Ad Blocking

Aleksandr Gritckevich, Zsolt Katona and Miklos Sarvary[1]

January 12, 2018

[1]Zsolt Katona is Associate Professor of Marketing at the Haas School of Business at UC Berkeley. Aleksandr Gritckevich is a Ph.D. Candidate and Miklos Sarvary is the Carson Family Professor of Business at Columbia Business School.

Abstract

In recent years, “ad blocking” has become a significant threat to advertising supported publishers. Ad blockers’ business models generally fall in two categories: they can “sell” their software to consumers directly (e.g. through the app store or using a freemium model). More often, ad blockers negotiate with publishers, allowing some ads to go through in return for payment, a practice called “whitelisting”. We build an analytic model to find the conditions under which these models are likely to emerge and understand the incentives behind various contractual arrangements between publishers, ad blockers and consumers. Furthermore, we are interested in the effect of contracting on equilibrium quality levels chosen by publishers as well as their welfare implications. Our results also help a transaction platform (e.g. a social network or a mobile operating system) to ‘regulate’ ad blocking for the overall benefit of its diverse user base.

Keywords: advertising, game theory, media competition, media platforms.

1 Introduction

"Ad blocking" - the practice of installing a third-party software or app to block advertisements from loading on visited websites - has grown steadily since 2013 to reach well over 200 million Internet users worldwide (Garrahan and Kuchler, 2015). In some countries (e.g. Greece or Poland) over 30% of Internet users installed ad blocking software. The user base of such software amounts to 20% in Germany, Sweden, UK and represents over 43 million active users in the US alone. Most installations (over 98%) concern desktop computers but mobile devices are increasingly affected as well (Cookson, 2016): some ad blocking applications can block ads within smartphone apps. For example, in September 2016, Apple started allowing iPhone users to block ads in its Safari web browser and a number of ad blocking apps (e.g. Crystal or Purify) have become some of the most popular apps in its App Store. Elsewhere, mobile operators (e.g. Digicel) started blocking advertisements on their network forcing Internet companies (including Google, Yahoo! and Facebook) to pay for access to customers (Cookson, 2015).

Ad blocking is a major threat to those content providers (publishers) whose revenue model is based on advertising. In the US, where digital ad spend is estimated to surpass $100 billion in 2017, ad blocking-related costs to publishers may reach $20 billion for the year (Cookson, 2016). To respond to the threat to their revenues some publishers (e.g. Axel Springer, a large German publishing group, The Washington Post or the UK's ITV and Channel 4) have denied content to their readers who blocked ads encouraging them to stop doing so or to purchase a subscription (Vasagar, 2015). These strategies provided mixed results. In the case of Springer, rather than paying for a subscription, most users switched off their ad blocker. It is also clear however, that such strategies are less likely to work for lesser-known brands or content providers who do not have enough resources to manage a subscription system.

Another approach by publishers to mitigate the impact of ad blockers, which is often encouraged by the ad blockers themselves, is to engage in direct negotiations with ad block software providers. The idea is that the ad blocker allows for a limited amount of, presumably higher quality ads to pass the blockade for a fee from the publisher. This practice is known as "whitelisting" in the trade. In fact, more recently, some ad bockers (e.g. Eyeo, the owner of the most popular ad block software, Ad Block Plus or ABP) have gone even further by considering the introduction of their independent advertising exchanges that place whitelisted ads on pages controlled by ad blocking software. The ad blocker keeps a percent of the transaction between the publisher and the advertiser (Kastrenakes, 2016). A similar move was recently introduced by the Israeli ad blocker, Shine (Fildes, 2017).

The goal of this paper is to assess the impact of the ad blocking phenomenon on advertising supported digital publishing. In a first stage, we build a model to understand what arrangement emerges between ad blockers, publishers and consumers that is sustainable in the long run. Clearly, while ad blockers want to maximize the surplus extracted from publishers, they need to make sure that publishers survive, otherwise ad blockers lose their revenue source. Thus, we are interested in understanding how market characteristics affect the choice of the *long-term* arrangement between ad blockers and publishers. Subsequently, we also seek to understand how such arrangements will impact the quality of content provided by the publishers and the resulting welfare implications.

Our dynamic model assumes heterogeneous consumers both in terms of their valuation for content quality as well as in their disutility for advertising. Publishers choose costly quality levels and commit to these for the long-run. We assume that publishers monetize their content by deciding on advertising volume.[1] Consistently with practice, ad blockers can "charge" consumers for their products or, alternatively, they can charge publishers in return for reduced ad volumes. Interestingly, we find that the equilibrium contract has an ambiguous effect on the choice of quality investment by the publisher and, as a result, on overall welfare. Our results suggests that ad blocking can be beneficial to consumers by improving their content consumption experience, but the ad blocker has a strong incentive to try to "extort" the publisher. When

[1] It is easy to show that when publishers can offer a subscription at no cost then ad blocking is not an issue. In fact, a two-tier service with either advertising or subscription without advertising is an efficient discrimination tool for publishers between ad sensitive and non-sensitive consumers.

potential advertising revenues are high, the ad blocker negotiates a fee that the publisher will have no choice but to pay in order to maintain some of the advertising revenue. If the extent of such extortion is high, the publisher's profitability will shrink and quality declines, hurting consumers and total welfare.

We build on the above results to explore the nature of competition between publishers and ad blockers when they interact on a transaction platform that fully controls ad traffic. This scenario is increasingly relevant as most ad supported media content is consumed on such platforms (e.g. on a dominant smartphone operating system such as iOS or Android or on a social network like Facebook or YouTube). Importantly, platforms have broad discretion in regulating the transactions between their customers. In this setup, we ask: what is the optimal strategy of the platform in 'regulating' the contractual arrangements between the two parties. Our main finding here is that the platform wants to limit the entry of ad blockers to situations where it can expand the market by allowing highly ad sensitive users to consume content that would otherwise have too much advertising for their taste. The platform also wants to avoid the scenario where the ad blocker negotiates and possibly extorts the publisher as this decreases content quality and consumer welfare.

The rest of the paper is organized as follows. In the next section, we survey the relevant literature. Section 3 describes and analyses the base model. Section 4 explores a game that features a "transaction platform" that controls the interaction between consumers, advertisers and publishers. The paper ends with a general discussion and conclusion. To improve readability, all proofs are relegated to the Appendix.

2 Relevant literature

The emergence of online advertising has led to a great amount of annoyance on the part of consumers from the early days of digital ads, eventually leading to the success of ad blockers. The academic literature recognized the disutility inherent in advertising on the Internet and began to explore the reasons behind online advertising avoidance. In an early paper, Cho and Cheon (2004) find the most important reason to be that advertising gets in the way of users' goals, but past negative experiences and the perceived ad clutter also contribute.

Consumers naturally try to mitigate the disutility by avoiding ads which raises a number of important questions that can be addressed analytically. Anderson and Gans (2011) study the impact of ad avoidance technologies and their effects on content providers. They find that the reaction to ad avoidance leads to increased ad clutter and reduced content quality and decreases welfare. In another paper, Johnson (2013) considers how better targeted ads interact with ad avoidance, showing that consumers may underutilize ad avoidance tools. Eventually, consumers and content providers end up in a game where each player uses some methods to avoid or push through ads (Vratonjic et al., 2013).

On the empirical side, Goldstein et al. (2014) examine the costs of placing ads that are annoying. They find that such ads can cause users to abandon the website and negatively effect the process of consuming the site's content, potentially costing more than the revenue from the ads. Shiller, Waldfogel, and Ryan (2017) make the first attempt at thoroughly documenting the impact of ad blocking on online publishers. Using web traffic and ranking data for a broad set of sites, they show that, as larger proportion of consumers use ad blockers, publisher sites decline in their rankings (e.g. their rank becomes larger) and this effect is larger for worse-ranked sites. They conclude that "ad blocking poses a substantial threat to the ad-supported web".

More recent theoretical work in marketing challenges this general view. In particular, Despotakis, Ravi, and Srinivasan (2017) show that under some conditions, ad blockers can benefit a competing publishing platform as they serve to differentiate consumers based on their different tolerance for ads. Our focus is elsewhere, namely on the business model of ad blockers and how this affects their complex relationship with publishers.

On the conceptual front, our paper relates to several streams of literature. The base model of publishers providing content to consumers in return for showing advertisements is reminiscent of typical tow-sided markets (Rochet and Tirole, 2006), although for simplicity, the base model assumes exogenous advertising prices. We need this simplification because with the ad blocker, the model extends to a three-sided platform. In a later extension, we again build on this platform literature when we assume that the publisher and the ad blocker are both hosted on a multi-sided transaction platform that has full control over access. We assume a standard consumer model with heterogeneity at multiple levels. In particular, in terms of consumers' valuation for content quality, we adopt the standard approach of the literature on vertical differentiation (see Shaked and Sutton, 1982 and Moorthy, 1988). Furthermore, the ad blocking ecosystem exhibits characteristics of double marginalization (Spengler, 1950; Bresnahan and Reiss, 1985).

At the high level, ad blocking shows similarities with online music and software piracy as a third party (in our case the ad blocker) allows users to consume content without proper compensation (advertising eyeballs in case of ad blocking). Piracy has been extensively studied in the marketing and economics literature. Prominent work includes Jain (2008) who investigates digital piracy from a competitive perspective, showing that weaker copyright protection may be beneficial to firms selling content, because it softens price competition by allowing price sensitive consumers to pirate the content. Danaher et al. (2010) show how reducing the availability of content on paid channels increases the demand for the pirated version of the same content, whereas Vernik, Purohit, and Desai (2011) look at Digital Rights Management, finding that piracy might decrease even if usage restrictions are removed from digital products.

Perhaps the most important policy question regarding piracy is the effect on welfare and how consumer and content provider surpluses change. Bacache, Bourreau, and Moreau (2015) study the effects of piracy in the music industry. Their work shows that piracy has a mixed effect on musicians, depending on how they earn their income. Those that rely more on performances are less concerned since they benefit from increased exposure. Kim, Lahiri, and Dey (2017) study the interaction between piracy and double marginalization, finding that some level of piracy may benefit the manufacturer, retailer and consumer of content.

A distinct, but fairly small body of literature shows interesting resemblances to our research questions. As ad blockers essentially hold content publishers "hostage", there is a clear connection to the economic analysis of extortion. Several important details of ad blocking relate to the mechanics underlying extortion. For example, Konrad and Skaperdas (1997) examine the credibility of threats related to extortion. They find that an equilibrium with extortion only exists if there is a large enough number of victims. Choi and Thum (2004) investigate the dynamics of repeated extortion. In their setting, government officials demand bribes in a repeated fashion which shows similarities to ad blocking and they also find that a pure strategy equilibrium does not exist. In a fascinating setting, Olken and Barron (2009) study extortion in the field by analyzing the bribes demanded from truckers by corrupt officials. The findings demonstrate how multiple layers of extortion interact with higher amounts of bribes closer to the end of the journey.

Overall, our contribution lies in uncovering the dynamics of the strategic interaction between ad blockers, content publishers and the platform in control of the ecosystem. Our results help in understanding the forces that allow ad blocking to thrive without destroying the content providers who sustain themselves from advertising revenues and do not charge a fee to consumers.

3 The model

Our base model assumes three agents: consumers, a monopolistic publisher and a monopolistic ad blocker. Advertisers are not explicitly modeled; we assume a competitive advertising market with a large number of advertisers who pay an exogenous ad rate (price), denoted p_a.

Consumer utility depends on the quality of content provided by the publisher, q and the amount (volume) of advertising presented to them, A, both of which are assumed to be non-negative.[2] Consumers are heterogeneous in their valuation of quality; without loss of generality

[2]We ignore the situation when publishers can charge consumers a subscription fee. We discuss this case later and argue that such a capability largely mitigates the adverse impact of ad blocking.

and following Shaked and Sutton (1982), we assume that there is a continuum of consumers and each consumer's valuation for quality is captured by the parameter θ, uniformly distributed in $[0, 1]$. We also assume that consumers are averse to advertising and exposure to advertising decreases consumer utility. Specifically, there are two types of consumers: high- and low-sensitivity types with advertising sensitivities γ_h and γ_l respectively. Formally, we define consumer utility as

$$u_i(q, A) = \theta q - \gamma_i A, \quad i = h,\ l,$$

if they visit the publisher's site and 0 otherwise. We assume that $0 < \alpha < 1$ is the *a priori* probability that a consumer is of high-type ($i = h$) while low-type ($i = l$) consumers are present in proportion $1 - \alpha$. We also assume that advertising disutility and sensitivity to content quality are independently distributed. In this setup, $\bar{\gamma} = \alpha\gamma_h + (1 - \alpha)\gamma_l$ measures the *average* ad sensitivity of consumers while $\gamma_l/\gamma_h \in [0,\ 1]$ is one possible measure of (the inverse of) consumer heterogeneity w.r.t. ad sensitivity.

The publisher maximizes his profit by choosing content quality and advertising quantity. For simplicity, we assume that the costs of providing quality is quadratic: $c(q) = cq^2$. Consistently with practice, we also assume that the choice of quality represents a long-term commitment. While it is relatively easy for a publisher to adjust the amount of advertising shown on a page, adjusting quality (hiring the appropriate editorial staff, redesigning the format of the publication, building a reputation for a brand, etc.) is much harder. Thus, the publisher maximizes the following profit function:

$$D(q, A)p_a A - cq^2$$

by choosing q in the first stage and A in the second stage, where $D(q, A)$ is the demand function obtained from consumers' maximization problem.[3]

The final player is the ad blocker, which we assume to be endowed with a technology that, once installed by the consumer, can block advertising while still allowing consumers to benefit

[3]For the publisher alone, the distinction between the two stages in irrelevant. However, it becomes crucial in the presence of the ad blocker.

from the content of the publisher. We assume that the ad blocker can monetize her technology in two distinct ways. On the one hand, she can sell it to consumers for a price p. This strategy is often easier to implement because it can rely on existing infrastructure (e.g. an app store) to reach consumers and does not require the ad blocker to contact and negotiate with publishers. Another monetization strategy employed by (typically larger) ad blockers involves negotiating a fee, f per unit of ad volume from the publisher in exchange for not making the ad blocker's software available to consumers.[4] In what follows, we will consider these two strategies and evaluate them against the benchmark case without the presence of the ad blocker. Importantly, we assume that the ad blocker only moves in stage 2, once the publisher has chosen the level of quality, q.

3.1 Benchmark: No ad blocker

First, we analyze a benchmark case without the presence of the ad blocker. In this case, consumers' choice is simple. They visit the publisher's website if $\theta q - \gamma_i A \geq 0$, where $i \in \{h, l\}$ respectively for the high- and low-sensitivity consumers. This means that only consumers for whom $\theta \geq \gamma_i A/q$ will visit the publisher's site. Hence, the demand function (that is, the fraction of people who desire to visit the publisher's website) for any segment is given by:

$$D_i(q, A) = \Pr\left(\theta \geq \frac{\gamma_i A}{q}\right) = \begin{cases} 1 - \frac{\gamma_i A}{q}, & \text{if } \frac{\gamma_i A}{q} \in [0, 1] \\ 0, & \text{if } \frac{\gamma_i A}{q} > 1, \end{cases} \tag{1}$$

where $i = l,\ h$. Given this demand, the outcome can be summarized as follows.

Proposition 1 *The publisher's problem has the following solution.*

- *If $\frac{\gamma_l}{\gamma_h} \geq \frac{1-\alpha}{2-\alpha}$, then both high- and low-type consumers consume the publisher's content and*

$$q_B^\star = \frac{p_a}{8c\bar{\gamma}}, \quad A_B^\star = \frac{p_a}{16c\bar{\gamma}^2}, \quad \pi_B^\star = \frac{p_a^2}{64c\bar{\gamma}^2}. \tag{2}$$

[4] This assumption is a simplification: in reality ad blockers still make the software available, but let whitelisted advertising through if publishers pay the fee. However, for these publishers the outcome is equivalent to consumers not having access to ad blocking.

- *If $\frac{\gamma_l}{\gamma_h} < \frac{1-\alpha}{2-\alpha}$, then only low-sensitivity consumers consume the publisher's content and*

$$q_B^\star = \frac{(1-\alpha)p_a}{8c\gamma_l}, \quad A_B^\star = \frac{(1-\alpha)p_a}{16c\gamma_l^2}, \quad \pi_B^\star = \frac{(1-\alpha)^2 p_a^2}{64c\gamma_l^2}. \tag{3}$$

This result is intuitive. When consumer heterogeneity is low and different types of consumers are similar to each other (i.e. γ_l/γ_h is large), the publisher serves both segments of consumers. Rearranging the condition shows that when the size of the sensitive segment (α) is large, the publisher cannot ignore this group and decides on ad volumes that still make this segment consume the content. In the opposite case (when consumers are very different in terms of ad sensitivity or the size of the ad sensitive segment is small) the publisher only caters to the low-sensitivity segment. Intuitively, in both cases, content quality, advertising quantity and the publisher's profit depend positively on the price of advertising, p_a and depend negatively on the cost of quality, c and consumers' sensitivity to advertising, $\bar{\gamma}$ or γ_l depending on whether both groups are served or only the low-sensitivity group is served.

3.2 Introducing ad blocking

To assess the impact of ad blocking we consider the two setups that are relevant in practice for the online publishing industry. We start by looking at the case when the ad blocker charges consumers directly. This setup fits many of the existing ad blocking products that find it easier to reach consumers. Charging consumers can take different forms. Some ad blockers simply charge for the app using a popular app store. Others employ a freemium model and require payment for unlocking features. Finally, some ad blockers charge an implicit payment by collecting and selling consumers' data.

Next, we look at a case where the ad blocker negotiates with the publishers directly. This case would apply to ad blockers who target ad revenues from larger publishers that would be easier to reach and negotiate with. Ultimately, we also want to explore the choice of the ad blocker w.r.t. these two business models. In this last case, we assume that the ad blocker chooses her revenue model endogenously.

3.2.1 Ad blocker charging consumers directly

We assume that in stage 2, the ad blocker gets all her profit from charging consumers price p that allows them to block ads from the publisher. First, we establish that the one-shot game in the second stage has no (meaningful) equilibrium in pure strategies.

Lemma 1 *In the unique pure-strategy Nash equilibrium,* $q^* = 0$*,* $A^* = 0$ *and* $p^* = 0$*.*

The lemma clearly illustrates the basic nature of the strategic interaction between the publisher and the ad blocker. Given the investment of the publisher, that is sunk in stage 2, the ad blocker has an incentive to extract all surplus from the publisher.[5] Anticipating that in the second stage all profits will be taken by the ad blocker, the publisher chooses $q = 0$ to avoid the cost of quality investment. Games of this structure are called "expropriation games" (see Angelucci and Meraglia, 2016 for a recent example) and in practice, publishers tend to think about ad blocking as such.

The dilemma for the ad blocker is to extract surplus from publishers without bankrupting them. To reflect this reality, we set up the model as follows: In stage 1 (as before), q is chosen by the publisher at cost cq^2 and cannot be changed subsequently. The second stage consists of a simultaneous, repeated game with an infinite number of periods. Specifically, in each period, the publisher chooses A and the ad blocker chooses p simultaneously. We denote by $\delta < \bar{\delta} < 1$ the discount factor used by both the publisher and the ad blocker. We also assume that consumers live for only one period and, therefore, they are non-strategic. Figure 1 summarizes the structure of the game, for which we seek a stationary equilibrium.

The above setup is consistent with the ad blocking phenomenon and in particular, the way we think about publishers' quality. *The New York Times* for example, has built its reputation

[5]Interestingly, there is no equilibrium where the two firms extract revenue from separate segments. In a candidate equilibrium where the ad blocker charges only the highly ad sensitive consumers, the publisher raises advertising volume to a level where it makes sense for the ad blocker to sell ad blocking technology to the low sensitivity segment as well.

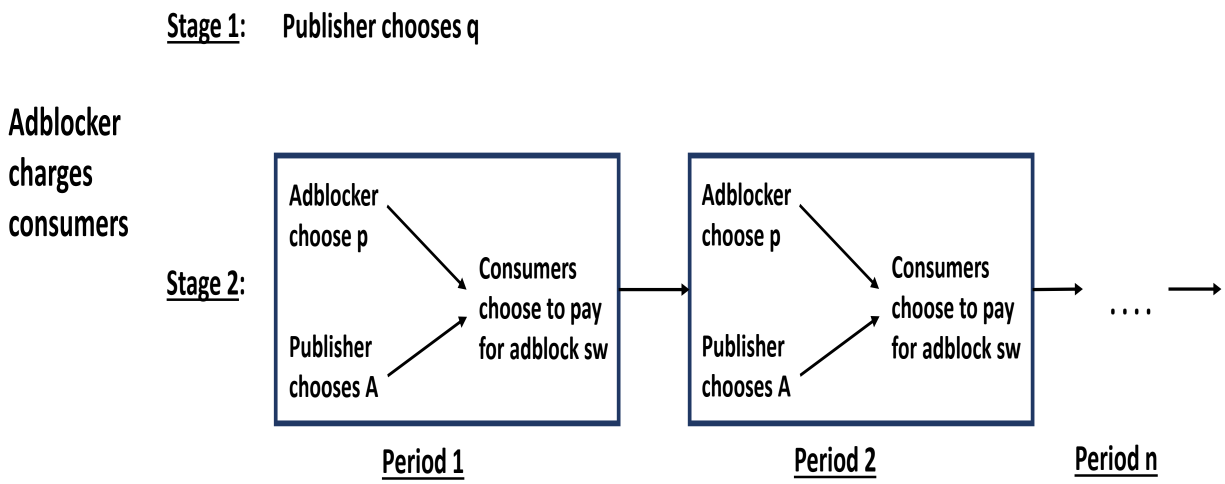

Figure 1: Structure of the game when the ad blocker charges consumers (Section 3.2.1).

well before ad blocking appeared. In contrast, its advertising decisions are flexible.[6] Similarly, it is reasonable to assume that consumers are non-strategic in the sense of long-term dynamic strategies and simply respond to the content and advertising presented to them.

As in any infinitely repeated game, one can find many equilibrium outcomes. However, our context helps select a unique, "natural" equilibrium. First, in looking for a stationary outcome with tacit coordination between the players, it makes sense to assume that the punishment strategy is the one that reverts to the outcome of the static game with 0 payoff to both the ad blocker and the publisher. This guarantees an *efficient* outcome in the sense that the maximum profit is extracted from consumers. We also look for an outcome where the division of payoffs is *feasible*, in the sense that there are no explicit transfers between the publisher and the ad blocker (a case that we model in detail subsequently). We will call the equilibrium with the above features an *"efficient stationary equilibrium"* and show below that it exists as long as δ is large enough.

Proposition 2 *Assume that the publisher has chosen quality q in the first stage. Then, if and*

[6]Note that, in reality, while the publisher commits to a quality level for the long-run, it incurs some fixed cost of quality every period. For the sake of simplicity, in our model, the cost of quality is sunk in the first stage of the game. Importantly, these two setups are equivalent however, because (i) the fixed cost is identical in each period and, (ii) it is independent of all other variables.

only if $\delta \geq 1-\alpha$ and either one of the following conditions are fulfilled:

$$\delta \geq \frac{(2-\alpha)\gamma_l\gamma_h - (1-\alpha)\gamma_l^2 - (1-\alpha)\gamma_h^2}{(2-\alpha)\gamma_l\gamma_h - (1-\alpha)\gamma_l^2} \quad or \quad \frac{\gamma_l}{\gamma_h} \leq \frac{2-\alpha-\sqrt{\alpha(4-3\alpha)}}{2(1-\alpha)}, \tag{4}$$

there is a unique efficient stationary equilibrium of the second-stage (multi-period simultaneous) game, in which the equilibrium advertising level is $A_C^\star = q/(2\gamma_l)$ and the price of the ad blocker's product is $p_C^\star = q/2$ and players' trigger strategies are $A = 0$ and $p = 0$ respectively. In this equilibrium, only high-type (γ_h) consumers buy the ad blocker's product. The players' long-term profits are:

$$\pi_C^{\star,pub} = \frac{1}{1-\delta}\frac{(1-\alpha)qp_a}{4\gamma_l} - cq^2, \quad \pi_C^{\star,adb} = \frac{1}{1-\delta}\frac{\alpha q}{4}.$$

This result is driven by the notion that the ad blocker is willing to leave the low sensitivity consumers to the publisher only to keep it alive in order to ensure future revenues. Consequently, only high ad-sensitivity consumers buy the ad block software, the publisher has no choice, but to essentially hand over these consumers to the ad blocker. The equilibrium only exists under certain conditions: first, δ needs to be large enough for the future to matter. Moreover, the two segments need to be different enough, i.e. γ_l/γ_h needs to be small enough (or, equivalently, α, the size of the ad sensitive segment needs to be small enough). Note that since the publisher only benefits from low sensitivity consumers, quality is limited by the profit earned from this segment only. Finally, in equilibrium, the maximum surplus is extracted from both segments. We can now solve for q in the first stage to fully characterize the equilibrium of the game.

Proposition 3 *If $\delta \geq 1-\alpha$ and either one of the conditions in (4) are fulfilled then, in equilibrium, only high-type (γ_h) consumers buy the ad blocker's product and firms' strategies are:*

$$q_C^\star = \frac{(1-\alpha)p_a}{8c\gamma_l(1-\delta)}, \quad A_C^\star = \frac{(1-\alpha)p_a}{16c\gamma_l^2(1-\delta)}, \quad p_C^\star = \frac{(1-\alpha)p_a}{16c\gamma_l(1-\delta)}.$$

Firms' long-term profits are:

$$\pi_C^{\star,pub} = \frac{(1-\alpha)^2p_a^2}{64c\gamma_l^2(1-\delta)^2}, \quad \pi_C^{\star,adb} = \frac{\alpha(1-\alpha)p_a}{32c\gamma_l(1-\delta)^2}.$$

From the publisher's perspective this equilibrium is similar to the benchmark case, in which the publisher only serves the low-sensitivity consumers. It is important to note, however, that this equilibrium exists in a broader range. As a result, the publisher is clearly worse off with the ad blocker in the market if on its own the publisher would serve both segments in the benchmark case, i.e. when $\frac{\gamma_l}{\gamma_h} > \frac{1-\alpha}{2-\alpha}$. In this case, the publisher would also choose higher quality and a higher advertising level without the ad blocker. That is, whenever $\frac{\gamma_l}{\gamma_h} > \frac{1-\alpha}{2-\alpha}$, quality goes down in the presence of the ad blocker, as does the volume of advertising. Conversely, when the publisher only serves the low-sensitivity consumers in the absence of an ad blocker, the latter's entry does not have much impact on the publisher. In this case, the entry of the ad blocker expands the market because it allows the high ad-sensitivity consumers using the ad block software to access the content. The so-created consumer surplus is (partially) captured by the ad blocker through the price consumers pay for the ad block software.

3.2.2 Ad blocker charging the publisher a fee

Here we solve a game with the publisher and the ad blocker negotiating a contract where a fee, f is paid by the publisher to the ad blocker that is proportional to the publisher's level of advertising, A.[7] Thus, the total fee transferred to the ad blocker is fA. To make this game comparable to the one in the previous section, where consumers pay the ad blocker, we assume a similar two-stage game with a dynamic second stage. As before, in the first stage, the publisher chooses quality, q at cost cq^2 that cannot be changed subsequently. The second stage consists of a multi-period repeated game where in each period the ad blocker first sets fee, f, the publisher then knowing this fee chooses the level of advertising, A and, given A and q, consumers choose whether to visit the website or not.[8] Figure 2 illustrates the game structure and Proposition 4 describes which segment is served in equilibrium.

[7]The implicit threat of the ad blocker here is that he makes the ad block technology available to consumers for free.

[8]Note, that in this case, we do not need a dynamic game, as the one-period game also has a meaningful (unique) equilibrium in pure strategies. We do specify a dynamic game to be able to compare profits with those of the previous section. This will be particularly useful when we endogenize the ad blocker's revenue model.

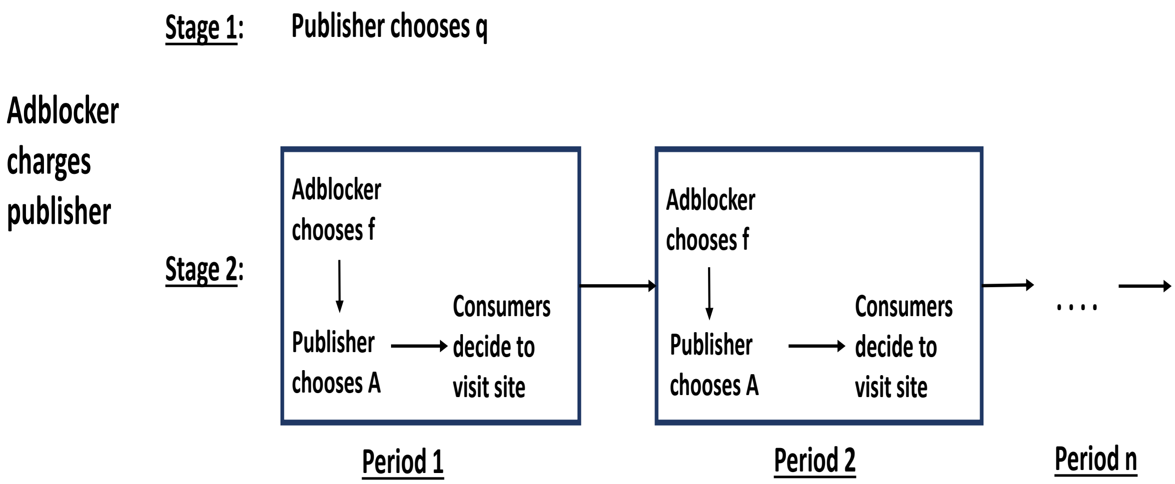

Figure 2: Structure of the game when the ad blocker charges the publisher (Section 3.2.2).

Proposition 4 *Define $x^\star(\alpha)$ as the unique solution to the equation $\frac{1+4\alpha}{\sqrt{1-\alpha}}x + \sqrt{1-\alpha}x^3 - 2x^2 - 4\alpha = 0$ on the interval $\left[\frac{\sqrt{1-\alpha}}{1+\alpha}; \sqrt{1-\alpha}\right]$. Then, in equilibrium we have the following two possible outcomes:*

1. *If $\frac{\gamma_l}{\gamma_h} \geq \frac{\alpha x^\star(\alpha)^2}{1-(1-\alpha)x^\star(\alpha)^2}$ then both segments visit the publisher's site.*

2. *If $\frac{\gamma_l}{\gamma_h} < \frac{\alpha x^\star(\alpha)^2}{1-(1-\alpha)x^\star(\alpha)^2}$ then only the low sensitivity consumers will visit the publisher's site.*

Figure 3 depicts the two regions in the $(\alpha,\ \gamma_l/\gamma_h)$ space. As before, low sensitivity consumers are always served and high sensitivity customers are served if and only if they are not too different from low sensitivity counterparts (i.e. γ_l/γ_h is high). The results are similar to the benchmark, but the region where both segments are served expands as a result of ad blocking that helps highly ad sensitive consumers enter the market. Generally, the higher the proportion of high sensitivity consumers (i.e. the higher is α), the more likely that these consumers will also be served although for small α this is surprisingly not the case. Indeed, for small α, as α increases, highly ad sensitive consumers are less likely to be served. This is due to the strategic interaction between the publisher and the ad blocker. Specifically, to maximize its profit the ad blocker charges a fee in such a way that the publisher chooses to serve only the low ad sensitive consumers leading to high advertising volume. As α increases, the ad blocker needs

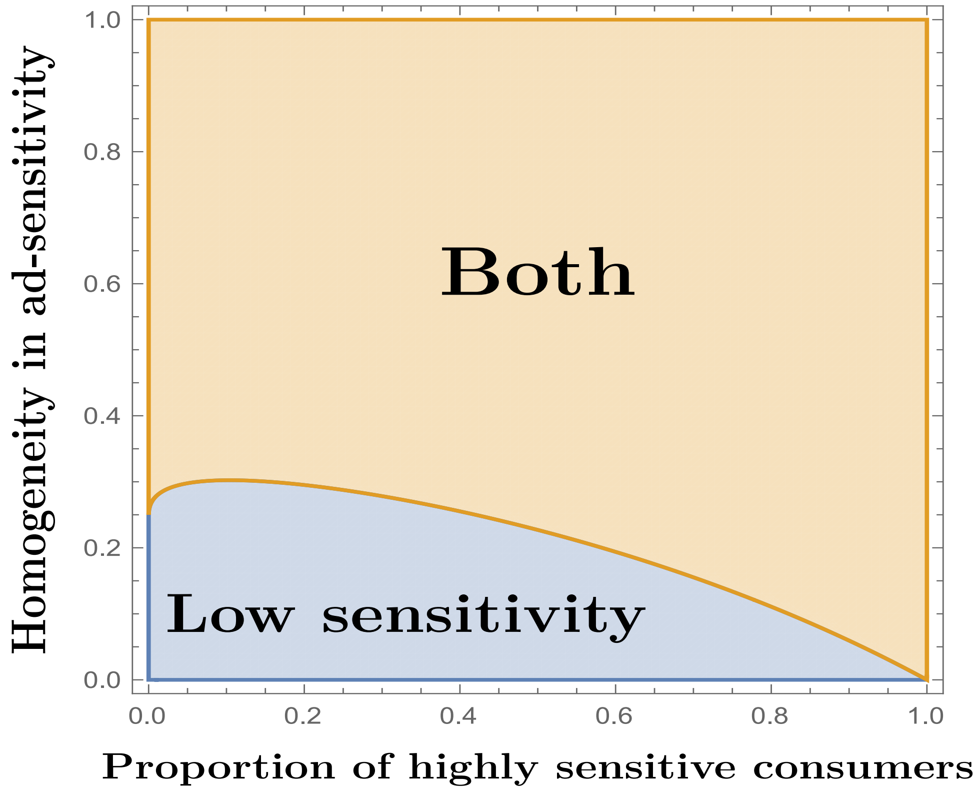

Figure 3: Segment(s) served in equilibrium in the $\left(\alpha, \frac{\gamma_l}{\gamma_h}\right)$ space when the ad blocker charges the publisher.

to lower its fee to a degree that more than compensates the publisher for keeping the high volume advertising regime (with only the low sensitivity consumers served). Beyond a certain threshold proportion of highly ad sensitive consumers however, this is not sustainable.

We have also derived firms' detailed equilibrium strategies for setting quality, advertising volume and the ad blocker's fee. These have closed form solutions (see Appendix - proof of Proposition 4) although the expressions are rather complex. However, it is easy to plot them graphically to understand the intuition behind the outcome(s). Figure 4 illustrates the fee f set by the ad blocker, quality q and advertising level A set by the publisher, and the two players' profits as the proportion of high ad sensitivity consumers, α changes while other parameters are fixed (Figure 4 provides a cross section of Figure 3's lower region at $\gamma_l/\gamma_h = 1/6$). When α is small (i.e. the proportion of ad-sensitive consumers is low) all of the variables shown tend to decrease. The reason is that, here, the publisher only wants to serve the low sensitivity consumers and their share is decreasing with α. Moreover, since the ad blocker's profits depend on how well the publisher is doing (and in particular, what quality he sets), the fee the ad blocker charges to the publisher and her profits decrease as well. At the other extreme, at high values of α, the publisher cannot avoid serving both segments. However, to do so, he needs to drastically reduce A (see the large drop in A). To compensate for the lost volume of advertising, the ad blocker raises her fee. Clearly, both of them end-up with lower profits.

The most interesting part of Figure 4 is at moderate values of α. Here, quality, advertising and the publisher's profits actually increase even as the size of the segment served, i.e. the proportion of low ad sensitivity consumers is decreasing. This happens because without the ad blocker, the publisher would consider serving both segments. However, the ad blocker is trying to prevent this and force the publisher to stay in the regime where only the low sensitivity consumers are served - see how the fee set by the ad blocker is declining in this middle region. As a result, the publisher can increase ad volume and consequently his quality level as well. While the ad blocker's profits are declining in this region, the publisher's increase.

3.3 Ad blocker chooses revenue model

In this section, the ad blocker endogenously chooses her revenue model. Specifically, we assume that charging consumers directly is always possible for the ad blocker. This boils down to also assuming that δ is large enough, which - as per Proposition 3 - ensures the existence of such an equilibrium. Then, the two players negotiate under the threat that the ad blocker charges the consumers directly.[9] We need to evaluate under what condition the publisher chooses to accept the ad blocker's offer to negotiate a contract and under which conditions the ad blocker offers negotiation in the first place. This meta game then has the following structure. In stage 1, the publisher chooses his quality level at cost cq^2 (as mentioned earlier, we assume that quality choice is a long-term commitment). In stage 2, the ad blocker decides whether she wants to strike a deal with the publisher or not. If she decides not to, then she charges price p

[9]As opposed to the threat that the ad blocker makes the ad block technology available to consumers for free.

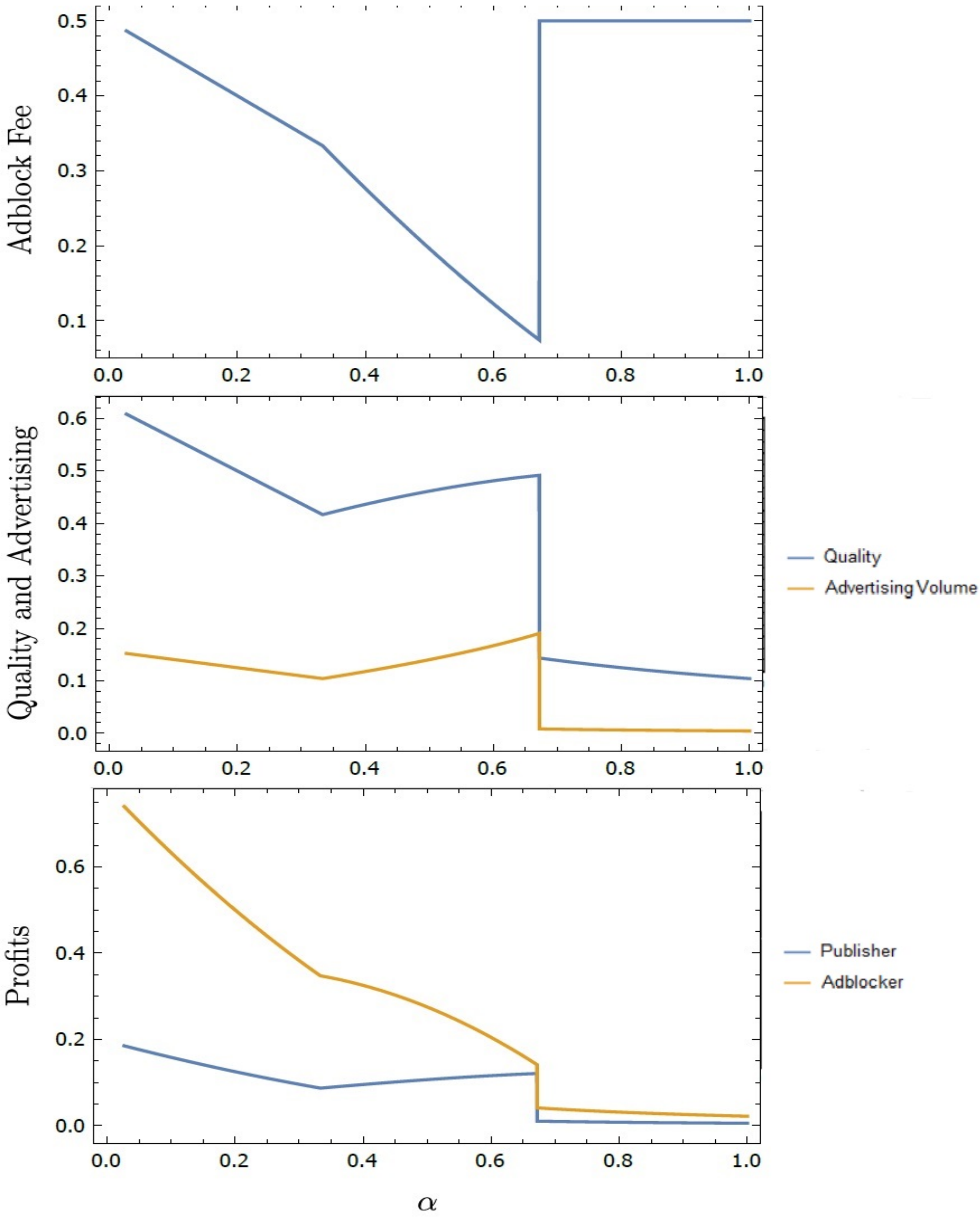

Figure 4: Equilibrium outcomes as functions of α when the ad blocker charges the publisher. Other parameters are fixed at $p_a = 1$, $c = 0.5$, $\gamma_h = 6$, $\gamma_l = 1$ and $\delta = 0.9$.

to consumers directly for the ad blocking service. If the ad blocker decides to offer a contract, f then the publisher decides whether to accept it or not. If he does not accept the offer then, again, the ad blocker charges the consumers directly. If the publisher accepts the offer then he chooses the optimal level of advertising, A given f as in section 3.2.2. Given this structure, we can formulate the following proposition.

Proposition 5 *Assume that δ is large enough, such that, the condition in (4) is fulfilled. If $\alpha < 1/2$ or $\frac{\gamma_l}{\gamma_h} < \frac{4\alpha(1-\alpha)}{(3-2\alpha)(2\alpha-1)}$ or $p_a \leq 2\alpha\bar{\gamma}$, then the parties do not negotiate and the ad blocker charges consumers directly. Low sensitivity consumers see ads and highly sensitive consumers use ad blocking. Firms' equilibrium strategies are*

$$q_C^\star = \frac{(1-\alpha)p_a}{8c\gamma_l(1-\delta)}, \quad A_C^\star = \frac{(1-\alpha)p_a}{16c\gamma_l^2(1-\delta)}, \quad p_C^\star = \frac{(1-\alpha)p_a}{16c\gamma_l(1-\delta)},$$

and their long-term profits are

$$\pi_C^{\star,pub} = \frac{(1-\alpha)^2 p_a^2}{64c\gamma_l^2(1-\delta)^2}, \quad \pi_C^{\star,adb} = \frac{\alpha(1-\alpha)p_a}{32c\gamma_l(1-\delta)^2}.$$

If $\alpha > 1/2$ and $\frac{\gamma_l}{\gamma_h} \geq \frac{4\alpha(1-\alpha)}{(3-2\alpha)(2\alpha-1)}$ and $p_a > 2\alpha\bar{\gamma}$, then the ad blocker charges the publisher. In equilibrium, both groups of consumers watch ads and the equilibrium strategies are

$$q_N^\star = \frac{p_a}{32c\bar{\gamma}(1-\delta)}, \quad f_N^\star = \frac{p_a}{2}, \quad A_N^\star = \frac{p_a}{128c\bar{\gamma}^2(1-\delta)}.$$

Firms' long-term profits are

$$\pi_N^{\star,pub} = \frac{p_a^2}{1024c\bar{\gamma}^2(1-\delta)^2}, \quad \pi_N^{\star,adb} = \frac{p_a^2}{256c\bar{\gamma}^2(1-\delta)^2}.$$

Under an endogenous revenue model for the ad blocker, the game has a surprisingly simple solution. First, when the exogenous value of advertising is too low compared to the general level of ad-sensitivity in the consumer base ($p_a \leq 2\alpha\bar{\gamma}$), the ad blocker always charges consumers directly and only low sensitivity consumers are served ads. If the value of advertising is higher, then the revenue model depends on how heterogeneous the customer base is in terms of ad sensitivity. If the proportion of sensitive consumers is very low or the difference is small between

γ_l and γ_h (low level of heterogeneity) then, again, the ad blocker charges consumers directly and only low sensitivity consumers are served ads in equilibrium. In the opposite case, i.e. when ad sensitivity is more heterogeneous, then the publisher and ad blocker negotiate and we observe whitelisting practices. Importantly, under both scenarios the quality of the publisher is lower than in the benchmark case. We conclude that ad blocking is detrimental to publishers' quality.

In terms of consumer surplus however, the impact of the ad blocker is more complex. We can summarize the impact of the ad blocker on consumer surplus in the following corollary.

Corollary 1 *Assuming that (4) is fulfilled, consumer surplus is higher in the presence of the ad blocker, if and only if,* $\frac{\gamma_l}{\gamma_h} < \frac{1-\alpha}{2-\alpha}$.

In other words, consumers are better off with the presence of the ad blocker only when there was a segment previously not served. If both segments were served without the ad blocker, then, when the ad blocker enters, it reduces consumer welfare. In the case when the ad blocker charges consumers, this happens because both parties will focus on their respective segments and extract as much surplus as possible from them (they basically have monopoly power over those segments). In the negotiation case, it happens because the quality of the publisher's content is significantly reduced in equilibrium.

4 Publishing and advertising on a content platform

We now explore the incentives of the platform controlling the channel through which users consume content from publishers and through which advertising takes place. The ultimate interface is typically a browser or an application, but the entity in control varies: for desktop browsing there is typically less stringent control, but the developer of the web browser has the power; for mobile environments apps generally have to be approved by the firm maintaining the operating system (such as in the case of iOS or Android). We model this ecosystem as a three-sided platform, where the platform operator decides whether or not to allow the ad blocker to function. The platform mediates and profits from transactions between consumers, the publisher and - potentially - the ad blocker. Since the platform's revenues will be related to the total surplus generated by the transactions on it, we assume that the platform's goal is to maximize the total surplus generated across the three groups of participants. For simplicity, we do not elaborate on the "taxation" mechanism that the platform employs, rather, we assume that this is efficient not to distort the interactions between the three groups of participants. In this sense, the platform can be seen as an efficient central planner.

Total surplus is defined as the sum of the consumer surplus and the profits of the publisher and the ad blocker. We also assume that δ is high enough such that (4) is fulfilled. The following proposition summarizes our findings.

Proposition 6 *The platform will allow the ad blocker to operate, if and only if, one of the following conditions holds:*

- $\frac{\gamma_l}{\gamma_h} < \frac{1-\alpha}{2-\alpha}$ *or*
- $\alpha \leq \frac{1}{2}$ *and* $\frac{\gamma_l}{\gamma_h} \geq \frac{1-\alpha}{2-\alpha}$ *and* $p_a \leq \frac{(1-\alpha)(1+2\alpha)\gamma_l\bar{\gamma}^3-\alpha\gamma_l^2\gamma_h^2-(1-\alpha)\gamma_l^4}{\gamma_l^2\bar{\gamma}-(1-\alpha)^2\bar{\gamma}^3}$ *or*
- $\alpha > \frac{1}{2}$ *and* $\frac{1-\alpha}{2-\alpha} \leq \frac{\gamma_l}{\gamma_h} \leq \frac{4\alpha(1-\alpha)}{(2\alpha-1)(3-2\alpha)}$ *and* $p_a \leq \frac{(1-\alpha)(1+2\alpha)\gamma_l\bar{\gamma}^3-\alpha\gamma_l^2\gamma_h^2-(1-\alpha)\gamma_l^4}{\gamma_l^2\bar{\gamma}-(1-\alpha)^2\bar{\gamma}^3}$.

In all three cases, the ad blocker charges consumers directly.

The first scenario corresponds to the situation when the publisher serves only low sensitivity consumers in the benchmark case (i.e. without the presence of the ad blocker). Clearly, here, the ad blocker adds to total surplus by making sure that the high sensitivity consumers are also served by the publisher, even if it captures most of this surplus. In the second and third scenarios, the publisher serves both groups in the benchmark case. Here, the small advertising price ensures that the publisher's profits without the ad blocker are not as important as the consumer surplus generated. Again, most of this surplus is recaptured by the ad blocker.

In all three cases when the ad blocker is allowed on the platform, it would want to charge consumers directly, which is easy to implement in an "app store" induced pricing system.

5 Discussion and concluding remarks

We have developed a model to examine how the presence of an ad blocker changes the interaction between publishers and consumers. Focusing on the possible revenue models that the ad blocker can adopt, we found that when consumers are highly heterogeneous with respect to their advertising sensitivity, then ad blockers are more likely to charge consumers directly. When consumers are fairly homogeneous, the ad blocker is likely to attempt to extort the publisher by collecting a fee from the publisher, an outcome that captures the practice of whitelisting. Our model also examined how the presence of an ad blocker may change the quality choice of the publisher as well as the welfare implications of this new industrial structure. We found that, while the publisher's quality unambiguously declines with the presence of the ad blocker, the welfare implications are more ambiguous. Consumer surplus increases with the entry of an ad blocker when previously unserved, highly ad sensitive consumers start consuming content by taking advantage of ad blocking. On the flipside, consumer welfare decreases in cases where both segments consume the publisher's content without ad blocking. In this case, consumer surplus decreases either because it is transferred directly to the ad blocker or implicitly through the reduced content quality caused by the extortive pressure put on the publisher by the ad blocker.

Importantly, we have also considered the interaction of the publisher and the ad blocker on a content platform. This setup is increasingly relevant for content consumption and, therefore, also for advertising. The problem of the platform mirrors that of a social planner willing to maximize total welfare assuming that the platform can efficiently "tax" all three parties interacting on it (consumers, the publisher and the ad blocker). We find that the platform allows the ad blocker to participate in its ecosystem either when the customer base is very heterogeneous in terms of ad sensitivity or when consumer heterogeneity is moderate but the price of advertising is low. In both cases, the ad blocker is inclined to charge consumers directly. This result is consistent with the practice employed by mobile platforms who are hesitant to authorize free ad blocking software.

Our model considers a monopolist ad blocker and a monopolist publisher. The latter assumption is driven by the notion that content providers always have some monopoly power resulting from copyright laws or differentiated content. On the ad blocker side however, we can observe intense competition as entry costs are relatively low and differentiation is hard to implement. Assuming perfect competition between ad blockers means different outcomes depending on contractual arrangements. If the ad blockers charge consumers, then, in our dynamic setting, ad blockers have no incentive to push prices to or below the level when low-sensitivity consumers buy the ad block software. If this were the case, publishers would go out of business and the market would disappear. Nevertheless, under competition, prices go down to the level where low-sensitivity consumers are indifferent between using ad block software or not. In this case, surplus is transferred to high ad-sensitivity consumers. It is easy to see that if competition is intense between ad blockers, then this revenue model is problematic because, there is a large incentive to undercut prices below the critical level when the market unravels, i.e. when low sensitivity consumers are also interested in buying the ad block software. This incentive comes from the short-term gain of switching all high-sensitivity consumers to become customers. Again, on a platform, which controls the level of competition between players (e.g. the allowed set of competitors) this problem can be mitigated.

In the more likely case of negotiation between the ad blockers and the publisher, the situation is somewhat different. Now, with ad blocker competition, the threat of non-negotiation becomes less valuable for the ad blockers as charging consumers directly is at a lower price as argued above. In equilibrium, this leads to a lower negotiated fee by the ad blocker(s), which means that surplus is transferred to the publisher. Under this scenario, competition also has a secondary effect on quality, which should increase together with advertising levels.

Another simplification of our model is that we ignored the possibility for publishers to use a subscription model instead (or in parallel) to an advertising model. Many content providers have a two-tier system where the content is free from advertising if the consumer pays a subscription fee. With the appearance of ad blockers, some publishers have indeed asked consumers who used the ad block software to pay the subscription fee (Vasagar, 2015). If such a technology is available to the publisher, then ad blocking is largely a non-issue. However, running a subscription system is associated with considerable fixed costs that can only be justified by a critical size. Also, a subscription model may not be compatible with certain types of content (e.g. when content is not available on a regular schedule). Our model is clearly more relevant to publishers with a purely advertising-driven revenue model, which nevertheless applies to a very significant portion of online content.

References

Anderson, Simon P. and Joshua S. Gans (2011). "Platform Siphoning: Ad-Avoidance and Media Content". *American Economic Journal: Microeconomics* 3, pp. 1–34.

Angelucci, Charles and Simone Meraglia (2016). "Trade, Law and order, and Political Liberties, Theory and Application to English Medieval Boroughs". *Working Paper.*

Bacache, Maya, Marc Bourreau, and Francois Moreau (2015). "Piracy and Creation: The Case of the Music Industry". *European Journal of Law and Economics* 39.2, pp. 245–262.

Bresnahan, Timothy F. and Peter C. Reiss (1985). "Dealer and Manufacturer Margins". *RAND Journal of Economics* 16.2, pp. 253–268.

Cho, Chang-Hoan and Hongsik John Cheon (2004). "Why Do People Avoid Advertising on the Internet?" *Journal of Advertising* 33.4, pp. 89–97.

Choi, Jay Pil and Marcel Thum (2004). "The Economics of Repeated Extortion". *RAND Journal of Economics* 35.2, pp. 203–223.

Cookson, Robert (2015). "Digicel first mobile group to block ads in battle against Google". *Financial Times* September 30.

— (2016). "Media companies worried as ad blocking goes mobile". *Financial Times* February 22.

Danaher, Brett, Samita Dhanasobhon, Michael D. Smith, and Rahul Telang (2010). "Converting Pirates Without Cannibalizing Purchasers: The Impact of Digital Distribution on Physical Sales and Internet Piracy". *Marketing Science* 29.6, pp. 1138–1151.

Despotakis, Stylianos, R. Ravi, and Kannan Srinivasan (2017). "The Beneficial Effects of Ad Blockers". *Working Paper.*

Fildes, Nic (2017). "Ad-blocker Shine switches roles to become ad platform". *Financial Times* February 24.

Garrahan, Matthew and Hannah Kuchler (2015). "Adblocking unleashes anxiety across the advertising industry". *Financial Times* October 5.

Goldstein, Daniel G., Siddarth Suri, Preston McAfee, Matthew Ekstrand-Abueg, and Fernando Diaz (2014). "The Economic and Cognitive Costs of Annoying Display Advertisements". *Journal of Marketing Research* 51.6, pp. 742–752.

Jain, Sanjay (2008). "Digital Piracy: A Competitive Analysis". *Marketing Science* 27.4, pp. 610–626.

Johnson, Justin P. (2013). "Targeted advertising and advertising avoidance". *RAND Journal of Economics* 44.1, pp. 128–144.

Kastrenakes, Jacob (2016). "Adblock Plus now sells ads". *The Verge* September 13.

Kim, Antino, Atanu Lahiri, and Debabrata Dey (2017). "The 'Invisible Hand' of Piracy: An Economic Analysis of the Information-Goods Supply Chain". *Working Paper.*

Konrad, Kai A. and Stergios Skaperdas (1997). "Credible threats in extortion". *Journal of Economic Behavior & Organization* 33, pp. 23–29.

Moorthy, K. Sridhar (1988). "Product and Price Competition in a Duopoly". *Marketing Science* 7.7, pp. 141–68.

Olken, Benjamin A. and Patrick Barron (2009). "The Simple Economics of Extortion: Evidence from Trucking in Aceh". *Journal of Political Economy* 117.3, pp. 417–452.

Rochet, Jean-Charles and Jean Tirole (2006). "Two-sided markets: a progress report". *RAND Journal of Economics* 37.3, pp. 645–67.
Shaked, Avner and John Sutton (1982). "Relaxing price competition through product differentiation". *Review of Economic Studies* 49, pp. 3–13.
Shiller, B., J. Waldfogel, and J. Ryan (2017). "Will Ad Blocking Break the Internet?" *National Bureau of Economic Research* Working Paper 23058.
Spengler, Joseph J. (1950). "Vertical Integration and Antitrust Policy". *Journal of Political Economy* 58.4, pp. 347–352.
Vasagar, Jeevan (2015). "Axel Springer winning fight against adblockers". *Financial Times* November 4.
Vernik, Dinah A., Devavrat Purohit, and Preyas S. Desai (2011). "Music Downloads and the Flip Side of Digital Rights Management". *Marketing Science* 30.6, pp. 1011–1027.
Vratonjic, Nevena, Mohammad Hossein Manshaei, Jens Grossklags, and Jean-Pierre Hubaux (2013). "Ad-Blocking Games: Monetizing Online Content Under the Threat of Ad Avoidance". *The Economics of Information Security and Privacy.* Ed. by Rainer Böhme. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 49–73.

Appendix

5.1 Proof of Lemma 1

Proof: Consumers make two decisions: whether to visit the publisher's website at all and whether to use the ad blocker or not. If a consumer of type i ($i \in \{l; h\}$) chooses to visit the website, then he will use the ad blocker whenever $\theta q - \gamma_i A \leq \theta q - p$, or $p \leq \gamma_i A$. Depending on p, 3 possibilities can arise: both groups use the ad blocker ($p \leq \gamma_l A$); only high sensitivity group uses the ad blocker ($\gamma_l A \leq p \leq \gamma_h A$); nobody uses the ad blocker ($p \geq \gamma_h A$). The demand of group i for the publisher is given by:

$$D_i^P(q, A) = \Pr\left(\theta \geq \frac{\gamma_i A}{q} \mid p \geq \gamma_i A\right) = \begin{cases} 1 - \frac{\gamma_i A}{q}, & \text{if } \gamma_i A \leq \min(p, q) \\ 0, & \text{otherwise.} \end{cases}$$

The demand of group i for the ad blocker is given by:

$$D_i^A(q, A) = \Pr\left(\theta \geq \frac{p}{q} \mid p \leq \gamma_i A\right) = \begin{cases} 1 - \frac{p}{q}, & \text{if } p \leq \gamma_i A \leq q \\ 0, & \text{otherwise.} \end{cases}$$

The first observation is that the publisher (ad blocker) will never choose A (p) in such a way that $p < \gamma_l A$ ($p > \gamma_h A$) because it implies getting zero profit and by setting A (p), for instance, very close to zero but strictly positive he (she) will get a better outcome. Therefore, if the equilibrium exists it must be true that $p \in [\gamma_l A, \gamma_h A]$.

Consider the publisher's decision. Since the game is simultaneous he takes his opponent's choice of p as given and chooses optimal A. Assume he chose A in such a way that $p \in [\gamma_l A, \gamma_h A]$. Then only low sensitivity group of consumers is served. For this problem to make sense it must be also true that $\gamma_l A \leq q$, otherwise the demand is zero since the utility of every consumer is negative. The publisher's profit maximization problem is, therefore:

$$\max_A \left\{(1-\alpha)\left(1 - \frac{\gamma_l A}{q}\right) p_a A\right\} \quad \text{s.t. } A \in \left[\frac{p}{\gamma_h}, \frac{p}{\gamma_l}\right] \text{ and } A \leq \frac{q}{\gamma_l}.$$

Computing first order conditions and taking the constraints into account we get the following best response function of the publisher:

$$A^{\star} = \begin{cases} \frac{q}{2\gamma_l}, & \text{if } \frac{q}{2\gamma_l} \in \left[\frac{p}{\gamma_h}, \frac{p}{\gamma_l}\right] \\ \frac{p}{\gamma_l}, & \text{if } \frac{q}{2\gamma_l} > \frac{p}{\gamma_l} \\ \frac{p}{\gamma_h}, & \text{if } \frac{q}{2\gamma_l} < \frac{p}{\gamma_h}. \end{cases}$$

Consider the ad blocker's decision. Assume she chose $p \in [\gamma_l A, \gamma_h A]$. The participation constraint of a consumer looks like: $\theta q - p \geq 0$ or $\theta \geq \frac{p}{q}$. Therefore, it must hold that $p \leq q$, otherwise there are no consumers for whom the utility is positive. The ad blocker solves the following problem:

$$\max_{p} \left\{ \alpha \left(1 - \frac{p}{q}\right) p \right\} \quad \text{s.t. } p \in [\gamma_l A, \gamma_h A] \text{ and } p \leq q.$$

Computing the first order conditions and taking the constraints into account, we get the following best response function of the ad blocker:

$$p^{\star} = \begin{cases} \frac{q}{2}, & \text{if } \frac{q}{2} \in [\gamma_l A, \gamma_h A] \\ \gamma_l A, & \text{if } \frac{q}{2} < \gamma_l A \\ \gamma_h A & \text{if } \frac{q}{2} > \gamma_h A. \end{cases}$$

There are only 2 points where the best responses intersect: $(A, p) = (0, 0)$ and $(A, p) = \left(\frac{q}{2\gamma_l}, \frac{q}{2}\right)$. The point $(0, 0)$ is clearly an equilibrium of the game. Neither party has a profitable deviation.

Let us show that the point $\left(\frac{q}{2\gamma_l}, \frac{q}{2}\right)$ is not an equilibrium. Assume the ad blocker reduces her price by a very small quantity. Then, she will get both groups of consumers. Her profit from this deviation can be made arbitrarily close to $\frac{q}{4}$ while her profit from not deviating is $\frac{\alpha q}{4}$. Therefore, she always would like to deviate unless $q = 0$ but this case is already included into $(0, 0)$. Hence, the only pure strategy Nash equilibrium of this game is $(A, p) = (0, 0)$. □

5.2 Proof of Propositions 2 and 3

Proof: Here we show that the profile of strategies $\left(\frac{q}{2\gamma_l}; \frac{q}{2}\right)$ is a Nash equilibrium of an infinitely repeated game given that the parties are patient enough (their discount factor, δ, is high) and they play grim trigger strategies such that if any one of them deviates in one period then the other one retaliates by setting $p = 0$ or $A = 0$ for the rest of the game. For this strategy profile to be an equilibrium neither party should consider it profitable to deviate and capture the whole market at the expense of all future interactions.

It is also important to note that this is the unique "efficient stationary equilibrium" as defined in the main text. In every stationary equilibrium without side-transfers between players it must be true that the publisher serves only the low sensitive group of consumers while the ad blocker serves the high sensitive group of consumers. Neither one of them can serve both groups because the other party will have incentives to deviate and capture one-period profits by slightly undercutting the other party. Thus the only "efficient" equilibrium is the one we consider because here, both parties are monopolists on their respective segments, hence there is no possibility to extract more surplus from the consumers.

The profits which both parties can get from playing $\left(\frac{q}{2\gamma_l}, \frac{q}{2}\right)$ are given by:

$$\pi^{\star}_{pub} = \frac{(1-\alpha)p_a q}{4\gamma_l}\frac{1}{1-\delta}, \quad \pi^{\star}_{adb} = \frac{\alpha q}{4}\frac{1}{1-\delta}.$$

The publisher only wants to deviate to a regime where he serves both groups. He solves:

$$\max_{A}\left\{\alpha\left(1-\frac{\gamma_h A}{q}\right)p_a A + (1-\alpha)\left(1-\frac{\gamma_l A}{q}\right)p_a A - cq^2\right\} \text{ s.t. } A \in \left[0, \frac{q}{2\gamma_h}\right] \text{ and } A \leq \frac{q}{\gamma_h},$$

where the first constraint ensures that both groups of consumers find it better to watch ads instead of buying the ad blocker and the second one ensures that some consumers in both groups receive non-negative utility from visiting the website. As we see the second constraint is redundant. By solving the first order conditions we can find that the constraint is binding, that is, $A = \frac{q}{2\gamma_h}$. Hence, the best deviation the publisher can make is to set $A = \frac{q}{2\gamma_h}$. If the publisher deviates, the ad blocker retaliates with a grim trigger strategy leading to 0 profits for both parties in all subsequent periods. Therefore, by deviating the publisher will only get one-period profit which equals to $\frac{p_a q}{4\gamma_h}\left(2-\alpha-\frac{(1-\alpha)\gamma_l}{\gamma_h}\right)$.

If the ad blocker ever deviates she will also switch into a regime where she serves both groups of consumers. She solves:

$$\max_p \left\{ \left(\alpha \left(1 - \frac{p}{q}\right) + (1-\alpha)\left(1 - \frac{p}{q}\right) \right) p \right\} = \max_p \left\{ p\left(1 - \frac{p}{q}\right) \right\} \text{ s.t. } p \in \left[0, \frac{q}{2}\right] \text{ and } p \leq q,$$

where the first constraint ensures that both groups of consumers find it better to use the ad blocker rather than watch ads and the second one ensures that some consumers in both groups receive non-negative utility from visiting the website. The second constraint is again redundant. By solving first order conditions we can find that the constraint is binding, that is, $p = \frac{q}{2}$. Hence, the best deviation the ad blocker can make is to set $p = \frac{q}{2}$. If the ad blocker deviates, the publisher retaliates with $A = 0$ leading to 0 profits for both parties in all subsequent periods. Therefore, by deviating the ad blocker will only get one-period profit which equals to $\frac{q}{4}$.

To ensure the existence of an equilibrium we need to make sure that the set of equilibrium profits is greater than the set of profits obtained from deviating for both parties. There are 2 possible cases which could happen for the publisher. First, the one-shot profit he obtains from deviating is smaller than the one-shot equilibrium profit. In this case, the publisher will never deviate regardless of the value of δ. It happens whenever the following holds:

$$\frac{(1-\alpha)p_a q}{4\gamma_l} \geq \frac{p_a q}{4\gamma_h}\left(2 - \alpha - \frac{(1-\alpha)\gamma_l}{\gamma_h}\right)$$

or, after simplification, when $\alpha \leq \frac{(\gamma_h - \gamma_l)^2}{(\gamma_h - \gamma_l)^2 + \gamma_l \gamma_h}$.

If the one-shot profit from deviating is bigger than the one-shot equilibrium profit then in order to sustain the equilibrium the publisher should value future strongly enough (*i.e.* have δ high enough). This happens when:

$$\frac{(1-\alpha)p_a q}{4\gamma_l}\frac{1}{1-\delta} \geq \frac{p_a q}{4\gamma_h}\left(2 - \alpha - \frac{(1-\alpha)\gamma_l}{\gamma_h}\right)$$

or, after simplification $\delta \geq \frac{(2-\alpha)\gamma_l\gamma_h - (1-\alpha)\gamma_l^2 - (1-\alpha)\gamma_h^2}{(2-\alpha)\gamma_l\gamma_h - (1-\alpha)\gamma_l^2}$.

The one-shot profit from deviation is always bigger than the one-shot equilibrium profit for the ad blocker. Hence, to sustain the equilibrium, the ad blocker should be patient enough:

$$\frac{\alpha q}{4}\frac{1}{1-\delta} \geq \frac{q}{4}$$

or after simplification $\delta \geq 1-\alpha$.

To prove proposition 3 we will now compute the quality choice by the publisher. Given his equilibrium profits as a function of q, he solves the following problem:

$$\max_q \left(\frac{1}{1-\delta} \frac{(1-\alpha)qp_a}{4\gamma_l} - cq^2 \right).$$

The first order condition gives us the expression for quality: $q^\star = \frac{(1-\alpha)p_a}{8c\gamma_l(1-\delta)}$. All other equilibrium values are obtained by substituting q into the formulas from Proposition 2. □

5.3 Proof of Propositions 1 and 4

PROOF: Given consumer demand and knowing the per-unit advertisement fee, f set by the ad blocker, the publisher chooses advertising volume, A. There are two cases to consider: both groups of consumers are served; only low sensitivity consumers are served.

Case 1: $\gamma_h A \leq q$. Both groups of consumers are served and the publisher solves:

$$\max_A \left\{ \alpha \left(1 - \frac{\gamma_h A}{q}\right) p_a A + (1-\alpha)\left(1 - \frac{\gamma_l A}{q}\right) p_a A - fA \right\} \quad \text{s.t. } 0 \leq A \leq \frac{q}{\gamma_h}.$$

Taking F.O.C.s we obtain the following optimal choice of A with the corresponding profits:

$$(A^\star;\pi^\star) = \begin{cases} \left(\frac{q(p_a-f)}{2p_a\bar{\gamma}}; \frac{q(p_a-f)^2}{4p_a\bar{\gamma}}\right), & \begin{cases} \text{if } \left\{ f \in \left[p_a\left(1-\frac{2\bar{\gamma}}{\gamma_h}\right), p_a\right], \alpha < \frac{\gamma_h-2\gamma_l}{2(\gamma_h-\gamma_l)}, \gamma_h \geq 2\gamma_l \right\} \text{ or} \\ \{\gamma_h < 2\gamma_l, p_a \geq f\} \text{ or } \left\{\gamma_h \geq 2\gamma_l, \alpha \geq \frac{\gamma_h-2\gamma_l}{2(\gamma_h-\gamma_l)}, p_a \geq f\right\} \end{cases} \\ \left(\frac{q}{\gamma_h}; \frac{q}{\gamma_h}\left(p_a - f - \frac{p_a\bar{\gamma}}{\gamma_h}\right)\right), & \text{if } f \in \left[0, p_a\left(1-\frac{2\bar{\gamma}}{\gamma_h}\right)\right), \alpha < \frac{\gamma_h-2\gamma_l}{2(\gamma_h-\gamma_l)}, \gamma_h \geq 2\gamma_l \\ (0;0), & \text{if } p_a < f. \end{cases}$$

Case 2: $\gamma_l A \leq q < \gamma_h A$. Only the low sensitivity group is served and the publisher solves:

$$\max_A \left\{ (1-\alpha)\left(1 - \frac{\gamma_l A}{q}\right) p_a A - fA \right\} \quad \text{s.t. } \frac{q}{\gamma_h} < A \leq \frac{q}{\gamma_l}.$$

The first-order conditions and the constraint imply:

$$(A^\star;\pi^\star) = \begin{cases} \left(\frac{q((1-\alpha)p_a-f)}{2p_a(1-\alpha)\gamma_l}; \frac{q((1-\alpha)p_a-f)^2}{4p_a(1-\alpha)\gamma_l}\right), & \text{if } f \leq (1-\alpha)p_a\left[1-\frac{2\gamma_l}{\gamma_h}\right], \gamma_h \geq 2\gamma_l \\ \left(\frac{q}{\gamma_h}; \frac{q}{\gamma_h}\left((1-\alpha)p_a\left[1-\frac{\gamma_l}{\gamma_h}\right] - f\right)\right), & \text{if } \left\{ f > (1-\alpha)p_a\left[1-\frac{2\gamma_l}{\gamma_h}\right], \gamma_h \geq 2\gamma_l \right\} \text{ or } \gamma_h < 2\gamma_l \end{cases}$$

Directly comparing profits (calculations omitted), the aggregate profit for both cases is:

$$\pi^\star = \begin{cases} \frac{q((1-\alpha)p_a - f)^2}{4p_a(1-\alpha)\gamma_l}, & \text{if } f < \frac{(1-\alpha)\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}} p_a \\ \frac{q(p_a - f)^2}{4p_a\bar{\gamma}}, & \text{if } f \in \left[\frac{(1-\alpha)\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}} p_a, p_a\right] \\ 0, & \text{if } f > p_a. \end{cases}$$

Now we are ready to prove Proposition 1. Define $\hat{\gamma} = \sqrt{\frac{\gamma_l}{\bar{\gamma}}}$. We substitute $f = 0$ into the profit function above:

$$\pi^\star = \begin{cases} \frac{(1-\alpha)qp_a}{4\gamma_l}, & \text{if } \hat{\gamma} < \sqrt{1-\alpha} \\ \frac{qp_a}{4\bar{\gamma}}, & \text{if } \hat{\gamma} \geq \sqrt{1-\alpha}. \end{cases}$$

The publisher chooses quality by maximizing $\frac{1}{1-\delta}\pi^\star$ (that is, the long term profits) minus the cost cq^2. This is a simple second degree polynomial. The optimal q is:

$$q^\star = \begin{cases} \frac{(1-\alpha)p_a}{8c\gamma_l(1-\delta)}, & \text{if } \hat{\gamma} < \sqrt{1-\alpha} \\ \frac{p_a}{8c\bar{\gamma}(1-\delta)}, & \text{if } \hat{\gamma} \geq \sqrt{1-\alpha}. \end{cases}$$

All other key quantities listed in Proposition 1 are obtained by substituting q.

Now, let us go back to Proposition 4. The ad blocker chooses its fee, f, anticipating the level of advertising, A by the publisher. It is easy to see that the ad blocker will never choose $f > p_a$ since it will make the publisher choose zero advertising and, hence, will lead to zero profits for the ad blocker. Hence, there are only two cases left which we consider separately.

Case 1. $f \in \left[\frac{(1-\alpha)\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}} p_a, p_a\right]$. The ad blocker solves:

$$\max_f \left\{ f \frac{q(p_a - f)}{2p_a\bar{\gamma}} \right\} \quad \text{s.t.} \quad \frac{(1-\alpha)\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}} p_a \leq f \leq p_a.$$

The first order conditions give us the following expression for the optimal value of f and the corresponding profits π. If $\sqrt{(1-\alpha)\bar{\gamma}} > \sqrt{\gamma_l}$ (that is, $\frac{(1-\alpha)\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}} p_a > 0$), then:

$$(f^\star; \pi^\star) = \begin{cases} \left(\frac{p_a}{2}; \frac{p_a q}{8\bar{\gamma}}\right), & \text{if } \sqrt{\gamma_l} \geq \frac{1-2\alpha}{\sqrt{1-\alpha}}\sqrt{\bar{\gamma}} \\ \left(\frac{(1-\alpha)\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}} p_a; \frac{\alpha q p_a \left[(1-\alpha)\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}\right]}{2\sqrt{\bar{\gamma}}\left(\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}\right)^2}\right), & \text{if } \sqrt{\gamma_l} < \frac{1-2\alpha}{\sqrt{1-\alpha}}\sqrt{\bar{\gamma}}. \end{cases}$$

If $\sqrt{(1-\alpha)\bar{\gamma}} \leq \sqrt{\gamma_l}$ (that is, $\frac{(1-\alpha)\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}} p_a \leq 0$), then the constraint cannot be binding and we get:

$$(f^\star; \pi^\star) = \left(\frac{p_a}{2}; \frac{p_a q}{8\bar{\gamma}}\right).$$

Case 2. $0 \le f \le \frac{(1-\alpha)\sqrt{\bar{\gamma}}-\sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}}-\sqrt{(1-\alpha)\gamma_l}}p_a$. The ad blocker solves:

$$\max_f \left\{ f\frac{q((1-\alpha)p_a - f)}{2p_a(1-\alpha)\gamma_l} \right\} \quad \text{s.t. } 0 \le f \le \frac{(1-\alpha)\sqrt{\bar{\gamma}}-\sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}}-\sqrt{(1-\alpha)\gamma_l}}p_a.$$

This case is well defined if and only if:

$$\frac{(1-\alpha)\sqrt{\bar{\gamma}}-\sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}}-\sqrt{(1-\alpha)\gamma_l}}p_a \ge 0 \Leftrightarrow \sqrt{\gamma_l} \le \sqrt{(1-\alpha)\bar{\gamma}}.$$

The first order conditions lead to the following choice of f and the corresponding profits:

$$(f^\star;\pi^\star) = \begin{cases} \left(\frac{(1-\alpha)p_a}{2};\frac{(1-\alpha)p_a q}{8\gamma_l}\right), & \text{if } \sqrt{\gamma_l} \le \frac{\sqrt{1-\alpha}}{1+\alpha}\sqrt{\bar{\gamma}} \\ \left(\frac{(1-\alpha)\sqrt{\bar{\gamma}}-\sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}}-\sqrt{(1-\alpha)\gamma_l}}p_a;\frac{\alpha q p_a\left[(1-\alpha)\sqrt{\bar{\gamma}}-\sqrt{(1-\alpha)\gamma_l}\right]}{2\sqrt{(1-\alpha)\gamma_l}\left(\sqrt{\bar{\gamma}}-\sqrt{(1-\alpha)\gamma_l}\right)^2}\right), & \text{if } \sqrt{\gamma_l} > \frac{\sqrt{1-\alpha}}{1+\alpha}\sqrt{\bar{\gamma}}. \end{cases}$$

The combined profit function is the maximum of the two profit functions above. When $\frac{(1-\alpha)\sqrt{\bar{\gamma}}-\sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}}-\sqrt{(1-\alpha)\gamma_l}}p_a < 0$, the combined profit is $\pi^\star = \frac{p_a q}{8\bar{\gamma}}$ since Case 2 is not defined.

Now assume that $\frac{(1-\alpha)\sqrt{\bar{\gamma}}-\sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}}-\sqrt{(1-\alpha)\gamma_l}}p_a \ge 0$. Then, we can have 3 regions depending on the parameters. We consider those regions one by one.

Region 1. $\sqrt{\gamma_l} < \frac{1-2\alpha}{\sqrt{1-\alpha}}\sqrt{\bar{\gamma}}$.

$$\frac{\alpha q p_a\left[(1-\alpha)\sqrt{\bar{\gamma}}-\sqrt{(1-\alpha)\gamma_l}\right]}{2\sqrt{\gamma_E}\left(\sqrt{\bar{\gamma}}-\sqrt{(1-\alpha)\gamma_l}\right)^2} \text{ vs. } \frac{(1-\alpha)p_a q}{8\gamma_l}.$$

Tedious calculations yield that $\frac{(1-\alpha)p_a q}{8\gamma_l}$ is always bigger.

Region 2. $\frac{1-2\alpha}{\sqrt{1-\alpha}}\sqrt{\bar{\gamma}} \le \sqrt{\gamma_l} \le \frac{\sqrt{1-\alpha}}{1+\alpha}\sqrt{\bar{\gamma}}$.

$$\frac{p_a q}{8\bar{\gamma}} \text{ vs. } \frac{(1-\alpha)p_a q}{8\gamma_l},$$

which is equivalent to:

$$\sqrt{\gamma_l} \text{ vs. } \sqrt{(1-\alpha)\bar{\gamma}}.$$

The right hand side is bigger by assumption (otherwise, one of the optimization problems is ill defined). Therefore, in this region, the profit of the ad blocker is $\frac{(1-\alpha)p_a q}{8\gamma_l}$.

Region 3. $\frac{\sqrt{1-\alpha}}{1+\alpha}\sqrt{\bar{\gamma}} < \sqrt{\gamma_l} \leq \sqrt{(1-\alpha)\bar{\gamma}}$.

$$\frac{p_a q}{8\bar{\gamma}} \text{ vs. } \frac{\alpha q p_a \left[(1-\alpha)\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}\right]}{2\sqrt{(1-\alpha)\gamma_l}\left(\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}\right)^2}$$

Define $x^{\star}(\alpha)$ as the solution to the equation:

$$\frac{1+4\alpha}{\sqrt{1-\alpha}}x + \sqrt{1-\alpha}x^3 - 2x^2 - 4\alpha = 0.$$

We can show that this equation always has the unique root within Region 3. Also, the polynomial is positive whenever $\hat{\gamma}$ is greater than the root and negative whenever it is smaller than the root. This implies that the optimal profit of the ad blocker is $\frac{p_a q}{8\bar{\gamma}}$ when $\sqrt{\gamma_l} \geq x^{\star}(\alpha)\sqrt{\bar{\gamma}}$ and $\frac{\alpha q p_a \left[(1-\alpha)\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}\right]}{2\sqrt{(1-\alpha)\gamma_l}\left(\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}\right)^2}$ when $\sqrt{\gamma_l} < x^{\star}(\alpha)\sqrt{\bar{\gamma}}$.

Now we can finally write down the optimal fee and profit of the ad blocker:

$$(f^{\star}; \pi^{\star}_{adb}) = \begin{cases} \left(\frac{p_a}{2}; \frac{p_a q}{8\bar{\gamma}}\right), & \text{if } \sqrt{\gamma_l} \geq x^{\star}(\alpha)\sqrt{\bar{\gamma}} \\ \left(\frac{(1-\alpha)p_a}{2}; \frac{(1-\alpha)p_a q}{8\gamma_l}\right), & \text{if } \sqrt{\gamma_l} \leq \frac{\sqrt{1-\alpha}}{1+\alpha}\sqrt{\bar{\gamma}} \\ \left(\frac{(1-\alpha)\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}}p_a; \frac{\alpha q p_a \left[(1-\alpha)\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}\right]}{2\sqrt{(1-\alpha)\gamma_l}\left(\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}\right)^2}\right), & \text{if } \frac{\sqrt{1-\alpha}}{1+\alpha}\sqrt{\bar{\gamma}} < \sqrt{\gamma_l} < x^{\star}(\alpha)\sqrt{\bar{\gamma}}. \end{cases}$$

Interestingly, this third fee (i.e. $\frac{(1-\alpha)\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}}p_a$) leads to the intriguing result that can be observed on Figure 3: for small α the region where only the low sensitive segments gets served actually increases in α. This happens because the ad blocker chooses the fee in such a way as to keep the publisher indifferent between serving both segments of consumers or only the low sensitive segment (we assume the publisher serves only the low sensitive segment in such a case). If the publisher swithced to serving both segments it would mean that A will fall which is detrimental to the ad blocker. However, to keep the publisher from serving both groups the ad blocker has to compensate the publisher with the lower fee (see Figure 4). After some point, though, this becomes unsustainable.

Now let us go a step back and substitute the choice of f into the publisher's profit:

$$\pi^{\star}_{pub} = \begin{cases} \frac{q p_a}{16\bar{\gamma}}, & \text{if } \sqrt{\gamma_l} \geq x^{\star}(\alpha)\sqrt{\bar{\gamma}} \\ \frac{q p_a (1-\alpha)}{16\gamma_l}, & \text{if } \sqrt{\gamma_l} \leq \frac{\sqrt{1-\alpha}}{1+\alpha}\sqrt{\bar{\gamma}} \\ \frac{q\alpha^2 p_a}{4\left(\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}\right)^2}, & \text{if } \frac{\sqrt{1-\alpha}}{1+\alpha}\sqrt{\bar{\gamma}} < \sqrt{\gamma_l} < x^{\star}(\alpha)\sqrt{\bar{\gamma}}. \end{cases}$$

With these expressions, we can finally solve for quality to find all equilibria of the game.

Case 1: $\sqrt{\gamma_l} \geq x^\star(\alpha)\sqrt{\bar{\gamma}}$. The publisher solves:

$$\max_q \left\{ \frac{1}{1-\delta} \frac{qp_a}{16\bar{\gamma}} - cq^2 \right\}.$$

The first order conditions give us $q^\star = \frac{p_a}{32c\bar{\gamma}(1-\delta)}$.

Case 2: $\sqrt{\gamma_l} \leq \frac{\sqrt{1-\alpha}}{1+\alpha}\sqrt{\bar{\gamma}}$. The publisher solves:

$$\max_q \left\{ \frac{1}{1-\delta} \frac{qp_a(1-\alpha)}{16\gamma_l} - cq^2 \right\}.$$

The first order conditions give us: $q^\star = \frac{(1-\alpha)p_a}{32c\gamma_l(1-\delta)}$.

Case 3: $\frac{\sqrt{1-\alpha}}{1+\alpha}\sqrt{\bar{\gamma}} < \sqrt{\gamma_l} < x^\star(\alpha)\sqrt{\bar{\gamma}}$. The publisher solves:

$$\max_q \left\{ \frac{1}{1-\delta} \frac{q\alpha^2 p_a}{4\left(\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}\right)^2} - cq^2 \right\}$$

The first order conditions give us: $q^\star = \frac{\alpha^2 p_a}{8c\left(\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}\right)^2(1-\delta)}$.

This provides all three equilibria of the negotiation game. □

5.4 Proof of Proposition 5

PROOF: Let us see when the publisher chooses to negotiate. First, assume that the fee in the negotiation subgame happened to be $\frac{(1-\alpha)\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}} p_a$. Then we compare:

$$\frac{1}{1-\delta} \frac{q\alpha^2 p_a}{4\left(\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}\right)^2} \quad \text{vs.} \quad \frac{1}{1-\delta} \frac{(1-\alpha)qp_a}{4\gamma_l},$$

which boils down to $\sqrt{\gamma_l}$ vs. $\sqrt{(1-\alpha)\bar{\gamma}}$. The left hand side is always smaller than the right hand side (otherwise, $\frac{(1-\alpha)\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}}{\sqrt{\bar{\gamma}} - \sqrt{(1-\alpha)\gamma_l}} p_a$ cannot be an equilibrium price). Hence, in this case, the publisher chooses not to negotiate.

Now assume that the equilibrium fee happens to be $f = (1-\alpha)p_a/2$. Then, we compare:

$$\frac{1}{1-\delta} \frac{qp_a(1-\alpha)}{16\gamma_l} \quad \text{vs.} \quad \frac{1}{1-\delta} \frac{qp_a(1-\alpha)}{4\gamma_l}.$$

The publisher never chooses to negotiate either.

Finally assume that the equilibrium fee happens to be $p_a/2$. Then, we compare:

$$\frac{1}{1-\delta}\frac{qp_a}{16\bar{\gamma}} \quad \text{vs.} \quad \frac{1}{1-\delta}\frac{qp_a(1-\alpha)}{4\gamma_l} \Leftrightarrow \hat{\gamma} \text{ vs. } 2\sqrt{1-\alpha}.$$

The only set of parameters under which the publisher would like to negotiate is $\hat{\gamma} \geq 2\sqrt{1-\alpha}$. The ad blocker compares:

$$\frac{1}{1-\delta}\frac{p_a q}{8\bar{\gamma}} \quad \text{vs.} \quad \frac{1}{1-\delta}\frac{\alpha q}{4}$$

and the ad blocker chooses to negotiate when $p_a \geq 2\alpha\bar{\gamma}$.

To summarize we can say that there are only 2 equilibria in the metagame. Negotiation occurs when $\hat{\gamma} \geq 2\sqrt{1-\alpha}$ and $p_a \geq 2\alpha\bar{\gamma}$. There is no negotiation otherwise. $\square$

5.5 Proof of Corollary 1

Proof: In what follows (and in Proposition 6 as well), we assume that δ is high enough that Proposition 2 holds. We first compute the consumer surpluses for 4 possible situations: benchmark with only low sensitivity group served; benchmark with both groups served; negotiation; no-negotiation.

The consumer surplus of the group $i \in \{l, h\}$ that does not use the ad blocker is:

$$\int_{\frac{\gamma_i A}{q}}^{1} (\theta q - \gamma_i A) d\theta = \frac{q\theta^2}{2} - \gamma_i A\theta|_{\frac{\gamma_i A}{q}}^{1} = \frac{q}{2} - \gamma_i A + \frac{\gamma_i^2 A^2}{2q}.$$

The consumer surplus of the group $i \in \{l, h\}$ that uses the ad blocker is:

$$\int_{\frac{f}{q}}^{1} (\theta q - p) d\theta = \frac{q\theta^2}{2} - p\theta|_{\frac{p}{q}}^{1} = \frac{q}{2} - p + \frac{p^2}{2q}.$$

The total consumer surplus in **baseline case with both groups** served is

$$\frac{p_a}{16c\bar{\gamma}(1-\delta)}\left(\frac{\alpha\gamma_h^2}{4\bar{\gamma}^2} + \frac{(1-\alpha)\gamma_l^2}{4\bar{\gamma}^2}\right).$$

The total consumer surplus in **baseline case with only low sensitivity group** served:

$$\frac{(1-\alpha)^2 p_a}{64c\gamma_l(1-\delta)}.$$

The total consumer surplus in the **negotiation case**:

$$\frac{p_a}{64c\bar{\gamma}(1-\delta)}\left(\frac{1}{2}+\frac{\alpha\gamma_h^2}{16\bar{\gamma}^2}+\frac{(1-\alpha)\gamma_l^2}{16\bar{\gamma}^2}\right).$$

The total consumer surplus in the **no-negotiation** case:

$$\frac{(1-\alpha)p_a}{64c\gamma_l(1-\delta)}.$$

An important observation is that if the parties negotiate in the meta game then in the benchmark case both groups of consumers must be served ($\hat{\gamma} > 2\sqrt{1-\alpha} \Rightarrow \hat{\gamma} > \sqrt{1-\alpha}$). This eliminates one out of 4 scenarios. Another straightforward comparison shows that the no-negotiation case is always preferable to the benchmark case with only the low sensitivity group served.

Compare the negotiation case with the benchmark where both groups are served:

$$\frac{p_a}{16c\bar{\gamma}(1-\delta)}\left(\frac{\alpha\gamma_h^2}{4\bar{\gamma}^2}+\frac{(1-\alpha)\gamma_l^2}{4\bar{\gamma}^2}\right) \text{ vs. } \frac{p_a}{64c\bar{\gamma}(1-\delta)}\left(\frac{1}{2}+\frac{\alpha\gamma_h^2}{16\bar{\gamma}^2}+\frac{(1-\alpha)\gamma_l^2}{16\bar{\gamma}^2}\right) \Leftrightarrow$$

$$\Leftrightarrow 15(\alpha\gamma_h^2+(1-\alpha)\gamma_l^2) \text{ vs. } 8\bar{\gamma}^2.$$

We can show that the left hand side is bigger than the right hand side. Therefore, banning the ad blocker is better in this case for the consumers.

Finally, the no-negotiation case and the benchmark case with both groups of consumers served. We compare:

$$\frac{p_a}{16c\bar{\gamma}(1-\delta)}\left(\frac{\alpha\gamma_h^2}{4\bar{\gamma}^2}+\frac{(1-\alpha)\gamma_l^2}{4\bar{\gamma}^2}\right) \text{ vs. } \frac{(1-\alpha)p_a}{64c\gamma_l(1-\delta)} \Leftrightarrow$$

$$\Leftrightarrow \alpha\gamma_l\gamma_h^2+(1-\alpha)\gamma_l^3 \text{ vs. } (1-\alpha)\bar{\gamma}^3.$$

Again, it can be shown that the left hand side is always bigger than the right hand side. That means that consumers, again, win from having the ad blocker banned. □

5.6 Proof of Proposition 6

PROOF: The total surplus (including the publisher's profit is) in the **baseline case where both groups** are served:

$$\frac{p_a}{16c\bar{\gamma}(1-\delta)^2}\left(\frac{\alpha\gamma_h^2}{4\bar{\gamma}^2}+\frac{(1-\alpha)\gamma_l^2}{4\bar{\gamma}^2}+\frac{p_a}{4\bar{\gamma}}\right).$$

The total surplus in the **baseline case where only the low sensitivity group** is served:

$$\frac{(1-\alpha)^2 p_a}{64c\gamma_l(1-\delta)^2}\left(1+\frac{p_a}{\gamma_l}\right).$$

The total surplus in the **negotiation** case:

$$\frac{p_a}{64c\bar{\gamma}(1-\delta)^2}\left(\frac{1}{2}+\frac{\alpha\gamma_h^2}{16\bar{\gamma}^2}+\frac{(1-\alpha)\gamma_l^2}{16\bar{\gamma}^2}+\frac{p_a}{16\bar{\gamma}}+\frac{p_a}{4\bar{\gamma}}\right).$$

The total surplus in the **no-negotiation** case:

$$\frac{(1-\alpha)p_a}{64c\gamma_l(1-\delta)^2}\left(1+\frac{(1-\alpha)p_a}{\gamma_l}+2\alpha\right).$$

There are again only 3 cases due to the inconsistency between inequalities and the no-negotiation case is always preferable to the benchmark case with only the low sensitivity group served.

Comparing the negotiation case with the benchmark where both groups are served yields:

$$\frac{p_a}{16c\bar{\gamma}(1-\delta)^2}\left(\frac{\alpha\gamma_h^2}{4\bar{\gamma}^2}+\frac{(1-\alpha)\gamma_l^2}{4\bar{\gamma}^2}+\frac{p_a}{4\bar{\gamma}}\right) \text{ vs. } \frac{p_a}{64c\bar{\gamma}(1-\delta)^2}\left(\frac{1}{2}+\frac{\alpha\gamma_h^2}{16\bar{\gamma}^2}+\frac{(1-\alpha)\gamma_l^2}{16\bar{\gamma}^2}+\frac{p_a}{16\bar{\gamma}}+\frac{p_a}{4\bar{\gamma}}\right) \Leftrightarrow$$

$$\Leftrightarrow p_a \text{ vs. } \frac{8\bar{\gamma}^2-15(\alpha\gamma_h^2+(1-\alpha)\gamma_l^2)}{11\bar{\gamma}}.$$

We can show that the left hand side is always bigger than the right hand side. Therefore, banning the ad blocker is better in this case.

Finally, consider the no-negotiation case and the benchmark case with both groups of consumers served. We compare:

$$\frac{p_a}{16c\bar{\gamma}(1-\delta)^2}\left(\frac{\alpha\gamma_h^2}{4\bar{\gamma}^2}+\frac{(1-\alpha)\gamma_l^2}{4\bar{\gamma}^2}+\frac{p_a}{4\bar{\gamma}}\right) \text{ vs. } \frac{(1-\alpha)p_a}{64c\gamma_l(1-\delta)^2}\left(1+\frac{(1-\alpha)p_a}{\gamma_l}+2\alpha\right) \Leftrightarrow$$

$$\Leftrightarrow p_a \text{ vs. } \frac{(1-\alpha)(1+2\alpha)\gamma_l\bar{\gamma}^3 - \alpha\gamma_l^2\gamma_h^2 - (1-\alpha)\gamma_l^4}{\gamma_l^2\bar{\gamma} - (1-\alpha)^2\bar{\gamma}^3}.$$

First, if $\hat{\gamma} > 2\sqrt{1-\alpha}$, then the fraction above is negative meaning that prohibiting the ad blocker is always better. Second, if $\hat{\gamma} \leq 2\sqrt{1-\alpha}$, then the above inequality determines which regime is better, that is, allowing the ad blocker is better when:

$$p_a \leq \frac{(1-\alpha)(1+2\alpha)\gamma_l\bar{\gamma}^3 - \alpha\gamma_l^2\gamma_h^2 - (1-\alpha)\gamma_l^4}{\gamma_l^2\bar{\gamma} - (1-\alpha)^2\bar{\gamma}^3}.$$

$\square$