Problem Set #3

Doing these problems is optional. The solutions to these questions will be posted by on Thursday, September 24th and discussed in section Friday, September 25th. As always, the educational value of these exercises will be maximized if you attempt to answer these questions before you look at the answers.

Question 1

Your esteemed boss Hugo has just leased a new nut-grinding machine that has the capacity to produce 100 units of hugonuts per day. The daily lease cost is $405. The market price for a unit of hugonuts is $120, and no one expects this to change in the foreseeable future. When running the machine, you observe that daily total costs follow the pattern: \( C(Q)=405+20Q+5Q^2 \), where \( Q \) is the number of hugonuts produced and \( C(Q) \) is in $.

a) Hugo has decided to minimize his total costs. How much should he produce? What are his total profits?

b) Hugo is back in the shop. He understands the importance of amortizing the daily lease cost of the machine over a large production run. His brother-in-law the VP of marketing has convinced Hugo of the importance of dominating the market and getting as much market share as he can with his existing machine. Hugo decides to follow this friendly advice, believing that not only will he make great heaps of money but he will also achieve much more pleasant conditions at the next gathering with his in-laws. How much does he instruct you to produce in order to maximize sales? What are his total profits (i.e. his total revenues minus total costs)?

c) You notice Hugo has sprouted a few more gray hairs. Coincidentally, the marketing VP was called away to investigate a potential new client in Tierra del Fuego. Hugo has given the matter more thought, and instructs you to minimize the average cost of production. How much do you produce? What are your profits?

d) Hugo is looking a bit better. He almost smiles now, especially when he shows everyone those spectacular post-cards of the fog and ice his brother-in-law keeps sending from Tierra del Fuego. You spot your opportunity and recommend that the time has come to maximize profits. Hugo is feeling so
good, that he forgives you your MBA and follows your advice. How much do you produce? What are your profits?

Question 2

You own the only carrot juice bar in Berkeley, which appears to be a valuable franchise. The daily demand you face for carrot juice is \( Q = 100 - P \). The total daily cost of operating the juice bar is \( TC = 100 + 10Q \). (The daily fixed cost of operating is 100. That is an avoidable fixed cost if the juice bar is not open).

a) What is your firm’s marginal revenue schedule. (Remember that marginal revenue (MR) is the change in revenue associated with a one unit change in quantity, not a one unit change in price)? How much should you produce in order to maximize profits? How much will your daily profit be?

b) You have brought in a hot-shot production consultant with a Stanford MBA who explains to you that your cost function is not what you thought. She says that your cost function is actually a bit more complicated. Your daily fixed cost is indeed 100, but your marginal cost depends on the quantity you produce:

\[
MC = \begin{cases} 
10 & \text{for } Q \leq 20 \\
8 & \text{for } 20 < Q \leq 50 \\
6 & \text{for } Q > 50 
\end{cases}
\]

If this is so, how much should you produce in order to maximize profits? How much will your daily profit be?

c) Finally, you bring in a Berkeley-Haas MBA, who is confident without an attitude! She explains that your cost function is actually \( TC = 100 + Q^2 \), so your marginal cost rises as you produce more, \( MC = 2Q \). She also says that the demand function you face is actually \( Q = 20 - P \). If this is so, how much should you produce in order to maximize profits? How much will your daily profit be?

d) Finally, you recognize three things. First, your initial estimate of costs were correct so that when you produce a total of \( Q \) units then our costs are \( TC = 100 + 10Q \). Second, your initial estimate of Haas demand was true, and is equal to \( Q_H = 100 - P \) (where \( Q_H \) represents “Haas generated demand”). Third, the recent student and faculty walkout has generated a huge amount of tourism right outside the Haas school, and these tourists form another source of demand given by \( Q_T = 75 - P \). Since these tourists do not have Berkeley IDs, you realize that you can set a price \( P \), and offer a “Cal Discount” to your original customers. Would you like to do this? If so, what price would you charge, what would the discount be, and what would your profits be? Compared to part (a), has the discount made your Haas clientele better off? Why?
Question 3

Toscanini’s, a Boston firm, has the patent on gingersnap molasses ice cream. It decides to introduce it to the Bay Area, where it proves to be an enormous hit in both San Francisco County and Marin County. Ice cream does not travel well, and the congestion on the Golden Gate Bridge is so bad that it is impossible to transport the ice cream across the bridge. Consequently, Toscanini’s builds two identical plants, one in San Francisco and the other in Sausalito. For either plant, the daily cost of Q gallons of ice cream is 2Q. The daily demand for gingersnap molasses ice cream in San Francisco County is \( Q_{SF} = 35,000 - 5000p \). The daily demand for gingersnap molasses ice cream in Marin County is \( Q_{M} = 25,000 - 2500p \).

a) A California state law prohibits charging different prices in adjacent counties. What single price \( p \) should Toscanini’s charge to maximize its profit from gingersnap molasses ice cream? What is its profit?

b) Responding to a suit brought by Toscanini’s, a federal judge rules that San Francisco and Marin counties are not adjacent because they are separated by the Golden Gate. Toscanini’s is now free to charge different prices in the two counties, \( p_{SF} \) and \( p_{M} \). What prices \( p_{SF} \) and \( p_{M} \) maximize Toscanini’s profits? What are the resulting profits?

c) If the suit had not been brought by Toscanini’s, which county government would have had an incentive to bring the suit?

Question 4

Your software company has just completed the first version of SpokenWord, a voice-activated word processor. As marketing manager, you have to decide on the pricing of the new software. You commissioned a study to determine the potential demand for SpokenWord. From this study, you know that there are essentially two market segments of equal size, professionals and students (one million each). Professionals would be willing to pay up to $400 and students up to $100 for the full version of the software. A substantially scaled-down version of the software would be worth $50 to students and worthless to professionals. It is equally costly to sell any version. In fact, other than the initial development costs, production costs are zero. Assume that you cannot use third-degree price discrimination. In other words, you cannot identify students separately from professionals other than by the prices they are willing to pay.

a) What are the optimal prices for each version of the software?

Suppose that instead of the scaled-down version, the firm develops an intermediate version that is valued at $200 by professionals and $75 by students.

b) What are the optimal prices for each version of the software? Is the firm better off by selling the intermediate version instead of the scaled-down version?