}
\end{aligned}
$$

where

$$
\begin{aligned}
\hat{r}^2_{a,t} &= \left(\frac{Q_{a,t}}{\tau_v+Y_t}-\frac{Q_{a,t-1}}{\tau_v+Y_{t-1}}\right)^2(\tau_v+Y_{t-1})+\left(\frac{Q_{a,t}}{\tau_v+Y_t}\right)^2 y_t\\
&= \left(\frac{Q^2_{a,t}}{(\tau_v+Y_t)^2}+\frac{Q^2_{a,t-1}}{(\tau_v+Y_{t-1})^2}-2\frac{Q_{a,t}Q_{a,t-1}}{(\tau_v+Y_{t-1})(\tau_v+Y_t)}\right)(\tau_v+Y_{t-1})+\left(\frac{Q_{a,t}}{\tau_v+Y_t}\right)^2 y_t\\
&= \frac{Q^2_{a,t}}{\tau_v+Y_t}+\frac{Q^2_{a,t-1}}{\tau_v+Y_{t-1}}-2\frac{Q_{a,t}(Q_{a,t}-q_{a,t})}{\tau_v+Y_t}\\
&= \frac{Q^2_{a,t-1}}{\tau_v+Y_{t-1}}-\frac{Q^2_{a,t}}{\tau_v+Y_t}+2\frac{q_{a,t}Q_{a,t}}{\tau_v+Y_t}.
\end{aligned}
$$

Recalling that $q_{a,t}=\tau\Delta V_{a,t}$ and $Q_{a,t}=\tau V_{a,t}$, this leads to

$$
\gamma_a\Delta x_{a,t}=\sqrt{\tau\Delta V_{a,t}}\left(\omega_{a,t}+\frac{r_{a,t}\sqrt{\tau}}{\sqrt{\Delta V_{a,t}}}\xi_t\right),
$$

where $\omega_{a,t}\sim N(0,1)$ is independent of $\xi_t\sim N(0,1)$ and across agents,

$$
r^2_{a,t}=\frac{V^2_{a,t-1}}{\tau_v+Y_{t-1}}-\frac{V^2_{a,t}}{\tau_v+Y_t}+2\frac{\Delta V_{a,t}V_{a,t}}{\tau_v+Y_t},
$$

when $\Delta V_{a,t} > 0$, and

$$\gamma_a \Delta x_{a,t} = r_{a,t}\tau\xi_t,$$

when $\Delta V_{a,t} = 0$. Assuming that $\Delta V_{a,t} > 0$, we note that $\gamma_a \frac{\Delta x_{a,t}}{\tau\Delta V_{a,t}}\Big|\xi_t \sim N\left(\frac{r_{a,t}\sqrt{\tau}}{\sqrt{\Delta V_{a,t}}}\xi_t, 1\right)$. The law of large numbers, together with Lemma 8, then in turn implies that, conditioned on ξ_t,

$$\gamma_a \frac{1}{M}\sum_a |\Delta x_{a,t}| \to_{a.s.} \sqrt{\tau\Delta V_{a,t}}\left(\sqrt{\frac{2}{\pi}}e^{-\frac{\tau r_{a,t}^2\xi_t^2}{2\Delta V_{a,t}}} + \frac{r_{a,t}\sqrt{\tau}}{\sqrt{\Delta V_{a,t}}}\xi_t\left(1 - 2\Phi\left(-\frac{r_{a,t}\sqrt{\tau}}{\sqrt{\Delta V_{a,t}}}\xi_t\right)\right)\right).$$

It follows immediately that the unconditional expectation of the first term (not conditioning on ξ_t) is $\sqrt{\frac{2}{\pi}}\frac{\sqrt{\tau\Delta V_{a,t}}}{\sqrt{\frac{\tau r_{a,t}^2}{\Delta V_{a,t}}+1}} = \sqrt{\frac{2}{\pi}}\frac{\sqrt{\tau}\Delta V_{a,t}}{\sqrt{\tau r_{a,t}^2+\Delta V_{a,t}}}$. For the second term, we use the fact that $E[y\Phi(ay)] = \sqrt{\frac{1}{2\pi}}\frac{a}{\sqrt{a^2+1}}$, for a random variable $y \sim N(0,1)$, to get

$$\sqrt{\tau\Delta V_{a,t}}E\left[\frac{r_{a,t}\sqrt{\tau}}{\sqrt{\Delta V_{a,t}}}\xi_t\left(1 - 2\Phi\left(-\frac{r_{a,t}\sqrt{\tau}}{\sqrt{\Delta V_{a,t}}}\xi_t\right)\right)\right] = \sqrt{\frac{2}{\pi}}\sqrt{\tau\Delta V_{a,t}}\frac{\frac{r_{a,t}^2\tau}{\Delta V_{a,t}}}{\sqrt{1+\frac{r_{a,t}^2\tau}{\Delta V_{a,t}}}} = \sqrt{\frac{2}{\pi}}\sqrt{\tau}\frac{\tau r_{a,t}^2}{\sqrt{\tau r_{a,t}^2 + \Delta V_{a,t}}}.$$

Summing the two terms together, we get

$$E\left[\frac{1}{M}\sum_a |\Delta x_{a,t}|\right] \to \frac{\tau}{\gamma_a}\sqrt{\frac{2}{\pi}\left(r_{a,t}^2 + \frac{\Delta V_{a,t}}{\tau}\right)}.$$

We note that this formula also holds when $\Delta V_{a,t} = 0$, since $E|r_{a,t}\tau\xi_t| = \tau\sqrt{\frac{2}{\pi}r_{a,t}^2}$. This finally leads to (20)

$$\begin{aligned}
X_t &= \frac{\tau}{N}\sum_{a=1}^N \frac{1}{\gamma_a}\sqrt{\frac{2}{\pi}\left(r_{a,t}^2 + \frac{\Delta V_{a,t}}{\tau}\right)} \\
&= \frac{\tau}{N}\sum_{a=1}^N \frac{1}{\gamma_a}\sqrt{\frac{2}{\pi}\left(\frac{V_{a,t-1}^2}{\tau_v + Y_{t-1}} - \frac{V_{a,t}^2}{\tau_v + Y_t} + 2\frac{\Delta V_{a,t}V_{a,t}}{\tau_v + Y_t} + \frac{\Delta V_{a,t}}{\tau}\right)}.
\end{aligned}$$

Now, if one of the $\gamma_a \to 0$, then agent $V_{a,t}$ will determine A_t, y_t, and Y_t. In this case, we get

$$X_t = \frac{\tau}{N}\sum_{a=1}^N \frac{1}{\gamma_a}\sqrt{\frac{2}{\pi}\left(\frac{V_{t-1}^2\frac{\tau_u\tau^2}{\gamma_a^2}\Delta V_t^2}{(\tau_v + \frac{\tau_u\tau^2}{\gamma_a^2}\sum_{i=1}^{t-1}\Delta V_t^2)(\tau_v + \frac{\tau_u\tau^2}{\gamma_a^2}\sum_{i=1}^{t}\Delta V_t^2)} - \frac{2V_{t-1}\Delta V_t}{\tau_v + \frac{\tau_u\tau^2}{\gamma_a^2}\sum_{i=1}^t \Delta V_t^2} + \frac{\Delta V_t}{\tau}\right)}.$$

Using the fact that $V_t = \sum \Delta V_t$, leading to the inequality $V_t^2 \le t\sum_{i=1}^t \Delta V_i^2$ (from $E[x^2] \ge E[x]^2$), it follows that for large ΔV_t, the third term will be dominant, and therefore any sequence of X_t can be generated by choosing ΔV_t appropriately. This shows the second part of the theorem.

We are done. ∎

C.2 Noise traders

It may be difficult for the econometrician to separate the noise traders from the informed traders (the ones in the network) in the investor population. One approach is to simply not try to distinguish the two groups, and view the full network that includes noise traders as a perturbation of the true information network. Ozsoylev, Walden, Yavuz, and Bildik (2014) discuss how including noise traders, as well as how not having access to the full population, affects such network measurements. Their main conclusion is that the important network properties are quite robust to such perturbations. Their approach to identifying the network is different than ours, being based on trades. We therefore study the effects of including noise traders in the network measurement within our setting.

We use the preferential attachment model of Barabasi and Albert (1999) to randomly simulate a large number of networks and study the effect of including noise traders. The model is known to generate networks that are similar to those seen in many different real world contexts, and is therefore commonly used. Especially, it is based on the idea that nodes that are already well connected tend to be linked to by new agents who arrive to the network (in a network formation stage), possibly—and in line with our model—because of the value that these new agents associate with being connected with someone well connected.

We simulate 1,000 networks, each with $N = 300$ agents in the information network, and with N_{noise} noise traders randomly added to the network, together with "spurious links" that we also add randomly, leading to an extended network. The noise traders trade for exogenous reasons (contributing to the random supply of shares) and do not participate in the information network. The econometrician, however, includes the noise traders, as well as their spurious links when measuring the network, thus using the extended network. We choose the average number of spurious links for these noise traders to be the same as for the informed traders, but add the links completely randomly between the noise traders themselves and between noise traders and traders in the information network (same probability per link). The noise trader part of the network is thus completely random, whereas the true network, although randomly generated, has a structure that systematically makes some agents more well connected than others, in line with real-world networks.

We measure individual centrality, network centrality, and asymmetry—based on the Katz centrality proxy—in the true and extended network, and compare how closely related the two measures are. We do this in Figure 6 by comparing the average correlation between individual centrality in the two networks over all simulations, in the left panel of the figure, between network centrality, across simulations, in the middle panel, and between network asymmetry across correlations in the right panel. As seen in the figure, even when a significant majority of the nodes are made up by noise traders, (600 noise traders compared with 300 traders in the information network), the correlations are very high, above 0.9 for individual centrality, close to 0.9 for network centrality, and about 0.7 for asymmetry.

These results are robust to several variations, e.g., having different sizes of the true network, varying α in the Katz centrality measure, and using different network generation models. Similar results are also obtained when Spearman rank correlation is used.

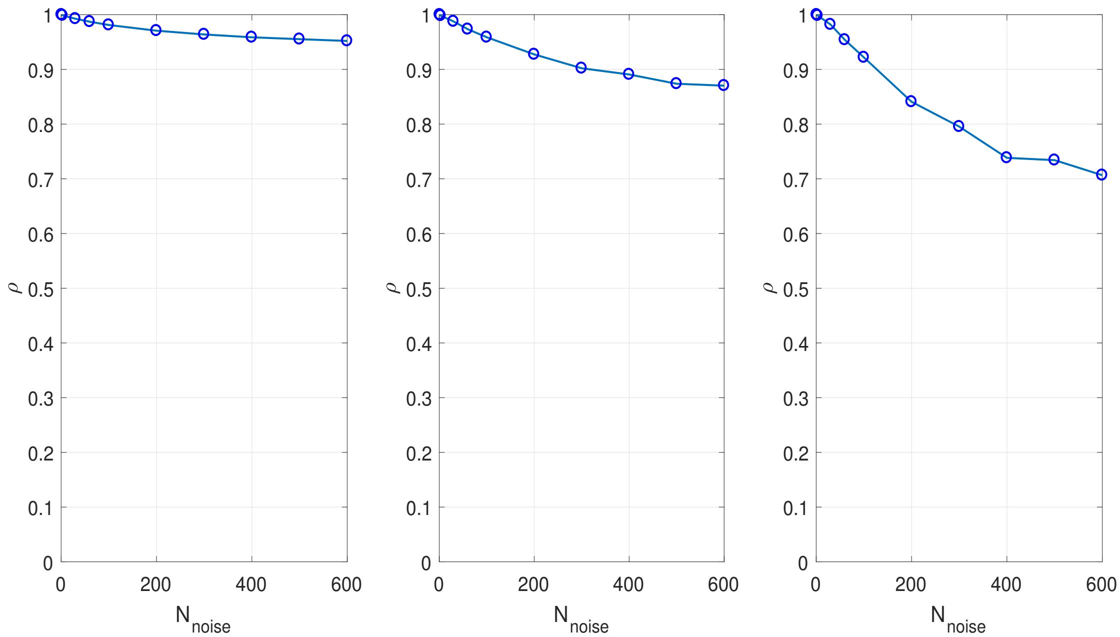

Figure 6: Correlation between actual and measured centrality and asymmetry, as a function of number of noise traders, N_{noise}. The left panel measures the average (over all simulations) correlation between individual centrality in the true and extended networks. The middle panel measures the correlation between network centrality in the true and expanded network, across simulations. The right panel measures the correlation between asymmetry of true and expanded network, across simulations. Number of simulations: 1,000 for each value of N_{noise}. Network size: $N = 300$. Katz centrality measure used, with $\alpha = \frac{\lambda^{-1}}{2}$.

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