Intertemporal Pricing with Strategic Customer Behavior

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This paper develops a model of dynamic pricing with endogenous intertemporal demand. In the model, there is a monopolist who sells a finite inventory over a finite time horizon. The seller adjusts prices dynamically to maximize revenue. Customers arrive continually over the duration of the selling season. At each point in time, customers may purchase the product at current prices, remain in the market at a cost to purchase later, or exit, and they wish to maximize individual utility. The customer population is heterogeneous along two dimensions: they may have different valuations for the product and different degrees of patience (waiting costs).

We demonstrate that heterogeneity in both valuation and patience is important because they jointly determine the structure of optimal pricing policies. In particular, when high-value customers are proportionately less patient, markdown pricing policies are effective because the high-value customers would buy early at high prices while the low-value customers are willing to wait (i.e., they are not lost). On the other hand, when the high-value customers are more patient than the low-value customers, prices should increase over time to discourage inefficient waiting. Contrary to intuition, we find that strategic waiting by customers may sometimes benefit the seller: when low-value customers wait, they compete for availability with high-value customers and thus increase their willingness to pay. Our results also shed light on how the composition of the customer population affects optimal revenue, consumer surplus, and social welfare. Finally, we consider the long-run problem of selecting the optimal initial stocking quantity.

Key words: dynamic pricing; strategic customer behavior; heterogeneity; intertemporal substitution; rationing

History: Accepted by Sunil Chopra, operations and supply chain management; received July 5, 2005. This paper was with the author 1 ½ months for 2 revisions.

1. Introduction

Pricing is one of the most fundamental but also most difficult decisions that firms have to make. An important reason is that businesses today are facing a generation of increasingly sophisticated customers. Whether firms adopt the most powerful category pricing software or the most data-intensive revenue management system, customers are becoming extremely adept at finding the best deals. Readers of this article may even have some personal experiences that they are proud to share. According to SmartMoney magazine, there is constantly a “cat-and-mouse game between retailers, who hope to charge full price for everything, and shoppers, who wait for a sale” (see Kadet 2004, pp. 90–91). The zero-sum nature of pricing makes this inevitable. Firms are constantly improving their pricing strategies to collect as much revenue as possible, and customers are constantly modifying their purchase plans to pay as little as possible.

Recently, a major electronics retailer, Best Buy, has expressed some strong opinions about this kind of strategic customer behavior. In an article that appeared on the front page of the Wall Street Journal, Best Buy Chief Executive Officer Brad Anderson openly labels some customers as “devils” (see McWilliams 2004). According to Anderson, these are the customers who wait for markdowns, respond to promotions, and apply for rebates. In contrast, Anderson also describes the “angels” as the customers who snap up high-end gadgets without hesitation. Best Buy estimates that approximately 100 million out of its 500 million customer visits each year are “undesirable.” Although these pejorative labels have attracted criticism (see Queenan 2005), Best Buy has implemented customer relationship management programs to better distinguish the “angels” from the “devils” (see Arndorfer and Creamer 2005). Apart from Best Buy, many other firms are also beginning to recognize that revenues are lost when customers wait for sales. Retailers, such as Bloomingdale’s, Ann Taylor, Gap, and Home Depot, are turning to price optimization software instead of blindly slashing prices toward the end of the selling season (see Schlosser 2004).

Although the importance of strategic customer behavior is recognized by many, its implications on intertemporal pricing strategies have not been widely studied. Therefore, in this paper, we set out with
two main research questions. First, we would like to understand the interplay between the firm’s pricing strategies and customers’ purchasing strategies. In this environment, should prices increase or decrease (or stay fixed) over time? What is the optimal timing and extent of the markups and/or markdowns? For the customers, how should they determine whether to wait for a sale, if the product may be sold out by then? We would like to model and understand these strategic interactions. Next, our second research question concerns the impact of strategic customer behavior. When customers delay purchases and wait for sales, does this behavior hurt or benefit the seller, and in what ways do these effects arise?

In our model, there is a monopolist who sells a finite inventory over a finite time horizon. The seller may charge different prices over time, and may also practice rationing by fulfilling only a portion of market demand. There is a continuous inflow of customers arriving into the market. If they are unwilling to purchase the product immediately, they may leave the system, or may wait for more attractive purchase opportunities in the future. Although prices may fall, there is also the possibility that the product will become unavailable. Furthermore, customers incur waiting costs. Within this environment, the seller seeks to maximize revenue, and customers wish to maximize individual utility.

Heterogeneity plays a key role in our model. We allow the customers to vary along two dimensions: they may have different valuations for the firm’s product and may have different degrees of patience (i.e., different waiting costs). Heterogeneous valuations imply that dynamic pricing is worthwhile because there is an opportunity to practice intertemporal price discrimination. Heterogenous waiting costs allow us to capture a wide variety of customer behavior. At one extreme, when waiting costs are infinitely large, customers are myopic and make a one-time buy-or-exit decision upon arrival; on the other hand, customers with finite waiting costs may delay their purchases strategically. Existing models consider customer populations that are either purely myopic or purely strategic, whereas we allow for arbitrary combinations of both. With these two dimensions of heterogeneity, we believe that we can capture a good representation of reality.

This paper generates two novel insights. First, we demonstrate that customer heterogeneity is a main driver determining the structure of optimal pricing policies. In particular, the two dimensions of heterogeneity (valuation and patience) jointly determine whether prices should increase or decrease over time. Markups such as end-of-season sales are preferred when low-valuation customers are sufficiently patient to wait for sales, while high-valuation customers are sufficiently impatient to buy early at higher prices. On the other hand, markups are preferred when high-valuation customers are more patient than low-valuation customers: this discourages inefficient waiting and also captures surplus from high-valuation customers who miss the initial low prices. In this way, our analysis endogenizes the structure of pricing policies and captures both markups and markdowns within a single framework.

Another managerial insight from this work is that strategic customer behavior may actually benefit the seller. This finding counters common intuition that customers who practice intertemporal substitution hurt seller revenues when they are able to find lower prices. While this is true, we point out another effect that to the best of our knowledge has not been identified before. That is, in an environment of scarcity (i.e., finite inventory), low-valuation customers who wait for discounts end up competing for availability with their high-valuation counterparts. This threat of stock-outs discourages the high-valuation customers from waiting and also increases their willingness to pay. Therefore, scarcity and heterogeneity interacts in a subtle way that puts strategic customer behavior in a new light.

The remainder of this paper is organized as follows. Section 2 provides a literature review. Section 3 describes the model. Section 4 develops some structural properties, uses them to characterize an upper bound on seller revenues, and shows that this upper bound can be attained. Section 5 presents the main results, characterizing the seller’s optimal policy. Section 6 examines the seller’s revenue, consumer surplus, and social welfare under the optimal regime. The long-run problem of selecting an optimal initial stocking quantity is analyzed in §7. Finally, §8 offers concluding remarks. All proofs are presented in the online appendix (provided in the e-companion).1

2. Literature Review

The three main issues examined in this paper are: (i) strategic customer behavior, (ii) price dynamics, and (iii) limited capacity. There are several streams of related literature, each addressing different subsets of these issues.

The revenue management literature on dynamic pricing of finite inventories is closely related to our work. This stream of papers focuses on price dynamics and limited capacity (the primary question is how to set prices as a function of remaining inventory). However, strategic customer behavior is absent from the earlier models. The first papers were by

1 An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.
Gallego and van Ryzin (1994, 1997). They model customer arrivals using Poisson processes and formulate the dynamic pricing problem as an intensity control problem; their 1997 paper generalizes the basic model to a network (multiproduct) setting. Federgruen and Heching (1999) combine pricing with inventory decisions. Feng and Gallego (1995, 2000) make a practically justified restriction: they consider a discrete menu of prices and policies involving at most one price change. Feng and Xiao (2000a, b) extend this to policies involving multiple and reversible (nonmonotonic) price changes. Similarly, Bitran and Mondshein (1997) consider periodic pricing policies that modify prices only at pre-specified times. Zhao and Zheng (2000) study dynamic pricing in more general situations with time inhomogeneous customer arrivals. For surveys of this literature, readers are referred to Bitran and Caldentey (2003), Elmaghraby and Keskinocak (2003), and McAfee and te Velde (2005). For a comprehensive coverage of revenue management, readers are referred to the book by Talluri and van Ryzin (2005). A common approach in this literature is to determine optimal prices dynamically by considering the option value of unsold units. The result is that optimal price paths are decreasing over time (on average) because the option value of unsold units decreases as the deadline approaches. However, this result requires the assumption that demand is exogenous and independent across time. This no longer applies in the current work because strategic purchase delays in our model imply that demand may spill over into the future. As a result, we obtain optimal price schedules that may both increase or decrease over time.

Recent papers in revenue management have begun to examine customer behavior more closely. However, the focus is on how customers choose between substitute products offered by the firm (rather than on intertemporal demand substitution). Talluri and van Ryzin (2004) use discrete choice models to describe how customers, in the context of airlines, choose among the set of fare classes offered; van Ryzin and Liu (2004) extends this analysis to the network setting. Shumsky and Zhang (2004) consider demand substitution, via upgrading, when inventory has been depleted. Netessine et al. (2006) consider cross-selling (i.e., offering customers a choice between their requested product and a package containing the requested product as well as another product). Cooper et al. (2006) show that neglecting substitution across products can lead to a spiral-down effect, in which the capacity allocation policy systematically performs worse and worse as the forecasting-optimization process continues. Zhang and Cooper (2005a) analyze a capacity allocation model with customer choice over parallel flights, and they extend this analysis (Zhang and Cooper 2005b) to incorporate pricing decisions. Maglaras and Meissner (2006) show that the dynamic pricing problem when customers choose between multiple products can be reduced to an equivalent one-dimensional problem. In all these papers, customers choose what to buy, whereas in our work, customers choose when to buy. Both aspects of strategic customer behavior are important, and they are addressed using different modeling techniques.

The papers on dynamic pricing of finite inventories that are most closely related to ours involve intertemporal demand. These papers explicitly model customers’ decisions regarding when to buy. Aviv and Pazgal (2003) assume that there is a single price reduction at a fixed time. Elmaghraby et al. (2004) also focus on markdown mechanisms, and customers with multi-unit demands choose how many units to purchase at each price step. These two papers explicitly assume that prices should decrease over time. On the contrary, Gallien (2006) demonstrates that optimal prices should increase over time, but his results apply to the infinite-horizon case; for the simpler case where customers do not wait, Arnold and Lippman (2001) and Das Varna and Vettas (2001) obtain similar conclusions. Unlike these papers, we consider a finite horizon, and we find that markups and markdowns may be optimal under different situations.

Instead of pricing decisions, Liu and van Ryzin (2005) consider quantity decisions in a two-period capacity rationing model with strategic customers. They assume that the seller pre-commits to prices in both periods. In another paper, Xu and Hopp (2004) use a martingale approach to endogenize strategic customer behavior via their price sensitivities over time; they assume that customers commit to a purchasing time right from the start and do not wait in the market after observing current prices. Unlike the above two papers, we do not require commitment, either on the buyer side or on the seller side. Instead, we analyze a dynamic principal-agent game in which both the seller and the customers continuously optimize their decisions over time. Rather than focusing on customers’ decisions, Ovchinnikov and Milner (2005) focus on firms’ pricing strategies when facing an exogenously specified profile of aggregate customer waiting behavior. This approach differs from our paper, in which we endogenously characterize customer waiting behavior as an equilibrium outcome. Zhou et al. (2005) analyze the purchasing strategies of a single customer facing dynamic prices; our approach additionally considers the interaction between customers, in the sense that they are competing for the same pool of inventory. While we assume that the initial inventory is publicly known, Gallego et al. (2004) and Liu and van Ryzin (2005) consider an alternative setting in which customers do not observe
capacity but learn about product availability through repeated interactions with the firm; Yin and Tang (2006) also analyze a model with unobservable inventory levels.

The aspect of strategic customer behavior analyzed in this paper (intertemporal demand) first appeared in the economics literature on durable goods monopoly. The general approach is based on rational expectations: customers anticipate future price changes and adjust their purchase timing in response. However, unlike the papers reviewed above, capacity constraints and time deadlines are not considered because the monopolist may sell as many units as desired over an infinite horizon. This literature has been inspired by the classic work of Nobel laureate Ronald Coase (1972); his main insight was that if customers strategically wait for price reductions, even a monopolist would be forced to price at marginal cost and earn zero profits. The earliest attempts to rigorously prove this result are by Stokey (1979, 1981) and Bulow (1982). Conlisk et al. (1984) introduce customer dynamics and show that the optimal price path involves periodic sales, with customers being willing to pay less as the next sale approaches. Gul et al. (1986), Ausubel and Deneckere (1989), and Sobel (1991) characterize a family of subgame perfect equilibria for the monopoly pricing game. Besanko and Winston (1990) present a dynamic programming procedure to compute the optimal price path. There are many variations of this basic setup; see Guth and Ritzberger (1998) for a survey. Unlike all these papers, we consider a fixed inventory and a finite time horizon. We show that these constraints influence customer expectations and thus have an important impact on optimal pricing strategies.

There are several other related papers that examine the relationship between capacity constraints (limited inventory) and pricing schemes. Harris and Raviv (1981) use a priority pricing mechanism to ration limited capacity. Wilson (1988) shows that when a monopolist sells a fixed quantity, it is optimal to post two prices and ration demand at the lower price. Lazear (1986) and Pashigian (1988) consider stochastic customer valuations and show how to mark-downs to extract high revenues during high-valuation realizations of demand. Desiraju and Shugan (1999) use a two-period model to investigate the profitability of various yield management practices, such as early discounting, overbooking, and rationing. Dana (1998, 1999a) analyzes pricing and rationing decisions when customers face uncertainty over their own demands, and Dana (1999b, 2001) also allows for aggregate demand uncertainty. The common theme across all these papers is that by rationing demand at lower prices, firms can stimulate demand at higher prices, although precise implementation details may vary across different model setups. In our current study, we find that it may sometimes be profitable to generate scarcity, but we also identify situations when this is not necessary. Furthermore, all the papers above adopt a static mechanism design approach: the seller first establishes some mechanism, and then customers make a static buying decision. We hope to add to this research by studying the dynamics of pricing and rationing (on the seller side) as well as the dynamics of purchase and consumption (on the customer side).

3. Model
There is a monopolist seller who operates over a finite time horizon, the length of which is normalized to one (time unit). At the start of the time horizon, the seller is endowed with an inventory of $Q$ units. By the end of the selling season, leftover units have zero value. This model is applicable to different industries, including travel (airplane seats and hotel rooms), retailing (fashion apparel and seasonal goods), and entertainment (concert and football tickets).

Customers are infinitesimally small and arrive continuously according to a deterministic flow of constant rate. This demand pattern is the same as that in the classical economic order quantity (EOQ) model. We normalize the customer arrival rate to one (customer unit per time unit). Each customer demands a single unit of the seller’s product. In other words, a mass of $t$ customer units would have arrived by time $t$, and the aggregate demand of these customers is $t$ units of the seller’s product.

The customer population is heterogeneous along two dimensions. The first dimension is valuation. A fraction $\alpha$ of the customers value the product at $V_H$ and the remaining $\bar{\alpha} \equiv 1 - \alpha$ value the product at $V_L$. We denote the difference using $\Delta \equiv V_H - V_L > 0$. We shall refer to these customers as “high-types” and “low-types,” respectively. We assume that $V_L \geq \alpha V_H$; otherwise, it is optimal to sell only to high-types (because selling to low-types not only depletes inventory at a faster rate, but also earns lower revenue per unit time). The second dimension of heterogeneity is patience. Customers who delay purchases incur different waiting costs, which may either be $b_P$ or $b_I$ per unit time, with $b_I > b_P \geq 0$. For concreteness, we may interpret waiting costs as monitoring costs: after arriving to the market (e.g., recognizing a need for the product), it is costly to continually monitor prices prior to buying. We shall adopt the terminology “patient” and “impatient” to distinguish these two cases. A fraction $\phi_H$ of the high-types are patient and the remaining $\bar{\phi}_H \equiv 1 - \phi_H$ are impatient; similarly, a fraction $\phi_L$ of the low-types are patient and the remaining $\bar{\phi}_L \equiv 1 - \phi_L$ are impatient. We shall refer to these four customer types...
as patient-high-types, impatient-high-types, patient-low-types, and impatient-low-types, denoted by \( \theta \in \{PH, IH, PL, IL\} \). It is convenient to denote the proportion of each customer type using \( f_{PH} = \alpha \phi_{PH} \), \( f_{IH} = \alpha \phi_{IH} \), \( f_{PL} = \bar{\alpha} \phi_{PL} \), and \( f_{IL} = \bar{\alpha} \phi_{IL} \) and to use \( V_{h} \) and \( b_{h} \) to denote the valuation and waiting cost of type-\( \theta \) customers. We assume that customer types are unobservable to the seller, and that the customer composition is stationary over time.

The seller has to decide on pricing and rationing policies \( [p(t), r(t)] \) and control policies \( [S(t), D(t)] \). These choices, collectively referred to as the selling policy, are announced at the start of the time horizon. The price schedule \( p(t) \) specifies the price charged at each time \( t \in [0, 1] \). The rationing policy \( r(t) \) specifies the fraction of current market demand that is fulfilled; we assume proportional rationing (see Tirole 1988). The control policies represent cumulative sales processes \( S(t) = \{S_{\theta}(t): \theta \in \theta \} \) and cumulative departure processes \( D(t) = \{D_{\theta}(t): \theta \in \theta \} \) planned by the seller. That is, according to the seller’s plans, by the end of time \( t \), \( S_{\theta}(t) \) units would have been sold to type-\( \theta \) customers, and \( D_{\theta}(t) \) type-\( \theta \) customers would have departed from the market (without buying). These control processes also jointly determine cumulative market demand

\[
Z_{\theta}(t) = A_{\theta}(t) - S_{\theta}(t) - D_{\theta}(t),
\]

where \( A_{\theta}(t) = f_{\theta} t \) denotes cumulative arrival processes. Here, the market demand \( Z_{\theta}(t) \) comprises of accumulated customers who have arrived and are waiting for a future purchase. When demand is rationed (i.e., \( r(t) < 1 \)), all the \( \sum_{\theta} Z_{\theta}(t) \) customers in the market have an equal \( r(t) \) chance of getting the product. Customers who are rationed remain in the market if and only if their continuation utility is positive. Finally, we make the technical requirement that the seller’s control policies \( [S(t), D(t)] \) are right continuous with left limits (RCLL) and have a finite number of discontinuities. These regularity conditions ensure that the controls are implementable, in the sense that each individual customer’s purchase or departure times (according to the controls) are well defined.

During the time horizon, based on the announced pricing and rationing policies \( [p(t), r(t)] \), customers decide whether or not to purchase the product and whether or not to leave the market. Staying in the market incurs waiting costs at a constant rate, buying the product reaps an instantaneous surplus (valuation minus price paid) at the time of purchase, and leaving the market yields zero continuation payoff. For a type-\( \theta \) customer with valuation \( V_{h} \) and waiting cost \( b_{h} \), his total utility may be zero (if he leaves immediately upon arrival), or \( V_{h} - p - b_{h}l \) (if he buys at price \( p \) after a time delay of length \( l \)), or \(-b_{h}l \) (if he waits around for time \( l \) but ends up not buying). Therefore, each customer faces a continuous time optimal stopping problem with two exit options. We assume that departed customers do not re-enter the market. Let \( I_{\theta}(t) \) denote the optimal continuation value to a type-\( \theta \) customer from staying in the market at the end of time \( t \). Then, the customer is willing to buy at time \( t \) if \( I_{\theta}(t) \leq V_{h} - p(t); \) if he does not buy, he is willing to leave if \( I_{\theta}(t) \leq 0 \).

There are two additional conditions that the selling policy must satisfy. First, the control policy \( [S(t), D(t)] \) must be incentive compatible because our customers are free to make their own utility maximizing choices. Specifically, cumulative sales \( S_{\theta}(t) \) may increase if and only if customers are willing to purchase and cumulative departures \( D_{\theta}(t) \) may increase if and only if customers are willing to leave. Second, the pricing and rationing policies \( [p(t), r(t)] \) must be credible. In other words, there must not exist any time \( t \) when it is to the seller’s advantage to deviate to a different price or rationing policy. Only then are customers willing to base their purchase decisions on these announcements.

Our goal is to determine the revenue-maximizing selling policy \( [p^{*}(t), r^{*}(t), S^{*}(t), D^{*}(t)] \), subject to incentive compatibility and credibility. At first glance, this is a control problem in continuous time. However, most existing methods do not apply directly because verifying incentive compatibility involves solving additional control problems (one for each customer type). Furthermore, verifying credibility requires us to analyze the continuation version of this problem at every point in time. Therefore, instead of tackling this problem directly, we shall first proceed to simplify it by exploiting some of its structural properties. This is done in the next section. Readers who prefer to first see the results and insights may proceed directly to §5, before returning to these technical and methodological details later.

4. Structural Properties and Upper Bounds

In this section, we show that the seller’s continuous-time control problem can be reduced to a single-variable nonlinear program, which we solve explicitly. We proceed in four major steps. In §4.1, we identify some key features that parameterize feasible solutions. In §4.2, for each set of policy parameters, we establish an upper bound on revenues. In §4.3, we construct candidate policies attaining the upper bounds above. Finally, in §4.4, we optimize over policy parameters to find the largest upper bound, and conclude that the corresponding candidate policy must be optimal.
4.1. Policy Parameters

We begin by proving some structural properties that an optimal solution must satisfy. Based on these properties, we show that all feasible policies \( p(t), r(t), S(t), D(t) \) are parameterized by several real-valued quantities. These parameters will be the focus of our analysis.

**Lemma 1.** For any feasible policy, we must have \( p(t) \geq V_L \).

Intuitively, prices strictly below \( V_L \) are not credible because when the time comes to honor these prices, the seller will increase prices to \( V_L \) because all customers are still willing to buy at this price. At the time of purchase, waiting costs are sunk and customers will contend with any price that leaves them with nonnegative surplus.

**Proposition 1.** For any feasible policy, there exist parameters \( \tau_0, \bar{\tau} \equiv \{ \tau_1, \ldots, \tau_N \}, \bar{\pi} \equiv \{ \pi_1, \ldots, \pi_N \} \), with \( \sum_{i=0}^{N} \tau_i = 1 \), such that at least the same revenues are earned under some policy satisfying

(i) \( p(t) = V_L \) and \( r(t) = 1 \) for every \( t \in [0, \tau_0] \).

(ii) \( p(t_k) = V_L \) and \( r(t_k) = \pi_k \) for \( t_k = \sum_{i=0}^{k} \tau_i \), for \( k = 1, \ldots, N \).

This result illustrates that every feasible policy is associated with a set of parameters, namely, \( \tau_0, \bar{\tau} \equiv \{ \tau_1, \ldots, \tau_N \}, \bar{\pi} \equiv \{ \pi_1, \ldots, \pi_N \} \). We may interpret \( \tau_0 \) as the length of an initial time interval with price \( V_L \). After this, “sale” prices of \( V_L \) occur only at discrete time points: each \( \tau_k \) is the length of time that transpires before the next “sale” occurs, during which a fraction \( \pi_k \) of demand is fulfilled. We shall refer to the time interval with length \( \tau_k \) as the \( k \)th time segment, for \( k = 0, 1, \ldots, N \).

4.2. Upper Bounds

Next, for each set of policy parameters \( \tau_0, \bar{\tau} \equiv \{ \tau_1, \ldots, \tau_N \}, \bar{\pi} \equiv \{ \pi_1, \ldots, \pi_N \} \), we proceed to find an upper bound on revenues. In other words, any policy associated with the corresponding parameters cannot earn more revenues than this upper bound. To obtain these bounds, we compute customers’ willingness to pay (WTP) at each time, and aggregate these WTPs over all individuals. We first focus on the case \( b_I > b_P = 0 \); that is, patient customers face zero waiting cost. The other case with \( b_I > b_P > 0 \) will be examined later.

We begin by considering the stopping problem faced by low-valuation customers. First, if waiting costs are strictly positive, the customer will either buy immediately upon arrival (if price is \( V_L \)) or leave the market forever (if price exceeds \( V_L \)). These customers never wait because they know that they can at best enjoy zero surplus because the lowest possible price is \( V_L \). On the other hand, low-type customers with zero waiting cost will wait for a price of \( V_L \) before purchasing. Therefore, in the current case with \( b_I > b_P = 0 \), patient-low-types strategically wait for the sale, but impatient-low-types behave myopically and make a one-time buy-or-exit decision at the time of arrival.

Next, we consider the decision problem faced by high-valuation customers, given policy parameters \( \tau_0, \bar{\tau} \equiv \{ \tau_1, \ldots, \tau_N \}, \bar{\pi} \equiv \{ \pi_1, \ldots, \pi_N \} \). First, we consider patient-high-types. Because waiting costs are \( b_P = 0 \), the WTP of patient-high-types arriving during the \( k \)th time segment, denoted \( W_{PH}^k \), can be characterized using backward induction. Specifically, we have

\[
W_{PH}^k = V_H - \pi_k \Delta, \tag{2}
\]

\[
W_{PH}^k = V_H - \pi_k \Delta - (1 - \pi_k)(V_H - W_{PH}^{k+1}), \tag{3}
\]

These expressions imply the following upper bound on the WTP of patient-high-types:

\[
W_{PH}^k \leq V_H - \pi_k \Delta. \tag{4}
\]

By a similar logic, we can characterize the WTP of impatient-high-types. Because waiting costs \( b_I > 0 \) are now relevant, the WTP depends on the precise time of arrival within each time segment. Let \( W_{IH}^k(s) \) denote the WTP of impatient-high-types arriving in the \( k \)th time segment, at time \( s \leq \bar{\tau}_k \), before the end of the time segment. We have the following upper bound on WTP:

\[
W_{IH}^k(s) \leq \min\{V_H - \pi_k \Delta + b_I s, V_H\}. \tag{5}
\]

This is because impatient-high-types have an option to wait (at a cost of \( b_I s \)) until the end of the time segment. If we assume that impatient-high-types are required to leave the market by the end of the current time segment, their WTP then would be \( V_H - \pi_k \Delta \), so (5) would hold with equality. However, because exit is not mandatory, we only have an upper bound above. It is convenient to rewrite (5) as

\[
W_{IH}^k(s) \leq V_H - [\pi_k \Delta - b_I s]^+. \tag{6}
\]

Aggregating these WTP expressions yields an upper bound on seller revenues. In Lemmas 2 and 3, we characterize this upper bound and derive a useful condition that it satisfies.

**Lemma 2.** Let \( b_I > b_P = 0 \). Consider any policy with parameters \( \tau_0, \bar{\tau} \equiv \{ \tau_1, \ldots, \tau_N \}, \bar{\pi} \equiv \{ \pi_1, \ldots, \pi_N \} \). The revenue collected under this policy cannot exceed the upper bound \( UB(\tau_0, \bar{\tau}, \bar{\pi}) \) given by

\[
\tau_0 V_L + \sum_{k=0}^{N} \left( f_{PL} \tau_k \pi_k V_L + f_{PH} \tau_k (V_H - \pi_k \Delta) \right) + f_{IH} \int_{0}^{\bar{\tau}_0} V_H - [\pi_k \Delta - b_I s]^+ ds. \tag{7}
\]
Lemma 3. Define $\bar{\tau} = \sum_{k=1}^{N_1} \tau_k$ and $\bar{\pi} = \sum_{k=1}^{N_1} \pi_k / \sum_{k=1}^{N_1} \tau_k$. Then, we have $UB(\tau_0, \bar{\tau}, \bar{\pi}) \leq UB(\tau_0, \bar{\tau}, \bar{\pi})$.

The preceding lemmas yield an important implication. Given any feasible policy with parameters $\tau_0, \bar{\tau}, \bar{\pi}$, we know that revenues do not exceed $UB(\tau_0, \bar{\tau}, \bar{\pi})$, which in turn is less than $UB(\tau_0, \bar{\tau}, \bar{\pi})$ with $\bar{\tau}, \bar{\pi}$ defined in Lemma 3. Therefore, we can start with using $UB(\tau_0, \bar{\tau}, \bar{\pi})$ as an upper bound on policy revenues. This allows us to make the following conclusion.

Proposition 2. Let $b_1 > b_p = 0$. Then, under any feasible policy, there exist parameters $\tau_0, \tau_1, \pi_1$ such that the revenues collected do not exceed the upper bound $UB(\tau_0, \tau_1, \pi_1)$.

4.3. Candidate Policies

Next, for each set of parameters $\tau_0, \tau_1, \pi_1$, we proceed to construct a feasible policy attaining the upper bound given above. As we vary the parameters, this class of policies serves as possible candidates for the optimal policy. We will specify the price schedule $p(t)$, rationing function $r(t)$, as well as the controls for cumulative sales and departures $[S(t), D(t)]$. The policy described in the following proposition attains revenues that are arbitrarily close to the upper bound.

Proposition 3. Let $\tau_0, \tau_1, \pi_1 \in [0, 1]$ with $\tau_0 + \tau_1 = 1$. For some small $\epsilon > 0$, under the policy

$$p(t) = \begin{cases} V_L, & t \in [0, \tau_0], \\ V_H - [\pi_1 \Delta - b_1(1 - \epsilon - t)]^+, & t \in (\tau_0, 1 - \epsilon), \\ V_H - \pi_1 \Delta, & t \in (1 - \epsilon, 1), \\ V_L, & t = 1, \end{cases}$$

$$r(t) = \begin{cases} 1, & t \in [0, 1), \\ \pi_1, & t = 1, \end{cases}$$

we have

$$S_\theta(t) = \begin{cases} f_\theta t, & \text{if } \theta = IH, \text{ or } (\theta = PH \text{ and } t \geq 1 - \epsilon), \\ f_\theta \min(t, \tau_0), & \text{if } \theta = IL, \text{ or } (\theta = PH \text{ and } t < 1 - \epsilon), \\ f_\theta (\tau_0 + \pi_1 \tau_1), & \text{if } (\theta = PL \text{ and } t = 1), \\ 0, & \text{if } \theta \in [IH, PH], \text{ or } (\theta = PL \text{ and } t < 1), \text{ or } (\theta = IL \text{ and } t \leq \tau_0), \\ f_\theta (1 - \pi_1), & \text{if } (\theta = PL \text{ and } t = 1), \\ f_\theta (\tau_0), & \text{if } (\theta = IL \text{ and } t > \tau_0), \end{cases}$$

$$D_\theta(t) = \begin{cases} f_\theta (\tau_0), & \text{if } (\theta = PL \text{ and } t = 1), \\ f_\theta (\tau_0), & \text{if } (\theta = IL \text{ and } t > \tau_0), \end{cases}$$

revenues collected are at least $UB(\tau_0, \tau_1, \pi_1) - \epsilon \Delta$.

The essence of this result is depicted in Figures 1 and 2, which plot the prices, rationing function, and controls over time. For convenience, we shall refer to this as the $(\tau_0, \tau_1, \pi_1)$-policy.

Next, observe that the seller must exhaust all available inventory $Q$ by the end of the horizon. This is because any policy with leftover inventory violates the credibility condition because the seller would always prefer to sell them to waiting customers at $t = 1$ rather than to discard them. Customers anticipate this move and these expectations would affect their WTP. Thus, the outcome would be as if the seller has planned on selling out right from the start. This implies that for the policy of Proposition 3 to be feasible (in particular, credible), we must have

$$\tau_0 + \tau_1 (f_{PH} + f_{IH} + \pi_1 f_{PL}) = Q, \tag{12}$$

where the left-hand side is the number of units sold. Together with the condition

$$\tau_0 + \tau_1 = 1, \tag{13}$$

we are left with only one degree of freedom among the parameters $\tau_0, \tau_1, \pi_1$. That is, specifying the value of one parameter pins down the value of the other two. In particular, we shall primarily use $\pi_1$ to uniquely characterize any $(\tau_0, \tau_1, \pi_1)$-policy. In this way, we have a continuum of candidate policies parameterized by $\pi_1$.

It is worthwhile to point out the two extreme cases in our continuum of candidate policies. These are shown in Figure 3. On one extreme, $\pi_1$ is minimized at $\pi_1 = 0$, with $\tau_0 = (Q - \alpha)/(1 - \alpha)$ and $\tau_1 = (1 - Q)/(1 - \alpha)$ (recall that $f_{PH} + f_{IH} = \alpha$). This corresponds to a pure markup policy because prices start at $V_L$ and then increase to $V_H$ at time $\tau_0$. At the other extreme, $\pi_1$ is maximized at $\pi_1 = \overline{\pi}_1 \equiv (Q - \alpha)/(\overline{\alpha} \phi_L)$ (recall that $f_{PL} = \overline{\alpha} \phi_L$), and the other parameter values are $\tau_0 = 0$, $\tau_1 = 1$. This describes a pure markdown policy because the zeroth time segment has length $\tau_0 = 0$ and prices decrease over all of the next time segment of length $\tau_1 = 1$ and culminate at $V_L$. At all intermediate policies with $\pi_1 \in (0, \overline{\pi}_1)$, the price schedule is not
In this subsection, we maximize the upper bound referred to as price sale, patient-low-types wait to buy at the final sale during the season, impatient-high-types buy immediately, patient-high-types buy right before the final sale, and (13). From Propositions 1 and 2, we know that this maximum value must be an upper bound on optimal prices. Based on the characterization given in Proposition 3, we can then construct an e-optimal (τ₀, τ₁, π₁) policy whose revenue is within εΔ of the optimal revenue. Because ε can be made arbitrarily small, we shall henceforth contend with e-optimality and refer to it as "optimality" for brevity.

We emphasize that the above limitation to e-optimality is a consequence of our restriction to posted price mechanisms. Alternatively, we can achieve optimality (i.e., ε = 0) with a priority pricing mechanism at the final time t = 1. This mechanism sells all remaining units at two prices; demand at the higher price receives priority and is guaranteed availability, whereas demand at the lower price may be rationed. The price difference screens the patient-high-types from the patient-low-types, and can be interpreted as a call option.

Before presenting the next result, let us define the constants

\[ A = \alpha(1 - Q)/1 - \alpha \cdot (1 - \phi_H/\phi_L) \cdot \Delta, \]
\[ B = -\alpha \cdot \phi_H \cdot (\Delta^2/b_1), \]
and
\[ K = - (A \phi_H / B) = (1 - Q/1 - \alpha) \cdot (\phi_L - \phi_H / (1 - \phi_H)) \cdot (b_1 / \Delta). \]

Also, let us define the function \( G(u) \equiv u(1 - \phi_1 u)^2 \) and note that its maximum value over \( u \in [0, 1] \) is 4/(27\( \phi_L \)). We are now ready to state our result.

**PROPOSITION 4.** Let \( b_I > b_P = 0 \). Then, the optimal policy is a \((\tau_0, \tau_1, \pi_1)\)-policy, with \( \pi_1 \) given below.

(i) When \( K \leq 0 \), we have \( \pi_1 = 0 \).

(ii) When \( K \in (0, 4/(27\phi_L)) \), we have either \( \pi_1 = \pi_1 \) or \( \pi_1 = \xi \in (0, \pi_1) \), where \( \xi \) is the smallest solution of \( G(\xi) = K \). In particular, if \( K \leq G(\pi_1) \leq 4/(27\phi_L) \), we have \( \pi_1 = \xi \). (iii) When \( K \geq 4/(27\phi_L) \), we have \( \pi_1 = \pi_1 \).

**Figure 2** Cumulative Sales and Departures for Each Customer Type Under the \((\tau_0, \tau_1, \pi_1)\)-Policy

**Figure 3** Pure Markup and Pure Markdown Policies
Finally, we return to the case with \( b_l > b_p > 0 \). In this case, recall that all low-types, having a positive waiting cost, behave myopically, so that sales at the end of the length \( \tau_l \) time segment benefit only high-types. Because all these high-types are willing to pay \( V_{H_t} \), it is optimal to set prices at \( V_{H_t} \) after time \( t = \tau_0 \), so \( \pi_t = 0 \), which uniquely determines the optimal policy. We thus have the following result.

**Proposition 5.** Let \( b_l > b_p > 0 \). Then, the optimal policy is a \((\tau_0, \tau_l, \pi_l)\)-policy, with \( \pi_l = 0 \).

In this section, we have simplified a dynamic principal-agent control problem in continuous time into a nonlinear program in a single variable. The latter problem is then solved to characterize the seller's optimal policy. In the next section, we proceed to identify the main drivers that determine the structure of the optimal policy.

### 5. Optimal Policy

Should the seller still hold sales when customers strategically wait for them? Under the revenue-maximizing price discrimination strategy, should the seller have low-priced transactions at the start or at the end of the selling horizon? Or both? In this section, we show that the answers to these questions depend critically on customer valuations and waiting costs, as well as on the interaction between these two dimensions of heterogeneity.

**Theorem 1.**

(a) The optimal policy is a pure markup policy if at least one of the following conditions hold:

(i) \( \phi_l \leq \phi_{H_t} \),
(ii) \( b_p > 0 \).

(b) The optimal policy is a pure markdown policy if all of the following conditions hold:

(i) \( \phi_l \geq \phi_{H_t} \),
(ii) \( b_p = 0 \) and \( b_l \) is sufficiently large (that is, \( b_l \geq 4(1-\alpha)/(27(1-Q)) \cdot (1-\phi_{H_t})/(\phi_l(\phi_l - \phi_{H_t})) \cdot \Delta \)).

(c) The optimal policy is an interior policy if all of the following conditions hold:

(i) \( \phi_l \geq \phi_{H_t} \),
(ii) \( b_p = 0 \) and \( b_l \) is sufficiently small (that is, \( b_l \leq (Q-\alpha)(1-Q)/(1-\alpha)^2 \cdot (1-\phi_{H_t})/(\phi_l(\phi_l - \phi_{H_t})) \cdot \Delta \)).

This theorem follows from Propositions 4 and 5. It indicates that strategic waiting on the part of customers is critical in determining the structure of optimal price schedules. Within our framework, we show that a full spectrum of pricing policies may be optimal, ranging from monotone increasing to monotone decreasing price paths. In case (i), pure markup policies are optimal, and the seller should concentrate all low-priced transactions at the start of the horizon, whereas in case (ii), the seller should use pure markdown policies and defer all sales to the end of the horizon. The intuition is as follows. Markups defend the seller against strategic delays because waiting does not pay off. This is important when the high-type population is patient and more likely to wait, compared to the low-types (i.e., when \( \phi_{H_t} \geq \phi_l \)). In this case, intertemporal price discrimination via skimming does not pay off. Another situation in which markups are appropriate is when waiting costs are high in general (i.e., high \( b_p \) and \( b_l \)). In this case, it is optimal to use markups to discourage waiting because waiting is inefficient and induces a deadweight loss. On the other hand, the success of price markdowns is contingent upon whether low-valuation customers will wait around to purchase at sales, after the (relatively more impatient) high-types have purchased at a higher price. This requires low waiting costs for patient-low-types and high waiting costs for impatient-high-types (i.e., low \( b_p \) and high \( b_l \)), as well as a relatively large segment of impatient customers among high-types (i.e., \( \phi_l \geq \phi_{H_t} \)). For other parameter values, neither of the opposing factors above dominate. Then, it may be optimal to use an interior policy. The promotional selling phase appeals to early customers who object to either paying high prices or waiting too long for a sale, and the regular selling season is short enough so that it is not too costly for self-selecting customers to wait for the sale, thereby achieving price discrimination.

**Corollary 1.** Consider the limiting case with \( b_p = 0 \) and \( b_l = \infty \). Then, the optimal policy is a pure markup policy if \( \phi_{H_t} \geq \phi_l \) and a pure markdown policy when \( \phi_l \geq \phi_{H_t} \).

In this limiting case, Corollary 1 tells us that pure markups are optimal when high-types are relatively more patient, pure markdowns are optimal when high-types are relatively less patient, and interior policies are no longer needed. The same intuition above still applies. Because this limiting case allows us to derive the same insights, while being analytically more convenient, we shall focus on this case henceforth. Furthermore, because impatient customers with \( b_l = \infty \) never wait, we may describe them as being myopic and the patient customers as being strategic. The results presented thus far apply to the case where the seller's initial inventory \( Q \) lies between \( \alpha \) and \( \alpha + \phi_l \alpha \). Equivalently, there is excess inventory after satisfying all high-type demand (of mass \( \alpha \)), but this excess is not sufficient to supply to all patient-low-types (of mass \( \phi_l \alpha \)). The other cases are not presented here because they do not yield any new insights. In the case where \( Q < \alpha \), the seller simply sets a high price of \( V_{H_t} \) throughout, and in the case where \( Q > \alpha + \phi_l \alpha \), the seller initially sets promotional prices of \( V_{H_t} \) so that inventory can be depleted.
models either explicitly impose structural restrictions as well as other nonmonotone price paths. Most existing solution. This includes markups and markdowns, as well as other nonmonotone price paths. Most existing models either explicitly impose structural restrictions (such as requiring monotonicity, or specifying the number of price changes), or do so implicitly. For instance, models based on stochastic customer arrivals tend to yield decreasing prices because the option value of unsold units decreases toward the end of the horizon, while models based on uncertain customer valuations tend to lead to increasing prices because customers who buy early need to be compensated for bearing additional risk. In contrast, our model does not impose any restrictions a priori.

Within the revenue management literature, most models generate price paths that, on average, only decrease over time. In a recent review paper on the airline industry, McAfee and te Velde (2005, p. 23) write, “a remarkably robust prediction of theories... (is that) prices are falling as takeoff approaches.” This can be understood by considering the option value of unsold units. As the end of the horizon approaches, it becomes less likely that a given unit would be sold, so its option value declines. In fact, Gallego and van Ryzin (1994) use their stochastic model with Poisson arrivals to establish the following structural properties: (i) for a fixed inventory level, the price should decrease over time, and (ii) at a fixed time, the price should increase as the remaining inventory decreases. This implies that along any sample path, the optimal price decreases continually, and jumps up whenever units are sold. McAfee and te Velde (2005) scrutinize the Gallego and van Ryzin (1994) model and conclude that the two structural properties above, when combined, lead to a decrease in expected prices over time. Therefore, these stochastic models and their corresponding option-value interpretations do not explain the use of markups (commonly observed in airlines); a notable exception is Zhao and Zheng (2000), whose model with time-inhomogeneous demand processes may lead to increasing price paths. Our game-theoretic model, in contrast, rationalizes both markups and markdowns by explicitly accounting for intertemporal customer utility. This suggests that in some practical pricing contexts, strategic considerations are no less significant than stochastic influences.

Nevertheless, the option value interpretations from stochastic models can be combined with the insights from our game-theoretic model. In the stochastic analogue of our model (e.g., customers arrive according to Poisson processes), we conjecture that our main findings will remain unchanged. The work of Gallego and van Ryzin (1994, 1997) lends support for this conjecture by showing that deterministic models provide good approximations because statistical fluctuations in demand average themselves out. In particular, they observed that compensating for demand uncertainty by adjusting prices dynamically achieves a minimal effect: their deterministic heuristic performs
almost as well as the optimal dynamic pricing policy. In the same way, one can view the current work as a deterministic solution to the general yield management problem under strategic customer behavior. In general, although optimal prices should decrease along each sample path as the deadline approaches (because the option value decreases), these decreases only have a second-order effect. In contrast, first-order price changes are triggered by the "regime switches" captured in our deterministic model, in which the seller changes his target group of buyers. Specifically, markups reflect the seller’s intention to restrict sales to a smaller group of high-valuation buyers, while markdowns reflect the opposite intention to include a larger set of potential buyers with lower valuations. In a stochastic setting, whether these first-order price changes should involve markups or markdowns depends on the composition of the customer pool in a similar fashion as before. However, all these statements are conjectures that remain to be verified by future research.

6. Seller Revenue, Consumer Surplus, and Social Welfare

We are interested in the seller’s optimal revenue \( R \), as well as the consumer surplus \( U \), of type-\( \theta \) customers (averaged over all customers of each type). The following proposition which is, proved in the online appendix, expresses these quantities of interest in terms of basic model parameters.

**Proposition 6.** Let \( b_\alpha = 0 \) and \( b_\gamma = \infty \). Under the optimal selling policy, the seller’s revenue and customers’ surplus are characterized below.

(i) When \( \phi_H \geq \phi_L \) (i.e., markups are used),

\[
R = \alpha V_H + \frac{Q - \alpha}{1 - \alpha} (V_L - \alpha V_H), \tag{14}
\]

\[
U_{PH} = U_{IL} = \frac{Q - \alpha}{1 - \alpha} \Delta, \tag{15}
\]

\[
U_{PL} = U_{HL} = 0. \tag{16}
\]

(ii) When \( \phi_H \leq \phi_L \) (i.e., markdowns are used),

\[
R = \alpha V_H + \frac{Q - \alpha}{\phi_L \Delta} (\phi_L \Delta V_L - \phi_H \alpha \Delta), \tag{17}
\]

\[
U_{PH} = \frac{Q - \alpha}{\phi_H \Delta} \Delta, \tag{18}
\]

\[
U_{HL} = U_{PL} = U_{IL} = 0. \tag{19}
\]

These expressions have intuitive interpretations. When the high-type population is relatively more patient (\( \phi_H \geq \phi_L \)), Theorem 1 tells us that the seller uses a markup. With a markup, the seller’s revenue in Proposition 6(i) is the sum of the base revenue \( \alpha V_H \) (obtained by charging \( V_H \) throughout), and the incremental revenue from charging the low introductory price of \( V_L \) for the first \((Q - \alpha)/(1 - \alpha)\) time units. For high-types, the consumer surplus is \( \Delta \) during the introductory period and zero at other times, so this yields \(((Q - \alpha)/(1 - \alpha))\Delta\) on average. For low-types, it is clear that the consumer surplus is always zero. In the other case where the high-type population is relatively less patient (\( \phi_H \leq \phi_L \)), Theorem 1 tells us that the seller uses a markdown. With a markdown, the seller’s revenue in Proposition 6(ii) is the base revenue of \( \alpha V_H \) (obtained if all high-types pay \( V_H \)), plus the incremental revenue from a fraction \(((Q - \alpha)/(\alpha \phi_H))\) of the patient-low-types who buy at the end of the horizon (the first term in parentheses), minus the revenue from patient-high-types that must be sacrificed to ensure that these customers will not choose the “deal” intended for the low-types (the second term in parentheses). This foregone revenue results in consumer surplus of \(((Q - \alpha)/(\alpha \phi_H))\Delta\) for all the patient-high-types, while all other customers earn zero surplus.

The expressions for seller revenue and high-type consumer surplus are plotted against \( \phi_H \) and \( \phi_L \) in Figure 4. (We omit the low-types because they receive zero surplus.) First, we look at the graphs on the left-hand side. As the proportion of patient customers among the low-types increases (i.e., as \( \phi_L \) increases), both the seller’s revenue and consumers’ surplus initially remain unchanged as long as \( \phi_L \leq \phi_H \). As soon as \( \phi_L \) increases beyond \( \phi_H \), there is a transfer of surplus from the impatient-high-types to the patient-high-types. This occurs due to the regime switch from markup pricing (positive surplus available to all high-types) to markdown pricing (positive surplus is exclusive to patient-high-types). As \( \phi_L \) continues to increase, the surplus for impatient-high-types stays at zero and the surplus for patient-high-types decreases, while the seller’s revenue increases. This transfer from patient-high-types to the seller occurs because a larger pool of patient-low-types (i.e., higher \( \phi_L \)) increases the competition for availability at the final markdown price and helps the seller to extract a larger part of the patient-high-type consumer surplus. Next, we look at the graphs on the right-hand side. As the proportion of patient customers among the high-types decreases (i.e., as \( \phi_H \) decreases), all quantities initially remain unchanged. As \( \phi_H \) falls below \( \phi_L \), the regime switch (from markups to markdowns) leads to a transfer from impatient-high-types to patient high-types, as discussed above. As \( \phi_H \) continues to decrease, consumer surplus remains unchanged but the seller’s revenue increases. The seller gains, even though each individual customer is unaffected, because there is an increased mass of impatient-high-types who pay the maximum price \( V_H \).
A useful observation is that each of the four customer segments in our model affects the seller in different ways. First, impatient-high-types benefit the seller because he is able to extract high revenues from these customers immediately when they arrive. Second, patient-high-types hurt the seller because strategic delays have an adverse effect on revenue. Third, patient-low-types benefit the seller because when they strategically delay purchase, they create competition with the other high-valuation customers for product availability at the end of the selling season; this discourages the high-types from waiting and increases their WTP. Finally, impatient-low-types are of minimal value to the seller because it is not possible for these customers to generate high profits. Therefore, as illustrated by the two upper graphs of Figure 4, the seller is better off when either $\phi_H$ increases or $\phi_L$ decreases.

Customers who arrive early during the selling season (early-birds) earn different levels of surplus from those who arrive later (late-comers). According to Theorem 1, when markups are used, early-birds (who arrive before time $t = (Q - \alpha)/(1 - \alpha)$) with high valuations enjoy a surplus of $\Delta$, but late-comers (who arrive after time $t = (Q - \alpha)/(1 - \alpha)$) do not earn any surplus. On the other hand, when markdowns are used, individual surplus does not depend on arrival time: all patient-high-types have an equal chance of earning positive surplus (regardless of arrival time) and all other customers earn zero surplus. This suggests that in an environment with $\phi_H \geq \phi_L$, the use of markups will encourage customers (or the high-valuation customers, at least) to arrive earlier. On the other hand, in the opposite environment ($\phi_H \leq \phi_L$), the use of markdowns implies that arriving early is of no value. Therefore, if customers were able to choose when to arrive to the market, their arrival times would be influenced by the composition of the customer pool (which determines whether markups or markdowns are used). These choices of arrival times generate nonstationary intensities and valuations in the arrival process, which in turn influence the seller’s optimal intertemporal pricing strategies. We leave the task of endogenizing customers’ choices of arrival times as a topic for future research. For a similar setting with endogenous arrivals to a queue, readers are referred to Lariviere and Van Mieghem (2004).

Finally, observe that the optimal selling policy is socially efficient. Whether the seller uses a markup or a markdown, the limited inventory is always used up and allocated to all the high-valuation customers and a portion of the low-valuation customers. The only difference is in terms of low-type allocation, i.e., which low-type customers receive the product? When markups are used, early-bird low-types receive the product, but when markdowns are used, patient-low-types receive the product. In either case, social welfare (i.e., the sum of the seller’s revenue and total...
consumer surplus) is maximized and equals \( \alpha V_{hi} + (Q - \alpha)V_L \). These implications can be tested empirically but we leave it as a topic for future research.

7. Initial Inventory Choice

So far, the seller’s inventory \( Q \) has been exogenously specified. Now, suppose that each unit can be procured at some cost, normalized to zero. (This normalization is valid after replacing the valuation parameters \( V_{hi} \) and \( V_L \) with \( V_{hi} - c, V_L - c \).) Given that inventory planning is now possible for the seller, how many units \( Q \) should he stock? By our previous analysis, we know that the seller must sell out his entire inventory (for credibility), and the optimal prices and optimal profits can be characterized for each initial inventory level \( Q \). Our present objective is to find the initial inventory \( Q \) that maximizes profits. The next proposition, which is proved in the online appendix, provides the answer.

**Proposition 7.** Let \( b_p = 0 \) and \( b_1 = \infty \). The seller’s optimal stocking quantity and selling policy are characterized below.

(i) When \( \alpha V_{hi} \geq V_i \) and \( \alpha \phi_{hi} V_{hi} \geq (\alpha \phi_{hi} + \bar{\alpha} \phi_{hi})V_L \), the seller should stock \( Q^* = \alpha \) units, charge the constant price \( p^*(t) = V_{hi} \), and satisfy all demand with \( r^*(t) = 1 \).

(ii) When \( \alpha V_{hi} \leq V_i \) and \( \alpha \phi_{hi} V_{hi} \leq (\alpha \phi_{hi} + \bar{\alpha} \phi_{hi})V_L \), the seller should stock \( Q^* = 1 \) unit, charge the constant price \( p^*(t) = V_L \), and satisfy all demand with \( r^*(t) = 1 \).

(iii) In all other cases, the seller should stock \( Q^* = \alpha + \bar{\alpha} \phi_{hi} \) units, use the markdown price schedule

\[
p^*(t) = \begin{cases} V_{hi}, & t < 1, \\ V_L, & t = 1, \end{cases}
\]

and satisfy all demand with \( r^*(t) = 1 \).

This proposition distinguishes between three cases. We refer to case (i) as the constant-high-price case because the seller stocks \( \alpha \) units, charges the high price \( V_{hi} \) throughout the horizon, and satisfies all high-type demand. Next, we refer to (ii) as the constant-low-price case because the seller stocks one unit, charges \( V_L \) throughout, and satisfies all demand at the low price. Finally, case (iii) involves a single markdown. The seller stocks \( \alpha + \bar{\alpha} \phi_{hi} \) units, sells to impatient-high-types (of mass \( \alpha \phi_{hi} \)) at the high-price \( V_{hi} \) throughout the horizon, and sells to all patient customers (of mass \( \alpha \phi_{hi} + \bar{\alpha} \phi_{hi} \)) at the marked-down price \( V_L \) at the end.

Figure 5 shows the regions (in parameter space) over which each of these three cases apply. On the \( x \)-axis, we have

\[
x = \alpha \phi_{hi} V_{hi} - (\alpha \phi_{hi} + \bar{\alpha} \phi_{hi})V_L,
\]

which represents the profit differential between selling to the patient-high-types at the high price \( V_{hi} \) and selling to all patient customers at the low price \( V_L \). Observe that \( x \) is increasing in \( \phi_{hi} \) but decreasing in \( \phi_i \). Therefore, we can interpret an increase in \( x \) as the high-type population becoming proportionately more patient (i.e., an increase in \( \phi_{hi} \)), and we can interpret a decrease in \( x \) as the low-type population becoming proportionately more patient (i.e., an increase in \( \phi_i \)). Next, on the \( y \)-axis, we have

\[
y = \alpha V_{hi} - V_L,
\]

which represents the profit differential between selling to all the high-types at the high price \( V_{hi} \) and selling to all customers at the low price \( V_L \). Therefore, we can interpret an increase in \( y \) as the high-type population becoming more valuable (i.e., an increase in \( \alpha V_{hi} \)), and we can interpret a decrease in \( y \) as the low-type population becoming more valuable (i.e., an increase in \( V_L \)). According to Proposition 7, cases (i) and (iii) may apply in the upper half-plane (i.e., \( y \geq 0 \)), and the other condition for (i) to hold here is \( \alpha \phi_{hi} V_{hi} \geq (\alpha \phi_{hi} + \bar{\alpha} \phi_{hi})V_L \) or \( x \geq 0 \). Similarly, in the lower half-plane (i.e., \( y \leq 0 \)), cases (ii) and (iii) may apply. Here, the condition for (ii) to hold is \( \alpha \phi_{hi} V_{hi} \leq (\alpha \phi_{hi} + \bar{\alpha} \phi_{hi})V_L \) or \( x \leq y \). This yields the three regions shown in the diagram.

In a similar model, Wilson (1988) shows that uniform prices are optimal if the seller can choose initial inventory levels. In his model, all the customers are strategic (i.e., \( \phi_{hi} = \phi_i = 1 \)), so his results apply to situations on the \( x = y \) line in Figure 5. In other words, our findings are consistent with earlier work. In fact, by considering a more general composition of the customer population, we show that apart from uniform pricing, markdowns may also be optimal when the initial inventory level is flexible.

Let us now draw some observations from the regions in Figure 5. First, when the low-type population is proportionately more patient than the high-type population, the seller should use markdown pricing. This agrees with our earlier findings when the initial inventory is fixed. On the other hand, when the...
high-type population is proportionately more patient, the seller should charge fixed prices: a constant high price should be used when there is a significant value differential between the high-type and low-type clientele, otherwise, a constant low price should be used. This result differs from our earlier case where the initial inventory is fixed; in that case, markup pricing is appropriate because it provides the optimal time mixture between the constant-low-price policy and the constant-high-price policy, subject to the inventory constraint that all available units must be sold. However, when the initial inventory is flexible, such time mixing is no longer needed because the seller is free to choose the constant price (and stock the associated quantity) that maximizes profits.

These results suggest that price increases, which are quite common in practice, are driven by factors that extend beyond the realm of our stylized model. For example, consider uncertainty and nonstationarity in demand patterns. In environments with aggregate demand uncertainty, initially low prices may help the firm learn about market potential. Furthermore, with time-inhomogeneous demand, increasing prices may even be necessary if higher-value customers arrive later. A more complete investigation of the joint pricing-inventory problem with strategic customers should incorporate these factors to better identify the forces favoring either markups or markdowns and understand their interplay with the initial inventory choice.

Our analysis has examined both short-term (when the initial inventory is fixed) and long-term (when the initial stock is flexible) solutions. Under some practical situations, the short-term solutions are also relevant in the long run. This may occur when there is uncertainty in demand during the inventory planning stage. The stock of $Q$ units must be ordered or produced in the absence of perfect demand information. Subsequently, after demand is realized, the inventory has already been ordered or produced, and the seller must choose prices according to the short-term solutions. Alternatively, there may also be scenarios where the initial inventory is used to serve different streams of demand (for example, in airlines, the inventory of seats on the same plane is used on flights with different demand characteristics). In this case, the stocking quantity $Q$ is not completely flexible even in the long run: there must be some situations when the seller faces a “fixed” inventory that has been optimized for some other stream of demand, and the short-term solutions must be used instead. These situations suggest that our short-term solutions in Theorem 1 may continue to persist in the long run.

8. Conclusion
The main contribution of this paper is to analyze the intertemporal pricing problem when the customer pool consists of four distinct groups: patient-high-types, impatient-high-types, patient-low-types, and impatient-low-types. This model sheds light on how the composition of the customer pool influences the optimal pricing regime, as well as the division of surplus between the seller and his customers. In most existing work, the customer pool is either entirely myopic (no one waits) or entirely strategic with homogeneous discount rates. In contrast, we find that heterogeneity in both valuations and waiting costs are crucial and they jointly determine the structure of optimal pricing policies.

This analysis provides a unifying framework that includes markups and markdowns, as well as other nonmonotone policies. We find that prices should decrease over time when the market is dominated by either impatient-high-types or patient-low-types. On the other hand, when the market is dominated by patient-high-types or impatient-low-types, prices should increase over time. Figure 6 summarizes these conclusions.

Another major finding is that strategic customer behavior need not always be detrimental to the seller. Although strategic waiting by high-types does indeed hurt revenues, we find that strategic waiting by low-types achieves the opposite effect: by generating competition for availability, the presence of strategic-low-types actually allows the seller to extract higher revenues from strategic-high-types. We stress that this effect is present only when there is a limited supply because it is driven by rationing and the threat of stock-outs. In contrast, in classical models of durable goods monopoly, there are no supply constraints and the Coase effect operates in full force against the interests of the seller. This suggests that depending on the demand composition, supply constraints may contribute to a “reverse Coase effect” such that strategic waiting enhances seller profits.
Beyond dynamic pricing, strategic customer behavior is also relevant in several other contexts. In ongoing work, Su and Zhang (2005) investigate the impact of strategic customer behavior on supply chain performance. The basic setup involves a newsvendor model, in which customers may either purchase at full price or wait for a markdown (which is applied to all leftover units after full-price demand has been satisfied). If customers rationally anticipate stock-outs and markdowns, how should ordering and pricing decisions be made in conjunction? What are the implications on decentralized decisions in a supply chain? Similarly, in a multiperiod setting, it is also important to understand the effect of intertemporal demand on multiperiod inventory replenishment in supply chains; see Ahn et al. (2005). Another area in which strategic customer behavior is also relevant is services. In such settings, the service provider sets prices dynamically, and customers strategically choose when to seek service, taking into account future prices as well as negative congestion externalities. A similar kind of customer behavior has been examined by Larivi`ere and Van Mieghem (2004) and Armony and Maglaras (2004a, b), but these papers do not consider pricing. In my opinion, each of these two areas present fruitful opportunities for future research.

9. Electronic Companion
An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

Acknowledgments
The author thanks Terry Hendershott, Teck Ho, Ganesh Iyer, Marty Larivi`ere, Tom Marschak, Priya Raghukir, Jose Silva, Garrett van Ryzin, Miguel Villas-Boas, and Candi Yano for several helpful discussions. He also thanks seminar participants at Stanford University, University of California at Berkeley, University of California at Irvine, and University of Southern California for their thoughtful comments. He is grateful to two anonymous referees, an associate editor, and Sunil Chopra, the department editor, for their constructive suggestions.

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