Social Transmission Bias and Investor Behavior*

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Abstract

Individual investors often invest actively and lose thereby. Social interaction seems to exacerbate this tendency. In our model, senders' propensity to discuss their strategies' returns, and receivers' propensity to be converted, are increasing in sender return. A distinctive implication is that the rate of conversion of investors to active investing is convex in sender return. Unconditionally, active strategies (high variance, skewness, and personal involvement) dominate the population unless the return penalty to active investing is too large. Thus, the model can explain overvaluation of 'active' asset characteristics even when investors have no inherent preference over them. It also has strong predictions for how adoption of active strategies depends on features of the investor social network. In contrast with nonsocial approaches, sociability and other features of the sending and receiving processes are determinants of the popularity of active investing and the pricing of active strategies.

Keywords: capital markets, behavioral finance, active investing, social networks, thought contagion, transmission bias

JEL Classification: G11, G12

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1 Introduction

A neglected topic in financial economics is how investment ideas are transmitted from person to person. In most investments models, the influence of individual choices on others is mediated by price or by quantities traded in impersonal markets. However, more direct forms of social interaction also affect investment decisions. As Shiller (1989) put it, "...Investing in speculative assets is a social activity. Investors spend a substantial part of their leisure time discussing investments, reading about investments, or gossiping about others' successes or failures in investing." In one survey, individual investors were asked what first drew their attention to the firm whose stock they had most recently bought. Almost all referred to direct personal contact; personal interaction was also important for institutional investors (Shiller and Pound 1989). Furthermore, an empirical literature finds that social interactions affect investment decisions by individuals and money managers, including selection of individual stocks.[1]

Our purpose here is to model how the *process by which ideas are transmitted* affects social outcomes, with an application to active versus passive investment behavior. We view the transmission process here as including both in-person and electronic means of conversation, and one-to-many forms of communication such as blogging and news media. We explore the idea that biases in conversation can favor superficially-appealing ideas about personal investing (see also Shiller (2000a, 2000b, 2017)).

It is a remarkable fact that individual investors trade actively and have invested in active investment funds for decades, and thereby have on average underperformed net of costs relative to a passive strategy such as holding a market index—the active investing puzzle.[2] In addition to underperforming relative to factor benchmarks, trading in individual stocks and investing in active funds adds idiosyncratic portfolio volatility. For example, the idiosyncratic risk exposure of Swedish households accounts for half of the return variance for the median household (Calvet, Campbell, and Sodini 2007). An investor belief that they can choose advisers to beat the market is also the basis for perennially occurring financial scams. A further notable aspect of active investing is that investors are attracted

[1]Shiller (1990, 2000b) discusses other indications that conversation matters for security investment decisions and bubbles. The empirical literature includes Kelly and O'Grada (2000), Duflo and Saez (2002, 2003), Hong, Kubik, and Stein (2004, 2005), Massa and Simonov (2005), Ivković and Weisbenner (2007), Brown et al. (2008), Cohen, Frazzini and Malloy (2008, 2010), Shive (2010), Gray, Crawford, and Kern (2012), and Mitton, Vorkink, and Wright (2015).

[2]On underperformance in individual trading, see Barber and Odean (2000a), Barber et al. (2009). Carhart (1997) and Daniel et al. (1997) find that active funds typically do not outperform passive benchmarks. French (2008) documents very large fees paid in the aggregate by investors to active funds.

to stocks with high skewness ('lottery' stocks) and volatility (Kumar 2009; Bali, Cakici, and Whitelaw 2011; Han and Kumar 2013; Boyer and Vorkink 2014).

The leading explanations for naive active investing are based on individual-level cognitive biases. For example, excessive individual investor trading is often attributed to investor overconfidence (DeBondt and Thaler 1995; Barber and Odean 2000a), the tendency of investors to overestimate their abilities. However, trading aggressiveness is greatly exacerbated by social interactions. For example, participants in investment clubs seem to select individual stocks based on reasons that are easily exchanged with others (Barber, Heath, and Odean 2003); select small, high-beta, growth stocks; turn over their portfolios very frequently; and underperform the market (Barber and Odean 2000b). There is evidence (mentioned in footnote 1) that stock picking by individuals and institutions, an active investing behavior, spreads socially. Furthermore, stock market participation increases with measures of social connectedness (Hong, Kubik, and Stein 2004; Kaustia and Knüpfer 2012).[3]

The leading explanations for the attraction of investors to lottery stocks have also uniformly been based on individual-level biases—specifically, nontraditional preferences (Brunnermeier and Parker 2005; Barberis and Huang 2008). One contribution of our paper is to describe a simple mechanism that can lead to attraction to skewness even if investors have conventional preferences. Also, our approach can explain why higher intensity of social interactions is associated with stronger attraction of investors to both high volatility and high skewness stocks (Kumar 2009).

Although individual-level bias is probably an important part of the explanation for the attraction to skewness, these facts suggest that social interaction is also important. But the sheer fact of contagion in investment choice, as documented in several empirical studies, does not explain a tilt toward active investing strategies, since either active or passive strategies can spread from person to person. In our model, systematic *biases* in the transmission process promote active over passive investing. Our model offers a rich set of further testable implications. These include convexity in the relation between conversion to a new strategy and its past returns, and an attraction of investors to high variance and high skew strategies that increases with sociability.

The key features of the model are the *sending schedule*, which gives the probability that the sender reports the sender's return outcome as a function of that return; and the *receiving schedule*, defined as the probability that a given reported return will convert the receiver

[3]During the millennial high-tech boom, investors who switched early to online trading subsequently began to trade more actively and speculatively, and earned reduced trading profits (Barber and Odean 2002; Choi, Laibson, and Metrick 2002). Early internet investors probably had greater access to and interest in online forms of social interaction, such as e-mail and investment chat rooms. Internet discussions rooms were, according to media reports, important in stimulating day trading.

to the strategy of the sender. The model shows how the interplay between the probability distribution of strategy return outcomes with the shapes of these schedules determine which investment strategies spread through the population. Our social framework also captures a third interpretation of active investing (apart from high volatility and high skewness), an attraction to stocks that are engaging to talk about with others.

As an illustration of an effect of the sending function, we find that high-volatility strategies spread because investors like to recount to others their investment victories more than their defeats, and that listeners do not fully discount for this. We call this sender behavior *self-enhancing transmission bias*, or *SET*. There is considerable evidence (see footnote 13) suggesting that self-enhancing thought processes influence financial behavior.

Both a rational concern for reputation and psychological bias can contribute to *SET*. Research on self-presentation and impression management finds that people seek to report positively about themselves, as constrained by the need to be plausible and to satisfy norms for modesty (Goffman 1961; Schlenker 1980). There is also extensive evidence of internal self-enhancing thought processes, such as the tendency of people to attribute successes to their own virtues, and failures to external circumstances or luck (Bem 1972; Langer and Roth 1975). Such processes encourage people to think more about their successes than their failures, as in the model of Benabou and Tirole (2002). Such self-enhancing thinking is likely to result in self-enhancing bias in conversation.

In the model, investors adopt either an Active (A) or Passive (P) investment strategy. We interpret A as the riskier option, or alternatively, the more engaging one (meaning that adopters are, all else equal, more likely to talk about it, perhaps because it is more novel, affect-laden, or arousing). *SET* creates an upward selection bias in the sender's reports to other investors about the profitability of the chosen strategy: they hear more often about good outcomes than bad ones. The bias increases with return variance; for example, if variance is zero the selection bias vanishes. Listeners do not fully discount for the biased sample of return reports they receive, and we also assume that they naively think that past performance is indicative of future performance. So if A has higher variance than P, receivers overestimate the value of adopting A relative to P, causing A to spread through the investor population.

As an illustration of the importance of the receiving schedule, suppose that receivers attend more to extreme outcomes. This makes the receiving function convex, so that extreme returns are incrementally more persuasive to the receiver (relative to a linear schedule; higher returns are still always more persuasive than lower returns). So high salience of extreme outcomes promotes the spread of high volatility strategies, because such strategies generate extreme returns more often.

As two illustrations that the *interaction* of the sending and receiving schedules is crucial, first suppose that there is both *SET* on the part of senders and salience of extreme returns on the part of receivers. This causes high skewness strategies to spread—even after controlling for volatility. The reason is that such strategies more often generate the extreme high returns which are most often reported, attended to, and are most influential. So A spreads through the population unless it has a strong enough offsetting disadvantage (lower expected return).

As a second illustration of how the sending and receiving schedules interact, consider again the more basic feature of these schedules—that a higher return encourages senders to send (*SET*), and is more persuasive to receivers. (The argument here can accommodate, but does not require, convexity of the receiving function.) Then conditioned on the sender's return, the probability that a receiver is transformed into the type of the sender is a convex function of the sender return. This effect derives from the multiplicative interaction between the increasing probability of the sender sending, and of the receiver being converted conditional upon a message being sent.

This convexity offers a new explanation for the well-known finding of convexity of fund flows. Furthermore, the model offers distinctive implications about the degree of convexity as determined by empirically measurable parameters of the social interaction process.

Finally, returning to an effect driven primarily by the sending schedule, if A is more engaging than P as a conversation topic (more *conversable*, in our terminology), then A is recommended and its return reported to current adopters of P more often than reports about P are made to adopters of A. This favors the spread of A.

In addition to market-wide implications, the model offers a rich set of predictions about the behaviors of specific investors embedded in a social network. The determinants of investor's strategy depends on who the investor is linked to, the performance of an investor's neighbors' strategies, the volatilities and skewnesses of neighbors' strategies, the sociability of the investor, and the investor's homophily (tendency to be linked to investors with similar strategies). We further derive implications for active investing of the *aggregate* homophily, and of aggregate connectivity in the network.

The interplay of investor sending and receiving functions provides a unified and fundamentally *social* explanation for a wide range of patterns in trading and return predictability. These include the convexity of new participation in investment strategies as a function of past performance;[4] the participation of individuals in lotteries with negative expected re-

[4]The convexity implication is consistent with evidence of disproportionate inflows to strongly-performing mutual funds. Kaustia and Knüpfer (2012) provide evidence of such convexity in new stock market participation as a function of neighbor's recent stock return. Our model predicts this effect as a result of interactions between the shapes of the sending and receiving functions. In particular, it predicts that the slope and convexity of flows to an investment strategy as a function of its past return will be greater when social interactions intensify and investors are more influenced by *SET*.

turn; the attraction of some investors to high variance and high skewness ('lottery') stocks, resulting in return anomalies; overvaluation of lottery-like categories of stocks, such as growth stocks, distressed firms, firms that have recently undertaken Initial Public Offerings (IPOs), and high volatility firms; and heavy trading and overvaluation of firms that are attractive as topics of conversation (such as sports, entertainment, and media firms, firms with hot consumer products, and local firms). There are alternative theories based upon individual-level biases that offer piecemeal explanations for subsets of these facts; our framework provides a unified explanation, as well as an extensive set of further distinctive empirical implications.

A key set of distinctive implications of our approach is that these effects tend to be intensified by social interactions, and are therefore stronger when there is higher sociability, appropriately measured—at both the individual level and in society at large. There is evidence supporting the hypothesis that these effects are associated with proxies for sociability.[5] Our approach offers the empirical predictions that sociability increases the slope and convexity of the schedule describing the adoption of active investing strategies by new investors as a function of the past returns of such strategies. Our framework also offers a distinctive set of further testable empirical implications derived from varying the parameters of sending and receiving schedules, such as *SET*, the sensitivity of receivers to reported returns, and the intensity of social interactions. In sum, our approach offers a new *social* approach to understanding investor behavior and security prices.

We are not the first to examine biases in the social transmission of behavior. The effects of social interactions on the spread of cultural traits have been analyzed in fields such as anthropology (Henrich and Boyd 1998), zoology (Lachlan, Crooks, and Laland 1998; Dodds and Watts 2005), and social psychology (Cialdini and Goldstein 2004). Economists have also modelled how cultural evolutionary processes affect ethnic and religious traits, and altruistic preferences (Bisin and Verdier 2000; Bisin and Verdier 2001). The focus here is on understanding investment and risk-taking behavior. Financial models have examined how social interactions affect information aggregation, and potentially can generate financial crises.[6] This paper differs from this literature in examining how social transmission biases such as *SET* affect the evolutionary outcome.

[5]See, e.g., Hong, Kubik, and Stein (2004) and Kaustia and Knüpfer (2012) for stock market participation, and Kumar (2009) for preference for high skewness stocks and high volatility stocks.

[6]Such models address how information flows in social networks affect asset markets (DeMarzo, Vayanos, and Zwiebel 2001), crises and herd behavior (Cipriani and Guarino 2002; Cipriani and Guarino 2008), and IPO allocations and pricing (Welch 1992). Brunnermeier (2001) and Hirshleifer and Teoh (2009) review the theory of herding in financial markets. Recent models of social networks explore information acquisition, cost of capital, liquidity, and trading volume (Ozsoylev and Walden 2011; Han and Yang 2013). Burnside, Eichenbaum, and Rebelo (2016) apply an epidemic model to explain booms and busts in the housing market; they do not examine transmission bias in conversation, which is the focus of our paper.

DeMarzo, Vayanos, and Zwiebel (2003) show that persuasion bias, the failure of receivers to account for possible repetition in the messages they hear from others, plays an important role in the process of social opinion formation. They find that network position is a key determinant of how influential an individual is, and that an individual's opinions across different issues will be highly correlated. Our paper differs in focusing on other transmission biases originating from both senders and receivers, and in exploring the spread of active investing.

Hong, Kubik, and Stein (2004) provide evidence of social influence in stock market participation. In their motivating model, it is assumed that social interaction causes participation, rather than nonparticipation, to spread from person to person. It follows that more social individuals participate more. However, contagion of nonparticipation is also possible. People who fear the market or view it as an unsavory gambling casino can spread negative attitudes to others. Our paper differs in modeling explicitly whether it is favorable or unfavorable information that is transmitted and used by others; and in studying the more general topic of whether active or passive investing strategies spread.[7]

2 The Model

2.1 Social Interactions in Network of Investors

Consider a population consisting of an even number of investors, N, who adopt either an Active (A) or Passive (P) type of investment strategy, with returns R_A and R_P. In this section the return distributions of these strategies are exogenously given. Section 3 derives return distributions endogenously.

The Social Network

Investors are connected in an undirected social network represented by the graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the set of investors and $\mathcal{E}$ is the set of edges connecting them. The set of investors $\mathcal{N} = \{1, \ldots, N\}$, and $(m, n) \in \mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ if investors m and n are connected through a social tie. By convention, the network is undirected, i.e., $(m, n) \in \mathcal{E} \Leftrightarrow (n, m) \in \mathcal{E}$, and investors are not connected to themselves ($(n, n) \notin \mathcal{N}$). Collectively, the investment strategies of all investors are summarized by the vector $z = (z_1, z_2, \ldots, z_N) \in \{A, P\}^N$, where $z_m \in Z = \{A, P\}$ is investor m's strategy. For now, we develop the model in a static (two-date) setting. We subsequently analyze a dynamic market with multiple dates

[7] Also, in Hong, Kubik, and Stein (2004), the knowledge and practices that social investors disproportionately acquire are *useful*. If socials are more sophisticated than others, they may be less prone to undesirable active investing strategies. In contrast, our approach implies that more social investors will make better decisions in some ways (participation) but worse decisions other ways (e.g., buying high-expense mutual funds, engaging in day trading, or trying to pick the best IPOs).

$t = 0, 1, 2, \ldots$, added as a superscript to these variables.

In the model, social ties could represent friendship, professional collaboration, membership in the same country club, or involvement with the same online community. If $(m, n) \in \mathcal{E}$, there is a chance that investor m tells n his investment strategy and performance. The set of investors that n is socially linked to is $\mathcal{D}_n = \{m : (n, m) \in \mathcal{E}\} \subset \mathcal{N} \backslash \{n\}$, and n's *degree* (number of connections) is $|\mathcal{D}_n|$. An investor with a higher degree is said to be more connected. We view degree as a proxy for how sociable investor n is.

We assume a standard simple network formation process between investors, according to the Erdös-Rényi-Gilbert random graph model (see Erdös and Rényi (1959), Erdös and Rényi (1960), and Gilbert (1959)). In this model, links between investors are formed randomly and independently. We adopt the Erdös and Rényi (1959) version of the model, in which the number of connections is fixed. Specifically, M connections out of the $Q = N(N-1)/2$ possible connections are randomly chosen sequentially without replacement. Here, M measures social connectivity in the economy. For tractability, we assume that a new network is independently formed in each time period.[8]

Senders and Receivers

In each period (generation), a pair of investors (m, n) is randomly selected, m being the potential *sender* and n being the potential *receiver*, with associated strategies $(z_m, z_n) \in Z \times Z = \{AA, AP, PA, PP\}$.[9] If the investors are connected, $(m, n) \in \mathcal{E}$ which occurs with probability M/Q, the sender with some probability reports his return to the receiver. Let $0 \leq h < 1$ represent *homophily*, the tendency to associate with similar individuals. If $h > 0$, then a link between investors of opposite type is sometimes inoperative. Specifically, a receiver who is sent a message from a sender of different type only considers the message with probability $1 - h$.

When an AA or PP pair is selected (i.e., $z_m = z_n$), population frequencies remain unchanged. When A and P meet (i.e., $z_m \neq z_n$, and the sender and receiver are linked), the probability that sender of type $i \in \{A, P\}$ reports his return performance to the receiver is $s(R_i)$, which is increasing in the sender's return.[10] Upon being sent this message, the receiver then converts to the type of the sender with probability $(1 - h)r(R_i)$, where the function r is also increasing in the sender's return. It is convenient to also define

[8] A similar, albeit less tractable, approach would be to assume that connections between investors are gradually and randomly severed and added over time.

[9] A standard model for allele (gene type) frequency change in evolutionary biology is the Moran process (Moran 1962), in which in each generation exactly one investor is born and one dies, leaving population size constant. Here we apply a Moran process to the spread of an investment strategy, viewed as a culture trait that can be adopted or rejected.

[10] In actual conversations, often both parties recount their experiences. The model's sharp distinction between being a sender and a receiver in a given conversation is stylized, but since either type can become the sender, is unlikely to be misleading.

$g = M(1-h)/Q$, which combines the probabilities that an actual link is chosen among potential links (probability M/Q) and that the sender's message is not disallowed owing to homophily (probability $1-h$).

We have assumed that for given sender return, the sending and receiving functions s and r are independent of whether the sender or receiver are A or P. Nevertheless, transformations do depend indirectly on the sender's type, as this affects the distribution of the sender's return.

We further assume that investors sometimes spontaneously switch their investment strategies even in the absence of conversations with others. Allowing for this ensures that there is a unique long-term distribution of A's in the dynamic version of the model. For simplicity, we capture this by assuming that with probability $q \ll 1$, a complete reset of strategies occurs such that $N/2$ of the investors randomly choose to be active and the other $N/2$ choose to be passive in the next period.[11] If the reset probability were zero, the states with 0 or N active investors would be absorbing, since the only way an investor can be persuaded by another to switch is if there is at least one investor of the opposite type.

Let N_A be the number of A's and f be the population frequency of A's at the start of a period before the meeting,

$$f \equiv \frac{N_A}{N}. \tag{1}$$

The probability that an A sender is paired with a P receiver in that period, given that the sender and receiver are actually connected and that there are N_A type A investors, is then χ_{N_A}, where

$$\chi_{N_A} = \frac{N_A}{N} \times \frac{N-N_A}{N-1}. \tag{2}$$

This is also the probability that a P sender is paired with an A receiver. It follows that $\chi_{N_A} = f(1-f)N/(N-1)$, so the probability of a mixed pairing is low when the fraction of A's is close to zero or one. Finally, the probability that an A sender who is paired with a P receiver converts that receiver is $T_A(R_A)$, and the probability that a P sender converts a P partner is $T_P(R_P)$.

The Sequence of Events

The overall sequence of events that determines how the number of A's at time t, N_A^t, changes to N_A^{t+1} at $t+1$ is shown in Figure 1. Our initial focus is not on resets and homophily, and we therefore assume for now that $q = 0$ and $h = 0$. We later show that almost all the results discussed here extend to $q, h > 0$. Allowing for resets results in a unique long-term distribution of A's and P's, regardless of their initial numbers (Proposition 2).

[11] This is for tractability. The reset can be viewed as a simplification of a model where each investor switches independently of the others.

The effects of homophily will be discussed in Section 2.8. The network formation process is ex ante symmetric; before the network is formed, the probability that any two given investors are paired is the same.

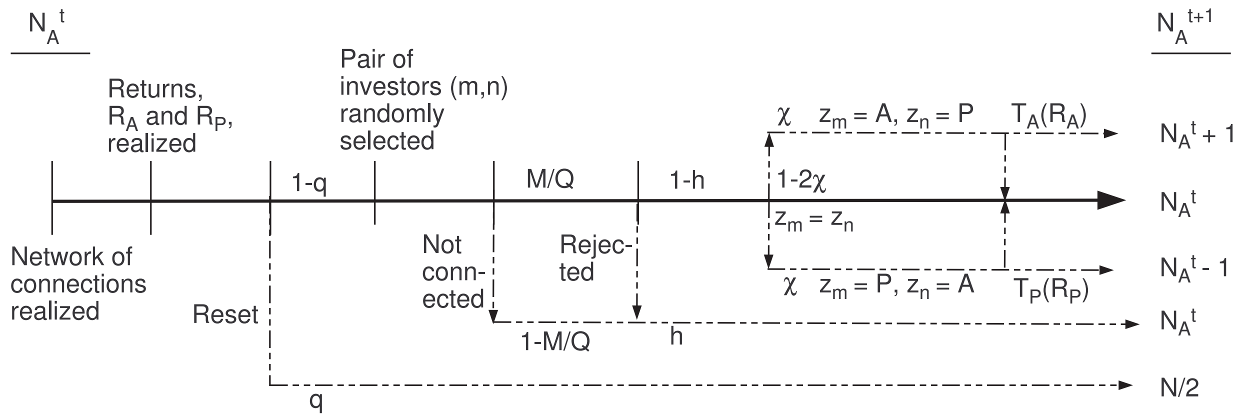

Figure 1: **Sequence of events:** *At time t there are N_A^t active investors. First, the network of connections, $\mathcal{G}$, and returns R_A and R_P are realized. With probability q there is then a reset so that $N_A^{t+1} = N/2$. If no reset occurs (probability $1-q$), a sender (m) and a receiver (n) are chosen, and if (1) they are connected (probability M/Q), (2) they have different strategies, and (3) the pairing is not disallowed by homophily (probability $1-h$), the sender converts the receiver to the senders style with probability $T_{z_m}(R_{z_m})$. The next period's number of active investors is then N_A^{t+1}.*

To derive the transformation probability function, in the next two subsections we describe the sending function and then the receiving function in more detail. These transition probabilities are not conditioned on the realized social network, $\mathcal{E}$. Such probabilities are relevant for many empirical settings in which the network is unobservable. However, we also derive network-conditioned empirical implications.

2.2 Self-Enhancement and the Sending Function

To capture self-enhancing transmission bias, we assume that $s'(R_i) > 0$; the probability that sender of type i sends a message describing the sender's strategy and performance is increasing in the sender's return, R_i. A sender may, of course, exaggerate or simply fabricate a story of high return. But if senders do not always fabricate, the probability of sending will still depend upon the actual return, and the reported return will tend to be increasing in the actual return.

We apply a linear version of *SET*,

$$s(R_i) = \beta R_i + \gamma, \qquad \beta, \gamma > 0, \tag{3}$$

where i is the type of the sender. The sending function is type-independent, so β and γ have no subscripts. To ensure that $0 \leq s(R_i) \leq 1$, we require that $-(\gamma/\beta) \leq R_i \leq (1-\gamma)/\beta$ with high probability, which can hold under reasonable parameter values for β and γ. Appendix A provides details for an endogenous derivation of this sending function.

The assumption that sending is stochastic and smoothly increasing reflects the fact that raising a topic in a conversation depends on both social context and on what topics the conversation partner happens to raise. High return encourages reporting of return, but sending is still uncertain, as senders are constrained by conversational norms against bragging. Similarly, conversational norms for responsiveness will sometimes lead to reporting of a low return.[12]

The positive slope β of the sending schedule reflects *SET*. As discussed in the introduction, *SET* can be driven by either internal biases or by incentives for positive self-presentation. In a review of the impression management field, Leary and Kowalski (1990) discuss how people tend to avoid lying, but, consistent with *SET*, selectively omit information "...to put the best parts of oneself into public view" (pp. 40-1). There is also evidence of *SET* in financial settings.[13]

Consistent with $\beta > 0$, in a database from a Facebook-style social network for individual investors, Simon and Heimer (2015) report that the frequency with which an investor contacts other traders is increasing in the investor's short-term return. Also potentially consistent with $\beta > 0$, Shiller (1990) provides survey evidence that people talked more about real estate in U.S. cities that have experienced rising real estate prices than those that have not. The more tightly bound is the sender's self-esteem or reputation to return performance, the stronger is *SET*, and therefore the higher is β.

[12]Reporting favorably about one's achievements and competence when doing so is not in response to a specific question often leads to negative reactions in observers (Holtgraves and Srull 1989). So owing to conversational norms, in some contexts a sender with high return may not get a graceful chance to raise the topic, and in others even a reluctant sender with poor return will feel pressured to report his performance.

[13]Karlsson, Loewenstein, and Seppi (2009) and Sicherman et al. (2012) find that Scandinavian and U.S. investors reexamine their portfolios more frequently when the market has risen than when it has declined. Consistent with *SET*, for a wide set of consumer products, positive word-of-mouth discussion of user experiences tends to predominate over negative discussion (see the review of East, Hammond, and Wright (2007)), perhaps because users want to persuade others that they are expert at product choice (Wojnicki and Godes 2008). Consistent with *SET* in investing behavior, in a database drawn from a Facebook style social network for individual investors, Simon and Heimer (2015) report that traders are more likely to initiate communication with others when they experience strong short-term gains. A one standard deviation higher weekly returns is associated with a 7% higher probability of contacting other traders in a given week. Using cross-industry stock-financed acquisitions as an instrument to establish causality, Huang, Hwang, and Lou (2016) provide further evidence of *SET* in investor communication about firms in different industries. Using spatial proximity as a proxy for social linkage and amount of trading in the acquirer industry (excluding the acquirer) as proxies for investor communication, they find that target investors are about twice as likely to communicate views about firms in the acquirer industry with their neighbors after experiencing above-median rather than below-median target announcement-day returns.

The constant γ reflects the *conversability* of the investment choice. When the investment is an attractive topic for conversation, the sender raises the topic more often. The sender also raises the topic more often when conversations are more extensive, as occurs when investors are more sociable (how much they talk and share information with each other). So γ also reflects investor sociability.

2.3 The Receiving Function

For notational convenience, we continue to focus on the case with no homophily, $h = 0$. All results in this section and Section 3 still hold when $h > 0$ (as shown in the internet appendix). For a mixed pair of investors, consider now the probability that a receiver of type j is converted to the sender's type i. Given a sender return R_i and that this return is indeed sent, the conditional probability that the receiver is converted is denoted $r(R_i)$. Two key psychological considerations motivate the assumed shape of the receiving function.

First, we expect incomplete discounting of receivers for the selection bias in the messages they receive. There is extensive evidence in various contexts, including financial markets, that observers do not fully adjust for selection bias in the data they observe, a phenomenon called selection neglect.[14] Selection neglect is to be expected when individuals with limited processing power automatically process data in fast intuitive ways rather than taking the effortful cognitive step of adjusting for selection bias.

Second, we expect that the receiver perceives the sender return to be substantially informative about the desirability of the sender's strategy. Regardless of whether this conclusion is correct, it is tempting, as reflected in the need for the boilerplate warning to investors that "past performance is no guarantee of future results." Investors overweight past performance as an indicator of future performance. One or a few recent observations of the performance of a trading strategy generally convey little information about its future prospects. But investors think otherwise. Such extrapolative expectations are consistent with the representativeness heuristic (Tversky and Kahneman 1974), and have been incorporated extensively in financial models (e.g., DeLong et al. (1990), Hong and Stein (1999), Barberis and Shleifer (2003), Barberis et al. (2015), and Hirshleifer, Li and Yu (2015)).[15]

The representativeness heuristic attenuates discounting by a receiver for a sender's upward selection in reporting returns. A receiver who thinks that even a single return

[14] See, e.g., Nisbett and Ross (1980) and Brenner, Koehler, and Tversky (1996). Koehler and Mercer (2009) find that mutual fund families advertise their better-performing funds, and that both novice investors and financial professionals suffer from selection neglect. Selection neglect is consistent with representativeness, a general psychological bias discussed below.

[15] There is evidence that investors have extrapolative expectations from experimental markets (Smith, Suchanek, and Williams 1988; Choi, Laibson, and Madrian 2010), as well as surveys of return expectations and field evidence on security and fund investing.

observation is highly informative will adjust less for sender suppression of bad news. (E.g., the report of a conclusive signal is still definitive, when received, even if sometimes the signal is suppressed). If a receiver believes that past performance is indicative of strategy value, and does not adequately adjust for SET as reflected in the sending function (3), then the receiver will tend to overvalue the sender's strategy. This tends to raise the probability of type switching. Together these two observations suggest that $r'(R_i) > 0$.[16]

We adopt a convex increasing shape for the receiving function, which we specify as the quadratic function

$$r(R_i) = a(R_i)^2 + bR_i + c, \qquad a, b, c > 0, \tag{4}$$

under appropriate parameter constraints ensuring that with probability close to 1, r is monotonically increasing and takes value between 0 and 1. This functional form is motivated by a combination of two effects: greater receiver attention to extreme return outcomes (inducing convexity), and, conditional upon paying attention, greater persuasiveness of higher return. Appendix A.2 provides details for an endogenous derivation of this receiving function.

Intuitively, $b > 0$ means that messages from a sender with strong performance are more persuasive than messages from a sender with weak performance. The parameter b reflects the degree to which the receiver tends to naively extrapolate past strategy returns, or at least to be persuaded by high returns.

The positive quadratic parameter a reflects the tendency, after allowing for the effect of b, for extreme returns to be more persuasive. It captures general evidence that extreme news is more salient than moderate news, and therefore is more often noticed and encoded for later retrieval (Fiske 1980; Moskowitz 2004; Morewedge, Gilbert, and Wilson 2005).[17] The assumption $a > 0$ is mainly needed for the model's skewness predictions, but also reinforces the variance predictions. When cognitive processing power is limited, a focus on extremes is a useful heuristic, as extreme news tends to be highly informative.

The parameter c measures the susceptibility of receivers to influence, deliberate or otherwise, of the sender's report. Such susceptibility can derive from a preference for conformity.

In this quadratic functional form, the probability that the receiver is converted is smoothly increasing in the sender return, and is positive even when the sender has a

[16] Also, our assumptions of increasing sending and receiving functions are compatible with the possibility of sender lying and exaggeration, and a degree of receiver skepticism about such behavior.

[17] High salience of extremes is consistent with the finding that individual investors are net buyers of stocks that experience extreme one-day returns of either sign (Barber and Odean 2008), and the finding that extreme gains or losses at other time horizons are associated with higher probability of both selling and of buying additional shares of stocks that investors currently hold (Ben-David and Hirshleifer 2012). It is also consistent with the salience theory of choice under risk of Bordalo, Gennaioli, and Shleifer (2012, 2013), wherein individuals' attention focuses upon atypical payoffs.

negative return. One reason for positivity in the negative range is that the sheer fact that another investor has adopted or recommends a trading strategy can make an investor aware of the strategy, and can persuade in favor of it. Furthermore, the receiver may have experienced an even lower return from the receiver's current strategy.

Indeed, more generally it is plausible that receivers sometimes make a comparison between the return reported by the sender and the return that the receiver has recently experienced. Such a specification of the receiving function makes the model algebraically more complicated, but generates similar results. As a robustness check, we have verified that similar results apply with an alternative specification where the receiver's switch decision depends on the *difference* in return between sender and receiver.

Also, in the model, investors decide whether to switch strategy based only on the most recent period's return. In principle, fully rational investors might eventually converge to the best action by observing a long history of returns. However, this can be slow since return realizations are noisy indicators about which strategy is better, and there is continual generational transition from experienced to inexperienced investors. Our model captures this by allowing investors to retain return messages for only single period.

2.4 Transformation Probabilities

The transformation probability $T_A(R_A)$ that a sender of type A with return R_A converts a receiver of type P that he is paired to is

$$\begin{aligned} T_A(R_A) &= r(R_A)s(R_A) \\ &= (aR_A^2 + bR_A + c)(\beta R_A + \gamma) \\ &= a\beta R_A^3 + BR_A^2 + CR_A + c\gamma, \end{aligned} \tag{5}$$

where

$$\begin{aligned} B &= a\gamma + b\beta \\ C &= b\gamma + c\beta. \end{aligned} \tag{6}$$

Similarly, the probability that a sender of type P converts a receiver of type A is

$$T_P(R_P) = a\beta R_P^3 + BR_P^2 + CR_P + c\gamma. \tag{7}$$

By assumption, $r', s' > 0$, so $T_A'(R_A), T_P'(R_P) > 0$. The unconditional expected transformation probabilities given that two investors of opposite type meet are denoted

$$T^A = E[T_A(R_A)], \qquad \text{and} \qquad T^P = E[T_P(R_P)]. \tag{8}$$

2.5 Evolution of Types Conditional on Realized Return

We first derive the relationship between the spread in active investing in the population and past returns. We examine both the expected net shift in the fraction of A's, which reflects both inflows and outflows, and the expected unidirectional rate of conversion of P's to A's, such as the rate at which investors who have never participated in the stock market start to participate.

Given returns R_P and R_A, we calculate the expected change in the fraction of type A in the population after one social interaction between two randomly selected connected investors. In the four possible sender-receiver pairings AA, PP, AP, or PA, the change in the frequency of type A given AA or PP is zero. The expected changes in the frequency of type A given a meeting AP or PA and realized returns are

$$\begin{aligned} E[\Delta f|AP, R_A] &= \left[T_A(R_A) \times \frac{1}{N}\right] + [(1 - T_A(R_A)) \times 0] = \frac{T_A(R_A)}{N} \\ E[\Delta f|PA, R_P] &= \left[T_P(R_P) \times \left(-\frac{1}{N}\right)\right] + [(1 - T_P(R_P)) \times 0] = -\frac{T_P(R_P)}{N}. \end{aligned} \tag{9}$$

Taking the expectation across the different possible combinations of sender and receiver types (AA, PP, AP, PA), by (2) and (9),

$$E[\Delta f|R_A, R_P] = \frac{\chi_{N_A}}{N}\left[T_A(R_A) - T_P(R_P)\right], \tag{10}$$

where as defined earlier, χ_{N_A} is the probability of pairing of P sender with a A receiver. So for given returns, the fraction of type A increases on average if and only if $T_A(R_A) > T_P(R_P)$.

Recalling that $T_A(R_A) = s(R_A)r(R_A)$, we derive some basic predictions from the features of the sending and receiving functions. If R_A and R_P are not perfectly correlated, we can calculate the effect of increasing R_A for given R_P. Partially differentiating (10) with respect to R_A twice and using the earlier conditions that $r'(R_A), s'(R_A) > 0$, that $s''(R_A) = 0$ by (3), and that $r''(R_A) > 0$ by (4), gives

$$\left(\frac{N}{\chi_{N_A}}\right)\frac{\partial E[\Delta f|R_A, R_P]}{\partial R_A} = \frac{\partial T_A(R_A)}{\partial R_A} = r'(R_A)s(R_A) + r(R_A)s'(R_A) > 0 \tag{11}$$

$$\left(\frac{N}{\chi_{N_A}}\right)\frac{\partial^2 E[\Delta f|R_A, R_P]}{\partial (R_A)^2} = \frac{\partial^2 T_A(R_A)}{\partial (R_A)^2} = r''(R_A)s(R_A) + 2r'(R_A)s'(R_A) > 0. \tag{12}$$

Since R_A affects T_A but not T_P, these formulas describe how active return affects both the expected net shift in the fraction of A's, and the expected unidirectional rate of conversion from P to A.

Furthermore, substituting for the sending function $s(R_A)$ from (3) and the receiving function $r(R_A)$ from (4) into (11) and (12) gives

$$\left(\frac{N}{\chi_{N_A}}\right)\frac{\partial E[\Delta f|R_A,R_P]}{\partial R_A} = (2aR_A+b)(\beta R_A+\gamma)+\beta(aR_A^2+bR_A+c) \tag{13}$$

$$\left(\frac{N}{\chi_{N_A}}\right)\frac{\partial^2 E[\Delta f|R_A,R_P]}{\partial (R_A)^2} = 2a(\beta R_A+\gamma)+2\beta(2aR_A+b). \tag{14}$$

The fact that sending and receiving functions and their first and second derivatives are all positive signs some of the terms in parentheses, so by it follows immediately from (13) that the sensitivity of the transformation rate of investors to A as a function of past active return is increasing with the parameters of the sending and receiving functions, β, γ, a, b, and c. By (14), a similar point follows immediately for convexity as well, with the exception that c does not enter into convexity.

Proposition 1 *Suppose that the returns to A and P are not perfectly correlated. Then under the parametric specifications of the sending and receiving functions:*

1. *The one-way expected rate of transformation from P to A and the expected change in frequency of A are increasing in return R_A.*

2. *The one-way expected rate of transformation from P to A and the expected change in frequency of A are strictly convex in return R_A.*

3. *The sensitivity of the expected transformation rate of investors to A as a function of past R_A, and the convexity of this relationship, are increasing with SET as reflected in β, sociability as reflected in γ, attention of receivers to extremes as reflected in a, and the extrapolativeness of receivers b.*

4. *The sensitivity of the expected transformation rate of investors to A as a function of past R_A (but not the convexity of this relationship) is increasing with the susceptibility of receivers c.*

This is a rich set of empirical implications, several as yet untested. The predictions of Parts 3-4 are distinctive to our model. For example, since past literature has provided empirical proxies for sociability, it will be valuable to test whether greater sociability is associated with greater slope and convexity of the transformation of investors to active investing as a function of past returns on active strategies.

It will also be valuable to test for the effects of variation in *SET* as reflected in β, which can be measured using psychometric testing, or by exploiting findings from cross-cultural psychology to test for differences in investment behaviors across countries or ethnic groups.

These predictions help further distinguish our model from possible alternative hypotheses. For example, it is possible that in a nonsocial setting with extrapolation, adoption of a strategy may be more sensitive to performance in the gain region than in the loss region. However, a basic extrapolation setting would not share the rich set of predictions of Parts 3 and 4 of Proposition 1.

Some important existing evidence is consistent with the first two empirical predictions. Chevalier and Ellison (1997) and Sirri and Tufano (1998) find that investor funds flow into mutual funds with better performance. This is a non-obvious effect since evidence of persistence in fund performance is very limited. Furthermore, the flow-performance relationship is convex; flows are disproportionately into the best-performing funds.

Lu and Tang (2015) find that 401(k) plan participants place a greater share of their retirement portfolios in risky investments (equity rather than fixed income) when their coworkers earned higher equity returns in the preceding period. Kaustia and Knüpfer (2012) report a strong relation between returns and new participation in the stock market in Finland in the range of positive returns. Specifically, in this range, a higher monthly return on the aggregate portfolio of stocks held by individuals in a zip code neighborhood is associated with increased stock market participation by potential new investors living in that neighborhood during the next month.[18] Their study also provides evidence that supports a prediction of Part 3 in Proposition 1 that the sensitivity of the one-way expected rate of transformation from P to A (stock market entry in their setting) increases with the intensity of social interaction.

The greater strength of the effect in the positive range is consistent with the convexity prediction. Our model does not imply a literally zero effect in the negative range, but a weaker effect within this range (as predicted by Proposition 1) would be statistically harder to detect.

In our setting, an increasing conversion of nonparticipants to participation derives from the combination of *SET* and overextrapolation of others' past returns. Part 1 of Proposition 1 captures *SET* by $s'(R_A) > 0$, and the greater willingness of receivers to convert when return is higher by $r'(R_A) > 0$.

Part 2 of the proposition delivers a more subtle effect, the convexity of the conversion/return relation. This effect arises naturally from the interaction of sending and receiving functions in our model. By (11), $s' > 0$ and $r' > 0$ together contribute to convexity of expected transformation as a function of R_A. Intuitively, multiplying two increasing functions generates rising marginal effects as the argument increases. A further contribu-

[18] Their test focuses on the conversion of new investors to stock market investing, i.e., the conversion of P's to A's. Their study does not test predictions in Proposition 1 about change in net shift from P to A, which accounts for possible shifts from A to P as well.

tor is the convexity of the receiving function, $r''(R_A)$, reflecting high salience of extreme outcomes.[19]

If we interpret A as active trading in the market for individual stocks, with a preponderance of long positions, then a high market return implies high average returns to A's. Proposition 1 therefore implies that when the stock market rises, volume of trade in individual stocks increases. This implication is consistent with episodes such as the rise of day trading, investment clubs, and stock market chat rooms during the millennial internet boom, and with evidence from 46 countries including the U.S. that investors trade more when the stock market has performed well (Statman, Thorley, and Vorkink 2006; Griffin, Nardari, and Stulz 2007). In Appendix C, we formally model market equilibrium with trading volume to verify that evolution toward A is associated with high trading volume.

2.6 Strategy Return Components and the Meaning of Active Investing

We now make exogenous assumptions about the distributions of strategy returns to derive implications about the spread of active investing. This partial equilibrium approach lets us interpret 'active investing' broadly as referring either to static actions such as holding a given risky asset, or to dynamic strategies such as day trading, margin investing, stock picking, market timing, sector rotation, dollar cost averaging, technical analysis, and so forth. The assumptions are relaxed in Section 3.

Let r be the common component of returns shared by A and P (e.g., the market porfolio), where $E[r] = 0$, and let ϵ_i be the strategy-specific component, $E[\epsilon_i] = 0$, $i = A, P$. We assume that r, ϵ_A and ϵ_P are independent, and write the returns to the two strategies as

$$\begin{aligned} R_A &= \beta_A r + \epsilon_A - D \\ R_P &= \beta_P r + \epsilon_P, \end{aligned} \tag{15}$$

where β_i is the sensitivity of strategy return to the common return component. We assume that the active strategy has higher systematic risk, $\beta_A > \beta_P \geq 0$. We further assume that $\sigma_A^2 > \sigma_P^2$, $\gamma_{1A} > 0$, $\gamma_{1P} \approx 0$, and $\gamma_{1r} \geq 0$, where σ_A^2, σ_P^2 are the variances of ϵ_A and ϵ_P, γ_{1r}

[19] This discussion makes clear that Parts 1 and 2 rely only on first and second derivative conditions rather than the specific polynomial specifications of the sending and receiving functions. Also, an examination of (13) and (14) clarifies the drivers of the basic findings (Parts 1 and 2). Part 1 holds even without *SET* (i.e., even if $\beta = 0$). Intuitively, a higher return is simply more persuasive to receivers, which causes conversion. *SET* provides another channel for the prediction in Part 1 by causing sending to increase after positive returns. For Part 2, attention to extremes ($a > 0$) promotes convexity, because as past returns increase, at first the marginal effect is weak (because of lack of attention to very low returns), and then becomes stronger (because of increasing attention to very high returns). But attention to extremes is not required for convexity. Even if $a = 0$, *SET* induces convexity, because when $\beta > 0$, the persuasive effect of higher return on receivers is reinforced multiplicatively by a stronger tendency of senders to send.

is the skewness of r, and γ_{1A}, γ_{1P} are the skewnesses of ϵ_A and ϵ_P. We also let σ_r denote standard deviation of the common factor r.

To summarize, active investing means choosing strategies with return distributions that have higher volatility and possibly also higher skewness. This corresponds fairly well with common parlance, but there are possible exceptions. For example, a long-short strategy that achieved low risk, or a dynamic hedging strategy that generated a riskfree payoff, would not be active in the sense we are using.

Since $E[r] = E[\epsilon_i] = 0$, (15) implies that $E[R_P] = 0$, and D is the return penalty (or if negative, premium) to active trading. We call D the return penalty rather than the 'cost' of active trading, because a major part of the welfare loss may come from lack of diversification and excessive idiosyncratic risk-bearing. So even if $D < 0$, the A's may be worse off than P's.[20]

2.7 The Unconditional Evolution of Investment Types

In our model, the evolution of types, as captured by the aggregate number and fraction of active investors over time, N_A^t and f^t, follows a Markov chain, as shown in Figure 2. When

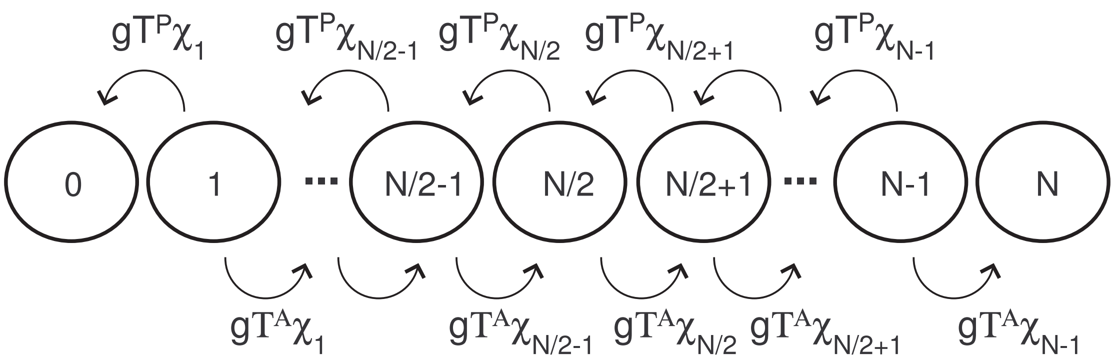

Figure 2: **Markov chain:** *Dynamics of N_A^t when $q = 0$.*

$q > 0$, there is also a chance that $N_A^{t+1} = N/2$ regardless of N_A^t, because of a reset event. Given the initial number of active investors, $N_A^0 = n$, the expected fraction of future A's is defined as $\phi^t = E[f^t | f^0 = n/N]$.

[20]Even when $D < 0$, if A's overvalue the risky asset and P's are rational, being an A rather than a P *decreases* an investor's true expected utility (owing to excessive risk-taking, and an insufficient reward for bearing risk). So the return penalty to active trading D underestimates the welfare loss from active trading. Greater transaction costs of active trading (not modeled here) would also be reflected in D.

When $q = 0$, the unconditional evolution of $\Delta f^{t+1} = f^{t+1} - f^t$ is

$$E_t[\Delta f^{t+1}] \;=\; \frac{\chi_{N_A^t}}{N} g(T^A - T^P). \tag{16}$$

where as defined earlier, g is the probability that an existing link out of all possible links is selected and is not disallowed by homophily, $g = M(1-h)/Q$. The expression above also combines the probabilities that investors of different type are chosen which occurs with probability 2χ), and that they choose to switch (probability T^A and T^P, from P to A and from A to P, respectively).

It follows from (16) that when $q = 0$, the fraction of A's, ϕ^t, increases on average if and only if $T^A > T^P$. It turns out that this condition is sufficient for ϕ^t to increase over time also when $q > 0$. In this case, the long-term distribution of f is independent of the initial number of A's.[21] Intuitively, the presence of a reset pulls the expected number of A's downward toward $N/2$, and thereby weakens the expected upward drift of Δf^{t+1} without completely eliminating it. The following proposition summarizes the result.

Proposition 2 *When $q > 0$, there is a unique long-term distribution of the fraction of A's, f^*, and an associated long-term expected fraction, $\phi^* = \lim_{t\to\infty} \phi^t = E[f^*] < 1$, that does not depend on N_A^0, the initial number of A's. Moreover, if $T^A > T^P$, and $N_A^0 = N/2$ so that $\phi^0 = 1/2$, then ϕ^t is strictly increasing in t, and $\mathbb{P}(f^t \geq 1/2) > 1/2$ for all $t \geq 1$. The reverse obtains if $T^A < T^P$.*

The uniqueness of the long term distribution when $q > 0$ follows from the Perron-Frobenius theorem for stochastic processes, as discussed in the proof of the proposition.

To determine how the return distributions of A versus P affect the relative survival of these strategies, we therefore need to see how these distributions affect whether $T^A > T^P$. For the remainder of the paper, we assume that half of the investors initially choose A, $N_A^0 = N/2$, $f^0 = 1/2$. An example of the evolution of the distribution of the fraction of A's is shown in Figure 3.

Since the only random variable that the r and s functions depend upon is the sender return, the expected change in relative frequency of A versus P is driven by how these strategies affect the distribution of sender returns R, as reflected in mean, variance, and skewness. By (15), direct calculation, and taking the expectation over r, ϵ_A and ϵ_P, the expected change in frequency over one period satisfies

$$\begin{aligned}
\left(\frac{2N}{g\chi_{N_A^t}}\right) E[\Delta f] \;&=\; T^A - T^P \\
&=\; a\beta[(\beta_A^3 - \beta_P^3)\gamma_{1r}\sigma_r^3 + \gamma_{1A}\sigma_A^3 - \gamma_{1P}\sigma_P^3] + \; B[(\beta_A^2 - \beta_P^2)\sigma_r^2 + (\sigma_A^2 - \sigma_P^2)] \\
&\quad + \; Da\beta(-3\sigma_A^2 - D^2 - 3\sigma_r^2\beta_A^2) + D^2B - DC,
\end{aligned} \tag{17}$$

[21] In contrast, when $q = 0$, the long-term distribution, f^*, depends on the initial number of A's.

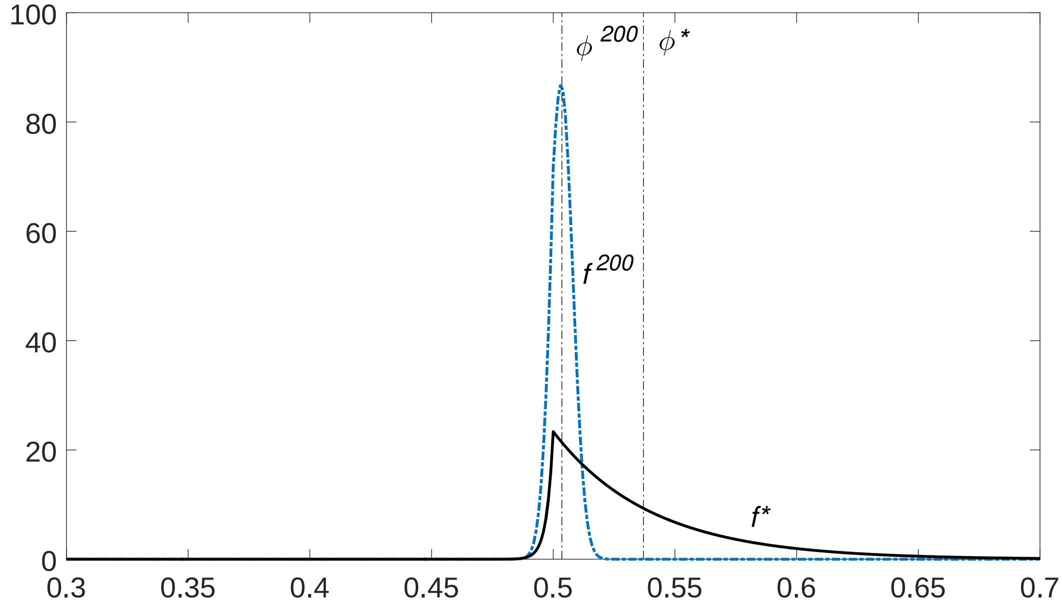

Figure 3: **Distribution of number of active investors.** *Dynamics of distribution and expectation of fractions of A's, f and ϕ in economy with $N = 1000$ investors. Dotted (blue) curve shows distribution at $t = 200$. Solid (black) line shows long-term distribution. Expected fraction of A's are $\phi^{200} = 503.6/1000 = 0.5036$ and $\phi^* = 537.0/1000 = 0.537$. Parameter values: $q = 0.0005$, $h = 0$, $T^A = 0.25$, $T^P = 0.175$, $M/Q = 0.5$.*

recalling that σ denotes standard deviation and $\gamma_{1\cdot}$ denotes skewness.

We now describe conditions under which evolution favors A or P. The next proposition follows immediately by (17), the parameter constraints of the model ($\beta_A > \beta_P \geq 0$, $\sigma_A^2 > \sigma_P^2$, $\gamma_{1A} > 0$, $\gamma_{1P} \approx 0$, and $\gamma_{1r} \geq 0$), and Proposition 2.

Proposition 3 *If the return penalty to active trading D is sufficiently close to zero, then under the parameter constraints of the model, ϕ^t is increasing in t, i.e., on average the fraction of active investors increases over time toward its steady state value.*

This comes from reinforcing effects. Owing to *SET*, the spread of A over P is favored by parameter values that increase the volatility of A relative to P: higher factor loading β_i and idiosyncratic volatility σ_i. A strategy that is more volatile (either because of greater loading on a factor or because of idiosyncratic risk) magnifies the effect of *SET* in persuading receivers to the strategy. Furthermore, the greater idiosyncratic skewness of A ($\gamma_{1A} \geq \gamma_{1P}$), promotes the spread of A. Owing to greater attention to extremes ($a > 0$), skewness (which generates salient and influential high returns) further reinforces the success of A, but *SET* promotes the spread of A even if $a = 0$.

An additional direct effect which does not rely on *SET* further promotes the spread of A. This effect only operates if $a > 0$ (salience of extreme news). Starting as benchmark with the case of $a = 0$, in the absence of *SET* ($\beta = 0$), and if the expected returns of the two strategies are the same, the transformation of P investors to A resulting from overextrapolation by receivers of high A returns is exactly offset by transformations in the other direction when returns are low. So the expected change in the fraction of A's from a meeting is zero.[22]

If instead $a > 0$, the receiving function is convex, so that high returns have a stronger effect on the upside than low returns have on the downside. Owing to its higher variance, A it generates extreme returns more often, which intensifies this favorable effect.

To see this algebraically, eliminate *SET* in the model by setting $\beta = 0$. Then the expected change in frequency of A is, up to a multiplicative constant,

$$T^A - T^P \quad = \quad a\gamma[(\beta_A^2 - \beta_P^2)\sigma_r^2 + (\sigma_A^2 - \sigma_P^2)] + D^2 a\gamma - Db\gamma. \tag{18}$$

Setting aside last two terms involving the mean return term D (which vanish when $D \approx 0$), we see that even without *SET*, there tends to be growth in the frequency of A if there is attention to extremes ($a > 0$). However, there is no inherent tendency for high skewness strategies to spread. This can also be seen from the comparatives statics of equations (B.2) and (B.3), in which the effects of skewness are eliminated when $\beta = 0$.

In summary, *SET* promotes A owing to its higher variance and (if $a > 0$), its higher skewness; without *SET*, the attention to extremes effect (in combination with extrapolation) also promotes A solely via a variance effect.

As compared with investment professionals, individual investors are almost surely more strongly influenced by casual social communication of performance anecdotes relative to independent analysis and investigation. This suggests that the predictions of Propositions 3 and 4 that social interaction favors active investing will apply more strongly to individual investors than to professionals.

2.7.1 Comparative Statics

To gain insights into the determinants of the reproductive success of A versus P strategies, we describe comparative statics effects on the growth in the active population fraction.

Proposition 4 *If $D \approx 0$, then under the parameter constraints of the model, the expected change in the fraction of A, $E_t[\Delta f^{t+1}]$:*

[22] More generally, whichever strategy has higher mean return will, all else equal, tend to spread owing to the persuasiveness of higher returns. However, in an equilibrium setting, growing popularity is self-limiting, as it drives the price of the A strategy up and its expected return down.

1. *Decreases with the return penalty to active trading D;*

2. (a) *Increases with factor skewness, γ_{1r};*

   (b) *Increases with active idiosyncratic skewness, γ_{1A};*

   (c) *The above effects are intensified by the salience of extreme returns as reflected in a, and SET as reflected in β.*

3. (a) *Increases with active idiosyncratic volatility, σ_A;*

   (b) *Increases with the factor loading of the active strategy, β_A;*

   (c) *Increases with the variance of the common factor, σ_r^2;*

   (d) *The above effects are intensified by greater sociability, as reflected in γ, and by the following other characteristics of the sending and receiving functions: salience of extreme returns as reflected in a, SET as reflected in β, and the extrapolativeness of receivers as reflected in b.*

4. *Increases with SET, β;*

5. *Increases with the extrapolativeness of receivers, b;*

6. *Increases with attention of receivers to extremes, a;*

7. *Increases with the conversability, γ, of trading strategies.;*

8. *Can either increase or decrease with the susceptibility of receivers, c; the relation is increasing when $D < 0$ and decreasing if $D > 0$.*

The proof of these claims follows directly by differentiation, and is provided in Appendix B.2.

The predictions in Proposition 4 about conversion of types translate into predictions about the popularity of active trading strategies. Based upon a simple assumption about pricing—that the higher the demand for a security, the higher its price and therefore the lower its expected long-run future return, we can interpret the comparative statics from Proposition 4 as comparative statics on the expected returns of active investments. This negative relation between number of A's and expected returns is derived as a formal equilibrium result in Section 3. We provide intuitions for the effects in the rest of this subsection.

Part 1 makes the fairly obvious point that if the average return penalty D to active trading is larger, A will be less successful in spreading through the population. Part 2a asserts that the advantage of A over P is increasing with factor skewness. Intuitively, extreme high returns are especially likely to be sent, to be noticed, and to convert the receiver when noticed. More positive skewness for the common factor implies that it is more likely to observe high realized return for the common factor. Because of the larger factor loading for active strategy, such high factor return is magnified in A relative to P, making A more contagious.

Part 2b on the effect of varying active idiosyncratic skewness, γ_{1A}, implies that conversation especially encourages demand for securities with high skewness. Mitton and Vorkink (2007) and Goetzmann and Kumar (2008) document that underdiversified individual investors (presumably naive investors—whom we would expect to be most subject to social influence) tend to choose stocks with high skewness—especially idiosyncratic skewness. Examples of skewed securities include options, and 'lottery stocks', such as real option firms that have a small chance of a jackpot outcome. As more investors favor positively skewed stocks, the expected returns of such stocks in the future would be depressed. This is consistent with the empirical finding that ex ante return skewness is a negative predictor of future stock returns (Conrad, Dittmar, and Ghysels 2013; Eraker and Ready 2015).[23]

The implications of the theory for the attraction of individual investors to lottery stocks are among this paper's key contributions. Existing explanations for this phenomenon have focused solely on an inherent individual characteristic—nontraditional preferences. In Brunnermeier and Parker (2005), agents who optimize over beliefs prefer skewed payoff distributions. In Barberis and Huang (2008), prospect theory preferences with probability weighting creates a preference over portfolio skewness, which induces a demand for 'lottery' (high idiosyncratic skewness) stocks that contribute to portfolio skewness. Surprisingly, we find that attraction to lottery stocks can instead derive from biases in the process of social interaction.

Existing preference-based theories are highly plausible, but there are indications that the tendency to favor lottery stocks does not derive solely from hard-wired psychological biases. Consistent with a possible effect of social contagion, individuals who live in urban areas buy lottery tickets more frequently than individuals who live in rural areas (Kallick et al. (1979)). Furthermore, there is evidence suggesting that the preference for high skewness stocks is greater among urban investors, after controlling for demographic, geographic, and personal investing characteristics (Kumar 2009).[24]

A key difference of our approach from approaches based upon inherent preferences over beliefs or over portfolio skewness, as in the theories just mentioned, is that biases in

[23]There is also evidence from initial public offerings (Green and Hwang 2012) and general samples (Bali, Cakici, and Whitelaw 2011) that lottery stocks are overpriced, and that being distressed (a characteristic that leads to a lottery payoff distribution) on average predicts negative abnormal returns (Campbell, Hilscher, and Szilagyi 2008). Boyer and Vorkink (2014) find that the ex ante skewness of equity options is a negative cross-sectional predictor of option abnormal returns.

[24]Kumar (2009) empirically defines lottery stocks as stocks with high skewness, high volatility, and low price, so his findings do not distinguish the effects of skewness versus volatility.

the transmission process cause the purchase of lottery stocks to be contagious. This can help explain the empirical association of high social interaction with gambling and lottery behaviors. In our setting, greater social interaction increases contagion, thereby increasing the holdings of lottery stocks.[25] For example, investors with greater social connection (as proxied, for example, by population density, participation in investment clubs, or self-reports of interactions with neighbors or regular church-going) will favor such investments more.

Barber and Odean (2008) find that individual investors are net buyers of stocks following extreme price moves, but that institutional investors behave in opposite fashion. So if naive individual investors are more affected by the salience of extreme returns, the attraction of individual investors to high skewness, as implied by Part 2c, is stronger than the attraction of institutional investors.

Part 3a implies that there is greater investor demand for more volatile stocks. Consistent with Part 3a, Goetzmann and Kumar (2008) document that underdiversified investors prefer stocks that are more volatile. A further empirical implication of Part 3a is that in periods in which individual stocks have high idiosyncratic volatility, all else equal there will be greater holding of and volume of trade in individual stocks. Intuitively, during such periods A's have higher returns to report selectively. This implication is in sharp contrast with the prediction of portfolio theory, which suggests that in periods of high idiosyncratic volatility, the gains to holding a diversified portfolio rather than trading individual stocks is especially large. There are theories of bubbles in which high return volatility might be associated with high stock trading because investors are experiencing especially strong sentiment or misperceptions. A distinctive implication of the prediction here is that when an increase in the volatility of fundamentals is the driver of an increase in return volatility, there will still be an increase in stock holding and trading volume.

The greater demand of investors for a higher-volatility stock implies that it will have a higher price, depressing its expected return. This is consistent with the idiosyncratic volatility puzzle that stocks with high idiosyncratic risk earn low subsequent returns (Ang et al. (2006, 2009)). This apparent overpricing is stronger for firms held or traded more heavily by retail investors (Jiang, Xu, and Yao 2009; Han and Kumar 2013), for whom we would expect conversational biases to be strong. Thus, the theory offers a possible *social* explanation for the idiosyncratic volatility puzzle: the high returns generated by volatile stocks are heavily discussed, which increases the demand for such stocks, driving up their prices.

[25] The effect is formalized in Section 2.8, Proposition 9, in which it is shown that the expected number of active investors—in this context investing in lottery stocks—increases with investors' social connectivity, M.

A plausible nonsocial explanation for these findings is that realization utility or prospect theory with probability weighting creates a preference for volatile portfolios and stocks (Barberis and Huang 2008; Boyer, Mitton, and Vorkink 2010). A distinctive aspect of our explanation is that the effect derives from social interaction. Consistent with social contagion playing a role, in tests using extensive controls, the preference for high volatility is greater among urban investors (Kumar (2009); see also footnote 24).

Part 3b implies that there is higher demand for high-beta stocks, pushing their price upward (and thereby depressing their expected returns). This is consistent with the anomaly that high beta stocks underperform and low beta stocks overperform (Baker, Bradley, and Wurgler 2011; Frazzini and Pedersen 2014). Frazzini and Pedersen (2014) propose a rational explanation of this effect based on leverage constraints. However, Bali et al. (2016) provide evidence suggesting that the beta anomaly is not incremental to the effect of lottery characteristics in predicting returns. Our model provides a new explanation for investor attraction to lottery-like stocks.

Part 3c indicates that greater volatility, σ_r of the common factor favors the spread of A. Greater factor volatility encourages the spread of the strategy with the greater loading, A, by creating greater scope for *SET* to operate. This implies that all else equal, there will be greater stock market participation in time periods and countries with more volatile stock markets. This contrasts with the conventional theory, in which greater risk, *ceteris paribus* reduces the benefit to participation.

Part 3d highlights a distinctive set of empirical implications, that demand for stocks with high beta or high idiosyncratic volatility will be strengthened by greater sociability as reflected in γ, and by other social psychological factors reflected in other parameters of the sending and receiving functions. These include the salience of extreme returns as reflected in a, *SET* as reflected in β, and the extrapolativeness of receivers as reflected in b. Such parameters can be measured, so these predictions are empirically testable.[26]

Proposition 4 also suggests direct effects of various characteristics of the social transmission process and the evolution toward A. First, in the Part 4 comparative statics on β, greater *SET* increases the evolution toward A, because *SET* causes greater reporting of the high returns that make A enticing for receivers. A generates extreme returns for *SET* to operate upon through higher factor loading, idiosyncratic volatility, or more positive idiosyncratic skewness. The link between performance and self-esteem could be estimated empirically using psychometric testing.

Second, in the Part 5 comparative statics on b, greater extrapolativeness of receivers

[26]For example, Barber and Odean (2008) estimate the effects of investor attention to extreme returns, and several papers estimate the extrapolativeness of return expectations using both survey approaches (Case and Shiller 1988; DeBondt 1993; Vissing-Jorgensen 2003) and field evidence (Greenwood and Shleifer 2014; Hoffmann, Post, and Pennings 2015).

helps A spread by magnifying the effect of *SET*. This suggests that active investing will be more popular when extrapolative beliefs are stronger (past returns are perceived to be more informative about the future); as mentioned above, extrapolativeness can be estimated empirically to test this hypothesis.

Third, in the Part 6 comparative statics on a, greater attention by receivers to extreme outcomes promotes the spread of A over P. This is because A generates more of the extreme returns which, when a is high, are especially noticed and more likely to persuade receivers. This effect is reinforced by *SET*, which causes greater reporting of extreme high returns.

Fourth, in the Part 7 comparative statics on γ, greater conversability can help the active strategy spread because of the greater attention paid by receivers to extreme returns ($a > 0$), which are more often generated by the A strategy. This is consistent with active trading becoming more popular when people become more talkative about their investment performance. Examples include the rise of communication technologies, media, and such social phenomena as ubiquitous computing, stock market chat rooms, investment clubs, and blogging. This raises the possibility that the rise of these phenomena—to the extent that this occurred for reasons other than a rising stock market, such as technological change—contributed to the internet bubble.

Also, trading outcomes are a trigger for conversation about trading, so over time as markets become more liquid and trading becomes more frequent, we expect conversation about outcomes to become more frequent. The trend toward greater availability of real-time reporting and discussion of financial markets on television and through the internet therefore can induce more rapid evolution toward more active investing.

If greater general sociability is associated with greater comfort in discussing performance information, then in any given conversation it increases the unconditional probability that the sender will discuss returns; i.e., it increases γ. So again, if the expected return of A is not too low, this will increase evolution toward active trading. Empirically, participation in online communities has been found to be associated with riskier financial decisions (Zhu et al. (2012)). Using field studies, the authors found greater risk-taking in bidding decisions and lending decisions by participants in discussion forums (Prosper.com) and in discussion boards and chat rooms (eBay.de), and that risk-taking increases with how active the participants are in the community.

There is also survey evidence that greater household involvement in social activities is associated with greater stock market participation both in the U.S. (Hong, Kubik, and Stein 2004) and in ten European countries (Georgarakos and Pasini 2011). Furthermore, Heimer (2014) documents that social interaction is more prevalent amongst active investors who buy and/or sell stocks than passive investors who hold U.S. savings bonds, thereby supporting our explanation for the active investing puzzle in which informal communication tends to promote active rather than passive strategies.

As discussed earlier, another reasonable way to interpret the active versus passive distinction is that active strategies are more conversable (less conventional, more affect-triggering, or more arousing). As documented by Berger and Milkman (2012), more arousing online content is more viral. This distinction could be incorporated formally by replacing γ in the sending function with γ_A and γ_P, where $\gamma_A > \gamma_P$. However, the model generates a survival advantage for A even without a conversability advantage. It is immediately evident that $\gamma_A > \gamma_P$ favors the spread of A (as we have verified), since a receiver cannot be converted unless he receives a message from the sender. Intuitively, $\gamma_A > \gamma_P$, *ceteris paribus*, causes adopters of A to evangelize to P's more often than the other way of around, which favors evolution of the population toward A. So we simply assert this conclusion while maintaining the simplicity of a single γ for the remaining analysis.

Since strategy A is more overpriced when the frequency of A in the population is higher, with $\gamma_A > \gamma_P$, the model further implies that overvaluation of stocks with 'glamour' characteristics that make them attractive topics of conversation. Such characteristics include growth, recent IPO, sports, entertainment, media, and innovative consumer products (on growth, see Lakonishok, Shleifer, and Vishny (1994); underperformance of IPO and small growth firms, see Loughran and Ritter (1995) and Fama and French (1993)). In contrast, there will be neglect and underpricing of unglamourous firms that are less attractive topics of conversation, such as business-to-business vendors or suppliers of infrastructure. Conversational transmission biases can therefore help explain several well-known empirical puzzles about investor trading and asset pricing.

Related predictions about the effects of investor attention have been made before (Merton 1987). A distinctive feature of our theory is that the effects derive from social interaction, and should therefore be stronger in times and places with greater sociability. This point provides additional empirical predictions about the effects on trading and return anomalies of population density, urban versus rural localities, pre- and post-internet periods, differences in self-reported degrees of social engagement, and popularity of investment clubs and chat rooms.

Lastly, the comparative statics on c in Part 8 of Proposition 4 implies that when there is a stronger preference for conformity (hence, greater susceptibility of receivers), there is a stronger tendency for the population to evolve toward A. Different ethnic and religious groups differ greatly in their exclusivity and the extent to which they place conformist pressures upon members (as reflected, for example, in the theory of club goods and religion; Iyer (2015)). The degree of ethnic or religious homogeneity is also likely to affect conformist pressures. So this implication is empirically testable using demographic data.

Proposition 4 provides implications about the expected change in the fraction of active investors over the next transaction. We perform comparative statics for the level of expected frequency of active investors in the population at any given future time.

Proposition 5 *Under the parameter constraints of the model, for D sufficiently close to zero, for any given time $t > 0$, the expected population frequency of A, ϕ^t:*

1. *Decreases with the return penalty to active trading D;*
2. *Increases with active idiosyncratic skewness, γ_{1A};*
3. *Increases with active idiosyncratic volatility, σ_A;*
4. *Increases with attention of receivers to extremes, a, when $\left(\frac{\beta_A}{\beta_P}\right)^3 \geq \left(\frac{\sigma_A}{\sigma_P}\right)^2$ or $\beta_P^3 \gamma_{1r} \sigma_r^3 \leq \frac{\gamma^2 c}{\beta^2 b}$;*[27]
5. *Increases with SET, β, when $\left(\frac{\beta_A}{\beta_P}\right)^3 \geq \left(\frac{\sigma_A}{\sigma_P}\right)^2$ or $\beta_P^3 \gamma_{1r} \sigma_r^3 \leq \frac{bc}{a^2}$.*

Since these results are similar to those of Proposition 4, we refer the reader there for discussion of intuition and empirical implications.

2.8 Investor Behavior in the Social Network

The model has strong empirical implications for how social connections and investor and neighbor characteristics influence investor behavior.

First, convexity in the transition dynamics, as described in aggregate in Proposition 1, also holds at the investor level.

Proposition 6 *Given a social network, $\mathcal{E}$, the probability that investor n changes type is strictly convex in each of the returns of the opposite-type investors that n is connected to.*

Second, investor transformation to the types of network neighbors is directly related to their strategies and performance.

Proposition 7 *Given a social network, $\mathcal{E}$, the probability that investor n changes to the opposite type is increasing in*

1. *The number of n's connections to investors of the opposite type, and*

[27]Under our standing parameter restrictions, the additional condition that $\left(\frac{\beta_A}{\beta_P}\right)^3 \geq \left(\frac{\sigma_A}{\sigma_P}\right)^2$ ensures that the ratio of the third moment of the A return to its second moment is larger than the same ratio for the return of P, which is similar to saying that the skewness of R_A is bigger than skewness of R_P. The alternative sufficient conditions in Parts 4 and 5 are not very restrictive. For example, they hold when factor skewness is close to zero, when the passive strategy has low sensitivity to the factor, or when susceptibility is high.

2. *The performance of n's connections that are of the opposite type.*

Third, related to Proposition 4, conversion to A tends to be encouraged by the skewness and volatility of the strategies of the investor's neighbors.

Proposition 8 *Given a social network, $\mathcal{E}$, the probability that investor n converts to A is increasing in the skewnesses of the portfolio returns of n's network connections, $\mathcal{D}_n$. If the return penalty is small, $D \approx 0$, then the probability that n converts is also increasing in the return volatilities of n's connections.*

Finally, there is a rich set of testable empirical implications about the relationship between network connectedness properties, such as being more well-connected or more homophilous, personal characteristics, such as attention to extremes and susceptibility, and the tendency for A to predominate in the population. In the following proposition, we extend the model to allow for individual differences in homophily, attention to extremes, extrapolativeness, SET, and conversability, in addition to differences in network connectedness.

Proposition 9

1. *The probability that investor n is an A at any time $t \geq 1$ increases in the investor's number of connections at time 0, $|\mathcal{D}_n^0|$.*

2. *The expected fraction of A's, $E[f^t]$, at any time $t \geq 1$ increases in the aggregate connectivity of investors, M (i.e., the total number of connections in the population).*

3. *The expected fraction of A's, $E[f^t]$, at any time $t \geq 1$ decreases in aggregate homophily, h.*

4. *The probability that investor n is an A at $t = 1$ decreases in that investor's homophily, h_n.*

5. *If $D \approx 0$, the probability that investor n is an A at $t = 1$ increases with that investor's attention to extremes, a_n, and extrapolativeness, b_n.*

6. *If $D \approx 0$, the probability that investor n is an A at $t = 1$ increases with each of that investor's neighbors' SET, β_m, and conversability, γ_m, $m \in \mathcal{D}_n^0$.*

7. *The sensitivity of the probability that a passive investor n switches to A between $t = 0$ and $t = 1$, as a function of R_A, and the convexity of this relationship, are increasing in that investor's attention to extremes, a_n, and extrapolativeness, b_n.*

8. *The sensitivity of the probability that passive investor n switches to A between $t = 0$ and $t = 1$, as a function of R_A, and the convexity of this relationship, are increasing in each of that investor's neighbors' SET, β_m, as well as their conversability, γ_m, $m \in \mathcal{D}_n^0$.*

These implications are empirically testable. Part 1 of Proposition 9 predicts that more sociable investors tend to be A's. This could be tested by relating social interaction proxies with a variety of active investing behaviors.

Parts 2 and 3 indicates that the expected prevalence of A is increasing with aggregate gregariousness and decreasing with homophily.

Part 4 turns to individual homophily of an investor, which decreases the investor's probability of being an A. Specifically, it is straightforward to allow for individual variation in homophily in the sequence of events described in Figure 1. Investors with higher homophily more often reject pairings with those of opposite types, and are therefore less prone to switch investment strategy. The effect goes both ways, i.e., P's switch to a lesser degree to A's, as do A's to P's, but because of the asymmetry in conversion probabilities the effect of homophily is stronger in hampering the conversion of P's than of A 's, leading to the result. Similarly, Part 5 shows that investors with higher individual attention to extremes and/or extrapolativeness have higher probability of becoming A's.

Part 6 indicates that the conversion probability of a P switching to A increases with the SET and conversability of each of that investor's neighbors.

Parts 7 and 8 are the direct network extensions of Proposition 1 Part 3. Part 7 indicates that the sensitivity of the probability of transformation of a receiving P investor who is linked to an A is increasing with key parameters of that investor's receiving schedule: attention to extremes, a_n, and extrapolativeness, b_n. Furthermore, these parameters increase the convexity of this relationship.

Finally, Part 8 indicates that the sensitivity and convexity of the conversion probability of a P to A as a function of R_A, increases with the neighbor characteristics which, as we know from before, encourage (biased) message sending: the neighbors' *SET*, β_m, and sociability, γ_m.

3 Equilibrium Dynamics

So far, we have modeled the economy in a partial equilibrium setting with exogenous return distributions for A and P, along with informal arguments that when there are more A's in the investor population, demand for this strategy increases, decreasing future returns. In practice, after extensive inflow of investors into active strategies, we expect the cost of acquiring strategy positions to rise, resulting in low expected future returns. So in equilibrium, we expect evolution toward A to be self-limiting. We now extend the model to capture such equilibrium effects.

We model the supply-side of the economy as a set of short-term investment opportunities with diminishing returns to scale, which implies imperfectly elastic supply. We assume that the output elasticity is lower for investments associated with active than for passive strategies, reflecting the idea that active strategies may be less scalable. For simplicity, we assume that investments associated with P's are perfectly elastic, whereas investments associated with A's are not. As a special case, the passive investment could, for example, represent a low-risk storage technology.

In contrast to the constant return distributions in (15), the one-period returns in this case depend on total active investments, X, as

$$R_A(N_A) = (\beta_A r + \epsilon_A + \kappa) \times (\rho X)^{-1/2} - \kappa, \tag{19}$$

$$R_P = \beta_P r + \epsilon_P, \tag{20}$$

where the $\kappa > 0$, $\rho > 0$ are parameters, and X in equilibrium will depend on N_A.[28]

Investors have mean-variance expected utility, $U = E[R] - \frac{\zeta}{2} Var(R)$, where for simplicity we set the risk aversion coefficient $\zeta = 1$. The riskfree asset has return r_f. Here, since we have normalized such that $E[r] = E[\epsilon_A] = E[\epsilon_P] = 0$, we assume that $r_f < 0$. The negative riskfree rate could, for example, represent a storage technology with some depreciation. This assumption could easily be modified, at the cost of greater algebraic complexity, by allowing for additional intercept components of returns in (19) and (20).

By assumption, P's maximize expected utility of investing in a portfolio of a risky investment alternative—that available to P investors—and the riskfree asset. Similarly, A's optimize a portfolio of the risky investment alternative available to A investors and the riskfree asset. Investors solve these problems rationally, but do not consider including both passive and active assets in their portfolios at the same time. In equilibrium, active investors total demand is X, where they optimize expected utility given the return distribution in (19).

In this specification, the return penalty, D_n, depends on n, the number of A's. We choose a specific value for the ρ parameter,

$$\rho = \frac{2(\beta_A^2 \sigma_r^2 + \sigma_A^2)}{N|r_f|},$$

[28] The return specification in (19) corresponds to a concave production function where input X leads to stochastic production $(\beta_A r + \epsilon_A + \kappa) \times \left(\frac{X}{\rho}\right)^{1/2} - \kappa X$. The parameters are such that a higher ρ is associated with a lower expected output, and a higher κ corresponds to a more concave production function.

which in equilibrium implies an initial return penalty of zero, $D_{N/2} = 0$, as we shall see. The case of a zero return penalty to active investing is a simple benchmark case that for identifying what influences the spread of competing investment strategies when the obvious effect of expected return differences is neutralized. In contrast with (8) in the partial equilibrium setting, it follows from the assumed technology that the transformation probability also depends on the number of A's,

$$T_n^A = E[T_A(R_A(n))]. \tag{21}$$

The following proposition provides conditions under which the results from Section 2 generalize to the equilibrium setting.

Proposition 10 *Under the parameter restrictions that $|r_f|$ is small, $\kappa \geq |r_f|$, $\gamma_{1P} \approx 0$, $\gamma_{1r} \geq 0$, and*

$$\beta_A > 2\beta_P \tag{22}$$

$$\sigma_A > 2\sigma_P, \tag{23}$$

the equilibrium return penalty, D_n, is small, and Propositions 1-3, Proposition 4.2-4.8, Proposition 5.2 and 5.3, and Propositions 6-9, continue to hold in equilibrium. Moreover, under the additional condition

$$\left(\frac{\beta_A}{\beta_P}\right)^3 \geq 2\left(\frac{\sigma_A}{\sigma_P}\right)^2, \tag{24}$$

Proposition 5.4 and 5.5 also continue to hold. Finally, $\mathbb{P}(D_{N_A^t} \geq 0) > 1/2$ for all $t \geq 1$, and the expected returns an agent receives from active investments is nonpositive and strictly decreasing over time.

The equilibrium return penalty is positive most of the time, since the number of A's tends to be greater than half the size of the population. The positive return penalty is thus an equilibrium outcome in this setting. The A's bear higher risk to achieve lower returns, thereby doing worse on average. Intuitively, transmission bias causes A to spread, putting a downward pressure on the returns to the A strategy, and thereby inducing a return penalty to active investing. In other words, owing to transmission bias, A investing persists despite needing to climb uphill against a return penalty.

The sufficient conditions on β_A and σ_A are stricter in the equilibrium setting, as seen by the extra factor 2 in (22) and (23). This factor arises because the restriction T_n^A depends on the number of active investors, n, and $T_n^A > T_n^P$ needs to be satisfied for all $1 \leq n \leq N$. Of course, these are just sufficient conditions.

The only results that do not extend to the equilibrium setting (Proposition 4 Part 1, and Proposition 5 Part 1) are the comparative statics with respect to the return penalty. Such comparative statics are not defined in the equilibrium model because the return penalty is endogenous.

We do not wish to overemphasize the implication that in equilibrium A has lower expected return than P (and associated comparative statics), because under a reasonable alternative assumption, this implication can be reversed. The model has assumed that the susceptibility of receivers, c, is the same regardless of whether the sender was an A or a P, so that the probability that a receiver is converted depends only on the sender's return. However, it is possible that a receiver who recognizes that A is riskier than P will be less willing to convert, for any given return, if the report comes from an A. For example, a report of a 4% annual return might be much more attractive if it is about a riskfree asset than about a risky tech IPO. So we would expect receivers to be less susceptible to messages that come from an A. This would be reflected by having the receiver susceptibility parameter c in the receiving function be lower if the sender was an A than a P, $c_A < c_P$. This would weaken the spread of the A strategy relative to P, reducing its price, so that in equilibrium the expected return of A could be higher than P.

In summary, the comparative statics in the equilibrium setting are similar to those derived in the partial equilibrium setting.

4 Concluding Remarks

We argue that success in the struggle for survival between investment strategies is determined by sending and receiving functions for the transmission of information about the strategies and their performance. In the model, owing to self-enhancing transmission, senders' propensity to communicate their returns is increasing in sender return. Owing to naive extrapolation, the propensity of receivers to be converted is also increasing in sender return. Owing to the salience of extremes, the propensity of receivers to attend to and be converted by the sender is convex in sender return. These shapes of the sending and receiving functions, together with the structure of the social network and the intensity of social interactions describe the social transmission process. The parameters of the sending and receiving function capture how a sender's return performance is communicated and how hearing it influences a receiver. The psychological traits of investors determine the parameters of this communication process.

We find that active strategies—those with high volatility, skewness, and personal engagement, spread after they experience high returns, and, more surprisingly, that this relationship is convex. We further find that the active strategies on average tend to spread through the population (as constrained by equilibrium price effects), providing a social explanation for anomalies such as the lottery, volatility, beta, and IPO effects. Again, these effects depend on empirically observable parameters of the social transmission process, leading to a rich set of additional empirical implications about investor trading and return anomalies.

More generally, we suggest that a fruitful direction for understanding how social interactions affect financial decisions is to study the factors that shape the sending and receiving functions, i.e., that cause an investor to talk about an investment idea, or to be receptive to such an idea upon hearing about it. Conversations are influenced by chance circumstances, subtle cues, and even trifling costs and benefits to the transactors. This suggests that small variations in social environment can have large effects on economic outcomes. For example, the model suggests that a shift in the social acceptability of talking about one's successes, or of discussing personal investments more generally, can have large effects on risk taking and active investing. This suggests a possible explanation for both secular and higher-frequency shifts in investor behavior.

Much of the empirical literature on social interactions focuses on *whether* information or behaviors are transmitted, and on what affects the strength of social contagion. Our approach suggests that it is valuable to understand how *biases in the transmission process* affect decision making and economic outcomes.

Our approach also offers a microfoundation for research on fluctuations in investor sentiment toward different kinds of investment strategies. For example, observers have often argued that social interactions contribute to bubbles (e.g., Shiller (2000b)). Notably, the millennial high-tech stock market boom coincided with the rise of investment clubs and chat rooms. If the sending and the receiving functions of our model depend on the sender's return over multiple periods (rather than just the most recent period return), there can be overshooting and correction. Alternatively, if a higher frequency of active investors makes it more socially acceptable to discuss one's investment successes, the popularity of active strategies will be self-reinforcing. So our model, and more generally the social finance approach, offers a possible framework for modeling how the spread of investment ideas cause bubbles and crashes.

Appendices

A Endogenizing the Receiving and Sending Functions

We now consider explicitly the determinants of the sending and receiving functions, and derive the assumed functional forms endogenously.

A.1 The Sending Function

To derive a sending function that reflects the desire to self-enhance, we assume that the utility derived from sending is increasing with own-return. Conversation is an occasion for an investor to try to raise the topic of return performance if it is good, or to avoid the topic if it is bad. Suppressing i subscripts, let $\pi(R,x)$ be the utility to the sender of discussing his return R,

$$\pi(R,x) = R + \frac{x}{\beta'}, \tag{A.1}$$

where β' is a positive constant that measures the relative weight in the individual's utility on conversational context versus the desire to communicate higher returns. The more tightly the investor's self-esteem is ties to return performance, the higher is β'. The random variable x measures whether, in the particular social and conversational context, raising the topic of own-performance is appropriate or even obligatory.

The sender sends if and only if $\pi > 0$, so

$$\begin{aligned} s(R) &= Pr\left(x > -\beta' R | R\right) \\ &= 1 - F(-\beta' R), \end{aligned} \tag{A.2}$$

where F is the distribution function of x. If $x \sim U[\tau_1, \tau_2]$, where $\tau_1 < 0, \tau_2 > 0$, then

$$\begin{aligned} s(R) &= \frac{\tau_2 + \beta' R}{\tau_2 - \tau_1} \\ &= \frac{\tau_2}{\tau_2 - \tau_1} + \beta R, \end{aligned} \tag{A.3}$$

where $\beta \equiv \beta'/(\tau_2 - \tau_1)$, and where we restrict the domain of R to satisfy $-\tau_2/\beta' < R < -\tau_1/\beta'$ to ensure that the sending probability lies between 0 and 1. This will hold almost surely if $|\tau_1|, |\tau_2|$ are sufficiently large. Equation (A.3) is identical to the sending function (3) in Subsection 2.2 with

$$\gamma \equiv \frac{\tau_2}{\tau_2 - \tau_1}.$$

In the sender's utility $\pi(R,x)$ of discussing return R, the parameter β' captures the value placed on mentioning one's high return experience, versus the appropriateness of doing so. The more tightly bound is the sender's self-esteem or reputation to return performance, the larger is the parameter β', and hence the stronger is *SET*, as measured by β in the sending function (3) which is proportional to β'.

The constant γ in the sending function (3) reflects the *conversability* of the investment choice. When investment is a more attractive topic for conversation or when conversations are more extensive, as occurs when investors are more sociable, higher γ shifts the distribution of x to the right (i.e., an increase in τ_2, for given $\tau_2 - \tau_1$, implies higher γ).

A.2 The Receiving Function

We derive a convex increasing shape for the receiving function as in equation (4) in Section 2 from the combination of two effects: greater receiver attention to extreme return outcomes (inducing convexity), and, conditional upon paying attention, greater persuasiveness of higher return.

Greater attention to extreme outcomes arises naturally in a setting with a quadratic loss function for estimating mean returns. As an illustrative example, suppose that the return on a strategy has unknown mean μ, $R = \mu + \epsilon$, where $E[\mu] = \mu_0$, $E[\epsilon] = 0$, and where μ, ϵ are normally distributed with variances $var(\mu), var(\epsilon)$. The receiver can, at cost C_1 pay attention to a single realization of R, and update about μ accordingly, or can refrain from incurring this cost, in which case the investor's belief remains at the prior belief μ_0. The cost of paying attention is stochastic (e.g., it depends on the situation and varies across investors). Assume that C_1 is uniformly distributed $C_1 \sim U(0, \overline{C}_1)$, and the receiver observes her private cost c_1 (a realized value of C_1) when deciding whether to pay attention.

Let $I(A)$ be an indicator for whether the receiver attends to R, incurring c_1. The receiver make the binary decision to attend or not attend to R with the objective of maximizing

$$U = -(E[\mu|R] - E[\mu|\phi])^2 - I(A)c_1, \tag{A.4}$$

where the first expectation is the best rational assessment of μ based on R; and where, if the receiver attends to R, $\phi = R$, and otherwise ϕ is equal to the null information set.

If the receiver incurs c_1 and attends to R, $U = -c_1$. The rational update is

$$E[\mu|R] = \mu_0 + \beta_1(R - \mu_0), \tag{A.5}$$

where

$$\beta_1 = \frac{var(\mu)}{var(\mu) + var(\epsilon)} > 0.$$

So when the receiver does not attend,

$$U = -\beta_1^2(R - \mu_0)^2. \tag{A.6}$$

The receiver therefore attends iff

$$\beta_1^2(R - \mu_0)^2 > c_1. \tag{A.7}$$

So for given R, since c_1 is uniformly distributed, the probability $A(R)$ that the receiver pays attention to a reported return R is

$$A(R) = \frac{\beta_1^2(R-\mu_0)^2}{\overline{C}_1}, \tag{A.8}$$

a positive quadratic function of R with positive first and third coefficients. This model of attention allocation is based on the idea that the human mind has been designed to deal with many kinds of decision environment, and that the attentional heuristics used by the human mind are those that work effectively in a range of decision environments based on natural selection over thousands of generations. So attention here is optimized for a general problem of assessing the mean value of a variable, rather than optimizing the payoffs of any very specific investment problem.

Conditional on the receiver attending to the reported return R, let $B(R)$ be the probability that the receiver adopts the sender's strategy. The receiver interprets sender return as providing information about the desirability of the sender's strategy. Assume the benefit of receiver switching strategy is proportional to the expected return of the sender's strategy $kE[\mu|R]$. Under normal distribution, the updated belief about expected return of the sender's strategy is linearly increasing in reported R (if it is attended to), as given by (A.5). Thus, the benefit of receiver switching strategy is also linearly increasing in reported return of the sender $k[\beta_1 R+(1-\beta_1)\mu_0]$. Further assume that the cost of switching strategy c_2 is drawn from a uniform distribution $U(0,\overline{C}_2)$. Then the receiver switches strategy iff

$$k[\beta_1 R + (1-\beta_1)\mu_0] > c_2. \tag{A.9}$$

This implies that conditional on attending to the reported return R of the sender, the probability that the receiver adopts the sender's strategy is

$$B(R) = \frac{k[\beta_1 R + (1-\beta_1)\mu_0]}{\overline{C}_2} \tag{A.10}$$

So $B(R)$ is an increasing linear function of sender return. That is, the higher the reported return, the greater the probability that receiver switches to the strategy of the sender.

With the A and B functions endogenously derived as in (A.8) and (A.10), the receiving function

$$r(R) = A(R)B(R)$$

is a positive cubic function of R with positive first and fourth coefficients. A quadratic Taylor approximation leads to a quadratic expression for $r(R)$, as in equation (4) in Section 2, where we assume that most of the probability mass of R is in the range where the coefficients of this quadratic approximation are positive, consistent with a convex increasing shape for the receiving function.

B Proofs

The proofs in this section depend heavily on work-horse Markov chain models that we analyze in the Internet Appendix I. Specifically, in the Internet Appendix, we consider the variable $w^t = w^t(a, b) \in \{0, 1, 2, \ldots, N\}$, $t \geq 0$, which evolves according to an $N + 1$ state Markov model with transition matrix $\Phi = \Phi(a, b) \in \mathbb{R}^{(N+1)\times(N+1)}$.

In the *base model*, $a = (a_1, a_2, \ldots, a_{N-1})'$, and $b = (b_1, b_2, \ldots, b_{N-1})$ are $(N-1)$-dimensional vectors, such that $0 < a_n \leq b_n < \frac{1}{2}$, $n = 1, \ldots, N-1$, and Φ is a tri-diagonal matrix with elements

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & \cdots & & \\ a_1 & 1 - a_1 - b_1 & b_1 & 0 & \cdots & 0 \\ 0 & a_2 & 1 - a_2 - b_2 & b_2 & \cdots & 0 \\ & \ddots & \ddots & \ddots & \ddots & \\ & \cdots & 0 & a_{N-1} & 1 - a_{N-1} - b_{N-1} & b_{N-1} \\ & & & & 0 & 1 \end{bmatrix}. \tag{B.1}$$

The model in the main part of the paper with $q = 0$, $h = 0$, and $M = Q$, corresponds to $\Phi(a, b)$, with $a_n = \chi_n T^P$ and $b_n = \chi_n T^A$.

First modification: For $0 < \alpha \leq 1$, we define the modified transition matrix

$$\Theta(a, b, \alpha) = (1 - \alpha)I + \alpha\Phi,$$

where I is the $(N + 1) \times (N + 1)$ identity matrix, and the associated modified Markov process. When $\alpha = 1$, the model reduces to the previous one, $\Theta(a, b, 1) = \Phi(a, b, 1)$. The case $\alpha < 1$ corresponds to the model in the main part of the paper with $q = 0$, but allowing for general M and h, corresponds to setting $a_n = \chi_n T^P$, $b_n = \chi_n T^A$, and $\alpha = g = \frac{M}{Q}(1-h)$.

Second modification: For $0 \leq q < 1$, define the modified transition matrix

$$\Psi = \Psi(a, b, \alpha, q) = (1 - q)\Theta + qR,$$

where $R \in \mathbb{R}^{(N+1)\times(N+1)}$, with elements $R_{ij} = 1$ when $i = N/2 + 1$, and $R_{ij} = 0$ otherwise. This stochastic matrix, R, represents a degenerate Markov chain which immediately moves to state $N/2$ in the next period. The modified model is thus one in which, with probability q, such a reset occurs, and with probability $(1 - q)$ the model propagates according to the Θ transition matrix. The general model in the main part of the paper with, allowing for arbitrary q, M and h, corresponds to $\Psi(a, b, \alpha, q)$, with $a_n = \chi_n T^P$, $b_n = \chi_n T^A$, and $\alpha = g = \frac{M}{Q}(1-h)$. The Internet Appendix provides several useful results for these stochastic matrices, which we use in the subsequent proofs

B.1 Proof of Proposition 2

The result follows from the Perron-Frobenius theorem for stochastic processes and the fact that the stochastic matrix Ψ is irreducible and aperiodic when $q > 0$. Specifically, it is easy to verify that for $k \geq N/2+1$, $\Psi^k_{i,j} > 0$ for all i, j, corresponding to the process starting at i, resetting to $N/2$ in the next period, moving to j over the next $|N/2 - j|$ periods, and then staying at j from there on. This implies that Ψ is irreducible and aperiodic. It follows that there is a unique long-term distribution for N^t_A, and thus also for f^t.

That ϕ^t is strictly increasing follows from Proposition I.1 in the Internet Appendix, and its extensions under the second modification, as discussed in Appendix I, and it follows from Proposition I.3 that $\mathbb{P}(\phi^t > 1/2) > 1/2$ for all $t \geq 1$. ∎

B.2 Proof of Proposition 4

To show Part 1, we differentiate (17) with respect to D to obtain that if $D < 0$ or D is positive but not too large,[29]

$$\left(\frac{N}{g\chi_{N_A}}\right)\frac{\partial E[\Delta f]}{\partial D} = -3a\beta(\beta_A^2\sigma_r^2 + \sigma_A^2) + D(-3aD\beta + 2B) - C < 0.$$

For Part 2a, differentiating with respect to factor skewness γ_{1r} gives

$$\begin{aligned}\left(\frac{N}{g\chi_{N_A}}\right)\frac{\partial E[\Delta f]}{\partial \gamma_{1r}} &= a\beta\sigma_r^3(\beta_A^3 - \beta_P^3) \\ &> 0, \end{aligned} \tag{B.2}$$

since $\beta_A > \beta_P$. Thus, the advantage of A over P is increasing with factor skewness.

For Part 2b, differentiating with respect to active idiosyncratic skewness γ_{1A} gives

$$\begin{aligned}\left(\frac{N}{g\chi_{N_A}}\right)\frac{\partial E[\Delta f]}{\partial \gamma_{1A}} &= a\beta\sigma_A^3 \\ &> 0. \end{aligned} \tag{B.3}$$

Thus, the advantage of A over P is increasing with the idiosyncratic skewness of A.

For Part 2c, it suffices to note that the right hand sides of (B.2) and (B.3) both increase with *SET* in the sending function as reflected in β and salience of extreme returns in the receiving function as reflected in a.

For Part 3a, differentiating with respect to active idiosyncratic volatility σ_A gives

$$\begin{aligned}\left(\frac{N}{g\chi_{N_A}}\right)\frac{\partial E[\Delta f]}{\partial \sigma_A} &= 3a\beta\gamma_{1A}\sigma_A^2 + 2(B - 3aD\beta)\sigma_A \\ &> 0 \end{aligned} \tag{B.4}$$

[29] The ambiguity for large D results from a spurious effect: for sufficiently large negative R, the slope of the quadratic receiving function turns negative. In consequence, a larger return penalty to active trading, D, can, perversely, help convert P's to A's by inducing larger losses.

if $D \approx 0$ or $D < 0$. Thus, if D is sufficiently small, the growth of A increases with active idiosyncratic volatility σ_A. Greater return variance increases the effect of *SET* on the part of the sender. Although high salience to receivers of extreme returns ($a > 0$) is not required for the result, it reinforces this effect. Indeed, even if there were no *SET* ($\beta = 0$), since $a > 0$ implies that $B > 0$, the result would still hold. Intuitively, high volatility generates the extreme outcomes which receive high attention.

For Part 3b, differentiating with respect to the factor loading of the active strategy, β_A, gives

$$\begin{aligned}\left(\frac{N}{g\chi_{N_A}}\right)\frac{\partial E[\Delta f]}{\partial \beta_A} &= 3a\beta\beta_A^2\gamma_{1r}\sigma_r^3 + 2\beta_A\sigma_r^2 B - 6a\beta\beta_A\sigma_r^2 D \\ &> 0 \qquad \text{(B.5)}\end{aligned}$$

if $D < 0$ or $D \approx 0$. So a greater factor loading for A increases the spread of A, since the greater dispersion of return outcomes encourages the sending of high, influential messages.

For Part 3c, differentiating with respect to the variance of the common factor, σ_r^2 gives

$$\begin{aligned}\left(\frac{N}{g\chi_{N_A}}\right)\frac{\partial E[\Delta f]}{\partial \sigma_r^2} &= 1.5a\beta(\beta_A^3 - \beta_P^3)\gamma_{1r}\sigma_r + B(\beta_A^2 - \beta_P^2) - 3Da\beta\beta_A^2 \\ &> 0 \qquad \text{(B.6)}\end{aligned}$$

if $D < 0$ or $D \approx 0$. So greater volatility of the common factor favors the spread of A. Greater factor volatility outcomes encourages the spread of the strategy with the greater loading, A, by creating greater scope for *SET* to operate.

For Part 3d, note that the right hand sides of equations (B.4), (B.5), and (B.6) increase with $B = a\gamma + b\beta$ (which is in turn positively related to γ by definition) as well as *SET* in the sending function as reflected in β and salience of extreme returns in the receiving function as reflected in a.

For Part 4, we differentiate with respect to β, the strength of *SET*. This reflects how tight the link is between the sender's self-esteem and performance. Recalling by (6) that B is an increasing function of β, gives

$$\begin{aligned}\left(\frac{N}{g\chi_{N_A}}\right)\frac{\partial E[\Delta f]}{\partial \beta} &= a(\gamma_{1A}\sigma_A^3 - \gamma_{1P}\sigma_P^3) + a\sigma_r^3(\beta_A^3 - \beta_P^3)\gamma_{1r} + b[(\beta_A^2 - \beta_P^2)\sigma_r^2 + \sigma_A^2 - \sigma_P^2] \\ &\quad + Da(-3\sigma_A^2 - 3\beta_A^2\sigma_r^2 - D^2) + D^2 b - Dc \\ &> 0 \qquad \text{(B.7)}\end{aligned}$$

if $D \approx 0$ or $D < 0$. So greater *SET* increases the evolution toward A, because *SET* causes greater reporting of the high returns that make A enticing for receivers. A generates extreme returns for *SET* to operate upon through higher factor loading, idiosyncratic volatility, or more positive idiosyncratic skewness.

For Part 5, differentiating with respect to how prone receivers are to extrapolating returns, b, gives

$$\begin{aligned} \left(\frac{N}{g\chi_{N_A}}\right)\frac{\partial E[\Delta f]}{\partial b} &= \beta[(\beta_A^2 - \beta_P^2)\sigma_r^2 + \sigma_A^2 - \sigma_P^2] + D(D\beta - \gamma) \\ &> 0 \end{aligned} \tag{B.8}$$

if $D \approx 0$ or $D < 0$. Greater extrapolativeness of receivers helps A spread by magnifying the effect of *SET* (reflected in β), which spreads A because of the higher volatility of A returns.

For Part 6, recall that the quadratic term of the receiving function a reflects greater attention on the part of the receiver to extreme profit outcomes communicated by the sender. Differentiating with respect to a gives

$$\begin{aligned} \left(\frac{N}{g\chi_{N_A}}\right)\frac{\partial E[\Delta f]}{\partial a} &= \beta\sigma_r^3\gamma_{1r}(\beta_A^3 - \beta_P^3) + \beta[\gamma_{1A}\sigma_A^3 - \gamma_{1P}\sigma_P^3] + \gamma[(\beta_A^2 - \beta_P^2)\sigma_r^2 + \sigma_A^2 - \sigma_P^2] \\ &\quad - 3D\beta(\beta_A^2\sigma_r^2 + \sigma_A^2) + D^2\gamma - D^3\beta \\ &> 0 \end{aligned} \tag{B.9}$$

if $D \approx 0$ or $D < 0$. So greater attention by receivers to extreme outcomes, a, promotes the spread of A over P because A generates more of the extreme returns which, when a is high, are especially noticed and more likely to persuade receivers. This effect is reinforced by *SET*, which causes greater reporting of extreme high returns.

For Part 7, differentiating with respect to conversability γ gives

$$\begin{aligned} \left(\frac{N}{g\chi_{N_A}}\right)\frac{\partial E[\Delta f]}{\partial \gamma} &= a[(\beta_A^2 - \beta_P^2)\sigma_r^2 + \sigma_A^2 - \sigma_P^2] - bD + aD^2 \\ &> 0 \end{aligned} \tag{B.10}$$

if $D \approx 0$ or if $D < 0$. Greater conversability γ can help the active strategy spread because of the greater attention paid by receivers to extreme returns ($a > 0$), which are more often generated by the A strategy. When $D < 0$, this effect is reinforced by the higher mean return of A. In this case an unconditional increase in the propensity to report returns tends to promote the spread of the sender's type more when the sender is A. On the other hand, if $D > 0$ is sufficiently large, A earns lower return than P on average, so greater conversability incrementally produces more reporting of lower returns when the sender is A than P, which opposes the spread of A.

Lastly, for Part 8 of the Proposition 4, differentiating with respect to the susceptibility of receivers c gives

$$\begin{aligned} \left(\frac{N}{g\chi_{N_A}}\right)\frac{\partial E[\Delta f]}{\partial c} &= -D\beta \\ &> 0 \end{aligned} \tag{B.11}$$

if $D < 0$; the inequality is reversed if $D > 0$. Greater susceptibility increases the likelihood that the receiver is transformed *given that* the sender sends. Owing to *SET* (as reflected in the β term above) the probability that A sends is increased relative to the probability that P sends when the returns of A are higher, i.e., $D < 0$. This condition will hold if there is a risk premium for the active strategy, even if the premium is not fully commensurate with the risk. ∎

B.3 Proof of Proposition 5

We first show the result for the case with no reset, $q = 0$. From the definition of the sending and receiving functions it follows that

$$\begin{aligned} T^A = E[T_A(R_A)] &= a\beta[\beta_A^3\gamma_{1r}\sigma_r^3 + \gamma_{1A}\sigma_A^3] + \ B[\beta_A^2\sigma_r^2 + \sigma_A^2] \\ &\quad + Da\beta(-3\sigma_A^2 - D^2 - 3\sigma_r^2\beta_A^2) + D^2B - DC + c\gamma, \end{aligned} \tag{B.12}$$

$$T^P = E[T_P(R_P)] = a\beta[\beta_P^3\gamma_{1r}\sigma_r^3 + \gamma_{1P}\sigma_P^3] + \ B[\beta_P^2\sigma_r^2 + \sigma_P^2] + c\gamma, \tag{B.13}$$

$B = a\gamma + b\beta$, $C = b\gamma + c\beta$. The transition matrix, Φ, as described in Figure 2 now has the same structure as in the Markov model in Appendix I, see (I.1), with $a_i = \chi_i g T^P$, $b_i = \chi_i g T^A$.

1. For small D, it follows from (B.12,B.13) that T^P is decreasing in D, whereas T^A does not depend on D, so the result follows from Proposition I.4:2.

2. It follows from (B.12,B.13) that T^A is increasing in γ_{1A}, whereas T^P does not depend on γ_{1A}, so the result follows from Proposition I.4:2.

3. It follows from (B.12,B.13) that T^A is increasing in σ_{1A}, whereas T^P does not depend on σ_{1A}, so the result follows from Proposition I.4:2.

4. It follows from (B.12,B.13) that Φ's coefficients are of the form

$$\begin{aligned} \frac{1}{g\chi_i} b_i &= a\left[\beta(\beta_A^3\gamma_{1r}\sigma_r^3 + \gamma_{1A}\sigma_A^3) + \gamma(\beta_A^2\sigma_r^2 + \sigma_A^2)\right] + \beta b(\beta_A^2\sigma_r^2 + \sigma_A^2) + c\gamma, \\ &\stackrel{\text{def}}{=} a q_A + r_A, \\ \frac{1}{g\chi_i} a_i &= a\left[\beta(\beta_P^3\gamma_{1r}\sigma_r^3 + \gamma_{1P}\sigma_P^3) + \gamma(\beta_P^2\sigma_r^2 + \sigma_P^2)\right] + \beta b(\beta_P^2\sigma_r^2 + \sigma_P^2) + c\gamma, \\ &\stackrel{\text{def}}{=} a q_P + r_P. \end{aligned}$$

It is straightforward to verify that if $\left(\frac{\beta_A}{\beta_P}\right)^3 \geq \left(\frac{\sigma_A}{\sigma_P}\right)^2$, then $\frac{q_A}{q_P} \geq \frac{r_A}{r_P}$. The result follows from the following standard Lemma

Lemma 1 *Consider strictly positive x, y, s, s_0, t, t_0, and assume that $\frac{s}{s_0} > \frac{t}{t_0}$. Then*

$$\frac{t}{t_0} < \frac{xs + yt}{xs_0 + yt_0} < \frac{s}{s_0}.$$

Proof: We note that $\frac{s}{t} > \frac{s_0}{t_0}$ and $\frac{t}{s} < \frac{t_0}{s_0}$, which leads to

$$\frac{t}{t_0} < \frac{t}{t_0} \times \frac{x\frac{s}{t} + y}{x\frac{s_0}{t_0} + y} = \frac{xs + yt}{xs_0 + yt_0} = \frac{s}{s_0} \times \frac{x + y\frac{t}{s}}{x + y\frac{t_0}{s_0}} < \frac{s}{s_0}.$$

∎

Define $v_A \stackrel{\text{def}}{=} \beta_A^2\sigma_r^2 + \sigma_A^2$, and $v_P \stackrel{\text{def}}{=} \beta_P^2\sigma_r^2 + \sigma_P^2$. It follows from Lemma 1 that

$$\frac{v_A}{v_P} \leq \max\left\{\frac{\beta_A^2}{\beta_P^2}, \frac{\sigma_A^2}{\sigma_P^2}\right\},$$

which, since $\beta_A > \beta_P$, under the assumption that $\left(\frac{\beta_A}{\beta_P}\right)^3 \geq \left(\frac{\sigma_A}{\sigma_P}\right)^2$ in turn implies that $\left(\frac{\beta_A}{\beta_P}\right)^3 \geq \frac{v_A}{v_P}$. Now, since $\gamma_{1P} \approx 0$, and $\gamma_{1A} >\geq 0$, and q_A is increasing in γ_{1A}, a sufficient condition for $\frac{q_A}{q_P} \geq \frac{r_A}{r_P}$ is that

$$\frac{\beta\beta_A^3\gamma_{1r}\sigma_r^3 + \gamma v_A}{\beta\beta_P^3\gamma_{1r}\sigma_r^3 + \gamma v_P} \geq \frac{\beta b v_A + c\gamma}{\beta b v_P + c\gamma}. \tag{B.14}$$

An application of Lemma 1, with $s = \beta_A^3$, $s_0 = \beta_P^3$, $t = v_A$, $t_0 = v_P$, $x = \beta\gamma_{1r}\sigma_r^3$, $y = \gamma$, implies that

$$\frac{\beta\beta_A^3\gamma_{1r}\sigma_r^3 + \gamma v_A}{\beta\beta_P^3\gamma_{1r}\sigma_r^3 + \gamma v_P} \geq \frac{v_A}{v_P},$$

and another application with $s = v_A$, $s_0 = v_P$, $t = 1$, $t_0 = 1$, $x = \beta b$, $y = c\gamma$ implies that

$$\frac{\beta b v_A + c\gamma}{\beta b v_P + c\gamma} \leq \frac{v_A}{v_P}.$$

So, it follows that $\frac{q_A}{q_P} \geq \frac{r_A}{r_P}$, which by Proposition I.6 immediately implies the result for Φ.

For the alternative sufficient condition, $Q_P \stackrel{\text{def}}{=} \beta_P^3\gamma_{1r}\sigma_r^3 \leq \frac{\gamma^2 c}{\beta^2 b}$, we argue as follows. Define $Q_A \stackrel{\text{def}}{=} \beta_A^3\gamma_{1r}\sigma_r^3$, and note that $Q_A \geq Q_P$, and $v_A \geq v_P$. A reformulation of (B.14), then yields that a sufficient condition for $\frac{q_A}{q_P} \geq \frac{r_A}{r_P}$ is that

$$\frac{\beta Q_A + \gamma v_A}{\beta Q_P + \gamma v_P} \geq \frac{\beta b v_A + c\gamma}{\beta b v_P + c\gamma},$$

which is equivalent to

$$(\beta Q_A + \gamma v_A)(\beta b v_P + c\gamma) \geq (\beta Q_P + \gamma v_P)(\beta b v_A + c\gamma),$$

and in turn to

$$v_P(\beta^2 b Q_A - \gamma^2 c) + \beta c\gamma(Q_A - Q_p) \geq v_A(\beta^2 b Q_P - \gamma^2 c) \tag{B.15}$$

Now, the second term on the left-hand-side of (B.15) is positive. Moreover, under the condition that $Q_P \leq \frac{\gamma^2 c}{\beta^2 b}$, the right-hand-side is negative. If $\beta^2 b Q_A - \gamma^2 c \geq 0$, (B.15) therefore follows immediately. Moreover, under the complimentary scenario, when $\beta^2 b Q_A - \gamma^2 c < 0$, then $0 < \gamma^2 c - \beta^2 b Q_A \leq \gamma^2 c - \beta^2 b Q_P$ (because $Q_A \geq Q_P$), and since $0 < v_P \leq v_A$, it follows that $v_P(\gamma^2 c - \beta^2 b Q_A) \leq v_A(\gamma^2 c - \beta^2 b Q_P)$, and thus again that (B.15) Via Proposition I.6 the result is again implied for Φ. We are done.

5. It follows from (B.12,B.13) that Φ's coefficients are of the form

$$\begin{aligned}
\frac{1}{g\chi_i} b_i &= \beta\left[a(\beta_A^3 \gamma_{1r}\sigma_r^3 + \gamma_{1A}\sigma_A^3) + b(\beta_A^2\sigma_r^2 + \sigma_A^2)\right] + \gamma a(\beta_A^2\sigma_r^2 + \sigma_A^2) + c\gamma, \\
&\stackrel{\text{def}}{=} \beta\hat{q}_A + \hat{r}_A, \\
\frac{1}{g\chi_i} a_i &= \beta\left[a(\beta_P^3 \gamma_{1r}\sigma_r^3 + \gamma_{1P}\sigma_P^3) + b(\beta_P^2\sigma_r^2 + \sigma_P^2)\right] + \gamma a(\beta_P^2\sigma_r^2 + \sigma_P^2) + c\gamma, \\
&\stackrel{\text{def}}{=} \beta\hat{q}_P + \hat{r}_P.
\end{aligned}$$

A similar argument as in 4. above, with applications of Lemma 1, implies that under the assumption $\left(\frac{\beta_A}{\beta_P}\right)^3 \geq \left(\frac{\sigma_A}{\sigma_P}\right)^2$,

$$\frac{aQ_A + bv_A}{aQ_P + bv_P} \geq \frac{v_A}{v_P} \geq \frac{a\gamma v_A + c\gamma}{a\gamma v_P + c\gamma},$$

which implies that $\frac{\hat{q}_A}{\hat{q}_P} \geq \frac{\hat{r}_A}{\hat{r}_P}$, again by Proposition I.6 leading to the result for Φ. Moreover, a similar argument as in 4. also implies that when $Q_P \leq \frac{bc}{a^2}$, then

$$\frac{aQ_A + bv_A}{aQ_P + bv_P} \geq \frac{a\gamma v_A + c\gamma}{a\gamma v_P + c\gamma},$$

and another application of Proposition I.6 leads to the result. We are done.

Finally, it follows from the extension to the second modification in Appendix I, that these results also hold in case when $q > 0$. ∎

B.4 Proof of Proposition 7

We focus on the case with $q = 0$, $h = 0$. The proof in the general case is very similar, the only difference being that it contains extra parameters. Consider investor n, who has adopted a passive investment strategy. Given return realizations, R_A and R_P, the transition probability for a sender from A to P is $T_A(R_A)$. Denote the subset of neighbors of investor n that are type A (resp. P) by $\mathcal{D}_n^A$ (resp. $\mathcal{D}_n^P$).

We prove the result for a more general case than our base model in which, even within the same class of investment strategies (A or P), investors may have different returns. Specifically, the return of an A investor $n \in D_m^A$ is assumed to be $R_{A,n}$. The main body considers the special case of in which $R_{An} = R_A$ for $n = 1, \ldots, N$.

For a type P investor n to convert to A, he must (i) be selected for communication, which occurs with probability d_n/Q, (ii) be selected to be receiver, which occurs with probability 1/2, (iii) communicate with an A, $m \in \mathcal{D}_n^A$, and finally (iv) be converted, which occurs with probability $T_A(R_{Am})$. So the probability that investor m switches from P to A is therefore

$$C = \frac{1}{2} \times \frac{|\mathcal{D}_n|}{Q} \times \frac{|\mathcal{D}_n^A|}{|\mathcal{D}_n|} \sum_{m \in \mathcal{D}_n^A} T_A(R_{Am}). \tag{B.16}$$

Clearly, this probability is increasing in the number of A connections, $|\mathcal{D}_n^A|$, in that if a new connection is added, all else equal, the probability for conversion increases; and also in the performance of these connections, since T_A is an increasing function of R_{Am}. ∎

B.5 Proof of Proposition 6

By (B.16),

$$\frac{\partial^2 C}{\partial R_{An}^2} = \frac{1}{2} \times \frac{|\mathcal{D}_m|}{Q} \times \frac{|\mathcal{D}_m^A|}{|\mathcal{D}_m|} \frac{\partial^2 T_A}{\partial R_{An}^2} > 0, \tag{B.17}$$

for $n \in D_m^A$, since T_A is a convex function, and

$$\frac{\partial^2 C}{\partial R_{An}^2} = 0, \tag{B.18}$$

for $n \in D_m^P$. So the probability is indeed (weakly) convex in the returns of all the investors that m is connected to, and strictly convex for a type A connection. ∎

B.6 Proof of Proposition 8

Assume that $R_A = r - D$, where $E[r] = 0$, $Var[r] = \sigma_A^2$, $E[r^3/\sigma_A^3] = \gamma_1$. It follows from (5) that

$$T^A = \gamma_1 \sigma_A^3 \alpha\beta + a\gamma\sigma_A^2 + \sigma_A^2 b\beta - 3D\sigma_A^2\alpha\beta - D^3\alpha\beta + c(\gamma - D\beta) + b(D^2\beta - D\gamma),$$

which when $D = 0$ simplifies to

$$T^A = \sigma_A^2(b\beta + a\gamma) + \sigma_A^3\gamma_1\alpha\beta + c\gamma.$$

The first expression is increasing in γ_1, and the second is also increasing in $\sigma_A > 0$.

From (B.16) and the law of iterated expectations, it follows that

$$C = \frac{1}{2} \times \frac{|\mathcal{D}_n|}{Q} \times \frac{|\mathcal{D}_n^A|}{|\mathcal{D}_n|} \sum_{m \in \mathcal{D}_n^A} E[T_A(R_{Am})],$$

and therefore also that the probability, C, is strictly increasing in the skewnesses of the A portfolios, as well as in their volatility when $D \approx 0$. Moreover, the probability is nondecreasing (flat) in the volatility of the P connections of n, and the result thus follows. ∎

B.7 Proof of Proposition 9

1.: We prove this claim by induction. From the proof of Proposition 7 (equation B.16), it follows that the more connected the agent is at $t = 0$, the higher the probability that he is an A at $t = 1$. Assume that the probability at time t for the agent to be an A is p_A. The probability that he is also A at time $t + 1$ is then

$$q\frac{1}{2}+(1-q)\left[\frac{N_A}{N(N-1)}gT^A + p_A\left(1 - \frac{g}{N}\left(\frac{N_A}{N-1}T^P + \left(1 - \frac{N_A}{N-1}\right)T^A\right)\right)\right], N_A = 1, \ldots, N-1,$$

which is increasing in p_A. Thus, the higher the probability that the investor is A at time t, the higher is the probability that he will be A at $t + 1$, regardless of the number N_A of active investors at t, and by induction the higher the probability at all later points in time.

2. and 3.: From Proposition I.5 it follows that ϕ^t is strictly increasing in g, and since $g = \frac{M}{Q}(1 - h)$, both the results follow.

4.: The investor's probability of rejecting a message is increasing in h_n. The same argument as in Proposition 7, applied at the individual level, implies that his probability of being an A at $t = 1$ is lower the higher is h_n.

5. and 6.: The proofs follow from the same argument as in Proposition 4, but applied to the specific investor, n, and his neighbors, $m \in D_n^0$, respectively. Specifically, given a network realization and a sender-receiver pair, (m, n), $m \in \mathcal{D}_n^0$, the partial derivatives of the probability that n switches to A is proportional to (B.7-B.10), respectively, and the results therefore follow.

7. and 8.: The proofs follow from the same argument as in Proposition 1, but applied to the specific investor, n, and his neighbors, $m \in D_n^0$, respectively. Specifically, given a

network realization and a sender-receiver pair, (m, n), $m \in \mathcal{D}_n^0$, the partial derivatives of the probability that n switches to A is proportional to (13,14), respectively, and the results therefore follow. ∎

B.8 Proof of Proposition 10

As shown in Appendix I, the results in Section 2 do not rely on T^A and T^P being the same regardless of N_A, but rather on $T^A > T^P$ regardless of N_A. In the equilibrium formulation, T^P does not depend on N_A. However, T^A must be determined by market clearing.

The total demand of N_A active investors, given a risky investment opportunity with expected return $E[R_A]$ and return variance $Var(R_A)$ is $X = N_A \frac{E[R_A]-r_f}{Var(R_A)}$, and market clearance, by (19), leads to

$$X = \frac{\rho\kappa^2 N_A^2}{(\rho\kappa N_A - \rho N_A|r_f| + (\beta_A^2\sigma_r^2 + \sigma_A^2))^2}. \tag{B.19}$$

We note that X is increasing in N_A, i.e., that total active investment demand increases with the number of active investors. Since output is concave in demand, this implies that returns are decreasing in the number of active investors.

Plugging in $\rho = \frac{2(\beta_A^2\sigma_r^2+\sigma_A^2)}{N|r_f|}$ gives us

$$R_A = (\beta_A r + \epsilon_A + \kappa)F_{N_A} - \kappa,$$

where

$$F_{N_A} = 1 + \frac{|r_f|}{\kappa}\left(\frac{N}{2N_A} - 1\right).$$

It is easy to verify that F_{N_A} is decreasing in N_A, that $F_{N/2} = 1$, and that $F_N \geq 1/2$. The equilibrium values of the variance factors and the return penalty are:

$$\begin{aligned}
\beta_A(N_A) &= \beta_A F_{N_A}, \\
\sigma_A(N_A) &= \sigma_A F_{N_A}, \\
\gamma_{1A}(N_A) &= \gamma_{1A}, \\
D(N_A) &= \kappa(1 - F_{N_A}) = |r_f|\left(1 - \frac{N}{2N_A}\right).
\end{aligned}$$

We note that $|D(N_A)|$ is small when $|r_f|$ is small.

We first note that Proposition 1 does not depend on the return distributions of active and passive investments, but only on return realizations. It is therefore immediate that it also holds in the equilibrium setting. Moreover, it follows that if $\beta_A > 2\beta_P$, and $\sigma_A > 2\beta_A$, then $\beta_A(N_A) > \beta_P$, and $\sigma_A(N_A) > \sigma_P$ for all $1 \leq N_A \leq N$. An identical argument as that following (17) in the main paper therefore implies that $T^A(n) > T^P(n)$ for all

$1 \leq n \leq N$. Thus, the condition for increasing ϕ^t over time in Proposition 2 is satisfied, and Proposition 3 therefore also holds.

The equilibrium version of Proposition 4 also follows immediately, being based on (17) where β_A is replaced by $\beta_A(N_A)$ and σ_A by $\sigma_A(N_A)$. The only exception is the comparative static with respect to D, which is not defined since D is determined endogenously in equilibrium. Identical arguments can also be used for Proposition 5: 2.-3., whereas for 4. and 5., the condition $\left(\frac{\beta_A(n)}{\beta_P}\right)^3 > \left(\frac{\sigma_A(n)}{\sigma_P}\right)^2$ leads to the stronger condition $F_n^3\left(\frac{\beta_A}{\beta_P}\right)^3 > F_n^2\left(\frac{\sigma_A}{\sigma_P}\right)^2$, which—since $F_n \geq \frac{1}{2}$—is satisfied when

$$\left(\frac{\beta_A}{\beta_P}\right)^3 > 2\left(\frac{\sigma_A}{\sigma_P}\right)^2.$$

The extension of Propositions 7-9 to the equilibrium version go through with identical arguments as in the partial equilibrium setting.

It also follows that $D_{N/2} = 0$, and $D_n > 0$ for $n > N/2$. That $\mathbb{P}(D_{N_A^t} \geq 0) > 1/2$ therefore follows from the fact that $\mathbb{P}(f^t \geq 1/2) > 1/2$, see Proposition 2. Finally, the expected return agents make from the active investment strategy at time t, taking into account that an agent is likely to be active when there are many other agents that are also active and expected returns are therefore low, is

$$E\left[-D_{N_A^t} f^t\right] = |r_f|\left(\frac{1}{2} - E[f^t]\right).$$

Since $E[f^0] = 1/2$, and $E[f^t]$ is strictly increasing over time, the expected return is therefore nonpositive and decreasing over time.

∎

C Trading Volume

Total active demand is given by (B.19). When an investor switches from P to A, he liquidates his passive portfolio position of

$$\frac{|r_f|}{\sigma_P^2},$$

the number of active investors increases from N_A to $N_A + 1$, and he invests

$$\frac{1}{N_A + 1} X_{N_A+1}$$

in the active investment. Here, in equilibrium,

$$X_{N_A} = \frac{2\kappa^2 N N_A^2 |r_f|}{(2N_A(\kappa - |r_f|) + N|r_f|)^2(\beta_A^2\sigma_r^2 + \sigma_A^2)}. \tag{C.1}$$

Moreover, the N_A investors that are already active rebalance from a total position of X_{N_A} to $\frac{N_A}{N_A+1}X_{N_A+1}$. The total trading volume is thus: $\frac{|r_f|}{\sigma_P^2} + Z_{N_A}$, where $Z_{N_A} \stackrel{\text{def}}{=} \frac{1}{N_A+1}X_{N_A+1} + N_A \left| \frac{X_{N_A}}{N_A} - \frac{X_{N_A+1}}{N_A+1} \right|$. It is easy to verify that when $\kappa + r_f \approx 0$, i.e., when $|r_f|$ is of similar size as κ, then $\frac{X_n}{n}$ increasing in n, and therefore

$$Z_{N_A} = X_{N_A+1} - X_{N_A}.$$

Moreover, when $\kappa = -r_f$,

$$Z_{N_A} = \frac{2\kappa}{N(\beta_A^2\sigma_r^2 + \sigma_A^2)}(1 + 2N_A), \tag{C.2}$$

which is strictly increasing in N_A. Therefore, by continuity, for $\kappa + r_f \approx 0$, total trading volume, is also strictly increasing in N_A.

An identical argument applies to the situation when an investor switches from A to P. Specifically, if there are initially $N_A + 1$ investors, and an investor switches from A to P, that investor invests $\frac{|r_f|}{\sigma_P^2}$ in the passive strategy, sells $\frac{1}{N_{A+1}}X_{N_{A+1}}$ in the active investment, whereas the other N_A investors in total rebalance from $N_A\frac{X_{N_{A+1}}}{N_{A+1}}$ to X_{N_A}. Again, the total trading volume is described by $\frac{|r_f|}{\sigma_P^2} + Z_{N_A}$.

I Internet Appendix — Markov Chain Model

We introduce a work-horse Markov chains model. The technical developments here are used in the proof of several of the results in the paper. Consider the variable $w^t = w^t(a,b) \in \{0,1,2,\ldots,N\}$, $t \geq 0$, which evolves according to an $N+1$ state Markov model with transition matrix $\Phi = \Phi(a,b) \in \mathbb{R}^{(N+1)\times(N+1)}$. Here, $a = (a_1, a_2, \ldots, a_{N-1})'$, and $b = (b_1, b_2, \ldots, b_{N-1})$ are $(N-1)$-dimensional vectors, such that $0 < a_n \leq b_n < \frac{1}{2}$, $n = 1, \ldots, N-1$, and Φ is a tri-diagonal matrix with elements

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & \cdots & & \\ a_1 & 1-a_1-b_1 & b_1 & 0 & \cdots & 0 \\ 0 & a_2 & 1-a_2-b_2 & b_2 & \cdots & 0 \\ & \ddots & \ddots & \ddots & \ddots & \\ & \cdots & 0 & a_{N-1} & 1-a_{N-1}-b_{N-1} & b_{N-1} \\ & & & & 0 & 1 \end{bmatrix}. \tag{I.1}$$

The model in the main part of the paper with $q = 0$, $h = 0$, and $M = Q$, corresponds to $\Phi(a,b)$, with $a_n = \chi_n T^P$ and $b_n = \chi_n T^A$.

Define the set of probability vectors $\mathcal{P} = \{p \in \mathbb{R}^{N+1} : \sum_k p_k = 1, p_n \geq 0, n = 1, \ldots, N+1\}$, $\mathcal{P}_0 = \{p \in \mathcal{P} : p_1 + p_{N+1} < 1\}$, and $\mathcal{P}_{00} \subset \mathcal{P}_0 = \{p \in \mathcal{P}_0 : p_n > 0, n = 1, \ldots, N+1\}$.

For some $p^0 \in \mathcal{P}_0$, interpret p^0 as the probability vector for the value of w^0, i.e., $\mathbb{P}(w^0 = n-1) = p_n$, $n = 1, \ldots, N+1$. It then follows that when the probability vector p^t is defined such that $p^t_n = \mathbb{P}(w^t = n-1|p^0)$, then $(p^t)' = (p^0)'\Phi^t$. Moreover, define the sequence $z^0, z^1, \ldots$, where $z^t = z^t(p^0, a, b) = E[w^t|p^0] \leq N$. It then follows that

$$z^t = (p^t)'v = (p^0)'\Phi^t v,$$

where $v \in \mathbb{R}^{N+1}$ is the counting vector, $v = (0,1,2,\ldots,N)'$. Also, let $z^* = \lim_{t\to\infty} z^t$, a limit which we will show to always exist.

We introduce the following partial orders on general vectors, $c \in \mathbb{R}^{N+1}$, $d \in \mathbb{R}^{N+1}$:

- $c \geq d$ if $c_n \geq d_n$ for all n,
- $c > d$ if $c_n \geq d_n$ for all n, and $c_n > d_n$ for some n,
- $c >> d$ if $c_n > d_n$ for all n.
- $c >>_0 d$ if $c \geq d$ and $c_n > d_n$ for $n = 2, \ldots, N$.

We also introduce first order stochastic dominance ordering between probability vectors $p, r \in \mathcal{P}$:

- $p \succeq r$ if $\sum_{k=1}^n p_k \leq \sum_{k=1}^n r_k$, $n = 1, \ldots, N$.

- $p \succ r$ if $p \succeq r$ and the inequality above is strict for some n.

Intuitively, p first order stochastically dominates r, $p \succeq r$, if the p-probability for w to be higher than n is at least as large as the r probability, for all n.

The following result holds:

Proposition I.1

1. *z^t is nondecreasing in t, z^* exists and is less than N.*

2. *If $b = a$, then $z^t = z^0$ for all t, i.e., z^t is a martingale.*

3. *If $b >> a$, then z^t is strictly increasing in t.*

4. *If $b > a$, then z^t is strictly increasing in t for $t \geq N$.*

*P*roof:
1. It is easy to check that $u \overset{def}{=} \Phi v$ has $u_1 = v_1$, $u_{N+1} = v_{N+1}$, $u_n = v_n + b_n - a_n \geq v_n$ and thus $\Phi v \geq v$. Consequently, $p'\Phi v \geq p'v$ for any probability vector, $p \in \mathcal{P}$, leading to

$$z^{t+1} = p_0'\Phi^{t+1}v = p_0'\Phi^t\Phi v = (p^t)'\Phi v \geq (p^t)'v = z^t.$$

Since z^t is nondecreasing and bounded above by N, it follows from the least upper bound property that z^* exists. Moreover, since $a_n > 0$, and $p \in \mathcal{P}_0$, there is always a strictly positive probability that w reaches the absorbing state $w = 0$ within N steps, i.e., $\mathbb{P}(w^N = 0) = \epsilon > 0$, and thus $z^* \leq N(1 - \epsilon) < N$.
2. It is straightforward to verify that when $a = b$, $\Phi v = v$, so $p'\Phi^t v = p'v = z^0$ for all t.
3. The argument is identical to 1., but with strict inequalities. Specifically, $p^t \in \mathcal{P}_0$ for all t, and $(\Phi v)_n > v_n$ for $n = 2, \ldots, N$. Thus,

$$z^{t+1} = (p^t)'\Phi v > (p^t)'v = z^t.$$

4. The argument is similar to that in 1. Since $a_n > 0$, $b_n > 0$, for $n = 1, \ldots, N-1$, and $p^0 \in \mathcal{P}_0$, it follows that $p^t \in \mathcal{P}_{00}$ whenever $t \geq N$, that is since there is always a positive probability for w to either increase or decrease by one in each period, from N periods and forward there is a strictly positive probability for each state that w is in that state.

Pick a k such that $b_k > a_k$. Then,

$$z^{t+1} = (p^t)'\Phi v \geq (p^t)'v + p_k^t(b_k - a_k) = z^t + p_k^t(b_k - a_k) > z^t.$$

This completes the proof. ∎

Define the class of (weakly) increasing vectors, $\mathcal{V} = \{u \in \mathbb{R}^{N+1} : u_{n+1} \geq u_n, n = 1, \ldots, N\}$, and $\mathcal{V}_0 \subset \mathcal{V}$ for the subset of u's such that the inequality is strict for all n. Moreover, given the vectors, a and b used in the definition of Φ, define $\mathcal{V}_0^{a,b} = \{u \in \mathcal{V}_0 : b_i(u_{i+2} - u_{i+1}) > a_i(u_{i+1} - u_i), i = 1, \ldots, N-1\}$. Thus, $V_0^{a,b}$ consists of increasing vectors with additional restrictions on how their elements grow. If $b = a$, the growth is strictly increasing, corresponding to convexity. When $b >> a$, the growth does not have to be strictly increasing—it is for example easy to check that $v \in \mathcal{V}_0^{a,b}$ in this case. Instead, the necessary growth rate is bounded below by $\frac{a_i}{b_i}$.

Proposition I.2

1. *If $u \in \mathcal{V}$, then $\Phi u \in \mathcal{V}$,*
2. *If $u \in \mathcal{V}_0$, then $\Phi u \in \mathcal{V}_0$.*
3. *If $u \in \mathcal{V}_0^{a,b}$, then $\Phi u \in \mathcal{V}_0^{a,b}$.*

Proof:
1. Define $\Delta u_n = u_{n+1} - u_n$, and note that

$$(\Phi u)_1 = u_1 \quad \leq \quad (\Phi u)_2 = a_1 u_1 + (1 - a_1 - b_1)(u_1 + \Delta u_1) + b_1(u_1 + \Delta u_1 + \Delta u_2).$$

Also, note that

$$\begin{aligned}
(\Phi u)_{N+1} &= u_{N+1} \\
&\geq (\Phi u)_N \\
&= a_{N-1}(u_{N+1} - \Delta u_N - \Delta u_{N-1}) + (1 - a_{N-1} - b_{N-1})(u_{N+1} - \Delta u_N) + b_{N-1} u_{N+1},
\end{aligned}$$

and for $n = 2, \ldots, N-1$,

$$\begin{aligned}
(\Phi u)_n &= a_n(u_n - \Delta u_{n-1}) + (1 - a_n - b_n)u_n + b_n(u_n + \Delta u_n) \\
&= u_{n-1} + (1 - a_n)\Delta u_{n-1} + b_n \Delta u_n, \qquad n = 1, \ldots, N-1,
\end{aligned}$$

$$(\Phi u)_{n+1} = u_{n-1} + \Delta u_{n-1} + (1 - a_{n+1})\Delta u_n + b_{n+1}\Delta u_{n+1},$$

and since $1 > 1 - a_n$, and $1 - a_{n+1} > b_n$, it follows that $(\Phi u)_{n+1} \geq (\Phi u)_n$. Thus $\mathcal{V}$ is closed under composition with Φ.
2. An identical argument as in 1 shows that $\mathcal{V}_0$ is also closed under composition with Φ.
3. Define $f = \Phi u$. Of course, $f_1 = u_1$, $f_{N+1} = u_{N+1}$, and for $i = 2, \ldots, N$ it follows that

$$f_i = u_i - a_{i-1}(u_i - u_{i-1}) + b_i(u_{i+1} - u_i).$$

It is easy to verify that for $i = 2, \ldots, N-2$

$$
\begin{aligned}
b_i(f_{i+2} - f_{i+1}) - a_i(f_{i+1} - f_i) &= (1 - b_i - a_i)(b_i(u_{i+2} - u_{i+1}) - a_i(u_{i+1} - u_i)) \\
&+ b_i(b_{i+1}(u_{i+3} - u_{i+2}) - a_{i+1}(u_{i+2} - u_{i+1})) \\
&+ a_i(b_{i-1}(u_{i+1} - u_i) - a_{i-1}(u_i - u_{i-1})) \\
&> 0.
\end{aligned}
$$

Moreover,

$$
\begin{aligned}
b_1(f_3 - f_2) - a_1(f_2 - f_1) &= (1 - b_1 - a_1)(b_1(u_3 - u_2) - a_1(u_2 - u_1)) \\
&+ b_1(b_2(u_4 - u_3) - a_2(u_3 - u_2)) \\
&> 0,
\end{aligned}
$$

and

$$
\begin{aligned}
b_{N-1}(f_{N+1} - f_N) - a_{N-1}(f_N - f_{N-1}) &= (1 - b_{N-1} - a_{N-1})(b_{N-1}(u_{N+1} - u_N) - a_{N-1}(u_N - u_{N-1})) \\
&+ a_{N-1}(b_{N-2}(u_N - u_{N-1}) - a_{N-2}(u_{N-1} - u_{N-2})) \\
&> 0,
\end{aligned}
$$

altogether implying that $f \in \mathcal{V}_0^{a,b}$. This completes the proof. ∎

Now, it is a standard result that if p and r are probability vectors such that $p \succeq r$, and $u \in \mathcal{V}$, then $p'u \geq r'u$. This can, for example, be seen from the following summation-by-parts result:

$$
p'u = \sum_{n=1}^{N+1} p_n u_n = F_{N+1} u_{N+1} - \sum_{n=1}^{N} F_n \Delta u_n,
$$

where $F_n = \sum_{k=1}^{n} p_n$, and $G_n = \sum_{k=1}^{n} r_n$. Since $F_{N+1} = G_{N+1} = 1$, and $F_n \leq G_n$, for $n = 1, \ldots, N$, the result follows. Moreover, for $p \succ r$ and $u \in \mathcal{V}_0$, the inequality is strict, $p'u > r'u$. It is also easy to check that if $c >> b$, $p \in \mathcal{P}_0$, such that $p \succ r$, then $(p'\Phi(a,c))' \succ (r'\Phi(a,c))'$, and by induction $(p'\Phi(a,c)^s)' \succ (r'\Phi(a,c)^s)'$, $s \geq 1$.

Define the vector $\zeta \in \mathcal{V}$, with $\zeta_m = -1$, $m = 1, \ldots N/2$, $\zeta_{N/2+1} = 0$, and $\zeta_m = 1$, $m = N/2+2, \ldots, N+1$, and $\nu^t = \nu^t(a,b) = p'\Phi^t(a,b)\zeta$. It then follows that $\nu^t = \mathbb{P}(w^t > N/2) - \mathbb{P}(w^t < N/2)$, i.e., that ν^t is the difference between the probabilities that w^t is greater than and less than $N/2$, respectively. The previous argument implies that if $c >> b$, then $\nu^t(a,c) > \nu^t(a,b)$, and as a direct consequence:

Proposition I.3 $\mathbb{P}(w^t \geq N/2) > 1/2$ *for all* $t \geq 1$.

Proof: Because of symmetry it follows that $\eta^t(a,a) = 0$, and therefore, since $b >> a$, that $\eta^t(a,b) > 0$. Now, $1 = \mathbb{P}(w^t > N/2) + \mathbb{P}(w^t < N/2) + \mathbb{P}(w^t = N/2) = 2\mathbb{P}(w^t < N/2) + \eta^t + \mathbb{P}(w^t = N/2)$, so $\mathbb{P}(w^t < N/2) < 1/2$, and the result therefore follows. ▮

We also have

Proposition I.4

1. *If $p \succ r$, then $z^t(p,a,b) > z^t(r,a,b)$ for all $t \geq 0$.*
2. *If $c >> b$, then $z^t(p,a,c) > z^t(p,a,b)$ for all $t > 0$.*
3. *If $c >> a$, then $z^t(p,c,b) < z^t(p,a,b)$ for all $t > 0$.*

Proof: 1. From Proposition I.2, it follows that $u = \Phi^t v \in \mathcal{V}_0$ for all t, and therefore that $p'u > r'u$.
2. First, note that $\Phi(a,c)v >>_0 \Phi(a,b)v$, since $(\Phi(a,c) - \Phi(a,b))v)_n = (c_{n-1} - b_{n-1})$, $n = 2, \ldots, N$. Moreover, from the results above it follows that $\Phi(a,c)^t v \in \mathcal{V}_0$, $\Phi(a,b)^t v \in \mathcal{V}_0$ for all t.

Now, for general $u >>_0 r$, $u \in \mathcal{V}_0$, $r \in \mathcal{V}_0$, define $h = u - r$ and $g = c - b$. It follows for $n = 1, \ldots, N-1$, that

$$\begin{aligned} (\Phi(a,b)r)_{n+1} &= a_n r_n + (1 - a_n - b_n) r_{n+1} + b_n r_{n+2}, \\ (\Phi(a,c)u)_{n+1} &= a_n(r_n + h_n) + (1 - a_n - b_n - g_n)(r_{n+1} + h_{n+1}) + (b_n + g_n)(r_{n+2} + h_{n+2}), \end{aligned}$$

and thus that

$$(\Phi(a,c)u - \Phi(a,b)r)_{n+1} = a_n h_n + (1 - a_n - c_n) h_{n+1} + g_n(r_{n+2} - r_{n+1}) + (b_n + g_n) h_{n+2} > 0.$$

So $\Phi(a,c)u >>_0 \Phi(a,b)r$, and by induction therefore $\Phi(a,c)^t v >>_0 \Phi(a,b)^t v$. It follows that $z^t(p,a,c) > z^t(p,a,b)$.
3. The result follows from an identical argument is in 2. The proof is complete. ▮

First modification. For $0 < \alpha \leq 1$, now define the modified transition matrix

$$\Theta(a,b,\alpha) = (1-\alpha)I + \alpha\Phi,$$

where I is the $(N+1) \times (N+1)$ identity matrix, and the associated modified Markov process. Also, define

$$x^t = x^t(p^0, a, b, \alpha) = (p^0)'\Theta(a,b,\alpha)v = (p^t)'v,$$

where $p^t = ((p^0)'\Theta^t)'$, as the expected value of the modified Markov process at time t, and $x^* = \lim_{t\to\infty} x^t$. When $\alpha = 1$, the model reduces to the previous one, $\Theta(a,b,1) = \Phi(a,b,1)$.

We note that the model in the main part of the paper with $q = 0$, but allowing for general M and h, corresponds to setting $a_n = \chi_n T^P$, $b_n = \chi_n T^A$, and $\alpha = g = \frac{M}{Q}(1-h)$.

It follows immediately for $t \geq 1$, that

$$x^t = (1-\alpha)x^{t-1} + \alpha(p^{t-1})'\Phi v \tag{I.2}$$

$$= \sum_{k=0}^{t} \binom{t}{k} \alpha^k (1-\alpha)^{t-k} z^k. \tag{I.3}$$

From these relationships, it moreover follows that the results in the previous section carry over to the modified model. Especially, for a fixed α, Propositions I.1, I.3 and I.4 immediately carry over with x^t replacing z^t. This can easiest be seen by noting that the adjusted model is equivalent to the base model, but replacing a and b with αa and αb, respectively.

In addition, the following result shows how x^t depends on α.

Proposition I.5 *If $\beta > \alpha$ and $b >> a$, then $x^t(p, a, b, \beta) > x^t(p, a, b, \alpha)$, for $t \geq 1$.*

Proof: The result follows from (I.3), and the fact that z^t is strictly increasing in t. Specifically, note that $r_{i+1} = \sum_{k=0}^{i} \binom{t}{k} \alpha^k (1-\alpha)^{t-k}$ is the cumulative distribution function of a $Binomial(\alpha, t)$ distributed random variable, which for each $0 \leq i < t$ is decreasing in α. In other words, the higher α is, the more probability weight is put on high states, as defined by first order stochastic dominance. Thus, if r is viewed as a $t+1$ vector with elements $r_1, \ldots, r_{t+1}$, then $r(\beta) \succ r(\alpha)$. It follows that $x^t(p, a, b, \beta) = r(\beta)'(z^0, \ldots, z^t)' > x^t(p, a, b, \alpha) = r(\alpha)'(z^0, \ldots, z^t)'$, from the strict monotonicity of z over time. This completes the proof. ∎

To allow for further comparative statics, we study the case when a and b are smooth, increasing functions of some underlying parameter, k. Specifically, we assume that $a_i = a_i(k)$, $b_i = b_i(k)$, where $b_i(k) > a_i(k) > 0$, $b_i'(k) > a_i'(k) > 0$, $i = 1, \ldots, N-1$. The transition matrix is then a function of k, $\Phi(k)$, $k > 0$. Note that Θ is obtained, with $\alpha = k$, when $a_i(k) = a_i k$, $b_i(k) = b_i k$ are chosen. The k-dependent time-t expectation is now $x^t(k) = p'\Phi(k)^t v$. We are interested in the comparative static $x^t(k)' = dx^t/dk$. We have

Proposition I.6 *If $\frac{b_n'(k)}{b_n(k)} \geq \frac{a_n'(k)}{a_n(k)}$, $n = 1, \ldots, N-1$, then $x^t(k)' > 0$ for all $t \geq 1$.*

Proof: Using algebra for matrix differentiation, and defining $c_i = a_i'(k)$, $d_i = b_i'(k)$, we get $x^t(k)' = p'Xv$, where

$$X = \sum_{s=1}^{t} \Phi(k)^{s-1} Y \Phi(k)^{t-s},$$

and

$$Y \in \mathbb{R}^{(N+1)\times(N+1)} = \frac{d\Phi}{dk} = \begin{bmatrix} 0 & 0 & 0 & \cdots & & \\ c_1 & -c_1-d_1 & d_1 & 0 & \cdots & 0 \\ 0 & c_2 & -c_2-d_2 & d_2 & \cdots & 0 \\ & \ddots & \ddots & \ddots & \ddots & \\ & \cdots & 0 & c_{N-1} & -c_{N-1}-d_{N-1} & d_{N-1} \\ & & & 0 & 0 & 0 \end{bmatrix}.$$

Now, from Proposition I.2 it follows that $\Phi(k)^s v \in \mathcal{V}_0^{a,b}$. For a general $u \in \mathcal{V}_0^{a,b}$, consider $g = Yu$. Obviously $g_1 = g_{N+1} = 0$. We also have for $2 \le n \le N$

$$\begin{aligned} g_n &= d_n(u_{n+2} - u_{n+1}) - c_n(u_{n+1} - u_n) \\ &= \left(\frac{d_n}{b_n}\right) b_n(u_{n+2} - u_{n+1}) - \left(\frac{c_n}{a_n}\right) a_n(u_{n+1} - u_n) \\ &\ge \left(\frac{c_n}{a_n}\right) \left(b_n(u_{n+2} - u_{n+1}) - a_n(u_{n+1} - u_n)\right) \\ &> 0, \end{aligned}$$

where the second to last inequality follows from the fact that $\frac{d_n}{b_n} > \frac{c_n}{a_n}$, and the last inequality from the fact that $u \in \mathcal{V}_0^{a,b}$. Therefore $r'Yu > 0$ for any $r \in \mathcal{P}_0$, and thus $p'\Phi(k)^{s-1} Y \Phi(k)^{t-s} v > 0$, and also $p'Xv > 0$. This completes the proof. ∎

Second modification. We introduce a second modification: For $0 \le q < 1$, define the modified transition matrix

$$\Psi = \Psi(a, b, \alpha, q) = (1-q)\Theta + qR,$$

where $R \in \mathbb{R}^{(N+1)\times(N+1)}$, with elements $R_{ij} = 1$ when $i = N/2 + 1$, and $R_{ij} = 0$ otherwise. This stochastic matrix, R, represents a degenerate Markov chain which immediately moves to state $N/2$ in the next period. The modified model is thus one in which, with probability q, such a reset occurs, and with probability $(1-q)$ the model propagates according to the Θ transition matrix. Also, let probability vector $\delta^k \in \mathbb{R}^{N+1}$, with $(\delta^k)_k = 1$, and $(\delta^k)_n = 0$, $n \ne k$. The vector $d = \delta^{N/2+1}$, represents the initial distribution of 100% chance that the state is $N/2$.

We note that the model in the main part of the paper with, allowing for general q, M and h, corresponds to $\Psi(a, b, \alpha, q)$, with $a_n = \chi_n T^P$, $b_n = \chi_n T^A$, and $\alpha = g = \frac{M}{Q}(1-h)$.

Define $\phi^t = \phi^t(a, b, \alpha, q) = d'\Psi^t v$, so that ϕ^t represents the expected value of the process (under the second modification) at time t, $\phi^* = \lim_{t\to\infty} \phi^t$, and $x^t = d'\Theta^t v$. Obviously, $\phi^0 = x^0$. Now, for any stochastic matrix, Ξ, $\Xi R = R$, and therefore $\Theta^s R = R$ for all $s \ge 0$. In words, the dynamics of w up until s is irrelevant when there is a reset at time $s+1$.

Also, it is easily seen that $d'R = d'$. It therefore immediately follows that

$$\begin{aligned}\Psi^t &= ((1-q)\Theta + qR)^t \\ &= (1-q)^t\Theta^t + q\sum_{s=0}^{t-1}(1-q)^s R\Theta^s,\end{aligned}$$

so for $t \geq 1$,

$$\begin{aligned}\phi^t &= (1-q)^t x^t + q\sum_{k=0}^{t-1}(1-q)^s x^s, &\text{(I.4)}\\ &= \phi^{t-1} + (1-q)^t(x^t - x^{t-1}). &\text{(I.5)}\end{aligned}$$

From (I.5), and the fact that the sequence x^t is increasing as previously shown, it follows that the sequence ϕ^t is increasing in t. Moreover, from (I.4), it follows that Propositions I.1, I.3, I.4, I.5, and I.6 carry over to the second modified Markov process, since they hold term-by-term for all x^s.

Finally, we have the following result for how the sequence ϕ^t depends on q.

Proposition I.7 *If $q' < q$ and $b >> a$, then $\phi^t(a, b, \alpha, q') > \phi^t(a, b, \alpha, q)$, $t \geq 1$, and $\phi^*(a, b, \alpha, q') > \phi^*(a, b, \alpha, q)$.*

Proof: Note that

$$(1-q)^t + q\sum_{k=0}^{t-1}(1-q)^s = (1-q)^t + q\frac{1-(1-q)^t}{1-(1-q)} = 1,$$

so ϕ^t is a weighted average of $x^0, x^1, \ldots, x^t$. Define the vector $r(q) \in \mathbb{R}^{t+1} = (q, q(1-q), q(1-q)^2, \ldots, q(1-q)^{t-1}, (1-q)^t)'$, representing the weights in the average on different x^s terms. Note that $r(q)$, having positive elements and summing to one, can be thought of as a probability vector, and since $\sum_{i=1}^k r(q)_i = 1-(1-q)^k$ (for $k \leq t$), which is increasing in q, it follows that $r(q') \succ r(q)$. Since x^s is increasing in s, it then follows that $\phi^t(a, b, \alpha, q') = r(q')'(x^0, \ldots, x^t)' > \phi^t(a, b, \alpha, q) = r(q)'(x^0, \ldots, x^t)'$. Finally, $\phi^* = q\sum_{k=0}^{\infty}(1-q)^s x^s$. Where x^s is a strictly increasing, bounded series. It is therefore easily verified that $\frac{d\phi^*}{dq} < 0$, since term-wise differentiation is allowed. This completes the proof. ∎

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