When and How Should Firms Differentiate? Quality, Advertising and Pricing Decisions in a Duopoly

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Abstract

One of the hallmarks of competitive interaction is each firm’s desire to differentiate from rivals. Although differentiation may be achieved through product related choices, advertising levels may constitute another key mechanism. In this paper, we examine under what conditions firms will elect to differentiate through product quality vs. advertising intensity and characterize the set of equilibria that emerge. Consumers can only purchase from the set of products they are informed about through advertising, and choose the alternative that maximizes their utility. Firms select product quality in a first stage, advertising levels in a second stage, and prices in the last stage. We study two forms of advertising—blanket and targeted. Under blanket advertising, firms communicate indiscriminately and a consumer’s probability of seeing an ad depends on the level of ad expenditure. We find that when advertising is ineffective, i.e., the additional awareness generated by a heavy level is modest relative to the cost, both firms choose a light ad spending. This allows them to minimally differentiate in qualities without concern of intense price competition, as each firm expects to have a segment of ‘captive’ consumers that are only aware of its product. When advertising is moderately effective, one firm shifts to expending heavily on advertising, hence all consumers are aware of its product. However, the rival prefers to differentiate by advertising lightly, while choosing the same maximal quality level. This strategy softens price competition by inducing the heavy-advertiser to price highly more often in order to capitalize on its captive segment and allows the light-advertiser to increase its average price. Interestingly, we show that even if advertising heavily entails no extra cost, one firm will choose to advertise lightly in equilibrium for strategic reasons. When advertising is very effective, both firms advertise heavily. In this scenario, firms must differentiate in qualities in order to achieve positive profits. Under targeted advertising, we let firms choose the segment(s) they wish to inform. We identify conditions such that both firms choose equally high quality products, but advertise to distinct segments; thereby achieving differentiation through ad targeting. We further show that this can result in a pocket of unserved consumers, even though consumers with lower willingness to pay purchase. Generally speaking, we show that allowing market awareness to be determined endogenously suggests far less product differentiation than previously suspected and reveals regions where advertising creates viable differentiation.

(Product Quality, Advertising, Differentiation, Pricing, Competition)
1 Introduction

In most markets, firms must figure out how to contend with competitors. The presence of rivals means that consumers have multiple offerings to choose from, and hence demand for each company is by no means guaranteed. As they seek to navigate this challenge, firms typically consider ways to set themselves apart from the competition in order to avoid a “race to the bottom” in prices that would erode any and all profits. A common approach to softening such detrimental competitive intensity is through product positioning. In particular, vertical product differentiation may help avoid the dreaded “head to head” battle with rivals and allow each firm to carve out its own demand in the marketplace. Several examples of such a strategy come to mind. In the hybrid-electric vehicle market between 2003-2013, for instance, Toyota’s Prius had several advantages over other hybrids, such as the Honda Civic hybrid (e.g., in miles per gallon, battery life, and handling).1 And, as assessed by Consumer Reports (on a 100-point scale), a host of categories such as vacuum cleaners, oven ranges, laptops and fitness trackers, exhibit a substantial amount of quality variation between high- and low-end models (see top panel of Table 1).

However, causal empiricism suggests that there are also numerous industry contexts in which firms’ products are quite similar in quality and yet they are able to achieve positive profits. Cordless drills are one such instance where several brands, such as Makita, DeWalt and Milwaukee, offer products with similar specs.2 Furthermore, as assessed by Consumer Reports, categories such as lawn mowers, snow blowers, washers, and dryers seem to exhibit a relatively small degree of quality dispersion across models (see bottom panel of Table 1). Thus, it is likely the case that these firms are withstanding competitive pressures by “differentiating” through other strategic choices, despite having the option of selecting dissimilar qualities.

Two prominent actions, aside from product, that firms have at their disposal are advertising and pricing. The former is presumably intended to help firms communicate their offering, while the latter affects the value they can extract from customers (vs. leaving more of the value created with customers). Yet as with product, here too one observes a range of behaviors. Specifically, some markets are characterized by firms selecting similar levels of advertising, whereas other mar-

1 See, for example, Reynolds, “Hybrid Sedan Comparison: 2006 Honda Civic Hybrid vs. 2006 Toyota Prius,” in MotorTrend, August 11, 2006; and Edmunds (https://www.edmunds.com/toyota/prius/2005/comparison-test.html).
2 For example, review website justcordlessreviews.com gives these three brands similar ratings of 4.8, 4.7, 4.9, respectively (accessed June 2018).
Table 1: Quality and Price Dispersion for Select Categories (Based on Consumer Reports, 2017)

<table>
<thead>
<tr>
<th>Product</th>
<th>Average Quality</th>
<th>Standard Dev. Quality</th>
<th>Average Price</th>
<th>Standard Dev. Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large Quality Dispersion</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computers, Laptop: 10- to 11-inch</td>
<td>55.4</td>
<td>13.3</td>
<td>$385</td>
<td>290.3</td>
</tr>
<tr>
<td>Ranges: gas and dual-fuel, single oven (30&quot;)</td>
<td>58.9</td>
<td>12.6</td>
<td>$1,437</td>
<td>788.7</td>
</tr>
<tr>
<td>Computers, Laptop: 11- to 15-inch</td>
<td>68.8</td>
<td>11.6</td>
<td>$843</td>
<td>318.7</td>
</tr>
<tr>
<td>Ranges: smoothtop, double oven (30&quot;)</td>
<td>72.8</td>
<td>10.7</td>
<td>$1,658</td>
<td>428.6</td>
</tr>
<tr>
<td>Dishwashers</td>
<td>70.4</td>
<td>9.7</td>
<td>$854</td>
<td>407.3</td>
</tr>
<tr>
<td>Fitness Trackers: built-in data readout</td>
<td>72.6</td>
<td>9.6</td>
<td>$153</td>
<td>56.3</td>
</tr>
<tr>
<td>Vacuum Cleaners: bagless upright</td>
<td>51.9</td>
<td>9.3</td>
<td>$215</td>
<td>112.8</td>
</tr>
<tr>
<td>Small Quality Dispersion</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Snow Blowers: two-stage gas</td>
<td>87.1</td>
<td>3.8</td>
<td>$1,235</td>
<td>266.7</td>
</tr>
<tr>
<td>Generators: large stationary</td>
<td>90.75</td>
<td>3.3</td>
<td>$3,950</td>
<td>1618.5</td>
</tr>
<tr>
<td>Dryers: gas</td>
<td>78</td>
<td>2.7</td>
<td>$1,127</td>
<td>227.9</td>
</tr>
<tr>
<td>Dryers: electric</td>
<td>78.1</td>
<td>2.6</td>
<td>$1,044</td>
<td>241.8</td>
</tr>
<tr>
<td>Washers: front loaders</td>
<td>82.7</td>
<td>1.5</td>
<td>$1,134</td>
<td>287.4</td>
</tr>
<tr>
<td>Lawn Mowers: gas, self-propel</td>
<td>21.2</td>
<td>1.2</td>
<td>$298</td>
<td>36.6</td>
</tr>
<tr>
<td>Lawn Mowers: battery-powered</td>
<td>19.5</td>
<td>0.7</td>
<td>$425</td>
<td>47.2</td>
</tr>
</tbody>
</table>

Kets exhibit asymmetric levels across firms. For example, home improvement retailers Home Depot and Lowe’s spent similar amounts on advertising in 2017, while major wireless carriers spent very different amounts (see Table 2 for these and other examples). Moreover, there can be considerable heterogeneity between industries in terms of the ball park level of advertising spending. For instance, the automotive industry expends the most on measured media ads (over $17 billion in 2017), while the household supplies category only spends about a third as much.³

Likewise with pricing, in some categories one observes fairly stable prices over time while in others frequent price promotions are common (Mohammed 2011). There can also be variations within industry— as several department store retailers tend to play every day low pricing (or EDLP) and offer discounts over a small range of prices (e.g., Walmart), while others tend to offer more frequent and variable price promotions (or Hi-Lo pricing, e.g., Kohl’s); and some computer makers

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are well known for maintaining price consistency and seldom discount (e.g., Apple, see Stuart 2011), whereas other players tend to cut prices frequently and deeply (e.g., Lenovo).

Table 2: Advertising Spending in 2017 in Select Categories*

<table>
<thead>
<tr>
<th>Category</th>
<th>Company</th>
<th>Total U.S. advertising spend (millions of $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Improvement Retail</td>
<td>Home Depot</td>
<td>911</td>
</tr>
<tr>
<td></td>
<td>Lowe’s</td>
<td>893</td>
</tr>
<tr>
<td>Cruise Lines</td>
<td>Norwegian Cruise Lines</td>
<td>223</td>
</tr>
<tr>
<td></td>
<td>Royal Caribbean Cruises</td>
<td>213</td>
</tr>
<tr>
<td>Wireless Carriers</td>
<td>AT&amp;T</td>
<td>3,520</td>
</tr>
<tr>
<td></td>
<td>Verizon</td>
<td>2,643</td>
</tr>
<tr>
<td></td>
<td>T-Mobile</td>
<td>1,800</td>
</tr>
<tr>
<td></td>
<td>Sprint</td>
<td>1,300</td>
</tr>
<tr>
<td>Insurance</td>
<td>Geico</td>
<td>1,503</td>
</tr>
<tr>
<td></td>
<td>Progressive Corp.</td>
<td>911</td>
</tr>
<tr>
<td></td>
<td>State Farm Mutual</td>
<td>860</td>
</tr>
<tr>
<td></td>
<td>Allstate Corp.</td>
<td>717</td>
</tr>
<tr>
<td></td>
<td>Liberty Mutual</td>
<td>451</td>
</tr>
<tr>
<td></td>
<td>Nationwide Mutual</td>
<td>283</td>
</tr>
</tbody>
</table>

* Dark-shaded categories exhibit similar ad levels across firms; light-shaded categories exhibit considerable heterogeneity. (Source: AdAge, Leading National Advertisers, June, 2018)

Complicating matters, there is sometimes an observed interaction between product quality and ad levels, with some studies finding that higher quality firms also advertise more extensively than lower quality rivals (e.g., Archibald et al. 1983, Tellis and Fornell 1988), while in other cases no such systematic empirical relationship was found (e.g., Kash and Miller 2009, Caves and Greene 1996). One must also bear in mind that in some cases firms can be selective in their advertising efforts. For example, by compiling a list of consumers that meet certain criteria firms can send targeted e-mails or direct mail, and in B2B settings firms can dispatch their sales force to a subset of potential customers with common characteristics. Digital advertising (e.g., on Facebook or Instagram) also provides a number of opportunities to define target segments in detail.4 In other cases, firms cast

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4Most social media platforms allow firms to specify certain filters based on a person’s profile, past brows-
a much wider net and use media outlets that preclude direct control over who sees their ads. For example, by running a commercial on national TV (e.g., on the Superbowl) or placing an ad on a general interest website (e.g., on CNN.com), a firm will potentially reach a broad set of consumers. Although academics and practitioners have suggested that the nature of communication (targeted vs. non-targeted) can affect ad spending (Soberman 2005, Eechambadi 1994), no clear distinctions have been drawn on the role each format can play in withstanding competition vis-à-vis product quality.

It is not intuitively obvious why such heterogeneity in industry patterns exists and how to reconcile the variety of strategies along the three decision variables of product quality, advertising, and price – in effect, three of the so-called marketing “4Ps”. The objective of this paper is to shed light on these matters and help understand firm behavior by addressing the following research questions.

* When should we expect firms to differentiate in product qualities and when in advertising levels, assuming both strategies are available to them?

* Is it possible for firms to sustain positive profits with undifferentiated levels of quality and advertising? What pricing approach is needed to enable this outcome?

* If firms can target their advertising efforts, under what conditions should we expect them to select distinct vs. overlapping segments, how does this decision impact their quality and pricing policies, and what is the nature of the demand structure that arises?

To address these questions we develop a duopoly model in which firms choose product quality in a first stage, select their advertising levels in a second stage, and set prices in the last stage. Consumers are heterogeneous with respect to their valuation for quality, yet in order for them to consider the purchase of a product they must first be aware of its existence and informed about its characteristics. Although there are several ways consumers can become informed about the products available in the marketplace, firm initiated communication is a primary vehicle.5

On a given purchase occasion, consumers evaluate the various products they are informed about

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5 In the U.S. alone, companies spent close to $200 billion in 2017 to advertise their offerings to consumers (Advertising Age, “200 Leading National Advertisers 2018,” June 26, 2018).
and choose the one that delivers them maximum utility. Consequently, the return on advertising for a firm will critically depend on how its offering compares to the other alternatives in the marketplace and on how aggressively those products are advertised. In this context, the decisions of what quality product to offer and then how heavily to promote it through advertising become intertwined, and further depend on the resulting pricing the firms pursue. This structure allows us to examine the research questions laid out above; thereby shedding light on which strategic lever(s) a firm should pull in an effort to effect differentiation from a rival.

Our analysis looks at two forms of advertising commonly used to inform consumers about products (for empirical support on the informative role of advertising see, e.g., Ackerberg 2001, Caves and Greene 1996, Bagwell 2007). The first type—*blanket advertising*—captures situations where communication is indiscriminate and the probability that consumers become informed about the firm’s product is a function of the advertising expenditure level. Specifically, a heavy ad spend guarantees that all consumers receive the firm’s message and consider its product, while a light ad spend only results in a likelihood (which is less than 1) that each consumer receives the message. We find that three equilibria can emerge depending on the effectiveness of advertising, which is defined as the extra impact on demand that a heavy ad spend has over a light ad spend relative to the cost of doing so. Specifically, when advertising is ineffective, firms are fully undifferentiated in their actions: they choose the same product quality, the same advertising level, and the same pricing strategy. Price competition is softened because through their advertising choices firms limit the expected number of consumers who will consider both products; hence they earn healthy profits by co-locating on the highest quality position, leading to minimal product and advertising differentiation. We further show that by choosing to co-locate in quality in the first stage, a firm can “prevent” its rival from shifting to the heavy ad spend level in part of this region, even though a monopolist would already find it profitable to do so to gain from greater consumer awareness.

When advertising is moderately effective, asymmetric advertising choices with no product differentiation occurs. Specifically, one firm always has an incentive to choose a high-quality product and advertise heavily. Interestingly, the rival still finds it optimal to select the same high quality level, but in order to soften price competition it must advertise lightly (otherwise, if both advertise heavily and are undifferentiated in the quality space then price competition intensifies and profits will be driven to zero). In doing so, the light advertiser concedes an entire segment of consumers to the heavily advertising firm, yet benefits from the higher average equilibrium prices that ensue.
We further show that this incentive can be so strong in this region that, even if advertising heavily entails no extra cost, one firm still prefers to select the same high product quality but differentiate by refraining from advertising heavily. Finally, when blanket advertising is very effective, in the sense that the bump in market awareness from advertising heavily is large and comes at relatively little cost, quality differentiation with no advertising differentiation is the equilibrium outcome. In this case, a firm cannot afford to advertise lightly because it will then be relinquishing a very large segment of consumers who will only be aware of its rival’s product. Instead, both firms advertise heavily with one firm choosing maximal quality and a high price, while the rival differentiates with a much lower quality product and a cheaper price.

Thus, our findings suggest that with blanket advertising firms tend to utilize at most one of the actions at their disposal to soften competition – (i) advertising differentiation: selecting the same product strategy yet using distinct advertising levels to endogenously segment the market in terms of awareness, or (ii) product differentiation: choosing dissimilar product positions, thereby leveraging customer heterogeneity in willingness to pay for quality, yet both electing to advertise heavily. In these instances, pricing strategies are also differentiated. In particular, in case (i) the heavy advertiser selects higher prices on average than the light advertiser, and in case (ii) the high-quality firm prices higher than its low-quality rival. No differentiation in product quality or advertising is feasible as well, as long as the advertising levels chosen are light; in which case the resulting prices are in similar mixed strategies. We further characterize how equilibrium profits depend on advertising effectiveness and uncover a non-monotonic pattern: firms’ profits can initially increase and then decrease, with profits rising again for the firm switching to the heavy advertising level.

The second type of advertising we examine—targeted advertising—captures situations where firms can be selective and communicate to specific segment(s) they wish to inform about their product. With this form of advertising we establish the conditions for a robust equilibrium outcome whereby advertising strategies are “differentiated”, in the sense that each firm targets its communication to a distinct segment. Because every consumer considers at most only one product, the firms do not directly compete in the pricing stage and hence can both choose equally high qualities—that is, their products are undifferentiated. Depending on segment sizes, firms may asymmetrically expend on advertising and it may be the case that the firm targeting the lower willingness-to-pay segment spends more yet prices below its rival.
Surprisingly, there exist conditions such that a set of consumers with moderate valuation for quality are advertised to but make no purchase, even though consumers in a separate segment with lower willingness to pay are served. Additionally, one might have expected that as long as firms choose the same product quality then how the market is split between them should not affect total demand. However, we show that the total number of consumers served in equilibrium follows an inverted-U shape as a function of the targeting criterion. Lastly, we discuss the possibility of an equilibrium in which firms differentiate in product qualities and targeted advertising strategies, with the high-quality firm covering the entire market and the low-quality firm forgoing the premium segment.

Collectively, these results uncover when and how firms strategically use product quality, advertising and pricing to effectively compete: in some cases differentiating in the advertising approach taken, while electing similar product qualities, and in some instances differentiating through quality divergence, while electing similar advertising approaches. There are also scenarios where both actions are similar across firms. The range of patterns that can arise in equilibrium, depending on certain parameter values, may thus help explain the variation observed in practice with respect to these key decision variables and the seemingly inconsistent empirical research findings on the connection between product quality and advertising levels.

The rest of the paper is organized as follows. The next section relates our work to the relevant literature and summarizes our contribution. Section 3 sets up the basics of the model in terms of demand and firm behavior. Section 4 solves for the case of blanket advertising and Section 5 solves for the case of targeted advertising. Finally, Section 6 offers some managerial and empirical implications, discusses extensions and limitations, and outlines opportunities for future research. To enhance readability, we postpone all proofs to the Appendix.

2 Related Literature

Our work is primarily related to two streams of literature, the first pertaining to quality differentiation and the second to the relationship between advertising and product quality.

Within the stream of literature on vertical product differentiation, the widely known model of Shaked and Sutton (1982) examines price competition between firms that first choose the quality of the products they sell. Since consumers are assumed to be fully informed about all products in
their model, in equilibrium firms choose different qualities in order to reduce price competition in the last stage.\footnote{It is worth noting that earlier work had identified several of the forces operating in Shaked and Sutton (1982). Specifically, Gabszewicz and Thisse (1979) build a model with two products of exogenously known quality levels. They characterize the pricing equilibrium of this game and show that the comparative statics are such that if the qualities get closer to each other then price is driven to zero, leading them to conclude that: “we have thus natural forces which lead the duopolists to maintain product differentiation.”} Moorthy (1988) relaxes Shaked and Sutton’s zero production cost assumption by introducing a quadratic cost function for quality. While this can result in an equilibrium where the firm choosing the lower quality is better off, it is still the case that in any equilibrium (simultaneous or sequential move) firms differentiate in qualities. Assuming that firms do not cover the market, Choi and Shin (1992) show that the low quality firm will choose a quality level that is a fixed proportion of the high quality firm’s choice. Wauthy (1996) gives a full characterization of quality choices, allowing market coverage to be endogenous. More recently, Choudhary et al. (2005) and Jing (2006) find that the higher quality firm can be worse off in equilibrium. Choudhary et al. (2005) allow personalized pricing, which can intensify competition to the detriment of the high quality firm. Jing (2006) identifies the conditions on the cost structure under which producing the low-quality good can be more profitable. These studies all show that quality differentiation is a robust equilibrium outcome.

However, in reality similar quality products are often observed in the marketplace. Rhee (1996) cites evidence for this and offers an explanation that incorporates consumer heterogeneity along unobservable attributes into the vertical differentiation model. As a result, if consumers are sufficiently heterogeneous on the extra dimensions, in equilibrium firms offer products that are identical on the observed quality dimension yet differentiated on the unobserved dimensions. Bester (1998) investigates the case of experience goods defined along two dimensions: an imperfectly known vertical quality level (that can either be high or low) for which there is no consumer heterogeneity, and an observable horizontal attribute for which consumers’ tastes are uniformly distributed. His analysis shows that both firms have an incentive to choose the high quality level and use sufficiently high prices to signal this choice, which allows them to minimally differentiate along the horizontal attribute. Banker et al. (1998) investigate the relationship between equilibrium quality levels and the intensity of competition between firms. They find that the degree of differentiation depends on the market structure and production cost differences.

Notably, the papers in this stream analyze firms’ quality positions under various price and cost assumptions, but presume that all consumers consider all the products available. Hence, these...
works ignore the fact that in many markets consumers are ex-ante uninformed (or need to be reminded) about the various offerings, with advertising serving as an additional lever firms might use to differentiate and soften competition.

The second stream of literature related to our work involves studies on the connection between advertising choices and product positions. It is theoretically well-established in economics and marketing that advertising levels can signal product quality (Milgrom and Roberts 1986); although little empirical evidence exists to conclusively support this idea (Bagwell 2007, and references therein). Such works ignore advertising’s role in informing consumers about which products exist (Nelson 1974, Butters 1977) and thus its effect on market size. Grossman and Shapiro (1984) model advertising as informative in a horizontal set up, whereby receiving an ad increases the probability a consumer finds a product that better matches his or her tastes. They find that firms select excessive advertising levels relative to a social planner and that more efficient advertising technology intensifies price competition. Iyer et al. (2005) investigate targeted advertising when consumers have horizontal tastes and find that firms advertise more often to consumers who have a strong preference for their product, arguing that this is a way to soften price competition and eliminate wasted advertising. Other papers (Chen et al. 2001, Zhang and Katona 2012) examine how individual targetability or targeting accuracy affects competition, showing that imperfect targetability can reduce competition by serving as an implicit differentiator. While some of these forces will be relevant in our context, much of the literature studying targeting aspects of advertising, such as the paper by Narasimhan (1988), treats qualities as given. By contrast, we add quality as an important decision variable and show that endogenizing both product quality and advertising level critically affects equilibrium outcomes.7

Our paper also contributes to the debate on whether higher quality products should be associated with higher levels of advertising and prices, with the empirical literature finding mixed evidence for such a relationship (Erdem et al. 2008, Kash and Miller 2009, Caves and Greene 1996, 7In Zhu and Dukes (2017), firms need to decide which product attribute(s) to make prominent (e.g., through advertising) when consumers have limited attention and heterogeneous horizontal tastes. They show that firms may choose to make the same attribute prominent and that this can impact endogenous quality choices. Our setup is different: we assume only one attribute and a firm’s communication level affects the fraction of consumers aware of its product, independently of its rival’s advertising efforts (in Section 6.2 we briefly discuss communication spillover effects, which is a characteristic of the modeling in Zhu and Dukes 2017). We note that Varian (1980) assumes the exogenous existence of two customer segments: uninformed and informed. Firms are undifferentiated in his model, hence they compete using mixed pricing strategies. Our paper can be interpreted as extending his work by allowing firms to select product qualities and endogenously create informed and uninformed segments.
Archibald et al. 1983, Tellis and Fornell 1988, Song et al. 2016). Our analysis can help reconcile these seemingly inconsistent observations by delineating conditions for when a positive relationship should hold between these actions and when it should not in equilibrium.

In summary, our contribution lies in extending the first stream of literature by exploring the strategic role of advertising as another action, in addition to product quality, that firms can take to withstand competitive pressures. We characterize when and how firms use each of these two actions to differentiate and uncover important interactions between the two variables. Relative to the second stream, we endogenize product qualities and analyze how this decision is impacted by foreseeing the need to advertise to inform consumers and by the ensuing pricing equilibrium that can be in mixed or pure strategies. We further show that, given the opportunity to use targeted advertising, firms may minimally differentiate in qualities. That is, if consumers are initially uninformed about products, there is no need to divide the market by choosing distinct quality positions, since a division is created by the targeted advertising choices.

3 Model Setup

We consider two competing firms that seek to sell a product in a given market. We index the firms by the numbers 1 and 2 or the letters $i$ and $j$, always assuming that $i \neq j$. If firms offer different quality products, we denote the firm offering the lower quality product by 1 and the one offering the higher quality product by 2. We assume that every consumer purchases at most one unit. We further assume that consumers are heterogeneous with respect to their valuation of quality, denoted by $\vartheta$. The parameter $\vartheta$ is uniformly distributed in the interval $[0, 1]$. A consumer with parameter $\vartheta$ gains utility $\vartheta s - p$ from a product with quality $s$ priced at $p$, and purchases the product for which his/her utility is greater (provided the net utility is non-negative). However, consumers can only purchase products they are informed about (Keller and Kotler 2011). By advertising, a firm communicates the existence and characteristics of its product, thereby affecting the likelihood that it enters a consumer’s consideration set. Our setup is thus consistent with the informative view of advertising (see, e.g., Tirole 1988, Bagwell 2007) and with consumer behavior research (e.g., Mitra and Lynch 1995).  

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Trivially, an individual cannot consider buying a product that s/he is not aware of. Yet the consideration set can be narrower than the set of products the consumer is aware of, since some alternatives may be excluded before carefully comparing the utilities derived from them (Hauser and Wernerfelt 1990). Because informative advertising impacts the salience of products and their characteristics in consumers’ memory, it affects which products are “top
We study two types of informative advertising mechanisms. In the first type, which we call \textit{blanket advertising}, the probability that a consumer is informed about firm $i$’s product is $\alpha_i$, independently for each consumer. This probability depends on how intensely firm $i$ advertises and is uncorrelated with firm $j$’s advertising level. In the second type, which we call \textit{targeted advertising}, firms can communicate to subsets of consumers that belong to distinct segments. If a consumer is in a firm’s targeted segment, s/he is informed about that firm’s product with probability 1. In Section 6.2, we discuss our modeling assumptions on advertising and mention possible extensions (such as persuasive and spillover effects), as well as comment on additional ways consumers can learn about products (e.g., through search).

\textit{Timing:} The timing of the game is as follows. First, firms choose their qualities $s_i$ (without loss of generality $s_1 \leq s_2$). Qualities are positive and have a maximum value that is normalized to 1. Second, firms make their advertising decisions. In the blanket case they set their advertising level and in the targeted case they choose the segment(s) they wish to advertise to. Third, firms set prices. Finally, consumers make their purchase decisions. This timing reflects the notion that the quality position choice tends to be a long-term decision, whereas prices can be easily changed. The time-scope of advertising decisions is somewhere in between. It is further common for advertising allocation decisions to come after product positioning has been determined but before prices are set (this timing assumption is consistent with several prior works, e.g., McAfee 1994, Roy 2000). In practice, while prices can be adjusted on a weekly, daily or even hourly basis, advertising budgets and media planning are typically set on a quarterly or annual basis.\(^9\)

\textit{Costs and Profits:} We assume no fixed entry costs, hence both firms participate in the market. Furthermore, we assume that variable production costs are constant and normalize them to zero.\(^{10}\)

\(^9\)It is important to note that our modeling and timing structure do not preclude the possibility that a firm may include price (or some discount/sale promotion) in a particular ad execution. The idea is that the amount of advertising and likely even the media buy intentions have been set prior to final prices being decided upon (and these final prices can later appear as part of the ad copy). In other words, the specific content of ads may be changed more easily than the decision on whether an aggressive or unaggressive approach to advertising will be taken. Our modeling is also consistent with papers that assume consumers can costlessly obtain information on price after observing an ad (see, e.g., Mayzlin and Shin 2011).

\(^{10}\)Normalizing variable costs of production to zero might seem as an oversimplifying assumption but is done for several reasons. First, it allows us to focus on the strategic incentives to differentiate in qualities when advertising
However, advertising costs, denoted \( c(\cdot) \), depend on the level chosen in the following way. In the blanket advertising case, we assume that \( c(a_i) \) is a non-decreasing function of \( a_i \), i.e., to achieve a higher likelihood of being considered by a consumer a firm has to spend more on advertising (we also examine the special case whereby a heavy ad level comes at no additional cost). In the targeted advertising scenario, we assume that the cost is a linear function of the segment size. Firms’ profits are therefore simply their revenues (price \( \times \) quantity sold) minus advertising costs:

\[
\Pi_i = p_i D_i - c(a_i).
\]

We solve for the sub-game perfect equilibria of the game. We focus on pure strategies for quality and advertising decisions, yet allow pure or mixed strategies for prices (reflecting the fact that in several markets firms tend to run occasional price promotions). Next, we characterize the equilibria in the two advertising cases: blanket and targeted.

### 4 Blanket Advertising

In this setup, firms communicate to the entire mass of consumers but the probability of a firm’s ad reaching (or being attended to by) a particular consumer depends on the advertising level it selects. For the sake of simplicity, let us assume that there are two levels of advertising: light \( (a_L = a) \) and heavy \( (a_H) \), with costs \( c_L \leq c_H \), respectively. The probabilities of being considered by consumers are then \( 0 < a_L = a < a_H \). Without loss of generality we set \( a_H = 1 \). That is, a firm can either choose to heavily advertise at a high cost, thereby ensuring every consumer is informed about its product, or to advertise lightly at a low cost, thus expecting only a portion of consumers to be informed. We use the notation \( \Delta c \equiv c_H - c_L \) for the cost differential and \( \Delta a \equiv a_H - a_L \) for the ad-impact differential between the two levels of advertising. We further assume that the ad-impact differential is not negligible, to ensure that firms don’t trivially always advertise lightly. Formally, this means \( \Delta a > a \).\(^{11}\)

\(^{11}\)One way to interpret the lower bound on \( \Delta a (\sim 0.13) \) is that there are always some consumers who require more active communication initiated by the firms to be informed. Note further that if \( \Delta a = 0 \) (i.e., \( a_L = 1 \)), by advertising lightly firms already ensure all consumers are informed about their products. In such a case, advertising decisions would no longer be a relevant action in creating differentiation and this would trivially correspond to the full awareness case assumed in prior work.
Once firms have advertised their products, consumers may not all have the same consideration set. Some consumers will be able to consider both products, and they constitute what we call the competitive segment. We also refer to consumers in this segment as comparison shoppers since they are able to compare both products (on quality and price) before making their purchase decision. The size of the competitive segment is endogenous and can be as small as $a^2$ when both firms advertise lightly or as large as 1 (i.e., the entire market) when both firms advertise heavily. In addition to the competitive segment, a firm might have what we call a captive segment, composed of consumers who only consider its product and not the competitor’s. The sizes of captive segments are also endogenous and depend on firms’ advertising levels, as summarized in Table 3. As can be seen from the table, only when a firm faces a rival that advertises lightly will it have a captive segment of its own. For completeness, we note that when both firms advertise lightly there exists another segment of consumers who are unaware of any offerings (of expected size $(1 - a)^2$).

Table 3: Existence and Expected Size of Captive Segments as a Function of Advertising Decisions

<table>
<thead>
<tr>
<th>Ad Level</th>
<th>Heavy ($a_j = 1$)</th>
<th>Light ($a_j = a$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy ($a_i = 1$)</td>
<td>No captive segment</td>
<td>Only Firm $i$ has a captive segment of size $1 - a$</td>
</tr>
<tr>
<td>Light ($a_i = a$)</td>
<td>Only Firm $j$ has a captive segment of size $1 - a$</td>
<td>Both firms have captive segments each of size $a(1 - a)$</td>
</tr>
</tbody>
</table>

A firm with a captive segment always has the option of focusing on these consumers and not competing for the comparison shoppers. In that case, the firm would act as a monopolist over its captive segment. Thus, as a benchmark, we first solve the monopolist case, i.e., when there is only one firm in the market.

A consumer who receives an ad for the monopolist’s product buys it if and only if $\theta s_m - p_m \geq 0$. That is, the monopolist’s demand consists of consumers who are informed about the product and who value quality sufficiently, satisfying the following condition: $\theta \geq p_m / s_m$. Therefore, the monopolist’s demand is:

$$D_m = (1 - p_m / s_m) a_m,$$

where $a_m \in \{a, 1\}$. Given $a_m$ and $s_m$, the monopolist chooses a price $p_m^* = s_m / 2$ to maximize its
revenue: $p_m(1 - p_m/s_m)a_m$. Its profit becomes

$$\Pi_m = s_m a_m/4 - c(a_m),$$

which is increasing in $s_m$ regardless of the advertising level. Hence the monopolist always sets $s^*_m = 1$ in the first stage, and chooses $a_m = 1$ in the second stage if and only if $(a_L/4 - c_L) < (1/4 - c_H)$ or $\Delta c < \Delta a/4$. Specifically, the monopolist advertises heavily rather than lightly if the extra revenue that results from this advertising level is greater than the additional cost $\Delta c$. Another way to characterize this decision is in terms of ad effectiveness, measured by the ratio $\Delta a/\Delta c$ (for $\Delta c > 0$). A larger ratio means a greater ‘bang-for-the-buck’, and the monopolist advertises heavily if the ratio is greater than $e_m = 4$. Consequently, in analyzing the duopoly case, if a firm chooses to advertise lightly when ad effectiveness is larger than $e_m$, then we are assured that this is a result of the strategic interaction between the firms.

**Duopoly**

Let us now turn to the duopoly case. We seek to understand under what conditions a firm will elect to use advertising vs. product quality to soften competition with its rival; also allowing for the possibility of no differentiation on these decision variables. We start by analyzing the advertising level firms choose in the event they have elected to co-locate on quality in the first stage, followed by an analysis of quality choice incentives. The next lemma provides the profits to each firm as a function of advertising choices when there is no product quality differentiation.

**Lemma 1** Under no product differentiation $(s_1 = s_2 = s)$, the profits of firm $i$ as a function of its advertising level $(a_i)$ and its competitor’s advertising level $(a_j)$ are:

<table>
<thead>
<tr>
<th>Ad Level</th>
<th>Heavy $(a_j = H)$</th>
<th>Light $(a_j = L)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy $(a_i = H)$</td>
<td>$\pi_i = -c_H$</td>
<td>$\pi_i = (1 - a) 4/1 - c_H$</td>
</tr>
<tr>
<td>Light $(a_i = L)$</td>
<td>$\pi_i = a(1 - a) 4/1 - c_L$</td>
<td></td>
</tr>
</tbody>
</table>

Before discussing the profits revealed in Lemma 1 in more detail, we introduce some notation. Let $(L, L)$, $(L, H)$, $(H, L)$ and $(H, H)$ denote the advertising strategies, whereby the first argument is the advertising level chosen by Firm 1 and the second is the level chosen by Firm 2. We also refer to Table 3 for the size of the captive segments for the different advertising levels. Turning
to Lemma 1, the \((H, H)\) case is straightforward: intense price competition is triggered by the lack of any kind of differentiation. Specifically, when every consumer is informed about both products that are of equal quality, the market is comprised of only a competitive segment and firms undercut each other in price. Profits are negative as firms still incur the cost of choosing the heavy ad level.

Next consider the scenario where firm \(i\) advertises lightly \((a_i = L)\), and either has no captive segment (when \(a_j = H\)) or a small captive segment of size \(a(1 - a)\) (when \(a_j = L\)). If Firm \(i\) chooses a given price with probability one, two scenarios could unfold: (i) if Firm \(i\)'s price is high, its rival is prompted to undercut it slightly to sell to all the comparison shoppers; but then Firm \(i\) would want to deviate and undercut its rival's price, thus triggering a “race to the bottom” to sell to the competitive segment, (ii) if Firm \(i\)'s price is very low, its rival would concede the comparison shoppers and focus on its own captive segment. But then Firm \(i\) would have an incentive to deviate and sell at a higher price (just below Firm \(j\)'s price). In either case, Firm \(i\) cannot choose a single price in equilibrium and hence plays a mixed pricing strategy. The best response for Firm \(j\) is also to play a mixed pricing strategy. Firm \(i\) competes for the comparison shoppers by trading off choosing a high price but having a low probability of selling vs. choosing a low price and earning less per customer yet having a high probability of selling by undercutting its competitor. In the end, Firm \(i\) has to be indifferent between all the possible prices in equilibrium, and its profit is equal to the monopolist rents from selling to a segment of size \(a(1 - a)\).

The last scenario to discuss is when Firm \(i\) advertises heavily but its rival does not \((H, L)\). The size of the captive segment for Firm \(i\) is now \((1 - a)\) and is relatively large, such that it might be profitable to sometimes focus exclusively on that segment to extract monopolist rents. Indeed, with probability \((1 - a)\) firm \(i\) sets its monopolist price and only sells to its captive customers. With remaining probability \(a\), and similar to the situation in the light advertising case, Firm \(i\) competes over the competitive segment facing the following tradeoff: choose a high price but have a low probability of selling or choose a low price and have a high probability of selling but earning less revenues per customer. In the end, Firm \(i\) has to be indifferent between all the prices it mixes over, and profits are thus equal to the monopoly rents over its captive segment.

One conclusion worth highlighting from Lemma 1 is that when a firm advertises lightly it earns the same profit of \(a(1 - a)\frac{\pi}{4} - c_L\), regardless of the advertising choice of its rival. This property

\[12\]The details of the distributions from which equilibrium prices are drawn are given in the Appendix.
implies that the low-quality Firm 1 can always consider a deviation to co-locate its product with
the higher-quality Firm 2 (resulting in no quality differentiation) and choose to advertise lightly.
Since a profitable deviation should not exist in equilibrium, the profit of Firm 1 has to be at least
as high as the profit from this potential deviation. In other words,

$$\pi_1^* \geq a(1 - a) \frac{s_2}{4} - c_L.$$  

Another conclusion arising from Lemma 1 is that firms with undifferentiated product qualities
can still sustain positive profits by strategically selecting their advertising intensity levels. Indeed,
by choosing a light advertising level a firm strategically lets its rival have a captive segment, which
softens the ensuing price competition because not all consumers will be aware of both products.

With these insights in hand, we can discuss the implications of Lemma 1 for the best response
advertising choices when product qualities are undifferentiated. In the \((H, H)\) case, as noted, full
awareness in conjunction with identical products triggers intense price competition that drives
product market profits to zero; yet both firms still incur the cost of advertising heavily. Thus, with
undifferentiated product qualities, Firm \(i\) is always better off advertising lightly if it expects its
rival to advertise heavily. However, if its rival advertises lightly, Firm \(i\) needs to compare the profits
from advertising heavily and ensuring that all consumers are aware of its product vs. advertising
lightly and reducing the number of consumers aware of both products (i.e., shrinking the size of the
competitive segment). Lemma 1 reveals that when 

$$(1 - a)^2 c_H \geq a(1 - a)^2 \frac{s_2}{4} - c_L,$$

or 

$$\Delta c \leq (\Delta a)^2 \frac{s_2}{4},$$

advertising heavily is profitable. In other words, when the ad-impact differential \(\Delta a = 1 - a\) and
the quality level \(s\) are high enough relative to the cost differential \(\Delta c\), firm \(i\) will prefer to advertise
heavily when its rival advertises lightly.

To summarize, if firms do not differentiate in product qualities, whether they would elect to
differentiate in advertising strategies (one spending lightly and the other heavily) depends on the ad-
impact differential \(\Delta a\) relative to cost differential \(\Delta c\), i.e., on the degree of advertising effectiveness.
Advertising can be considered ‘ineffective’ when selecting the heavy level \(H\) only provides a small
awareness lift over selecting the light level \(L\), or when doing so comes at a very costly expense.
Advertising is considered ‘effective’ when the reverse is true.

We now turn to characterizing the equilibrium of the entire game by solving for the subgame
perfect ad levels chosen in the second stage and the quality positions chosen in the first stage. We
start by presenting the typical case where advertising heavily is more costly than advertising lightly.
Figure 1: Equilibrium Strategies and Differentiation Type as a Function of Advertising Effectiveness

<table>
<thead>
<tr>
<th>Differentiation:</th>
<th>None</th>
<th>Advertising</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qualities:</td>
<td>$s_1 = s_2 = 1$</td>
<td>$s_1 = s_2 = 1$</td>
<td>$s_1 &lt; s_2$</td>
</tr>
<tr>
<td>Ad Levels:</td>
<td>$a_1 = a_2 = L$</td>
<td>$a_1 = L, a_2 = H$</td>
<td>$a_1 = a_2 = H$</td>
</tr>
<tr>
<td>Prices:</td>
<td>Same mixed strategy</td>
<td>Different mixed strategies</td>
<td>Different pure strategies</td>
</tr>
<tr>
<td></td>
<td>$(p_{1ave} = p_{2ave})$</td>
<td>$(p_{1ave} &lt; p_{2ave})$</td>
<td>$(p_1 &lt; p_2)$</td>
</tr>
</tbody>
</table>

Note: $p_{ave}$ is firm $i$’s average price in the mixed strategy equilibrium. The advertising cost differential is positive.

$(\Delta c > 0)$ and address the no cost differential case $(\Delta c = 0)$ in a subsequent Corollary. Formal details of the cutoff values in each region of the proposition are given in the Appendix.

**Proposition 1** When advertising heavily entails some extra cost $(\Delta c > 0)$, the type of differentiation arising in equilibrium is as follows:

1. When advertising effectiveness is low, both firms choose maximal quality coupled with advertising lightly: no product or advertising differentiation.

2. When advertising effectiveness is intermediate, both firms choose maximal quality with one firm advertising heavily while the other advertising lightly: only advertising differentiation.

3. When advertising effectiveness is high, firms choose distinct quality levels with both advertising heavily: only product differentiation.

The proposition uncovers an interesting general finding: firms never elect to concurrently differentiate both in product qualities and in advertising levels. It is optimal to either only differentiate in advertising levels or in product qualities, or to choose no differentiation in these decision variables. Graphically, Figure 1 illustrates the type of differentiation (in advertising, product quality, or none), along with equilibrium pricing strategies, as a function of ad effectiveness.

We now discuss the intuition behind the results in Proposition 1, starting from the ineffective advertising case, i.e., part 1 of the proposition (the left-most region marked ‘None’ in Figure 1). When selecting maximal product quality in this region, firms anticipate that a light ad spend by both will lead to the emergence of sizable captive segments. Light advertising levels can thus act
to mitigate price competition by, in essence, creating segments that possess distinct information sets. The mixed pricing strategies played result in each firm primarily catering to its endogenous captive segment and earning monopoly rents from these consumers. Alternatively, conditional on Firm 2 selecting the highest quality level and advertising lightly, Firm 1 could elect to differentiate in quality. This differentiation strategy turns out to be less profitable because it entails selling a product of lower quality at a much lower price and not realizing the full amount of profits from the captive segment. Conversely, Firm 2 might like to advertise heavily. But by selecting minimal product differentiation in the first stage, Firm 1 prevents this from occurring by making this an unprofitable move; thus inducing Firm 2 to advertise lightly. Notably, throughout this region of low ad effectiveness the firms are fully undifferentiated – they select the exact same quality level, the same advertising level, and play the same mixed pricing strategy.\footnote{In some respects, because the pricing equilibrium is in mixed strategies in this case (which can be interpreted as offering price promotions/discounts drawn from a distribution), probabilistically firms won’t select the same prices.}

Recall that when advertising effectiveness is less than $e_m$ a monopolist would choose to advertise lightly, and when it is greater than $e_m$ it would advertise heavily (this threshold is marked with a vertical dashed line labeled ‘$e_m$’ in Figure 1). Naturally, if a monopolist does not want to invest heavily in advertising, firms in a duopoly would not either. Yet Proposition 1 identifies a region where both firms select a light ad spend even where a monopolist would already shift to advertising heavily (the region to the right of the dashed vertical line and to the left of the first bold vertical line in Figure 1). This happens because the minimal product differentiation strategy embraced in the first stage lowers the benefits of advertising heavily, and prevents each firm from switching to this level even though a monopolist would.

As advertising becomes more effective (Proposition 1 part 2, the middle region marked ‘Advertising’ in Figure 1), one firm has an incentive to advertise heavily; thereby ensuring the entire market is aware of its offering and incurring the cost of doing so. If Firm 1 expects its rival to select the top quality ($s_2 = 1$) followed by heavy advertising, what are its optimal product and advertising strategies? Firm 1 can either co-locate on quality and advertise lightly ($L, H$) (it would never advertise heavily when co-locating since doing so would obviously lead to a loss of $-c_H$; see Lemma 1) or it can differentiate its offering by choosing a lower quality level followed by a heavy ad spend ($H, H$). Further note that when all consumers are expected to be informed of both
products, the best response quality is easily found to be $s_1 = 4/7$. The optimal strategy for Firm 1 depends on the implications of its decisions for the pricing stage. If Firm 1 co-locates in quality and advertises lightly, its rival Firm 2 that advertises heavily will still have captive consumers (of expected size $(1 - a) = \Delta a$) and can earn monopoly profits on this segment. Since Firm 2 will indeed select prices as high as the monopolist price quite often, and as prices are strategic complements, Firm 1’s average mixed-strategy price will be higher than what its single, pure-strategy price would be under product differentiation and advertising heavily. Thus, when advertising is moderately effective, quality co-location coupled with a light ad spend generates higher profits than product differentiation followed by heavy advertising. Said differently, by opting for no quality differentiation with its rival in the first stage, Firm 1 in effect commits to advertising lightly, since this level of consumer awareness for its product generates more profits. For Firm 2, conditional on Firm 1 co-locating and expected to choose the light ad level, the best response is to choose a heavy ad level, given that the size of the resulting captive segment will allow it to price higher on average. It is worth pointing out that this asymmetric equilibrium in advertising strategies holds despite the two firms being identical at the outset: they go on to choose the same product quality path but a differentiated advertising path.

The last case to analyze is when advertising is very effective (Proposition 1 part 3; the rightmost region marked ‘Product’ in Figure 1). If Firm 2 advertises heavily, Firm 1 has no captive segment. Firm 1 again compares two strategies: product co-location with its rival followed by light advertising $(L, H)$ vs. quality differentiation $(s_1 = 4/7)$ followed by a heavy advertising level $(H, H)$. Very effective advertising implies a relatively large ad-impact differential and means that the size of the competitive segment (which is equal to $a$) is quite small in this region. Hence, choosing the light advertising level results in only few consumers that can consider Firm 1’s product (less than 10% of the market) and thus in low profits despite having high quality. By contrast, if Firm 1 chooses lower quality and also advertises heavily, the entire market becomes “competitive” and informed about both products. Hence, it would have to price cheaply enough to generate

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14 This would mirror the result in Shaked and Sutton (1982) who assume full consumer awareness and only focus on product differentiation.

15 As we show in the proof of Lemma 1, when firms select the same qualities and at least one firm chooses to advertise lightly, the average equilibrium prices increase in advertising effectiveness. This is easily seen in the $(L, H)$ case. The heavy advertising firm caters to its captive segment and selects its monopoly price with probability $\Delta a$, which obviously increases in advertising effectiveness by definition. This, in turn, allows the light advertising firm to select higher prices more often and still attract many competitive shoppers. See also the figure in the Appendix depicting average prices in the various mixed strategy equilibria as a function of $\Delta a$ (Figure 3 at the end of the proof of Proposition 1).
positive profits. Given the huge disadvantage in demand when advertising lightly in this case, and if the cost of advertising heavily is sufficiently modest (low $\Delta c$), Firm 1 can earn greater profits by following the latter approach: differentiating in product quality and matching the heavy advertising level of its rival.

We point out that the result whereby firms differentiate in advertising strategies (Proposition 1 part 2) is quite robust, as it does not rely on the cost of advertising heavily deterring Firm 1 from choosing this level. Rather it is due to the strategic implications that such a choice entails. The next corollary formalizes this observation and highlights the somewhat counterintuitive finding that a firm may forgo a heavy advertising level even when such a choice is costless. In this setting, advertising effectiveness is measured by looking directly at the ad impact differential $\Delta a$.

**Corollary 1** When advertising heavily entails no extra cost ($\Delta c = 0$),

- If the ad-impact differential $\Delta a$ is not too high, firms opt for advertising differentiation with both choosing maximal quality.

- If the advertising-impact differential $\Delta a$ is high, firms opt for product differentiation with both advertising heavily.

This result highlights that a firm might seek a lower level of consumer awareness than that of its rival for purely strategic reasons, as it could have matched the rival’s advertising intensity at no extra cost. Consistent with the intuition for Proposition 1, the result is driven by how quality selections coupled with the advertising choices affect subsequent price levels. Advertising heavily like its rival can only work for Firm 1 if it differentiates in product quality upfront and then sets a very low price; a path that will be followed only if advertising is very effective ($\Delta a$ close to 1). But as long as light advertising creates some awareness ($\Delta a$ not too close to 1), Firm 1 can sustain higher prices and profits by pursuing advertising differentiation with no product differentiation.

We now discuss how profits change as advertising becomes more effective. For ease of exposition, we specifically look at what happens as the ad-impact differential $\Delta a$ increases while keeping the costs constant. The proposition below shows that the relationship is not necessarily monotonically increasing as one might have expected.

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16 Notably, in many game-theoretic models a primary way to achieve an asymmetric equilibrium with respect to an action is if the cost of taking that action is in an intermediate range; so that one firm finds it profitable to incur the cost, but profits if both take the action are too low for the rival to also undertake it (e.g., Iyer et al. (2005) with respect to investing in ad targeting capability or Lauga and Ofek (2009) with respect to market research).
Proposition 2 Firm profits can vary non-monotonically as advertising effectiveness increases. Specifically, Firm 1’s profits are increasing and then decreasing, while Firm 2’s profits are first increasing and may then decrease and increase again.

The proposition indicates that the profits of Firm 1, and possibly those of Firm 2, initially exhibit an inverted-U pattern as $\Delta a$ increases. The intuition for this intriguing result is as follows. When advertising effectiveness is low ($\Delta a$ very small), from Proposition 1 we know that firms do not differentiate in quality or advertising, with both opting for the light ad spend. Firms’ profits in this case correspond to a monopolist’s profits from the captive segment each has (of size $a(1-a) = (1-\Delta a)\Delta a$; per the profit expressions in Lemma 1). As the ad-impact differential $\Delta a$ initially increases, the size of each firm’s captive segment increases and with it the average prices and profits. If the firms still play the no-differentiation equilibrium as the ad-impact differential reaches $\Delta a = \frac{1}{2}$ (which occurs when $\Delta c$ is very large), then profits peak at this point with each firm expecting a quarter of the market to be captive (another quarter of the market is competitive and a quarter is uninformed of any product). But as advertising becomes more effective from then on, continuing to spend lightly informs fewer consumers; hence the size of the captive segments decreases and the firms see their profits shrink. Proposition 2 thus suggests that, conditional on being undifferentiated, firms might prefer an advertising technology that is neither too ineffective nor too effective, which allows them to endogenously create captive segments of sufficient size while expending lightly on advertising.

As ad effectiveness increases further, beyond a certain point Firm 2 prefers to switch to advertising heavily (per region 2 of Proposition 1). In that case, the size of Firm 2’s captive segment and its average price are directly proportional to the ad-impact differential ($\Delta a$), while Firm 1 no longer has a captive segment and must compete for comparison shoppers. Consequently, the profits of Firm 2 rise in this region, while those of Firm 1 keep declining as a function of $\Delta a$. When eventually firms switch to the product-differentiation equilibrium and both advertise heavily ($H, H$), their profits remain constant as $\Delta a$ increases further. Note that if firms switch to the advertising-differentiation equilibrium prior to $\Delta a = \frac{1}{2}$ (which occurs when $\Delta c$ is small), then Firm 2’s profits will never decline, while Firm 1’s profits will always exhibit the inverted-U pattern (increasing in the no-differentiation region and then decreasing in the advertising-differentiation region as $\Delta a$ increases).
To conclude the blanket advertising setting, we note that two ex-ante identical firms choose to differentiate in either product quality or advertising or not at all. When firms are minimally differentiated in qualities they avoid intense price competition by choosing a reduced level of consumer awareness (with at least one firm electing to advertise lightly). This leads to the endogenous creation of segments whereby not all consumers are informed about both products and some consumers are captive to at least one of the firms. This, in turn, softens price competition and allows the firms to derive positive profits. When firms differentiate, they either choose different advertising levels coupled with different mixed price distributions or different qualities and prices. The findings indicate that by endogenizing all three decisions (quality, advertising, and price), rather than just two as in most of the literature, we can characterize a much richer set of outcomes.

The specific equilibrium predicted was shown to depend in large part on the degree of ad effectiveness. In practice, several measures show considerable variance across industries in terms of how effective advertising is at informing consumers. For instance, online display ads for apparel are clicked over 4 times more often than for grocery and food retailers and about 9 times more often than for computer hardware. Similar variation is found in traditional media (see DoubleClick, “Display Benchmarks,” October 10, 2016 and Thomas et al. 2000). Such disparities in ad effectiveness might help explain why some industries exhibit minimal differentiation across firms in the quality of products offered, while other industries exhibit considerable differentiation in advertising levels or product qualities (consistent with Proposition 1). Furthermore, the results may help explain why some studies fail to find a significant positive correlation between quality and advertising (Kash and Miller 2009, Caves and Greene 1996). Specifically, if advertising plays an informative role as modeled here: when one firm advertises heavily while its rival advertises lightly both may offer similar quality products (e.g., a scenario common when national brands compete with virtually identical private labels, Kane 2014); yet when they choose different quality levels they are both expected to choose a heavy advertising level. Such behaviors when aggregated would in fact yield minimal observed correlation between these decision variables.

5 Targeted Advertising

In the previous section, blanket advertising reached consumers indiscriminately, without regard to their willingness to pay for quality. We now turn our attention to a different type of advertising
that allows firms to target specific customer segments. We again seek to understand whether firms will elect to differentiate in qualities or advertising; albeit here the main issue will be whether they will choose to target their communications at the same or different segments.

In this advertising setup, firms need to decide which subsets of consumers to target with their communication. A consumer can only consider buying a firm’s product if s/he is in a segment the firm has communicated to. In order to focus on the targeting aspect, we assume that advertising is perfectly effective, in the sense that every consumer in the targeted segment is informed about the firm’s product. For the sake of simplicity we consider a two-segment model, although the model could easily be extended to allow for more segments.\textsuperscript{17} Specifically, the market is comprised of a low-valuation segment (denoted $L$) and a high-valuation segment (denoted $H$). Let the size of the low-valuation segment be $t \in (0, 1)$ and the size of the high-valuation segment be $1 - t$. Therefore, consumers with a high willingness to pay for quality ($\vartheta \geq t$) belong to the high-valuation segment and consumers with a low willingness to pay for quality ($\vartheta < t$) belong to the low-valuation segment. We assume that advertising costs are positive and linear in the size of the segment(s) targeted. That is, targeting the $L$-segment costs $c_L t$, and targeting the $H$-segment costs $c_H (1 - t)$. Hence, a firm targeting both segments would pay $c_H$, which is assumed not to be too large to ensure that both firms find it affordable to advertise to at least one of the segments. The general structure and timing of the game are the same as those described in Section 3.

A good way to interpret this setup is that the parameter $t$ reflects the “targeting criterion” that firms can use to classify consumers as having a high vs. low willingness to pay for quality in this market. For example, certain variables (income, education level, profession, zipcode, etc.) can indicate which individuals are typically willing to pay more for certain products. In addition, customers who patron certain venues (e.g., events, or conventions), visit certain websites (e.g., upscale brands vs. bargain-hunting forums), or purchase particular items (e.g., on travel websites or Amazon.com) are often associated with willingness to pay for related items and can be identified.\textsuperscript{18} In B2B settings, several firm characteristics (e.g., number of employees, annual revenue, industry SIC code) may be linked to willingness and ability to spend on certain products and services.

\textsuperscript{17}This analysis is available from the authors upon request.
\textsuperscript{18}As explained in the Introduction, this form of advertising is meant to capture scenarios where firms have access to criteria or filters that allow identifying whether a consumer belongs to a given willingness-to-pay segment. For example, from past online purchases or certain characteristics on a person’s social media profile firms can use predictive analytics to classify consumers into various valuation-for-quality segments in a given market.
Competition creates the desire to pursue differentiated strategies and firms could achieve this in several ways. One approach would be to choose dissimilar product qualities and prices, while targeting overlapping segments. Low-valuation customers would then buy from the low-quality cheaper firm, while high-valuation consumers would buy from the higher-quality more expensive firm. Alternatively, firms could create differentiation through their advertising strategies, by having one firm target the $H$-segment and its rival target the $L$-segment. The following proposition relies on fairly general conditions – that neither segment be too small – and identifies the non-Pareto-dominated equilibria of the game. We show that under these conditions advertising differentiation arises in equilibrium. Specifically, firms choose to target different customer segments while selecting the same product quality.

**Proposition 3** If neither segment represents less than one third of the entire market, firms choose the same maximal product qualities and target different segments with their advertising campaigns. The firm serving the high-valuation segment prices higher and earns greater profits.

The intuition behind this result is that when firms target disjunct segments in the advertising stage they effectively “divide” the market between them. Doing so allows them to act as de facto monopolists in their chosen segments and avoid price competition in the final stage. Foreseeing this possibility, when the firms select quality positions in the first stage they both choose maximal quality. Alternatively, one can think of the result as follows. By selecting the same high qualities in the first stage, firms create a strong incentive to target disjoint segments in the advertising stage to avoid intense price competition that would otherwise ensue given they are undifferentiated in qualities. The firm serving the high-valuation segment can take advantage of the greater willingness to pay of its customers (for the same quality product) and hence prices higher than its rival. What is interesting in this equilibrium result is that two otherwise similar firms end up “coordinating” to serve different segments exactly because they choose to minimally differentiate in product qualities. Lastly, notice that when the low-valuation segment is greater in size (i.e., $t > \frac{1}{2}$), the firm serving this segment spends more on advertising (as $c_U t > c_U (1 - t)$) yet prices lower than the firm serving the “niche”, high-valuation segment.

In the next two corollaries, we highlight important features of the equilibrium characterized in Proposition 3.
Corollary 2 If the size of the low-valuation segment is between $\frac{1}{3}$ and $\frac{1}{2}$ of the total market, there exists a ‘pocket’ of unserved consumers with intermediate quality valuations, in the sense that consumers with higher or lower quality valuations purchase but those in the ‘pocket’ do not.

It is common in vertical differentiation models that consumers with the lowest willingness to pay for quality are unserved in equilibrium because prices are too high for them to purchase even the low-quality product; and this is true in our model as well. However, with targeted advertising consumers with intermediate valuations for quality may also be unserved (see Figure 2). The intuition for why such a “pocket” of non-buying consumers can exist is as follows. Based on the equilibrium in Proposition 3, these consumers are part of the $H$-segment targeted by Firm 2. Hence they only consider buying the product of the firm advertising to the high-valuation segment, and this firm charges a relatively high price. If this price is above their maximum willingness to pay they do not purchase. Note that these consumers do not consider the product of Firm 1, which has the same quality and a lower price, since they are not targeted by this firm.

We now turn to examining how total demand in the market changes as a function of $t$, the size of the low-valuation segment. One might expect that as long as the equilibrium calls for both segments to be targeted, and given the firms are of equal quality, then how the market is split between the firms should not matter for total demand. The next corollary shows that this is not the case.

Corollary 3 Total demand (i.e., consumers who purchase) is first increasing and then decreasing as a function of $t$. Firm 1’s demand is a monotonically increasing function of $t$ while Firm 2’s demand is a non-increasing function of $t$.

The corollary reveals that the location of $t$ in fact matters for total demand, which exhibits an inverse-U pattern as a function of the targeting criterion. This dependence arises because the optimal pricing level for the high-end firm is not symmetric to that of the low-end firm, and because of the existence of the unserved pocket (as described in Corollary 2). In particular, Firm 1’s equilibrium price results in demand of size $t/2$, and thus as $t$ increases Firm 1 sells to more and more consumers. However, from Corollary 2 we know that Firm 2 does not sell to a pocket of consumers with valuations between $t$ and $1/2$; hence its demand is initially constant (and equals $1/2$) and then declines rapidly as $t$ increases beyond $1/2$ (as the size of the high-valuation segment shrinks). The
Combination of the two demand schedules yields the non-monotonic variation of total demand with respect to $t$. Note that the firm serving the low-valuation segment sees its profits increase with $t$, whereas the firm serving the high-valuation segment sees constant and then decreasing profits. Notwithstanding, the firm serving the high-valuation segment makes greater profits even when its targeted segment is half the size of its rival’s (when $t \to 2/3$). That is, serving consumers at the “top of the pyramid” is more lucrative.

The findings in this section might help explain why some product categories exhibit “market polarization” (Knudsen et al. 2005), whereby two primary segments are catered to with respect to their valuation for quality—the premium group and the value-oriented group. Examples from a range of industries include: home appliances (Knudsen 2006), certain consumer electronics (Ries 2011), and hardware technology (Ofek and Hamid 2005). In these cases, we typically find that each firm focuses on a distinct segment and targets its communications and distribution efforts to that segment. Moreover, and consistent with our findings, even though in reality quality and production costs can be similar across various offerings, the prices of the products targeted at value-oriented customers are typically lower than those targeted at premium-oriented customers.\footnote{In the case of home appliances, outsourcing parts or complete production have resulted in similar variable costs, yet products aimed at the premium end are often 2-3 times more expensive. In the vodka market, blind taste tests consistently reveal the top brands as roughly equally rated and their ingredients and processes similar to those of less prestigious offerings. However, the average price of a 750ml bottle of Smirnoff is about half of that of Grey Goose. The former vodka is primarily targeted at casual consumption (low willingness to pay consumers), while the latter for more upscale vodka drinkers. In the market for SAN storage devices in the mid-2000s, firms were able to
Some evidence also suggests that trying to attract consumers with intermediate willingness to pay values may not be beneficial, consistent with the findings in Corollary 2. Indeed, management consulting practice has called on firms to “escape the middle-market trap” (Knudsen 2006). Lastly, the findings may help explain the lack of a consistent positive correlation in the empirical literature between advertising spending and prices. If advertising is targeted in nature, segment sizes affect advertising levels with prices determined by the average willingness to pay of each segment. So, for example, when the low-valuation segment is relatively large \( t > 1/2 \), we can expect the firm serving this group of consumers to advertise more extensively while setting lower prices than a rival serving a high-valuation segment.

We have focused in this section on situations where the low and high willingness to pay segments each comprises at least a third of the market. We believe this is a relevant case to analyze because, for many real world settings, targeting criteria are only meaningful if they result in segments with a sufficient proportion of consumers. That said, for completeness we now explain what happens when \( t \) does not satisfy the conditions in Proposition 3. If the low-valuation segment \( (L) \) is small \( t < 1/3 \), it might be too insignificant for either firm to want to advertise to it, and both may end up advertising only to consumers in the high willingness to pay interval \( (H) \). In this case, we will have product differentiation (with no advertising differentiation). Although one firm will set \( s_2 = 1 \), the rival will set a quality level higher than \( s_1 = 4/7 \) when its optimal demand is limited by the lower bound of the interval. On the other hand, if the high-valuation segment is small \( 2/3 < t \), at some point as \( t \rightarrow 1 \) neither firm can afford to forgo the low-valuation segment and both firms will include it in their target market. Indeed, it is possible to obtain an equilibrium in which the high-quality firm advertises to the entire market and chooses quality \( s_2 = 1 \), while the low-quality firm advertises only to the low-valuation segment and chooses quality \( s_1 = 4/7 \). In some sense, this is an instance whereby the firms are differentiated in their qualities and to an extent in their advertising strategies.

target separate segments despite selling equally advanced products (the technology used was very similar, the only difference was the configuration of the device) – one firm sold to high-end data centers and the other to customers with less data-intensive applications.
6 Conclusion

6.1 Summary and Managerial Implications

In this paper, our goal has been to characterize the type of differentiation strategies firms are expected to pursue, particularly the interaction between endogenous quality and advertising choices, to best withstand competitive pressures. We developed a model in which firms choose product quality in a first stage, advertising levels in a second stage, and prices in the last stage.

In general, we find that the equilibrium strategies depend on how effective advertising is. When advertising is relatively ineffective, firms minimally differentiate on all fronts (quality, advertising and pricing strategies). Even though product qualities are the same, mixed-strategy price competition is not intense due to the endogenous presence of captive segments. When advertising is moderately effective, firms still minimally differentiate in product qualities but switch to advertising differentiation: one firm elects to advertise heavily while the other advertises lightly. The only way for the light advertiser to make positive profits in this setting is to compete for comparison shoppers (who are aware of both products) by occasionally undercutting its rival’s price. This undercutting does not, however, need to be aggressive because the heavy advertiser has a relatively large captive segment and tries to extract monopolist rents from these customers by pricing high most of the time; thus keeping overall prices higher on average. Surprisingly, we find that even if advertising heavily comes at no extra cost, one firm still advertises lightly for strategic reasons as it prefers to commit in the first stage to being undifferentiated in quality. In other words, firms may replace quality differentiation with advertising differentiation when advertising plays an informative role. Finally, there is a scenario where both firms choose the heavy advertising spend and, in turn, the only way to avoid intense price competition is to select dissimilar qualities. This product differentiation result, which corresponds to the classic finding in the literature, holds only when advertising is highly effective. Otherwise, firms prefer to create captive customers by at least one of them advertising lightly. Minimal product differentiation serves a strategic purpose in achieving this outcome.

These findings have several important managerial and empirical implications as they bear on three of the so-called “4Ps” of marketing— product, promotion and pricing— in a single framework. First, they suggest that the conventional wisdom whereby a firm should seek a high degree of product differentiation from its rival to avoid head-to-head price competition is qualified, and
depends on whether or not there is a role for informative advertising that generates awareness of the products offered. It further matters what kind of communication vehicles can be used to reach consumers. If firms cannot identify consumers through targeting criteria, and hence must use blanket advertising, the type of differentiation strategy they should engage in depends on how effective advertising is. Most managerial treatments of advertising classify media based on their impact on consumers and their cost (see, e.g., Thomas et al. 2000, Gotter 2018). Our results thus tell managers how the characteristics of communication vehicles in their market or industry should affect the quality of products to offer and the level of advertising to select vis-à-vis rivals. Furthermore, the findings bear on the nature of pricing strategies that should be implemented. When product differentiation is minimal, firms should engage in “flexible” pricing, i.e., a mixed strategy where they price promote often. And if they are advertising at different levels, then the heavy advertiser should stick to the higher list price more frequently than its rival. From an empirical standpoint, a testable implication of our theory is that as advertising effectiveness decreases (and with it expected quality differentiation between competitors) we should observe greater variance or fluctuations in pricing.

When firms can decide which segments to target, our findings suggest that minimal differentiation in quality may actually work best, provided they can “stay out of each other’s turf” by differentiating their advertising strategies, with each firm focusing on a distinct segment. In these instances, we might observe different prices despite the similarity in objective qualities. Importantly, our analysis reveals that a firm need not feel forced to lower price so as to serve all the consumers who have been targeted with communications; it may wish to ignore consumers that have moderate willingness to pay, even though low-end consumers are being served by the rival firm. In other words, “avoiding the middle market” may be the best course of action. Anecdotal evidence from sources such as Consumer Reports suggests that indeed sometimes firms in the same category offer practically identical quality products (as rated on a number of key attributes) yet differ in the price quoted (see the Technical Appendix for a few examples). One could check to what extent these firms are using marketing vehicles to target distinct segments as a way to test the findings presented here.

Our findings also suggest that when setting innovation strategy, which is often aimed at improving the quality of existing products (e.g., better performance on key attributes), to ensure the best possible commercial outcome it is crucial to take into account how the new products will be
advertised. In their seminal work surveying managers on how firms appropriate returns from R&D efforts given competition, Levin et al. (1987) found that marketing and sales were cited as the primary vehicle (over such means as patents and trade secrets). This suggests that managers need to understand the strategic implications of communication actions on the products they choose to develop vis-à-vis the products their rivals are expected to develop, and that differentiation can be achieved through various approaches. For instance, learning that a rival is developing an equally advanced product may not spell disaster, as long as the firms can refrain from both advertising heavily or targeting the same segments with their promotion efforts.

6.2 Limitations and Future Research

Although our study encompasses several important aspects of the quality-advertising-pricing set of decisions in a competitive context, and we have justified many of our assumptions, we acknowledge several limitations. First, we analyzed the various advertising models separately (in Sections 4 and 5). Yet one could consider a more general setting that combines the two forms of advertising. In particular, advertising could be targeted but its impact within segment would depend on the level of spending, resulting in two advertising decision variables: which segment(s) to target and how aggressively to try and inform consumers in the chosen segment(s). This might lead to equilibria where firms target overlapping segments but select differentiated blanket levels in each of them.

Second, while advertising in our model impacts consumer product consideration, one could imagine situations where advertising plays a persuasive role. Persuasive advertising can influence the way consumers perceive product quality, thereby affecting their willingness to pay. In an extension presented in the Technical Appendix, we show how our modeling framework can incorporate this form of advertising. Our main finding is that firms may differentiate less in objective qualities, yet the divergence between their persuasive advertising levels results in the same degree of perceived quality differentiation as when consumers fully know objective product qualities. Hence, in this case there is differentiation in both objective qualities and in ad levels.

Third, given that we modeled quality and price as continuous decision variables (consistent with prior literature), we simplified the analysis by permitting two levels of advertising in the blanket case. This still allowed capturing relevant intuitions with respect to the issues we sought to explore. The fact that firms often decide on the general scale of their ad campaigns, for example an aggres-
sive vs. a small-scale effort, conforms to our “heavy” vs. “light” designations. In reality, of course, the exact budget allocated to advertising can be more granular. Our model can be relaxed to accommodate this fact by letting advertising have several more levels than the binary heavy/light choices; with the findings remaining qualitatively similar to those reported in the paper. Extending the model to allow continuous advertising levels would smooth out the discrete transitions from light to heavy ad spend, yet we believe the competitive forces driving firms to differentiate in quality or ad levels, and the dependence on advertising’s effectiveness, should continue to operate.

Fourth, we assumed that the only way for consumers to become informed about products was through advertising. This allowed us to focus on advertising’s role in affecting whether consumers consider a given product and, in turn, on firms’ strategic incentives to choose quality, expend on advertising, and price their offerings. In reality, consumers have other ways to become informed about available products, e.g., through word of mouth or search, and may be prompted to do so after seeing an ad (as in Mayzlin and Shin 2011). Assuming that each consumer has a positive probability of becoming informed about the firm’s product even without directly receiving an ad for it would not change our model findings, as long as this probability is not too high. In this case, we could interpret advertising in our model as increasing this base probability by \( \alpha \). Our results would also hold if consumer search costs are sufficiently high, so that not all consumers have an incentive to incur them to become fully informed. However, if these costs are only moderate and there is much to gain by engaging in search efforts, uninformed consumers may have an incentive to search, which could impact our findings. We leave for future research the investigation of how consumer search affects endogenous quality positions and advertising levels. The model could also be extended to incorporate spillover effects, that is, when consumers receive an ad from one firm there is some likelihood that they will become aware of the other firm’s product as well. This could capture a “category expansion” effect for advertising beyond the “specific brand” effect modeled in this paper. In that case, the competitive segment would be larger and the captive segments smaller than in our analysis, which could create a disincentive to advertise. The existence conditions for the

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20 This analysis is available from the authors upon request.
21 Note that retail sales assistance or trade incentives can also be regarded as part of a firm’s promotion strategy (as a firm can offer more or less retail incentives; choose the retailers it provides the product to, etc.) to make its offerings more salient to consumers. The same is true with respect to ads on search engines and bidding for top links.
22 Alternatively, we could re-formulate our model so that consumers have probability \( \alpha \) of being informed through non-advertising mechanisms and firms have to choose whether to advertise or not. If a firm chooses to advertise, all consumers are informed of its product; if it does not advertise then only a portion \( \alpha \) are informed. It is easy to see that this description would lead to an identical analysis.
different equilibria presented here would likely change, but our main results should remain valid.

Fifth, we employed stylized assumptions on firm costs in order to simplify the presentation and analysis. In particular, we assumed that products are manufactured at a zero variable cost. In this way we were able to focus on the strategic forces that drive the degree of quality differentiation and the interaction with advertising and pricing. This assumption is also reasonable in categories where variable costs are either negligible, e.g., digital goods or pharmaceutical drugs, or only weakly related to quality positions (e.g., some beverages and clothing). Future research could examine the implications of including variable costs that are a function of product quality. It is conceivable, for example, that if variable costs are a convex function of quality that the high-quality firm earns a smaller margin on its product than the lower-quality rival does, and hence has less incentive to advertise heavily than in the model presented here.

Lastly, we have focused on consumer heterogeneity in willingness to pay for quality. This allowed us to examine how vertical differentiation is impacted by the need to inform consumers about product offerings. In practice, consumers may also exhibit heterogeneity in horizontal tastes. It is possible that firms’ choice of product positioning along a horizontal continuum may be impacted by the need to advertise to make consumers aware of their offerings. For example, it would be interesting to see if an equilibrium can be sustained where firms co-locate in the product space as in the vertical setup. The interaction between product location and advertising with horizontal heterogeneity can be fruitful ground for future research.

Appendix: Proofs

Proof of Lemma 1

In this subgame, firms have selected the same quality level \( s \) and then advertising levels \( a_i \) and \( a_j \), they now need to select their prices. In the \((H, H)\) case, the competitive segment comprises the entire market and products are undifferentiated, resulting in undifferentiated Bertrand competition. Thus prices are \( p_i = p_j = 0 \) and profits are \( \pi_i = \pi_j = -c_H \).

Consider the case when firm \( i \) chooses \( a_i = L \) and chooses its price based on the CDF \( F_i \) with no mass point and \( F_i(s/2) = 1 \). Firm \( j \) mixes over a set of prices \( p \) only if all these prices lead to the same profit. Recall that only consumers aware of product \( j \) can buy from firm \( j \). They do so if \( \theta \) is larger than \( p/s \) and, if they are aware of product \( i \), only if the price of \( j \) is smaller than the price of \( i \). The profit of firm \( j \) when
selecting \( p < s/2 \) is thus: \( \pi_j(p) = a_j(1 - a)p(1 - \frac{p}{s}) + a_j ap(1 - \frac{p}{s})(1 - F_i(p)) - c_j. \)

If firm \( j \) selects the monopolist price \( s/2 \), its profit will be \( a_j(1 - a)\frac{s}{2} - c_j \). Firm \( j \) is indifferent between \( p \) and \( s/2 \) iff \( F_i(p) = \frac{1}{a}(1 - \frac{1-a}{4p/s(1-p/s)}) \). Thus firm \( i \) mixes over prices in \( \left[ \frac{1 - \sqrt{a}}{2}, \frac{s}{2} \right] \) according to \( F_i \) and firm \( j \) earns a profit equal to \( \pi_j = a_j(1 - a)\frac{s}{2} - c_j \) if it chooses any prices in \( \left[ \frac{1 - \sqrt{a}}{2}, \frac{s}{2} \right] \).

If firm \( j \) also chooses \( a_j = L \), both firms choose prices in \( \left[ \frac{1 - \sqrt{a}}{2}, \frac{s}{2} \right] \) according to the CDF \( F(p) = \frac{1}{a}(1 - \frac{1-a}{4p/s(1-p/s)}) \). This concludes the case \((L, L)\).

Let’s now consider the case when firm \( j \) chooses \( a_j = H \) and firm \( i \) chooses \( a_i = L \). We need to look for a mixed-strategy with the possibility of a mass point at \( s/2 \) for firm \( j \). Let’s call \( \beta \) the probability that firm \( j \) selects a price \( p_j < s/2 \). With probability \( \beta \), firm \( j \) plays according to the CDF \( F_j \) and with probability \( 1 - \beta \), it selects \( s/2 \). Then the profit of firm \( i \) is for \( p < s/2 \): \( \pi_i(p) = ap(1-\frac{s}{2})(1 - \beta F_j(p)) - c_L. \)

As the price gets closer to the monopolist price \( s/2 \), the profit converges to \( a(1 - \beta)\frac{s}{2} - c_L \). \( \pi_i(p) = a(1 - \beta)\frac{s}{2} - c_L \) and leads to \( F_j(p) = \frac{1}{\beta}(1 - \frac{1-\beta}{4p/s(1-p/s)}) \) for any prices in \( \left[ \frac{1 - \sqrt{a}}{2}, \frac{s}{2} \right] \). If firm \( j \) mixes over \( \left[ \frac{1 - \sqrt{a}}{2}, \frac{s}{2} \right] \) and its profit is constant only over \( \left[ \frac{1 - \sqrt{a}}{2}, \frac{s}{2} \right] \), then \( \beta \leq a \). If firm \( i \) mixes over \( \left[ \frac{1 - \sqrt{a}}{2}, \frac{s}{2} \right] \) and its profit is constant only over \( \left[ \frac{1 - \sqrt{a}}{2}, \frac{s}{2} \right] \), then \( a \leq \beta \). Thus \( \beta = a \) which concludes the cases \((L, H)\) and \((H, L)\).

At the end of the proof of Proposition 1, we provide a visual representation of the equilibrium pricing strategies as a function of advertising impact.

**Proof of Proposition 1**

**Preliminaries:** Before we turn to proving the actions firm select in the full game equilibrium, it will prove useful to establish a few preliminary results. We first establish the pricing equilibrium when \( s_1 < s_2 \); as this will be needed for the proof of the equilibrium of the full game in Proposition 1.

(1) The demand structure is very similar to the one in Lemma 1 for the consumers who consider both firms’ products. However, we have to take into account the consumers who only consider one of the two products. Let us introduce notations for three types of indifferent consumers. The consumer whose \( \vartheta \) parameter is \( t_2 \equiv \frac{p_2 - p_1}{s_2 - s_1} \) obtains the same utility from buying firm 1’s and firm 2’s product. The consumer with \( \vartheta = t_2 \equiv \frac{p_2}{s_2} \) obtains 0 utility from buying firm 2’s product, that is, s/he is indifferent between purchasing from firm 2 and not buying anything. Finally, \( t_1 \equiv \frac{p_1}{s_1} \) is the critical point of buying firm 1’s product versus not buying anything.
The demand schedules are as follows

\[ D_1(p_1, p_2) = \begin{cases} 
    a_1(1 - t_1) & \text{if } p_1 \leq p_2 - (s_2 - s_1) \\
    a_1(1 - a_2)(1 - t_1) + a_1a_2(t_2 - t_1) & \text{if } p_2 - (s_2 - s_1) \leq p_1 \leq \frac{p_2 s_1}{s_2} \\
    a_1(1 - a_2)(1 - t_1) & \text{if } \frac{p_2 s_1}{s_2} \leq p_1 
\end{cases} \]

\[ D_2(p_1, p_2) = \begin{cases} 
    a_2(1 - a_1)(1 - t'_2) & \text{if } p_1 \leq p_2 - (s_2 - s_1) \\
    a_2(1 - a_1)(1 - t'_2) + a_2a_1(1 - t_2) & \text{if } \frac{p_1 s_2}{s_1} \leq p_2 \leq p_1 + (s_2 - s_1) \\
    a_2(1 - t'_2) & \text{if } p_2 \leq \frac{p_1 s_2}{s_1} 
\end{cases} \]

Since the variable cost is zero, the revenue is simply the price multiplied by the demand. The revenues are continuous but not necessarily concave

\[ R_1(p_1, p_2) = \begin{cases} 
    a_1p_1(1 - \frac{p_1}{s_1}) + a_1a_2p_1(\frac{p_2}{s_2} - \frac{p_1}{s_1} - \frac{p_1}{s_1}) & \text{if } p_1 \leq p_2 - (s_2 - s_1) \\
    a_1(1 - a_2)p_1(1 - \frac{p_1}{s_1}) & \text{if } p_2 - (s_2 - s_1) \leq p_1 \leq \frac{p_2 s_1}{s_2} \\
    a_1(1 - a_2)p_1(1 - \frac{p_1}{s_1}) & \text{if } \frac{p_2 s_1}{s_2} \leq p_1 
\end{cases} \]

\[ R_2(p_1, p_2) = \begin{cases} 
    a_2(1 - a_1)p_2(1 - \frac{p_2}{s_2}) + a_2a_1p_2(1 - \frac{p_2}{s_2} - \frac{p_1}{s_1}) & \text{if } p_1 \leq p_2 - (s_2 - s_1) \\
    a_2(1 - a_1)p_2(1 - \frac{p_2}{s_2}) + a_2a_1p_2(1 - \frac{p_2}{s_2} - \frac{p_1}{s_1}) & \text{if } \frac{p_1 s_2}{s_1} \leq p_2 \leq p_1 + (s_2 - s_1) \\
    a_2p_2(1 - \frac{p_2}{s_2}) & \text{if } p_2 \leq \frac{p_1 s_2}{s_1} 
\end{cases} \]

(2) Let’s characterize the pure-strategy equilibrium if it exists. In equilibrium, as firms will not choose a price above the monopolist price, we consider \( p_1 \leq \frac{s_1}{2} \) and \( p_2 \leq \frac{s_2}{2} \). In the first region \( p_1 \leq p_2 - (s_2 - s_1) \), the profit of firm 1 increases, thus there exists no equilibrium interior to that region. In the third region \( \frac{p_2 s_1}{s_2} \leq p_1 \), the profit of firm 2 increases, thus there exists no equilibrium interior to that region either.

A pure-strategy equilibrium can only exist in the middle region \( p_2 - (s_2 - s_1) \leq p_1 \leq \frac{p_2 s_1}{s_2} \). The profit of firm 1 has three potential local maxima: \( b_1(p_2) = \frac{s_1}{2} \left(1 - a_2\right)\left(s_2 - s_1 + a_2 p_2\right) \), \( p_2 - (s_2 - s_1) \), and \( \frac{s_1}{2} \). The profit of firm 2 also has three potential local maxima: \( b_2(p_1) = \frac{s_2}{2} \left(\frac{s_2 - s_1 + a_1 p_1}{s_2 - s_1 + a_1 s_1}\right) \), \( \frac{p_1 s_2}{s_1} \), and \( \frac{s_2}{2} \). Furthermore, the local maximum \( p_2 = \frac{p_1 s_2}{s_1} \) cannot be part of an equilibrium because \( \frac{p_2 s_1}{s_2} \) is not a local maximum for the profit of firm 1, thus firm 1 would have a profitable deviation (to \( \frac{s_1}{2} \) if this is coupled with the local maxima \( b_1(p_2) = \frac{s_1}{2} \left(1 - a_2\right)\left(s_2 - s_1 + a_2 p_2\right) \) or \( \frac{s_1}{2} \); to \( b_1(p_2) \) if this is coupled with the local maximum \( p_2 - (s_2 - s_1) \)). Similarly the local maximum \( p_1 = p_2 - (s_2 - s_1) \) cannot be part of an equilibrium because \( p_1 + (s_2 - s_1) \) is not a local maximum for the profit of firm 2, thus firm 2 would have a profitable deviation. In the end the only candidate for a pure-strategy equilibrium is \( (p_1 = b_1(p_2), p_2 = b_2(p_1)) \).

Therefore, the only candidate to be an equilibrium is

\[ p_1^* = s_1(s_2 - s_1) \frac{2(1 - a_2)(s_2 - s_1 + a_1 s_1) + a_2 s_2}{4(s_2 - s_1 + a_2 s_1)(s_2 - s_1 + a_1 s_1) - a_1 a_2 s_1 s_2} \]

\[ p_2^* = s_2(s_2 - s_1) \frac{2(s_2 - s_1 + a_2 s_1) + a_1(1 - a_2) s_1}{4(s_2 - s_1 + a_2 s_1)(s_2 - s_1 + a_1 s_1) - a_1 a_2 s_1 s_2} \]

with profits equal to

\[ \pi_1^* = \frac{a_1(s_2 - s_1 + a_2 s_1)}{s_1(s_2 - s_1)}(p_1^*)^2 - c_1 \]

and

\[ \pi_2^* = \frac{a_2(s_2 - s_1 + a_1 s_1)}{s_2(s_2 - s_1)}(p_2^*)^2 - c_2. \]
This is an equilibrium if and only if no firm wants to deviate to its monopolist price $\frac{s_1}{s_2}$ for firm 1 and $\frac{s_2}{s_2}$ for firm 2, that is, if
\[ \pi_1^* \geq \frac{s_1}{2}D_1 \left( \frac{s_1}{s_2}, p_2^* \right) - c_1 \quad \text{and} \quad \pi_2^* \geq \frac{s_2}{2}D_2 \left( p_2^*, \frac{s_2}{2} \right) - c_2. \]

Since $\frac{s_2}{s_2} - \frac{s_1}{s_2} \leq \frac{s_1}{s_2}$, the demand at the local maximum is $D_1 \left( \frac{s_1}{s_2}, p_2^* \right) = \frac{a_1(1-a_2)s_1}{4}$. Firm 2 has a local maximum at $\frac{s_2}{s_2}$ only when $p_1^* + (s_2 - s_1) \leq \frac{s_2}{s_2}$ and in that case $D_2 \left( p_2^*, \frac{s_2}{2} \right) = \frac{a_2(1-a_1)s_2}{4}$. Thus the no-deviation conditions become:
\[ \frac{a_1(s_2 - s_1 + a_2 s_1)}{s_1(s_2 - s_1)}(p_1^*)^2 - \frac{a_1(1-a_2)s_1}{4} \geq 0 \quad \text{and} \quad \frac{a_2(s_2 - s_1 + a_1 s_1)}{s_2(s_2 - s_1)}(p_2^*)^2 - \frac{a_2(1-a_1)s_2}{4} \geq 0. \]

Let’s express these conditions in terms of the ratio $s = \frac{s_1}{s_2}$.
\[ a_1(1 - s + a_2 s)s(1-s)\left(\frac{2(1-a_2)(1-s + a_1 s) + a_2}{4(1-s + a_2 s)(1-s + a_1 s) - a_1 a_2 s}\right)^2 - \frac{a_1(1-a_2)s}{4} \geq 0, \quad (1) \]
\[ a_2(1 - s + a_1 s)(1-s)\left(\frac{2(1-s + a_2 s) + a_1(1-a_2)s}{4(1-s + a_2 s)(1-s + a_1 s) - a_1 a_2 s}\right)^2 - \frac{a_2(1-a_1)s}{4} \geq 0. \quad (2) \]

The LHS of (1) and (2) are concave functions of $s$ for $0 \leq s \leq 1$. The LHS of (1) is equal to 0 when $s = 0$ and negative when $s = 1$. The LHS of (2) is positive when $s = 0$ and negative when $s = 1$. Therefore there exists a unique cutoff function $\hat{\gamma}(a_1, a_2)$ in $(0,1)$ such that $\frac{s_1}{s_2}$ satisfies (1) and (2) iff $\frac{s_1}{s_2} \leq \hat{\gamma}(a_1, a_2)$. Note that $\hat{\gamma}(1,1) = 1$ and
\[ \frac{s_1}{s_2} \leq \hat{\gamma}(a_1, a_2) \Rightarrow \text{pure-strategy pricing equilibrium} \]
\[ \frac{s_1}{s_2} > \hat{\gamma}(a_1, a_2) \Rightarrow \text{mixed-strategy pricing equilibrium} \]

(3) Let’s now prove that, when firms play mixed strategies in the pricing stage, then regardless of the advertising levels (recalling that at least one firm must be advertising lightly), firm 1 is better off co-locating with firm 2 than selecting a lower quality level $s_1 \in (\hat{\gamma}s_2, s_2)$.

When, the pricing equilibrium is in mixed strategies, if firm 2 advertises lightly ($a_2 = a$), then firm 1 selects its monopolist price $\frac{s_1}{s_2}$ with positive probability. Firm 1’s profit is equal to its monopolist profit on its captive segment $\pi_1 = a_1(1-a)\frac{s_1}{s_2} - c_1 < a_1(1-a)\frac{s_2}{s_2} - c_1$ which is, per Lemma 1, the profit that firm 1 would get by co-locating with firm 2, keeping the advertising levels constant.

Thus the last case to investigate is $a_1 = a$ and $a_2 = 1$. The pure-strategy equilibrium ceases to exist when firm 2 has an incentive to deviate to its monopolist price $\frac{s_2}{s_2}$, which is the case when
\[ \frac{a_2(s_2 - s_1 + a_1 s_1)}{s_2(s_2 - s_1)}(p_2^*)^2 = \frac{(s_2 - s_1 + a_1 s_1)}{s_2(s_2 - s_1)}(s_2(s_2 - s_1)(1-(s_2 - s_1)(1-a)s_1)) \geq (1-a)\frac{s_2}{s_2} \]
which is equivalent to $\frac{s_1}{s_2} \geq 4.5 - 3a - \sqrt{1 - 3a} = \hat{\gamma}(a, 1)$. In the mixed-strategy equilibrium, firm 2 selects its monopolist price $\frac{s_2}{s_2}$, but firm 1 does not select its monopolist price $\frac{s_1}{s_2}$ on the equilibrium path. Recall that revenues are:
\[ R_1(p_1, p_2) = \begin{cases} ap_1(1 - \frac{p_1}{s_1}) & \text{if } p_1 \leq p_2 - (s_2 - s_1) \\ ap_1(\frac{p_2 - p_1}{s_2 - s_1} - \frac{p_1}{s_1}) & \text{if } p_2 - (s_2 - s_1) \leq p_1 \leq \frac{p_2 s_1}{s_2} \\ 0 & \text{if } p_2 s_1 \leq p_1 \end{cases} \]
\[ R_2(p_1, p_2) = \begin{cases} 
(1 - a)p_2(1 - \frac{p_2}{s_2}) & \text{if } p_1 + (s_2 - s_1) \leq p_2 \\
(1 - a)p_2(1 - \frac{p_2}{s_2}) + ap_2(1 - \frac{p_2 - p_1}{s_2 - s_1}) & \text{if } \frac{p_1 s_2}{s_1} \leq p_2 \leq p_1 + (s_2 - s_1) \\
p_2(1 - \frac{p_2}{s_2}) & \text{if } p_2 \leq \frac{p_1 s_2}{s_1} 
\end{cases} \]

The pricing strategy of firm 1 includes a finite number of prices and thus does not have a continuous support. The number of prices in the strategy increases as \( s_1 \) gets closer to \( s_2 \). In general, firm 1 mixes over \( n \) prices \( p_{11} < p_{12} < \ldots < p_{1n} \) with probabilities \( \alpha_1, \alpha_2, \ldots, \alpha_n \) while firm 2 mixes over \( n + 1 \) prices \( p_{21} < p_{22} < \ldots < p_{2n} < \frac{s_2}{2} \) with probabilities \( \beta_1, \beta_2, \ldots, \beta_n, \beta_{n+1} \). For example, when \( s_1 \) is close to \( \gamma(a, 1)s_2 \) then \( n = 1 \) which means that firm 1 selects \( p_{11} \) and firm 2 chooses \( p_{21} \) with probability \( \beta_1 \) and \( \frac{s_2}{2} \) with probability \( 1 - \beta_1 \). Firm 2 either chooses its best response to \( p_{11} \) which is \( p_{21} = b_2(p_{11}) = \frac{s_2 s_2 - s_1 + \alpha p_{11}}{s_2 - s_1 + \alpha s_1} \) (see preliminaries (2) for the definition of this function) or its monopolist price \( \frac{s_2}{2} \). Therefore its profit when playing either price on the equilibrium path is the same: \( \pi_2^* = \frac{(s_2 - s_1 + \alpha)(p_{21})^2}{s_2 - s_1 + \alpha s_1} - c_H = (1 - a)\frac{s_2}{4} - c_H \). Leading to \( p_{21} = \frac{s_2}{2} \sqrt{\frac{1 - a}{s_2 - s_1 + \alpha s_1}} \). From the best response of firm 2, we get

\[ p_{11} = \frac{1}{(1 - a)(s_2 - s_1) - (s_2 - s_1)} \]

The profit of firm 1 has 2 local maxima: one in \( p_{21} - (s_2 - s_1), \frac{s_2}{2} p_{21} \) and one at \( \frac{s_2}{2} - (s_2 - s_1) \). The expected profit of firm 1 when \( p_1 \in [p_{21} - (s_2 - s_1), \frac{s_2}{2} p_{21}] \) is \( \beta_1 a p_1 (\frac{p_{21} - p_1}{s_2 - s_1} + \frac{p_1}{s_2 - s_1}) + (1 - \beta_1) a p_1 (1 - \frac{p_1}{s_2 - s_1}) \) which is maximized at \( p_{11} = \frac{s_2}{2} \frac{(1 - \beta_1)(s_2 - s_1) + \beta_1 p_{21}}{s_2 - s_1 + \beta_1 s_1} \). The best response of firm 1 leads to

\[ \beta_1 = \frac{s_2 - s_1}{s_1} \frac{\frac{s_2}{2} - p_{11}}{p_{21} - (s_2 - s_1)} \]

These strategies form an equilibrium as long as firm 1 does not want to deviate to its other local maximum \( (1 - \beta_1) a \frac{s_2}{2} \frac{(s_2 - s_1)}{s_1} (s_2 - s_1) \) at \( p_1 = \frac{s_2}{2} - (s_2 - s_1) \). In the end the profit of firm 1 is \( \pi_1^* = a \frac{(s_2 - s_1 + \beta_1 s_1)}{s_1(s_2 - s_1)} (p_{11})^2 - c_L \).

Let’s define \( \alpha_+ = \Pr[p_1 < p_{1i}] \) and \( \alpha_+ = \Pr[p_1 > p_{1i}] \), so \( \alpha_- = 0, \alpha_2 = \alpha_1, \ldots, \alpha_n = 1 - \alpha_n \) and \( \alpha_1 = 1 - \alpha_1, \alpha_2 = 1 - \alpha_1 - \alpha_2, \ldots, \alpha_{n+1} = 0 \). Then if firm 2 sets a price \( p_2 \) in \( [s_2 p_{11}, p_{11} + (s_2 - s_1)] \), its profit is \( \alpha_-(1 - a) p_2 (1 - \frac{p_2}{s_2}) + \alpha_+ (1 - a) p_2 (1 - \frac{p_2}{s_2}) + a p_2 (1 - \frac{p_2 - p_{1i}}{s_2 - s_1}) + \alpha_+ p_2 (1 - \frac{p_2}{s_2}) \) which is maximized at \( p_2 = \frac{s_2}{2} \frac{(1 - \alpha_1 - \alpha_2 a)(s_2 - s_1) + \alpha_1 a s_2}{s_2 - s_1 + \alpha_2 a s_2} \) and yields \( \pi_2^* = \frac{(1 - \alpha_1 - \alpha_2 a)(s_2 - s_1) + \alpha_1 a s_2}{s_2 - s_1 + \alpha_2 a s_2} (p_{1i})^2 - c_H \). Since when firm 2 chooses its monopolist price \( \frac{s_2}{2} \) its profit is \( (1 - a)\frac{s_2}{4} - c_H \), \( p_{21} = \frac{s_2}{2} \sqrt{\frac{1 - a}{s_2 - s_1 + \alpha s_1}} \).

From the best response of firm 2, we get \( p_{1i} = \frac{1}{\alpha a} \sqrt{(1 - a)(s_2 - s_1)((1 - \alpha_1 - \alpha_2 a)(s_2 - s_1) + \alpha_2 a s_2)} - (1 - \alpha_1 - a)(s_2 - s_1) \).

Similarly let’s define \( \beta_{i+} = \Pr[p_2 > p_{2i}] \), so that \( \beta_1 = 1 - \beta_1, \beta_2 = 1 - \beta_1 - \beta_2, \ldots, \beta_{n+1} = \beta_n + 1 \). Then if firm 1 sets a price \( p_1 \) in \( [p_{2i} - (s_2 - s_1), \frac{s_2}{2} p_{2i}] \), its profit is \( \beta_1 a p_1 (\frac{p_{2i} - p_{1i}}{s_2 - s_1} - \frac{p_{1i}}{s_2 - s_1}) + \beta_{i+} a p_1 (1 - \frac{p_{1i}}{s_2 - s_1}) \), which is maximized at \( p_{1i} = \frac{s_2}{2} \frac{\beta_{i+}(s_2 - s_1) + \beta_{i+} p_{2i}}{s_2 - s_1 + \beta_{i+} s_2} \) and yields \( \pi_1^* = \frac{\beta_{i+}(s_2 - s_1) + \beta_{i+} p_{2i}}{s_2 - s_1 + \beta_{i+} s_2} (p_{1i})^2 - c_L \). Plugging the expression of \( p_{1i} \) in the best response, we get \( \beta_1 = \frac{s_2 - s_1}{p_{1i} - \frac{1}{2}(p_{2i} - (s_2 - s_1))} \), and \( \beta_{n+1} = 1 - \beta_1 - \ldots - \beta_n \). We have now expressed all the prices and \( \beta_1 \) in terms of \( \alpha_1, \alpha_2, \ldots, \alpha_n \). Given that \( \alpha_n = 1 - \alpha_1 - \ldots - \alpha_{n-1} \), we have \( n - 1 \) unknown. The \( n - 1 \) conditions
come from the fact that firm 1 is willing to mix over $p_{11} < p_{12} < \ldots < p_{1n}$ only if all these prices deliver the same expected profits which is true if $\sqrt{\beta_{1+}(s_2-s_1) + \beta_1 s_2 p_{1i}} = \sqrt{(s_2-s_1) + \beta_1 s_1 p_{1i}}$ for $i = 2\ldots n$. These strategies form an equilibrium as long as firm 1 does not want to deviate to $\beta_{n+1} a \frac{s_1}{\beta_1} \left(\frac{s_2}{2} - (s_2 - s_1)\right)$, its other local maximum at $\frac{s_2}{2} - (s_2 - s_1)$. We then checked numerically that $\pi^*_1 \leq a(1 - a)\frac{s_1}{4}$ for $s_1 \in [\gamma(a, 1)s_2, s_2)$ and $a \in (0, 1)$. In the Technical Appendix, Figure A-2 shows an example of the price strategies of the firms when product differentiation is small and also depicts the CDF of the pricing strategies under co-location. In addition, Figure A-3 shows the corresponding profit of firm 1 as a function of its quality $s_1$ when firm 1 advertises lightly and its rival advertises heavily.

Proof: We are now ready to prove Proposition 1. Let’s define $\bar{c} = \frac{(1-a)^2}{4}$, $c = \frac{1}{4a} - \frac{a(1-a)}{4}$, and recall that $\Delta a \geq a = 0.13$, i.e., $a \leq 0.87$. We now prove the following results by analyzing the best responses of the 2 firms: on the equilibrium path (1) if $\Delta c \geq \bar{c}$, firms co-locate on qualities with $s_1 = s_2 = 1$, and both advertise lightly $(L, L)$; (2) if $c \geq \Delta c \geq \bar{c}$, firms co-locate on qualities with $s_1 = s_2 = 1$, and firm 1 advertises lightly while firm 2 advertises heavily $(L, H)$; (3) if $\bar{c} > \Delta c$, firms differentiate in qualities, and both advertise heavily $(H, H)$.

What is the best response of firm 1 when firm 2 chooses $s_2 = 1$ followed by $a_2 = a$ if $\Delta c > \bar{c}$, and $a_2 = 1$ if $\Delta c \leq \bar{c}$. Let’s first look at the case $\Delta c > \bar{c}$ when $a_2 = a$. For any $s_1$ and $a_1$, the profit of firm 1 is $\pi_1 \leq a(1 - a)/4 - c_L$. Indeed if $a_1 = a$ and the price equilibrium is in pure strategy, the profit is $\pi_1 = a(1 - s_1 + as_1)s_1(1 - s_1)\left(\frac{2(1-a)(1-s_1+as_1)+a}{4(1-s_1+as_1)+a}\right)^2 - c_L \leq \frac{a(1-a)}{4} - c_L$ for any $s_1$ if $a < 0.877$. If the price equilibrium is in mixed-strategy, $\pi_1 = \frac{a(1-a)}{4} s_1 - c_L \leq \frac{a(1-a)}{4} - c_L$. Similarly, if $a_1 = 1$ and the price equilibrium is in pure strategy, $\pi_1 = (1 - s_1 + as_1)s_1(1 - s_1)\left(\frac{2(1-a)+a}{8(1-s_1+as_1)+a}\right)^2 - c_H \leq \frac{(1-a)}{4} - c_H$ for any $s_1$ if $a < 0.895$. If the price equilibrium is in mixed-strategy, $\pi_1 = \frac{(1-a)}{4} s_1 - c_H \leq \frac{(1-a)}{4} - c_H$. Therefore $\pi_1 \leq \frac{(1-a)}{4} - c_H < \frac{a(1-a)}{4} - c_L$ given that $\Delta c > \bar{c}$. We have $\pi_1 \leq a(1 - a)/4 - c_L$ for any $s_1$ and $a_1$, and $\pi_1 = a(1 - a)/4 - c_L$ for $s_1 = 1$ and $a_1 = a$ per Lemma 1. The best response of firm 1 is thus $s_1 = 1$ in stage 1 and then in stage 2 $a_1 = a$ after $s_1 = 1$ and $a_1 = \arg \max_{a_1 \in \{a, 1\}} \pi_1(a_1, a_2; s_1, s_2)$ elsewhere.

Let’s now look at the case $\bar{c} \geq \Delta c > \bar{c}$ when $a_2 = 1$. For any $s_1$ and $a_1$, the profit of firm 1 is $\pi_1 \leq a(1 - a)/4 - c_L$. Indeed if $a_1 = a$ and the price equilibrium is in pure strategy, $\pi_1 = a s_1(1 - s_1)\left(\frac{a}{8(1-s_1+as_1)-as_1}\right)^2 - c_L$ is maximized at $s_1 = \frac{4}{1 + 3a}$ so $\pi_1 \leq \frac{a^2}{38} - c_L$. Thus, $\pi_1 \leq \frac{a(1-a)}{4} - c_L$ if $a < 11/12 \simeq 0.917$. While if the price equilibrium is in mixed-strategy, $\pi_1 \leq \frac{a(1-a)}{4} - c_L$ per the preliminaries of the proof given above. Similarly, if $a_1 = 1$ then the price equilibrium is in pure strategy, $\pi_1 = s_1(1 - s_1)\left(\frac{1}{4s_1}\right)^2 - c_L \leq \frac{1}{38} - c_H < \frac{a(1-a)}{4} - c_L$ given that $\Delta c > \bar{c}$. We thus have $\pi_1 \leq a(1 - a)/4 - c_L$ for any $s_1$ and $a_1$,
and \( \pi_1 = a(1 - a)/4 - c_L \) for \( s_1 = 1 \) and \( a_1 = a \) per Lemma 1. The best response of firm 1 is thus to choose \( s_1 = 1 \) and \( a_1 = a \), and \( a_1 = \arg \max_{a_1 \in \{a,1\}} \pi_1(a_1, a_2; s_1, s_2) \) elsewhere. The last case to consider is \( \Delta c \leq \mathcal{C} \) when \( a_2 = 1 \). For any \( s_1 \) and \( a_1 \), the profit of firm 1 is \( \pi_1 \leq 1/48 - c_H \). Indeed just like in the previous case, if \( a_1 = a \), \( \pi_1 \leq a(1 - a)/4 - c_L \) if \( a < 11/12 \simeq 0.917 \). Given that \( \Delta c \leq \mathcal{C}, \pi_1 \leq 1/48 - c_H \). Then if \( a_1 = 1 \), the price equilibrium is in pure strategy, \( \pi_1 = s_1(1-s_1)(1-s_1)/4-c_H \leq a(1 - a)/4 - c_H \). We have \( \pi_1 \leq 1/48 - c_H \) for any \( s_1 \) and \( a_1 \), and \( \pi_1 = 1/48 - c_H \) for \( s_1 = 1/2 \) and \( a_1 = 1 \). The best response of firm 1 is thus to choose \( s_1 = 1/2 \) in stage 1 and then in stage 2 \( a_1 = 1 \) after \( s_1 = 1/2 \) and \( a_1 = \arg \max_{a_1 \in \{a,1\}} \pi_1(a_1, a_2; s_1, s_2) \) elsewhere.

We now need to check firm 2’s best response to firm 1’s strategy. Let’s look at the same three cases. First, when \( \Delta c > \mathcal{C} \), firm 2 has a profitable deviation only if it can get a profit \( > a(1 - a)/4 - c_L \). Given that \( s_1 = 1 \), for any \( s_2 \) and \( (a_1, a_2) \), the profit of firm 2 is \( \pi_2 \leq a(1 - a)/4 - c_L \). Hence, no profitable deviation exists for firm 2. Let’s now consider \( \mathcal{C} \geq \Delta c > \mathcal{C} \) firm 2 has a profitable deviation only if it can get a profit \( > (1 - a)/4 - c_H \). Given that \( s_1 = 1 \), for any \( s_2 \) and \( (a_1, a_2) \), the profit of firm 2 is \( \pi_2 \leq (1 - a)/4 - c_H \) and \( \pi_2 \leq (1 - a)/4 - c_L \) otherwise, so no profitable deviation exists for firm 2.

When \( \Delta c \leq \mathcal{C} \) and \( s_1 = 1/2 \), if firm 2 chooses \( s_2 = 1 \) then \((H, H)\) follows. A potential profitable deviation is to locate close enough to \( s_1 = 1/2 \) to generate \((L, H)\). The first question is whether firm 2 makes more profit in that case. If \( s_2 \in [1/2, 7/(12(1-a))] \), the profit of firm 2 after \((L, H)\) is not larger than \( 7/48 - c_H \). If \( s_2 \in (7/(12(1-a)), 1] \), the profit of firm 2 after \((L, H)\) is larger than \( 7/48 - c_H \). Note that \( 7/(12(1-a)) < s_1/\gamma(a,1) \), so \((L, H)\) leads to a mixed strategy equilibrium in price for \( s_2 \in (7/(12(1-a)), s_1/\gamma(a,1)) \). The next question is whether \((L, H)\) will arise after a deviation of firm 1 at the product stage. This will not be the case if the best response of firm 1 to \( a_2 = 1 \) is \( a_1 = 1 \). Let’s consider that firm 2 chose \( s_2 \in [1/2, 7/(12(1-a)), 1] \) and firm 1 \( s_1 = 1/2 \). The profit of firm 1 after \((L, H)\) is lower than \( a(1-a)(4/7)/4 - c_L \) when \( s_2 \in [7/(12(1-a)), s_1/\gamma(a,1)] \) per the preliminaries of the proof given above and is decreasing with respect to \( s_2 \) when \( s_2 \in [s_1/\gamma(a,1), 1] \). We also know that the profit of firm 1 after \((H, H)\) is increasing with respect to \( s_2 \) when \( s_2 \in [s_1, 1] \). Therefore a sufficient condition for the best response of firm 1 to \( a_2 = 1 \) is \( a_1 = 1 \) that the profit when \( s_1 = 1/2, s_2 = 7/(12(1-a)) \) and \((H, H)\) is larger than \( a(1-a)/4 - c_L \). So when \( \Delta c \leq \mathcal{C} \) firm 2 does not have any profitable deviation from \( s_1 = 1/2 \) and \( s_2 = 1 \). We note for completeness that in a small region of the product differentiation case (close to the interface with the advertising differentiation case), \( s_1 = 1/2, s_2 = 1 \) and \((H, H)\) might not be an equilibrium because firm 2 might deviate to a lower quality product in the first stage leading to \((L, H)\). In that case, the equilibrium might entail mixed strategies at
Figure 3: Average Price Levels (for firm 2 on the left and firm 1 on the right) in the Different Equilibria

the advertising stage and/or at the product location stage which falls beyond the scope of this paper.

Figure 3 depicts the average prices in the various sub-game pricing equilibria as a function of the advertising impact differential ($\Delta \alpha$), conditional on each firm selecting the corresponding quality and advertising levels of the No Differentiation, Advertising Differentiation and Product Differentiation cases.

Proof of Corollary 1

The proof of Proposition 1 covers the subcase $\Delta c = 0$. Specifically, when $\underline{c} < \Delta c = 0 < \overline{c} \iff \Delta \alpha < \frac{1}{2} + \frac{1}{\sqrt{6}}$ $\simeq 0.91$ then advertising differentiation occurs in equilibrium. When $\Delta c = 0 < \underline{c} \iff \Delta \alpha > \frac{1}{2} + \frac{1}{\sqrt{6}}$ then product differentiation occurs in equilibrium.

Proof of Proposition 2

The comparative statics of $\pi_1$ and $\pi_2$ wrt $\Delta \alpha$ are derived from the expressions of the profits in the three types of equilibria.

Proof of Proposition 3

Formally, the targeting strategy of firm $i$, denoted by $S_i$, can be $\emptyset$, L, H, or $L \cup H$. The costs associated with the four possible advertising strategies are $0, c_L, c_H, c_U$, respectively. First, we examine the pricing stage of the game given the quality levels and the advertising segments in 9 possible different cases. 1- the two segments are distinct, that is, if $S_1 \cap S_2 = \emptyset$. 2- both firms choose both intervals, that is, if $S_1 = S_2 = L \cup H$. 
3- the firms choose the same intervals $S_1 = S_2 = H$. 4- the firms choose the same interval $S_1 = S_2 = L$. 5- $S_1 = L$ and $S_2 = L \cup H$. 6- $S_1 = L \cup H$ and $S_2 = H$. 7- $S_1 = H$ and $S_2 = L \cup H$. 8- $S_1 = L \cup H$ and $S_2 = L$. 9- each firm is a monopolist in its segment or one firm does not advertise and the other is a monopolist. The details of the price equilibrium in each of these cases is provided in the Technical Appendix.

We now turn our attention to the advertising stage of the game and determine which segments firms choose to advertise to. Let us examine the different types of possible equilibria. An equilibrium is described by a pair $(S_1, S_2)$, where the first set denotes the segment(s) chosen by firm 1 and the second set denotes the segment(s) chosen by firm 2. First we fix $s_1 < s_2$ and determine which types of equilibria are possible if $2/3 \geq t \geq 1/3$. Note that we do not consider cases where a firm does not advertise. We analyze the following cases in the Technical Appendix: $(L, H)$, $(L \cup H, H)$, $(L, L \cup H)$, $(H, L)$, and $(0, .)$ or $(., 0)$. In the following cases, we show in the Technical Appendix that one of the firms always has an incentive to deviate, thus, they do not constitute equilibria. We assume that firms make positive profits, otherwise it would be profitable for them to not advertise at all: $(L \cup H, L \cup H)$, $(L, L)$, $(H, H)$, $(H, L \cup H)$, and $(L \cup H, L)$.

If now we fix $s_1 = s_2$, then in the cases where $|S_1 \cap S_2| > 0$, we show that firms always have an incentive to deviate. When the target segments are exactly the same, prices are zero, hence firms have a straightforward incentive to deviate. There are two other cases with an intersection and non-zero profits: (1) when one firm advertises to both segments and the other to only the low segment (say $(L, L \cup H)$, w.l.o.g). In this case, firm 2’s profits before advertising are exactly the same as in the case of $(L, H)$, hence firm 2 will deviate to this to save on advertising costs (because $c_U > 0$). (2) when non-identical targets are chosen (say $(L \cup H, H)$, w.l.o.g), firm 1’s profits before advertising are exactly the same as in the case of $(L, H)$, hence firm 1 will deviate to the latter to save on advertising costs. Therefore, in the case of $s_1 = s_2$, the only possible advertising equilibria are $S_i = L$ and $S_j = H$. In this case, firms do not have an incentive to deviate as long as $2/3 \geq t \geq 1/3$.

Fixing $s_1 < s_2$, we have different patterns of equilibria depending on the parameter values. In all cases, low enough quality levels will make the costs of advertising prohibitively high for at least one firm resulting in a (partial) no advertising equilibrium. If costs are sufficiently low, the following equilibrium pattern emerges: Starting with $1/2 \leq t < 2/3$, the unique advertising equilibrium is $(L, L \cup H)$ when $s_1/s_2$ is small as defined by (??). When $s_1/s_2$ is larger than that threshold, $(L, H)$ is an equilibrium. This is a unique equilibrium when $s_1/s_2$ is in a medium range such that $s_1/s_2$ is below the threshold defined by (??). Above this threshold, the $(L, H)$ equilibrium is not unique and $(H, L)$ is an additional equilibrium. When
$1/3 < t < 1/2$, the equilibrium structure is similar, but for low values of $s_1/s_2$ (as defined by (??)) the unique equilibrium is $(L \cup H, H)$. For medium and high values of $s_1/s_2$, we have a unique $(L, H)$ equilibrium and multiple $(L, H)$ and $(H, L)$ equilibria as before.

Now we can examine the first (quality choice) stage of the game. We first show that a quality choice pair whereby both qualities are set at 1 cannot be an equilibrium. Note that firm 2’s revenue is increasing in $s_2$ in all advertising subgames, therefore firm 2 always has an incentive to increase its quality to the maximum given the advertising equilibrium. Also, increasing quality never causes the advertising equilibrium to change in a way that hurts firm 2. Therefore, in equilibrium $s_2 = 1$ must hold. Firm 1’s situation differs depending on $t$. First, when firm 1 does not advertise, it always has the incentive to increase its quality and advertise.\footnote{Since we assume $c_U < 1/48$ there is always a way for firm 1 to make positive profits by advertising.}

Next, we analyze the cases where both firms advertise, starting with $2/3 > t \geq 1/2$. When $s_1$ is small the advertising equilibrium is $(L, L \cup H)$. In this case firm 1 has an incentive to increase its quality up to $4/7$. However, this advertising equilibrium only possibly holds for $t > \bar{t}(s_1/s_2)$, and the highest possible $s_1$ value is reached in this region as $\bar{t}(s_1) = 2/3$ at $s_1 = 3\sqrt{3} - 5 < 4/7$; therefore firm 1 wants to increase its quality. In the other advertising equilibria, $(L, H)$ and $(H, L)$, firm 1 acts as a monopolist and trivially wants to increase its quality as long as that player does not expect the advertising equilibrium to change.

One potential Nash-equilibrium exists when $s_1 = s_1^* < 1$ with the $(H, L)$ advertising strategies. This is a subgame perfect Nash-equilibrium if player 2 commits to setting $S_2 = H$ for all $s_1 > s_1^*$. However, this equilibrium is Pareto dominated by the one where $s_1 = s_2 = 1$ and $S_1 = H$ and $S_2 = L$.

When $1/2 > t > 1/3$, the analysis is similar. For low values of $s_1$, the advertising equilibrium is $(L \cup H, H)$. However, in this case firm 1 may reach $s_1 = 4/7$ to maintain this type of equilibrium. One can then check that the profit firm 1 makes in this equilibrium $1/48 - c_U$ is strictly less than when setting $s_1 = 1$ and attaining the $(L, H)$ equilibrium with profits of $t^2/4 - tc_U$ as long as $t > 1/3$. Therefore, the only equilibria are the ones without overlapping advertising segments and then the same logic applies as in the previous case. Finally, when $s_1 = s_2$, it is clear that neither firm has an incentive to decrease its quality given the above comparisons, completing the proof of the proposition.

**Proof of Corollary 2**

When $t \leq 1/2$, per Proposition 3, $s_1 = s_2 = 1$, $S_1 = L$ and $S_2 = H$ with firm 2 choosing the monopolist price for its target segment $p_2 = \frac{1}{7}$. A consumer in $S_2$ either buys the product of firm 2 or does not buy since that consumer is not aware of the existence of the rival’s product. Such a consumer buys if and only
if $\theta s_2 - p_2 \geq 0$ or $\theta \geq \frac{1}{2}$. Recall that $S_2$ corresponds to $\theta \in [t, 1]$. Therefore there exists a pocket $\theta \in (t, \frac{1}{2})$ of unserved consumers.

**Proof of Corollary 3**

Per Proposition 3, $s_1 = s_2 = 1$, $S_1 = L$, $S_2 = H$, $p_1 = \frac{t}{2}$ and $p_2 = \max(\frac{1}{2}, t)$. Thus $D_1 = \frac{t}{2}$ and $D_2 = \frac{1}{2}$ when $t \in [\frac{1}{2}, \frac{1}{2}]$ and $D_2 = 1 - t$ when $t \in [\frac{1}{2}, \frac{3}{2}]$. Finally $D_1 + D_2 = \frac{1 + t}{2}$ when $t \in [\frac{1}{2}, \frac{1}{2}]$ and $1 - \frac{t}{2}$ when $t \in [\frac{1}{2}, \frac{3}{2}]$.

The comparative statics for the demands follow from these expressions.

**References**


