Contextual Advertising

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January 22, 2011

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Abstract

This paper studies the strategic aspects of contextual advertising. Such advertising entails the display of relevant ads based on the topic of the content a consumer views and takes advantage of the possibility that consumers' content browsing preferences are indicative of their product preferences. The results show that contextual targeting impacts advertiser profit in two ways: first, advertising through relevant content topics helps advertisers reach consumers who have strong preferences for their products. Second, heterogeneity in consumers' content preferences can be leveraged to reduce product market competition, even when consumers are homogeneous in their product preferences. The contextual advertising intermediary's incentives to strategically design its content structure and the targeting precision are governed by the following forces. When product market competition is high, the intermediary offers homogeneous content and increases its targeting precision. This encourages each advertiser to bid for multiple keywords to preventing its competitors from advertising to the consumers. This may lead to an asymmetric equilibrium where one advertiser monopolizes all the advertising spaces to completely preempt competition. When product market competition is low, the intermediary offers heterogeneous content but intentionally decrease its targeting precision. This encourages each advertiser to bid for multiple advertising spaces in order to reach consumers who prefer its product.

1 Introduction

> " Google's toughest search is for a business model... In other words, can Google create a business model even remotely as good as its technology?" - NY Times, April 2002

Eight years later, Google's annual revenue has surpassed $20 billion thanks to its immensely successful AdWord search advertising and AdSense affiliate advertising programs. Years after the established success of Google, critics are casting similar doubts on the emerging social media sites such as Facebook, YouTube and Twitter [1] and questioning their ability to monetize their user base. Once again, advertising may be the answer. By 2010, YouTube, Facebook and Twitter have all implemented their advertising programs, and that of YouTube is already turning a profit.

Why do successful search engines, video sharing websites, micro-blogging sites and social networks all embrace advertising as their preferred business model? Other than their broad penetration among Internet users, all the above mentioned sites have offered *contextual targeting* as a major value proposition. Contextual advertising refers to the targeted delivery of advertisement according to the content a consumer views. Consider an example from YouTube. A video named 'park ride' features a stunt-performing cyclist. Viewers of the that video will see an overlay Flash advertising from the bike maker Lynskey, who makes customized performance bikes. Such precise targeting is made possible by YouTube's wealth of user-generated videos that cover a wide range of topics. Similarly, Google's Adsense[2] network attracts an enormous amount of Internet publishers who wish to monetize their websites. An ad from Dahon foldable bike is displayed on foldforum.com, which is a general

[1] For example, see the article "YouTube is Doomed." http://www.businessinsider.com/is-youtube-doomed-2009-4

[2] Google Adsense displays ads on third party publishers's sites. This program is not the same as Google Adwords that allows advertisers to bid to appear on Google's search page. However, the bidding process is similar as advertisers bid for keywords that are matched to the content provided by the third party publishers.

interest discussion forum for foldable bike lovers. An ad from Organic Bikes, a company who sells bicycles made of bamboo, is displayed on the biking advocacy section of bikeforums.com, a popular gathering spot for environmentally minded city commuters. It is the heterogeneity in member publisher's website contents that allow the AdSense network to deliver such targeted advertising.

The idea of targeted advertising has a long history in the advertising industry. Advertising agencies often offer media planning services to their clients, choosing the advertising medium (e.g., newspapers or TV channels) according to the type of products. However, the most exciting developments of contextual advertising have taken place in the on-line environment. The reason is twofold. First, online intermediaries such as Google AdSense and YouTube typically boast massively heterogeneous content bases. The content base can be leveraged to deliver finely targeted ads to a large Internet population. In addition, the development of sophisticated content analysis algorithm and targeting technology has made ad targeting extremely efficient. The goal of this paper is to study the general phenomenon of contextual advertising in both the traditional and the on-line industry. Given the significance of contextual advertising in the on-line environment, we place emphasis on modeling the institutional details that are particularly relevant to the on-line context. We focus on three unique aspects of contextual advertising:

- First, we focus on studying the two cornerstones of contextual advertising: the content base and the targeting technology. The precision of contextual targeting critically hinges on the correlation between consumers' preference for different content topics and their preferences for certain products. While a search engine has relatively good information about a consumer's product interests (based on the search terms they use), a consumer's viewing preference on YouTube is at best an imperfect indicator of his or her potential interest in a brand or product. On the Adsense network, Google has little precise knowledge about the publisher websites' content and has to rely on automatic page mining algorithms. As we move from search engines to user generated content sites or advertising networks, the intermediary has less control over the content and the precision of targeting also tends to decrease. We recognize the imperfectness in ad targeting and model how such content-product preference correlation impacts the advertiser's profits.

- Second, we capture an important institutional detail in the contextual advertising market: Content is usually hosted by the intermediary and advertising slots are allocated to advertisers by keyword auctions. We explicitly model the keyword auction process and explore the determinants of intermediary's profits. To illustrate the importance of the pricing mechanism, we also consider the case where the intermediary has the power to mandate prices for the keywords.

- Finally, we place a strong emphasis on analyzing the strategic decisions of the contextual ad intermediary. We consider the intermediary's incentives to optimize its content base, improve its targeting technology, and its incentives to implement the popular quality score system, which is currently in use by most contextual ad networks.

We set up an analytical model with horizontally differentiated firms. In order for the products to enter consumers' consideration set, firms have to communicate their product information to potential consumers through advertising. A contextual advertising intermediary hosts the (Internet) content which consumers browse. Competing firms bid for the rights to advertise through different content topics (keywords), such that the browsers of a certain content topic will see the ads from the winning bidder. Given advertising outcomes, firms engage in price competition. Consumer preferences are heterogeneous both for the products and for the Internet content. Our analysis reveals the following results.

First, consistent with the advertising targeting literature (Iyer, Soberman, and Villas-Boas 2005), we find that an important role of contextual advertising is helping competing advertisers reduce product market competition. However, when product firms advertise through different keywords, the perfect alignment of product preference and content preference does not necessarily maximize advertiser profits. Specifically, when product market competition is high, imprecise targeting can benefit the advertisers. When consumers have less heterogeneous product preferences, advertisers can leverage consumers' heterogeneous preference for Internet content and create a type of 'informational differentiation', where some consumers only see the ad for one product although they prefer both products equally. Such informational differentiation diminishes if all consumers who like both products also see both ads. This finding reveals two distinct roles of contextual advertising: reaching a firm's loyal customers by advertising through relevant keywords and creating informational differentiation among comparison shoppers.

We next analyze the equilibrium keyword allocation when advertisers bid for the keywords in a second price auction. In our duopoly setup, we find that when product market competition is high, each firm has a strong incentive to bid for her competitor's keyword in the advertising market. A firm is able to preempt her competitor from advertising to comparison shoppers. The incentive of competitive preemption leads to a 'keyword shelving' equilibrium where a firm advertises through both the relevant keyword and irrelevant keyword. Interestingly, in some cases, advertisers have stronger incentives to bid for keywords with fewer click-throughs.

One of our most important results pertains to the intermediary's optimal choice of content and targeting technology. We find that when the product market competition is low, the intermediary should offer minimally-overlapping content topics and decrease the precision of targeting. For example, an ad network like AdSense should design its targeting algorithm taking into product category information but do not distinguish ads from competing brands.

Since the publishers (or keywords) have minimally overlapping audiences, each advertiser has the incentive to bid for as many keywords as possible, thereby driving up the equilibrium prices in the second price auctions.

When product market competition is high, however, the intermediary should offer maximally-overlapping content topics and make the targeting technology an *ineffective* tool to help competing advertisers reduce their product market competition. When competing firms bid for different keywords, their ad will reach similar audience, which intensity the product market competition. In this way, competing firms have strong incentives to bid for their competitors' keywords in order to preempt the product market competition. This drives up the equilibrium prices and increase the intermediary profits.

From the contextual ad intermediary's perspective, we explore the profit implication of the widely adopted quality score system. We show that by incorporating ad-content relevance into the auction system, the quality score system prevents the keyword shelving outcome and may either raise or lower intermediary profit.

The rest of the paper is organized as follows. We summarize the related literature and our relative contribution in Section 2. Then, in Section 3 we present the model and conduct the basic analyses in Section 4. We discuss the intermediary's strategic decisions in Section 5. Next, we present two extensions in Section 6. Finally, we conclude in Section 7.

2 Related Research

This paper is broadly related to three literature streams. First, our paper is closely related to the previous works on advertising targeting. In an influential paper on this topic, Iyer et al. (2005) argued that targeted advertising can help advertisers mitigate product market competition and increase advertiser profits in a competitive industry. Similar to Iyer et al. (2005), we focus on the *competitive* implications of contextual advertising. In addition, we model three unique institutional details of the contextual advertising market not considered by Iyer et al. (2005). First, contextual advertising relies crucially on the existence of heterogeneous media content which consumers browse. We explicitly model consumers' correlated preference structures for the content and the products, and study content relevance as a key parameter. Second, we consider the role of a contextual advertising intermediary. We place strong emphasis on understanding the strategic choices made by the contextual advertising intermediary. Finally, we consider a keyword auction process which is typical for many on-line contextual advertising platforms.

Second, our paper is related to the growing literature on on-line search advertising and keyword auction (Chen and He 2006, Edelman, Ostrovsky, and Schwarz 2007, Katona and Sarvary 2010, Varian 2007). Most papers in this stream focused on understanding the properties of the widely adopted keyword auction mechanism, such as the Generalized Second Price auction. To our best knowledge, the competitive implication of advertising targeting is not studied in this literature stream. Our model of keyword auction is built upon the auction mechanisms and solution concept proposed by Edelman et al. (2007), Varian (2007). In addition to keyword auction, we also consider a case where intermediary sets prices for the advertising keywords, a setup which is closer to the business model of traditional advertising agencies.

Third, by explicitly considering advertising intermediary as an independent market player, our model is related to the studies on commercial media station (Dukes 2004a,b) and the channel literature in general. As in Dukes (2004a,b) and early works in the channel literature (see for example Coughlan (1985), Coughlan and Lal (1992)), we study the possibilities for product firms to reduce price competition via the differentiation created by other channel players (either media stations or retailers). Different from these works, we study content relevance, keyword auction and intermediary strategic decisions that are unique to the contextual advertising industry.

On a technical level, our model of price competition follows the widely adopted formulation by Narasimhan (1988), Varian (1980). Our conceptualization of advertising follows the informational advertising paradigm, wherein advertising helps a product enter a consumer's consideration set (Grossman and Shapiro 1984, Iyer et al. 2005, Nelson 1974).

3 The Model

We consider a market with two horizontally differentiated firms[3] offering their products to a unit mass of consumers. We assume that consumers learn about the existence of the products through informative advertising and they cannot buy a product unless they are aware of it. We assume that the potential consumers are primarily Internet users and firms can reach them while they browse content on the Web. Firms have the option of advertising through a contextual advertising intermediary. On behalf of the advertisers, the intermediary delivers targeted ads to consumers browsing certain content topics. Consumers are heterogeneous with respect to the content topics they browse and these preferences may be correlated with their product preferences as follows.

3.1 Consumer Preferences

We adopt a standard discrete horizontal differentiation model with a unit mass of consumers composed of two segments that are each loyal to one of the firms and a comparison shopper segment which prefers the two products equally (Narasimhan 1988, Varian 1980). Firm i's loyal consumers receive positive utility only from consuming firm i's product[4]. Loyal consumers' valuation for their preferred product is normalized to 1, whereas their valuation for the other product is 0. The comparison shopper segment values both firms' products at

[3] We use the terms 'firm' and 'advertiser' interchangeably.

[4] Being loyal refers to the consumer's intrinsic preferences. The consumers become aware of the products (either their existence of their attribute information) only if they receive advertising.

1 and simply maximizes utility by choosing the lower priced item. We assume a symmetric setup in which the loyal segments are of the same size and the fraction of comparison shoppers is α_p, yielding a loyal segment of size $\frac{1-\alpha_p}{2}$ for each firm. The α_p parameter, thus, captures the structure of consumer product preferences and essentially measures the competitiveness of the market.

In order for a product to enter a consumer's consideration set, the consumer needs to receive an ad for this product (as in Iyer et al. (2005)). We assume that a fraction λ of all the consumers browse the on-line content offered by the intermediary. Consumers encounter ads while browsing content and clicks on an ad if she has positive valuation for the product featured in the ad. In the basic setup, we assume that there are two different content topics [5]. Similarly to the product market, we assume that consumers have heterogeneous preferences for on-line content. Among the consumers who browse any of these topics, a fraction of α_c browse both content areas, whereas $\frac{1-\alpha_c}{2}$ consumers browse exclusively topic 1 and another $\frac{1-\alpha_c}{2}$ browse only topic 2.

Finally, to allow for contextual targeting, we assume that a consumer's interest in a particular content topic may be indicative of his or her preference for a particular product (brand). In order to measure the relationship between content and product preferences, we use s_{ij} to measure the number of customers who have a preference for product i and content area j. The indices i and j can take the values of 1, 2 and b, where b indicates that a consumer has positive valuation for both products or that she is browsing both content areas. For example s_{b1} is the number of consumers who have positive valuation for both products, but only browse content area 1, whereas s_{2b} is the size of the segment that is loyal to product 2, but browses both content areas. Collectively, the nine s_{ij} values capture the relationship

[5] In the case of YouTube, the 'content topics' corresponds to user-generated videos on different topics. In the case of Google AdSense, the 'content topics' corresponds to the affiliated websites in the AdSense network. These websites have heterogeneous themes and attract different viewers. In Section 6, we extend this to a case of three content topics

between consumer preference distribution in the product and content markets. For example, when consumer preferences for content topics are independent of preferences for products, we have $s^I_{11} = s^I_{21} = s^I_{12} = s^I_{22} = \lambda\frac{1-\alpha_p}{2} \cdot \frac{1-\alpha_c}{2}$, $s^I_{b1} = s^I_{b2} = \lambda\alpha_p\frac{1-\alpha_c}{2}$, $s^I_{1b} = s^I_{2b} = \lambda\frac{1-\alpha_p}{2}\alpha_c$, and $s^I_{bb} = \lambda\alpha_p\alpha_c$,. In other words, a consumer's content preference is totally uninformative of her brand preference. On the other extreme, when $\alpha_p = \alpha_c = \alpha$ and the preference for content topics is perfectly aligned with the preference for products, we have $s^A_{11} = s^A_{22} = \lambda\frac{1-\alpha}{2}$, $s^A_{bb} = \lambda\alpha$, and $s^A_{12} = s^A_{21} = s^A_{b1} = s^A_{b2} = s^A_{1b} = s^A_{2b} = 0$. Put differently, a consumer is interested in brand i if and only if she is interested in content area i. Thus, a consumer's content preference is maximally informative of her brand preference, and precise contextual targeting is possible. When $\alpha_p \neq \alpha_c$, product and content preferences cannot be perfectly aligned, since a different percentage of consumers are interested in both products than in both content areas. We define maximally aligned product and content preferences, using $s^A_{11} = s^A_{22} = \lambda\min\left(\frac{1-\alpha_c}{2}, \frac{1-\alpha_p}{2}\right)$, $s^A_{bb} = \lambda\min(\alpha_p, \alpha_c)$, $s^A_{b1} = s^A_{b2} = \lambda\frac{\max(\alpha_c-\alpha_p,0)}{2}$, $s^A_{1b} = s^A_{2b} = \lambda\frac{\max(\alpha_p-\alpha_c,0)}{2}$, and $s^A_{12} = s_{21} = 0$. To simplify notation and measure alignment between product and content preferences using a single parameter, we introduce ρ and place constraints on the possible values of s_{ij}.

$$s_{ij} = (1-\rho)s^I_{ij} + \rho s^A_{ij}. \tag{1}$$

When $\rho = 0$, content and product preferences are independent, whereas when $\rho = 1$, they are maximally aligned. The formula above simply establishes a convex combination of the two extremes, allowing us to capture the relationship between the product and content preferences of consumers. Essentially, ρ captures the precision of contextual targeting.

Thus, λ and ρ model two key aspects of the 'relevance' of content areas to a firms products. On the one hand, we consider the overall relevance of the content areas to the product category (λ), on the other hand, we capture the alignment between the individual content areas and product brands (ρ). Consider an example where the firms are producers of different types of bicycles (e.g., foldable bikes vs mountain bikes). The content areas correspond to videos about biking on YouTube. Among these videos, some are more related to foldable bikes (for example a video about a clear city commuter) while some others are more relevant to mountain bikes (for example a video about national park biking adventure). These different *types* of biking videos correspond to the two content topics in our model. λ measures the fraction of potential consumers who view any types of biking videos on YouTube. ρ measures whether a consumer's preference for a particular type of biking videos on YouTube is indicative of his product preference for foldable bikes vs mountain bikes. While ρ determines the precision of contextual targeting, λ is a scaling factor determining the overall usefulness of the intermediary's content.

3.2 Advertisers and the Publisher

The firms have to advertise through a contextual advertising intermediary in order to make consumers aware of their products. They purchase the right of advertising (i.e., keyword) in one or both of the aforementioned content areas. If an advertiser obtains a keyword, the intermediary will deliver the ad from this advertiser to all the consumers browsing in the content area associated with this keyword. We assume that advertising is priced according to the popular pay-per-plick model. Put differently, the advertiser only pays the intermediary when a consumer clicks on the ad.

In reality, the contract between the advertisers and the intermediary can be of various sorts. We consider two typical types of contracts: second price auction and price setting by the intermediary. We model second price auction according to the most popular form of auction used in the industry: Pay-per-Click auction with click-through rate correction. The intermediary sets up an auction where the advertisers submit their bids in a pay-per-click (PPC) system, and the intermediary corrects for expected click-through rates (CTR) to determine the winner[6]. In the case of price setting, the advertiser sets prices for the different

[6]Specifically, the intermediary determines the winner by ranking $PPC_i * CTR_i$. Suppose $PPC_i * CTR_i >$

content areas and the advertiser that accepts the offer receive the advertising slot. While the second price auction is widely adopted by most on-line contextual ad intermediaries, the pricing case is a relevant business model of traditional advertising agencies.

To solve the keyword auction, we extend the so-called envy-free equilibrium concept (Edelman et al. 2007, Varian 2007) to multiple keywords. This type of equilibrium is a widely used concept for sponsored link auctions with multiple slots. The basic idea is that when a bidder considers deviating from her equilibrium strategy, and possibly acquiring a different position, she evaluates the deviation by using the price that is currently paid for that slot and not necessarily what she would pay eventually. This is a stronger condition for profitable deviations than in a simple Nash equilibrium as the currently paid price may be higher than what the deviating bidder will eventually pay due to the possible change in order. Thus, the set of envy-free (or symmetric) equilibria is a subset of the Nash-equilibria. We generalize this concept to a setting with simultaneous auctions for multiple items (keywords) using the same deviation criteria. A bidder will deviate from a potential equilibrium if and only if the deviation is profitable using the price that is currently paid for a set of items (keywords) that she wished to acquire. The novelty in our definition is that we consider deviations that entail acquiring and/or giving up multiple keywords at the same time, since our bidders submit multiple bids simultaneously.

Once the advertising decisions have been made and the advertising slots have been allocated, firms set prices for their products. We normalize the marginal cost of the products to zero.

$PPC_{-i} * CTR_{-i}$. Then the winning advertiser will pay $\frac{PPC_{-i} * CTR_{-i}}{CTR_i}$ for each click she receives. In total, she will pay $PPC_{-i} * CTR_{-i}$. This is the auction procedure used by many on-line contextual advertising intermediaries, such as Google AdSense. When click-through rates can be perfectly estimated, bidding by PPC is equivalent to bidding by impression. In Section 5.2, we explore the consequence of not correcting for click-through rates.

3.3 Timing

The timing of the game is described as follows:

- Intermediary Strategies: the intermediary makes strategic decisions such as targeting precision or quality score.
- Keyword Allocation: The contextual advertising intermediary organizes a second price auction (or sets prices) to allocate the keywords (i.e., right to advertise through various content topics) to the advertisers;
- Pricing: Advertisers set prices for their products.
- Browsing and Advertising: The ads are delivered according to the outcome of stage 2. Consumers browse the content topics they are interested in and see the displayed ads while browsing. They learn about the products featured in the ads.
- Shopping: Consumers make purchase decisions and profits are realized.

In the next section, we analyze the last four stages of the game and examine the equilibrium keyword allocations given the intermediaries actions. Then, in Section 5 we study the intermediary's strategies and their profit implications.

4 Analysis

We first study the impact of contextual targeting on advertisers' profit. We start by determining the equilibrium prices and profits in the basic case where each firm advertises through the more relevant keyword. As in Narasimhan (1988) andVarian (1980), we determine equilibrium prices by the number of consumers who only buy from one firm and the number of consumers who compare prices. A consumer can only purchase a firm's product if s/he receives an ad for that firm's product. Furthermore, the price of the product has to be below his/her reservation price, that is, loyal consumers will only consider one product.

Combining the effects of loyalty and advertising we can determine that $s_{11} + s_{1b} + s_{b1}$ consumers will only consider buying product i, either because they are loyal to it or because they are not aware of any other product. Similarly, $s_{22} + s_{2b} + s_{b2} (= s_{11} + s_{1b} + s_{b1})$ consumers will only consider product 2 and s_{bb} consumers will consider both products, choosing the one with the lower price. The solution of such a game is well know in the literature (Narasimhan 1988, Varian 1980): As both firms act as monopolist in their effective loyal segments, but they also compete for comparison shoppers, there is no pure strategy equilibrium. In the mixed strategy equilibrium both firms mix in the interval $\left[\frac{s_{11}+s_{1b}+s_{b1}}{s_{11}+s_{1b}+s_{b1}+s_{bb}}, 1\right]$ with expected profits of

$$\Pi_{duopoly} = s_{11} + s_{1b} + s_{b1} \tag{2}$$

for each firm. This leads to the following result.

Proposition 1 (Advertiser Revenues) *When advertisers advertise through the more relevant keywords, their revenues (net of advertising cost) are:*

$$\Pi_{duopoly} = \lambda(1-\rho)\frac{1 - 3\alpha_p\alpha_c + \alpha_p + \alpha_c}{4} + \lambda\rho \max\left(\frac{1-\alpha_p}{2}, \frac{1-\alpha_c}{2}\right).$$

Furthermore, advertiser revenues are decreasing in ρ iff $\min(\alpha_p, \alpha_c) > \frac{1}{3}$. Otherwise, revenues are increasing in ρ.

The above proposition highlights the complex link between the content relevance ρ (i.e., targeting precision) and advertiser revenues. We find that the advertiser revenues can either increase or decrease as consumers' product preferences becomes more aligned with the content topics. When consumers have heterogeneous preferences for both products and content topics (both α_p and α_c are small), advertiser profits increase with more precise contextual targeting. However, when there is a high enough proportion of consumers that are interested in both products and a high proportion interested in both content areas, more alignment between product preferences and content topics reduces advertiser revenues.

The intuition behind these rather surprising results can be best understood in light of the different roles of contextual advertising. First, by advertising through a content area that is more relevant to a firm's product, an advertiser can precisely deliver its product information to the consumers who are loyal to its product. This corresponds to the term $s_{11}+s_{b1}$ in (2). Second, contextual advertising creates *informational differentiation* in addition to preexisting product market differentiation. Although some consumers may equally prefer the advertisers' product, they have heterogeneous preferences for content topics. Consequently, they will consider only one product if they have browsed only one content topic and have only seen one ad. This corresponds to the s_{1b} term in (2). Put differently, the advertisers can leverage the differentiation in the content market to reduce product market price competition.

Interestingly, such informational differentiation is created precisely when product preferences are not totally aligned with content preferences. In a case where a consumer is interested in brand i if and only if she is interested in content topic i, all the comparison shoppers will also browse both content topics, and receive both ads. Consequently, the firms have to compete for these consumers. Figure 4 illustrates the above intuition: The ability to target loyal customers is always increasing in ρ while informational differentiation is always decreasing in ρ. When competition is less intense (small α_p and α_c), targeting loyal customers becomes more important and advertiser revenue is increasing in ρ. When the advertisers' loyal customer segments are small in size (large α_p and α_c), the effect on informational differentiation dominates. These forces illustrate how advertisers should be aware of the various competitive effects when bidding for advertising slots. We next examine how advertisers' bidding strategies are affected and what the equilibrium allocations are.

Proposition 2 (Equilibrium Keyword Allocation) *The equilibrium keyword allocation*

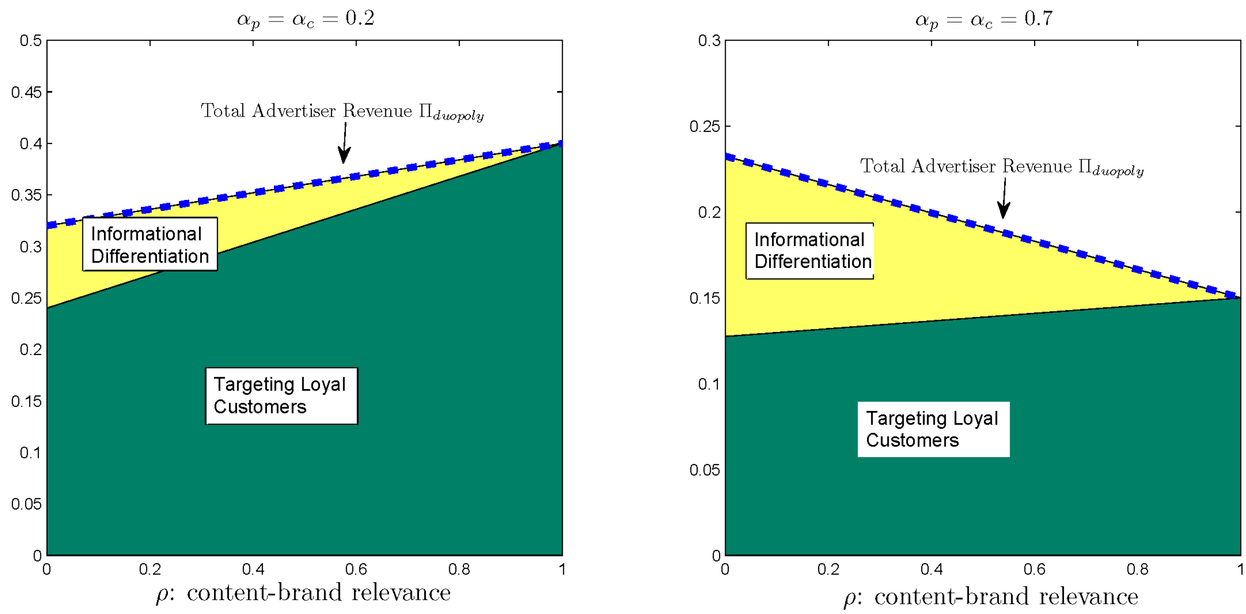

Figure 1: Decomposing Advertiser Revenue: The shaded are shows the revenue from targeting loyal customers, whereas the light area measures the benefits of informational differentiation.

outcome is independent of the keyword allocation mechanism (auction versus pricing):

- *Each advertiser gets one keyword when*

$$\alpha_p \leq \max\left(\frac{1}{3}, \frac{\alpha_c(1-\rho)+(1-2\alpha_c)\rho}{3\alpha_c(1-\rho)+\rho}\right).$$

- *Otherwise, one advertiser get both keywords.*

When product market competition is low (e.g., α_p is small), each advertiser wins the more relevant keyword. When product market competition is high (e.g., α_p is large), however, one firm will get both keywords in equilibrium. We refer to this type of equilibrium as the 'keyword shelving' equilibrium. Since advertising through both keywords preempts a firm's competitor to deliver its product information to the consumers, 'keyword shelving' can be considered as a form of competitive preemption. In the 'keyword shelving' equilibrium, one firm will buy its competitor's keyword even if the click through rate from that keyword is low. In fact, when $\frac{1}{3} < \alpha_p < 1 - 2\alpha_c$, firm i's valuation for keyword $-i$ is increasing in α_c. Put differently, the firm's valuation for the keyword *increases* as the unique click-throughs it gets from the keyword *decrease*. This is because when α_c is higher, the two keywords tend to deliver ads to the same consumers. This intensifies product market competition when both firms advertise and the need for competitive preemption increases although click-throughs decrease.

The above findings resonate with the empirical observations presented in Shin (2009). Shin observed that in some product categories advertisers will bid for their competitor's brand name, which is arguably the most relevant 'content topic' for that firm. Similarly, we observe a plethora of 'keyword spying' services emerging on the Internet during the recent years [7]. These companies provide their clients technologies to analyze the keyword bidding behavior of their *product market* competitors. Some keyword spying companies even provide consulting services on what is the best bidding strategy to win these keywords from a competitor. The reason why an advertiser might be interested in her competitor's keyword is multi-fold. Our analysis suggests that competitive preemption might be one explanation.

Although the allocation of the advertising space does not depend on the keyword allocation mechanism, the intermediary profit is strongly affected. Proposition 3 describes the equilibrium intermediary profits.

Proposition 3 (Intermediary Profit) *When keyword shelving occurs, the intermediary profit is*

$$R_{\text{shelving}} = \lambda \frac{1+\alpha_p}{2}$$

regardless of the keyword allocation mechanism. When each advertiser obtains one keyword, the intermediary profit is

$$R^{\text{auction}}_{\text{duopoly}} = \lambda(1-\rho)(1-3\alpha_p)\alpha_c + \lambda\rho\min(2\alpha_p, \alpha_p + \alpha_c)$$

[7] See for example keycompete.com, spyfu.com, or keywordspy.com

under a second price auction and

$$R_{\text{duopoly}}^{\text{price setting}} = \lambda(1-\rho)\frac{1-3\alpha_p\alpha_c+\alpha_p+\alpha_c}{2} + \lambda\rho\max(1-\alpha_p, 1-\alpha_c)$$

when keyword prices are set by the intermediary.

The results highlight the difference between price setting and second price auction as keyword allocation mechanisms. When the intermediary sets prices for keywords, she is always able to extract all the surplus from the advertiser who has the highest valuation for that keyword. For example, when each advertiser wins the more relevant keyword, the intermediary profit is $2\Pi_{duopoly}$, where $\Pi_{duopoly}$ is the advertiser revenue specified in Proposition 1. In contrast, in a second price auction, the auctioneer profit is determined by the bidder who has the second highest valuation. For example, when each firm advertises through the more relevant keyword, advertiser i's willingness to pay for keyword i is $\Pi_{duopoly}$. Advertiser $-i$'s willingness to bid for the keyword i is determined by the incremental benefit of winning both keywords, i.e., $\Pi_{monopoly} - \Pi_{duopoly}$. When $\Pi_{monopoly} > \Pi_{monopoly} - \Pi_{duopoly}$, the second highest bid for each keyword equals $\Pi_{monopoly} - \Pi_{duopoly}$. The intermediary profit equals $2(\Pi_{monopoly} - \Pi_{duopoly})$ and is decreasing in advertiser revenue $\Pi_{duopoly}$.

When each advertiser wins one keyword, the equilibrium second price bids $\Pi_{monopoly} - \Pi_{duopoly}$ can be decomposed into two components: the willingness to pay for *additional traffic* and the willingness to pay for *competition reduction*. Formally, for firm 1, $\Pi_{monopoly} - \Pi_{duopoly} = (s_{11} + s_{12} + s_{1b} + s_{b1} + s_{b2} + s_{bb}) - (s_{11} + s_{1b} + s_{b1}) = (s_{12} + s_{b2}) + s_{bb}$. The component $(s_{12} + s_{b2})$ are the consumers who are informed of firm 1's product if firm 1 also advertises through content area 2. s_{bb} corresponds to the comparison shoppers when both firms advertise. These consumers will only consider firm 1's product when firm 1 advertise through both content areas[8]. Thus, $(s_{12} + s_{b2})$ corresponds to firm 1's willingness to pay

[8] Loosely put, these comparison shoppers are 'converted' into loyal customers and firm 1 no longer has to compete for them.

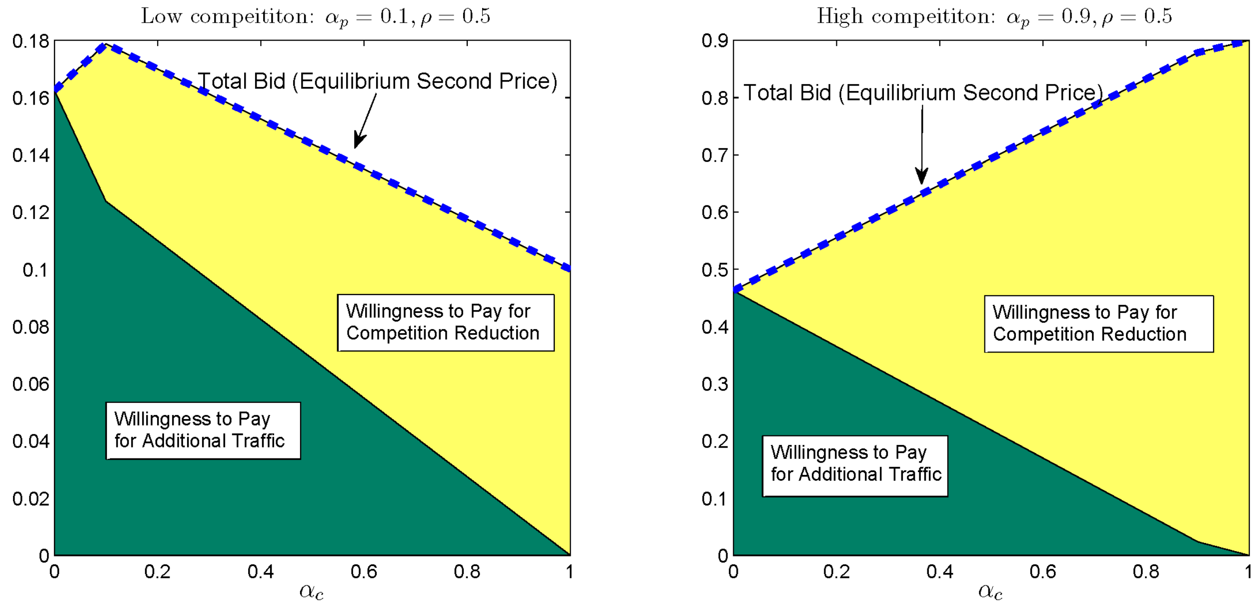

Figure 2: Decomposing the Equilibrium Bids

for additional traffic (i.e., additional click-throughs from keyword 2) and s_{bb} corresponds to firm 1's willingness to pay for competition reduction. Figure 4 illustrates $\Pi_{monopoly} - \Pi_{duopoly}$ as a function of α_c under high and low levels of product market competition (α_p). It is clear that the equilibrium second price bidding for the keywords may be either increasing or decreasing in α_c (depending on the competitiveness of the product market, α_p). A larger α_c implies greater overlap of the content topics, which decreases advertisers' willingness to pay for additional traffic. On the other hand, since more consumers will receive the ads from both advertisers, price competition is intensified. Thus, a larger α_c increases the advertisers' willingness to pay for competition reduction. The overall effects of α_c on the willingness to bid and on intermediary profit depend on the competitiveness of the product market. When product market is competitive, the need for competitive reduction dominates and intermediary profit is maximized at $\alpha_c = 1$. When product market is less competitive, the need for additional traffic dominates and the intermediary profit is maximized at $\alpha_c = \alpha_p$ or $\alpha_c = 0$, depending on the value of ρ.

In the next section, we further explore the determinants of intermediary profits and study the intermediary's incentives to choose its content structure and targeting precision.

5 Intermediary Strategies

5.1 Optimal Targeting Precision and Content Structure

In this section, we take the intermediary's perspective and consider the endogenous choice of λ, α_c and ρ. In Section 3, we introduced these parameters as exogenous properties of the intermediary's content base. In many cases, however, the intermediary can endogenously change these parameters by designing its content structure and targeting technology. Consider the following specific examples that are just a few of the different actions sites can take to influence the above parameters:

- Increasing α_c by cross-linking content: Recall that α_c measures the fraction of consumers who browse both content topics. On video-sharing websites such as YouTube, viewers of a video clip will receive a 'suggestion' list of related videos. Web sites usually do this to entrap consumers so that they spend more time at the site, but they have different options regarding internal linking. The website can either link to very similar videos that are in the content are to keep α_c low or it can increase the fraction α_c by cross-linking different types of user-generated videos.

- Decreasing ρ by reducing targeting precision: Recall that ρ measures the alignment of consumer brand preference and content topic preference. The intermediary can also influence ρ by setting its targeting precision. For example, Google's Adsense network relies on page analysis algorithm to determine the matching between a publisher page and a piece of ad. Google could set the algorithm to a rough precision such that only product category information (e.g., bicycles) is taken into account. As such, different types of biking websites are not distinguished. The ads from competing bicycle brands are matched to the biking websites at random. This corresponds to a low ρ. Alternative, the algorithm could distinguish between mountain biking forums and city commuter forums and target the viewers of these websites with ads from different bike brands. This corresponds to a higher ρ.

Proposition 4 describes the optimal intermediary decisions as a function of product market competitiveness. For simplicity, we assume that that the intermediary does not incur any cost when changing the values of α_c, ρ, or λ. In reality, these costs might prevent the intermediary from reaching the optimal values that we derive below. Nonetheless, our results provide important directions to the intermediary in addressing this problem.

Proposition 4 (Optimal Content Structure and Targeting Precisions) *When the advertising slots are sold via a second price auction, the intermediary always sets λ as large as possible. The intermediary's optimal decisions on α_c and ρ can be characterized as follows:*

- *When $\alpha_p > \frac{1}{3}$, $\rho * = \alpha_c^* = 1$ is an optimal intermediary choice.*
- *When $\alpha_p < \frac{1}{3}$, $\alpha_c^* = \rho^* = 0$ is an optimal intermediary choice.*

The above results on ρ and α_c reverse for the case of pricing setting intermediary.

The findings of Proposition 4 are consistent with the intuitions discussed in Section 4. Observe that in the keyword shelving equilibrium, neither α_c nor ρ is relevant for the intermediary profit. When each advertiser wins the more relevant keyword, the intermediary maximizes its profit by increasing the advertiser's willingness to bid for the less relevant keyword. When the product market competition is low, the contextual advertising intermediary should maximize the advertisers' willingness to bid for *additional traffic.* This is achieved by minimizing α_c such that the content topics target minimally-overlapping consumer segments, and minimizing ρ such that every keyword is relevant to every advertiser. This motivates all the advertisers to bid for all the keywords, and therefore raises the keyword prices.

When the product market competition is high, the contextual advertising intermediary should offer maximally overlapping keywords (maximize α_c) and choose a level of target precision such that the targeting through different keywords is *ineffective* in reducing the product market competition. This motivates the advertisers to bid for their competitors' keywords in order to *reduce competition*, and therefore raises the keyword prices.

As expected, the intermediary should always increase λ such that the content topics are maximally relevant to the product category. Although it is sometimes helpful to decrease the precision of targeting on the brand level (ρ), this should not be done at the cost of decreasing λ.

5.2 Quality Score

One important component of the auction mechanism that intermediaries use is the widely adopted 'quality score' system implemented by Google Adsense, Yahoo! Advertising (called the Quality Index) and a number of other contextual advertising networks. In a simple second price auction, the winner is determined solely based on the bids, and she pays the bid placed by the second highest bidder. The quality score system changes both the ranking rule and the payment amount. Suppose two advertisers ($i \in \{1, 2\}$) bid for one keyword in a pay-per-click setting. Each advertiser's bidding is weighted with a composite measure QS_i usually based on the expected click-through rate if the advertiser's ad is displayed through the keyword. Then the values $Bid_i \times QS_i$ will determine the winner of the auction. The winner (say, advertiser i) will pay an amount equal to $\frac{Bid_{-i} \times QS_{-i}}{QS_i}$ which can be higher or lower then the second highest bid, Bid_{-i}.

Google's Adsense network has been weighting advertiser bids with expected click-through rates since its inception. Yahoo! Advertising, in contrast, has not implemented the Quality Index system until 2007. Nowadays, the quality score system is gradually becoming the industry standard among contextual advertising intermediaries. In this section, we compare the intermediary profit in the baseline model, where the quality score system is implemented and a counterfactual scenario where the quality score system does not exist and the PPC bids solely determine the outcome of the auction and the amount paid. We consider the simplest type of quality score: $QS_i = CTR_i$, where the click through rates are known in advance by every player. In the following, let $CTR^s = (1-\rho)\frac{1+\alpha_p}{2} + \rho \min\left(1, \frac{1+\alpha_p}{1+\alpha_c}\right)$ denote the click-through an advertiser receives from the more relevant keyword and let $CTR^d = (1-\rho)\frac{1+\alpha_p}{2} + \rho\left(1 - \frac{1-\alpha_p}{1+\alpha_c}\right)$ denote the click-through an advertiser receives from the less relevant keyword.

Proposition 5 (Quality Score) *If there is no quality score adjustment in the second price auction, the keyword shelving equilibrium takes place when* $\frac{\Pi_{monopoly}-\Pi_{duopoly}}{CTR^d} > \frac{2\Pi_{duopoly}}{CTR^s}$*. Otherwise, each firm advertises through the more relevant keyword.*

- *If* $\frac{\Pi_{monopoly}-\Pi_{duopoly}}{CTR^d} < \frac{\Pi_{duopoly}}{CTR^s}$*, the equilibrium profit is lower when quality scores are implemented. In this case, each firm advertises through the more relevant keyword regardless of whether the quality score system is implemented.*

- *If* $\Pi_{monopoly} - \Pi_{duopoly} > \Pi_{duopoly}$*, the equilibrium profits are higher when the quality score system is implemented. In this case, keyword shelving takes place regardless of whether the quality score system is implemented.*

- *If* $\frac{\Pi_{duopoly}}{CTR^s} < \frac{\Pi_{monopoly}-\Pi_{duopoly}}{CTR^d} < \frac{\Pi_{duopoly}}{CTR^d}$*, the equilibrium profit may be either higher or lower when quality score is implemented. In this case, keyword shelving takes place only when the quality score system is not implemented.*

The above results reveal that quality scores can both increase or decrease intermediary profits. Interestingly, both scenarios are largely consistent with the early industry debates about the quality score system. By rewarding the most relevant bidder, a high quality score is considered a discount for the most likely winner of a keyword. Voices have been heard that such discount will lower the intermediary's profit. In our analysis, this scenario indeed takes place when each firm advertises through the more relevant word. In this case, the bidder who receives more click-throughs from a keyword also has a higher willingness-to-pay for each click. Thus, the quality score system reduces the price paid by the winning bidders and lowers intermediary profit. Formally, $\frac{Bid_{-i} \times QS_{-i}}{QS_i} < Bid_{-i}$ when $QS_i > QS_{-i}$ where i denotes the winning firm.

Arguments favoring the quality score insist that such a system essentially ranks the bidders according to $PPC \times CTR$, or their overall willingness to pay for an advertising position. Without quality score, an advertiser may have a high willingness-to-pay for each click and a low click-through rate. In this case, the keyword will generate less profit despite of a high price paid for each click. However, it is not entirely clear why an advertiser who expects *fewer* click-throughs from an ad would at the same time expect *higher* profit from each click. Our model of competitive keyword shelving offers a potential explanation: When keyword shelving takes place, a firm has higher valuation for its less relevant keyword because of the need for competitive preemption. At the same time, it derives fewer click-throughs from the keyword. When the quality score system is implemented, it is more expensive for the firm to purchase the less relevant keyword and intermediary profit increases as a result. Formally, $\frac{Bid_{-i} QS_{-i}}{QS_i} > Bid_{-i}$ when $QS_{-i} > QS_i$ where i denotes the winning firm.

6 Extensions

In our basic model, we assumed a duopoly market structure, where two advertisers compete for two keywords and the contextual advertising intermediary only displays one ad for each keyword. Here, we relax these restrictions and consider a case where the intermediary auctions away multiple ad slots for each keyword and another case with three keywords.

6.1 Two Ad Slots

We assume that for both keywords there are two advertising slots made available by the intermediary. Advertisements are allocated to the slots via a generalized second price auction where each bidder pays the next highest CTR corrected bid. In order to capture competition for the second slot, we introduce a third bidder who has a reservation valuation of R for the second ad slot for each keyword[9]. We assume that the two focal advertisers always value both ad slots higher than the third bidder. Therefore, in equilibrium, only the focal bidders will win any ad slot. To model the effect of ad position on consumer click-through behavior, we simply assume that a θ fraction of consumers will only click on the first ad and the remaining $1-\theta$ fraction of consumers will click on the second ad as well. Thus, a higher ranked ad receives more traffic[10]. In the following, let Π_1 denote the profit of a firm that wins the first positions for both keywords, let Π_2 denote the profit of a firm that acquires the second positions for both keywords, and let $\Pi_{duopoly}$ denote the profit when a firm wins the first position for its more relevant keyword and the second position for the other.

[9]This player can be thought of as the rest of the market that does not value these keywords as high as the two focal advertisers.

[10]We do not lose any generality by assuming that there are no consumers who only click on the second ad, since this would be captured by a lower α_c.

Proposition 6 (Two Ad Slots) *The advertiser profits in the different cases are:*

$$\Pi_1 = \lambda \left[\theta \left(\frac{1-\alpha_p}{2} + \alpha_p\right) + (1-\theta)\frac{1-\alpha_p}{2}\right]$$
$$\Pi_2 = \Pi_1 \frac{(1-\theta)\alpha_p + (1-\theta)((1-\alpha_p)/2)}{(1-\theta)\alpha_p + \theta((1-\alpha_p)/2+\alpha_p) + (1-\theta)((1-\alpha_p)/2)}$$
$$\Pi_{duopoly} = \lambda \left[\theta \left((1-\rho)\frac{1-3\alpha_p\alpha_c+\alpha_p+\alpha_c}{4} + \lambda\rho \max\left(\frac{1-\alpha_p}{2}, \frac{1-\alpha_c}{2}\right)\right) + (1-\theta)\frac{1-\alpha_p}{2}\right]$$

The equilibrium keyword allocation and the intermediary profits are:

- *When* $\Pi_1 - \Pi_{duopoly} \geq \Pi_{duopoly} - \Pi_2$*, one firm wins the first ad position on both keywords. The intermediary's profit is* $2(\Pi_1 - \Pi_{duopoly}) + 2R$.

- *When* $\Pi_1 - \Pi_{duopoly} < \Pi_{duopoly} - \Pi_2$*, each firm wins the first ad position for the more relevant keyword and the second ad position for the less relevant keyword. The intermediary's profit is* $\Pi_1 - \Pi_2 + R$.

The results are very similar to the basic model with a single slot for each keyword. When the relative benefit of winning the first position for both keywords over just getting it for the more relevant keyword is high enough compared to the benefit of winning the first position for at least one over getting the second slot for both, one player will be in the first position for both keywords. This is similar to the keyword shelving outcome in the basic model and one can identify the same forces that govern the incentives in reaching more of the loyal customers and informational differentiation. Not surprisingly, when θ is low and many consumers click on both slots, the keyword shelving outcome is less likely.

6.2 Three Keywords

Here, we consider a case where two advertisers bid for the ad slots for three different keywords. As before, the first and second keywords are more relevant to one brand each. Together, these two keywords cover λ_1 of the potential market. The third keyword is equally relevant to the competing brands and covers a λ_2 fraction of the potential market. The parameters ρ, α_p and α_c are defined for the λ_1 segment of consumers. We assume that the probability that consumers view the first and second keywords are independent of whether she views the third keyword. Let i denote the keyword that is more suitable for firm i ($i = 1, 2$), whereas keyword 0 is equally suitable for both firms. Below, let $\Pi_{monopoly}$ denote the profits of a firm that acquires all keywords, Π_l a firm's profits that acquires its own keyword and keyword 0, whereas Π_s the firm's profits that only acquires its own keyword. Furthermore, let Π_y denote a firm's profits that acquires keyword 1 and 2, but not 0 and Pi_x the profits of a firm that only gets keyword 0.

Proposition 7 (Three Keywords) *The advertiser revenues in the different cases are:*

$$\begin{aligned}
\Pi_{monopoly} &= (\lambda_1 + \lambda_2 - \lambda_1\lambda_2)\frac{1+\alpha_p}{2} \\
\Pi_l &= \lambda_1(1-\lambda_2)s + \lambda_2(1-\lambda_1)\frac{1+\alpha_p}{2} + \\
&\quad \lambda_1\lambda_2\frac{1-\alpha_p}{2} + \lambda_2\lambda_1\alpha_p\left((1-\rho)\frac{1-\alpha_c}{2} + \rho\max(0, \frac{\alpha_p-\alpha_c}{2})\right) \\
\Pi_s &= \Pi_l\frac{\alpha_1(1-\alpha_2)s + \lambda_1(1-\lambda_2)\left((1-\rho)\alpha_c\alpha_p + \rho\min(\alpha_c,\alpha_p)\right)}{\Pi_l + \lambda_1(1-\lambda_2)\left((1-\rho)\alpha_c\alpha_p + \rho\min(\alpha_c,\alpha_p)\right)} \\
\Pi_y &= \begin{cases} \lambda_1(1-\lambda_2)\frac{1+\alpha_p}{2} & \text{when} \lambda_1 > \lambda_2 \\ \lambda_2(1-\lambda_1)\frac{1+\alpha_p}{2}\frac{\lambda_2(1-\lambda_1)\frac{1+\alpha_p}{2}+\lambda_1\lambda_2\alpha_p}{\lambda_1(1-\lambda_2)\frac{1+\alpha_p}{2}+\lambda_1\lambda_2\alpha_p} & \text{when} \lambda_1 < \lambda_2 \end{cases} \\
\Pi_x &= \begin{cases} \lambda_2(1-\lambda_1)\frac{1+\alpha_p}{2} & \text{when} \lambda_2 > \lambda_1 \\ \lambda_1(1-\lambda_2)\frac{1+\alpha_p}{2}\frac{\lambda_1(1-\lambda_2)\frac{1+\alpha_p}{2}+\lambda_1\lambda_2\alpha_p}{\lambda_2(1-\lambda_1)\frac{1+\alpha_p}{2}+\lambda_1\lambda_2\alpha_p} & \text{when} \lambda_2 < \lambda_1 \end{cases}
\end{aligned}$$

where

$$s = (1-\rho)\frac{1 - 3\alpha_p\alpha_c + \alpha_p + \alpha_c}{4} + \rho\max\left(\frac{1-\alpha_p}{2}, \frac{1-\alpha_c}{2}\right).$$

The equilibrium keyword allocation and intermediary profits are:

- *One advertiser gets all three keywords iff* $2\Pi_s \leq \Pi_y$*,* $2\Pi_l \leq \Pi_y + \Pi_x$*, and* $\Pi_x + \Pi_s \leq \Pi_{monopoly}$*. The intermediary's profit is* $\Pi_{monopoly}$*.*

- *Advertiser i gets keyword i and 0 whereas advertiser $-i$ gets keyword $-i$ iff* $2\Pi_s \geq \Pi_y$, $2\Pi_l \geq \Pi_{monopoly} + \Pi_x$, *and* $\Pi_l + \Pi_s \geq \Pi_{monopoly}$. *The intermediary's profit is* $\Pi_l - \Pi_s + 2\max(\Pi_{monopoly} - \Pi_l, \Pi_y - \Pi_s)$.

SOME DISCUSSION

7 Conclusion

In this paper, we study the contextual advertising business model, which is being embraced by many successful UGC websites, search engines and on-line ad network as the preferred means to monetize their vast content base. We focus on the idea that advertising targeting helps competing firms reduce their product market price competition, and discuss the implications for advertisers as well as contextual ad intermediaries. Our analysis focuses on the importance of content relevance and targeting accuracy, and reveal that targeting through more relevant content is not always beneficial for the advertisers. Competing advertisers have incentives to bid for less relevant keywords in order to preempt their competitor from reaching the consumers. The intermediary's optimal content choice and targeting precision decisions strongly depend on the level of product market competition. Under high and low level product market competition, the intermediary should pursue starkly different strategies.

The contextual advertising industry is a fast evolving industry with many exciting developments. Our stylized model takes a first step in understanding this phenomenon, leaving many interesting questions to future research. First, despite the immense popularity of the AdSense-type content analysis algorithm, many alternative means are being developed to match ad with the relevant on-line content. For example, the ADSDAQ ad exchange sets up an open market where individual publishers can directly sell the ad space on their websites to advertisers. Such open markets exploit the private information each player holds, and may provide better matching than even the most sophisticated page analysis algorithm. A second issue unexplored in this paper is the competition between contextual ad intermediaries. The competition in the contextual advertising industry is becoming increasingly fierce as more players enter this arena. What is the implication of competition on the optimal content and targeting precision decisions of the intermediaries? What is the implication for advertisers? Finally, the coexistence of contextual advertising mediums and conventional (non-targeted) advertising mediums is an interesting issue. Will different types of advertisers self select into different advertising models? When one intermediary owns multiple advertising channels (for example, Google owns AdWord, AdSense, DoubleClick ad exchange and YouTube video sharing site.), how could positive synergies be created among these services? We believe the above mentioned issues are relevant to the industry and are fruitful avenues for future research.

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Appendix

PROOF OF PROPOSITION 1: When each firm advertises through one keyword, four groups of consumers will consider firm 1's product (same is true for firm 2 because of symmetry):

- s_{11}: Firm 1's loyal customers who have only received firm 1's ad.
- s_{b1}: The comparison shoppers who have only received firm 1's ad.
- s_{1b}: Firm 1's loyal customers who have received ads from both firms.
- s_{bb}: The comparison shoppers who have received both firm's ads.

Thus, each firm has a consumer segment of size $s_{11} + s_{b1} + s_{1b}$ that will consider its product, while s_{bb} consumers will consider both firms' product. From Narasimhan (1988), the pricing equilibrium has both firms playing mixed strategy in the interval $\left[\frac{s_{11}+s_{b1}+s_{1b}}{s_{11}+s_{b1}+s_{1b}+s_{bb}}, 1\right]$. The equilibrium profits are $s_{11}+s_{b1}+s_{1b}$ for each firm. Under the case where each firm advertises through the more relevant keyword, the profits are

$$s_{11} + s_{b1} + s_{1b} = (1-\rho)(s^I_{11} + s^I_{b1} + s^I_{1b}) + \rho(s^A_{11} + s^A_{b1} + s^A_{1b}), \tag{3}$$

where $s^I_{11}+s^I_{b1}+s^I_{1b}=\lambda\frac{1-3\alpha_p\alpha_c+\alpha_p+\alpha_c}{4}$, and $s^A_{11}+s^A_{b1}+s^A_{1b}=\lambda\max\left(\frac{1-\alpha_p}{2},\frac{1-\alpha_c}{2}\right)$. To check how advertiser revenues change with ρ, we consider the sign of the derivative

$$\frac{\partial\Pi_{duopoly}}{\partial\rho}=\lambda\frac{(1-3\min(\alpha_p,\alpha_c))(1-\max(\alpha_p,\alpha_c))}{4}.$$

Since $1-\max(\alpha_p,\alpha_c)>0$, the sign is negative iff $\min(\alpha_p,\alpha_c))>1/3$.

In order to later derive the equilibrium bids, we examine the equilibrium revenues in the case where each firm advertises through the less relevant keyword. We show that each advertiser makes lower revenue in this case. Similarly to (3), the equilibrium profits are

$$s_{11}+s_{b1}+s_{1b}=(1-\rho)(s^I_{11}+s^I_{b1}+s^I_{1b})+\rho(s^A_{12}+s^A_{b2}+s^A_{2b}), \tag{4}$$

where $s^A_{12}+s^A_{b2}+s^A_{2b}=\max\left(\frac{1-\alpha_p}{2},\frac{1-\alpha_c}{2}\right)-\min\left(\frac{1-\alpha_p}{2},\frac{1-\alpha_c}{2}\right)$. Thus,

$$\Pi^r_{duopoly}=\lambda(1-\rho)\frac{1-3\alpha_p\alpha_c+\alpha_p+\alpha_c}{4}+\lambda\rho\left(\max\left(\frac{1-\alpha_p}{2},\frac{1-\alpha_c}{2}\right)-\min\left(\frac{1-\alpha_p}{2},\frac{1-\alpha_c}{2}\right)\right).$$

It follows immediately that $\Pi^r_{duopoly}\le\Pi_{duopoly}$ with an equation iff $\rho=0$. □

Proof of Proposition 2: We first prove the proposition in the case of a second price auction. We seek the minimal envy-free equilibrium a la Varian (2007). The extended envy-free equilibrium defines a function $p():\{\{1\},\{2\},\{1,2\}\}\to\Re$ that assigns a price to any subset of keywords and a keyword allocation, such that no firm has any incentives to deviate given the current allocation and keyword prices. We always restrict $p^*(\{1,2\})=p^*(\{1\})+p^*(\{2\})$. This captures the important institutional detail that most contextual advertising intermediary does not introduce combinatoric auction due to its complexity. Deviation is defined as obtaining a keyword that is not won by this firm in the current allocation, or giving up a keyword that is won by this firm in the current allocation.

First observe that due to symmetry, there are three possible types of outcomes. One advertiser wins both keywords, each advertiser wins her more relevant keyword, and each advertiser wins her less relevant keyword. It is easy to prove that when it is an equilibrium where each advertiser wins the less relevant keyword, it is also an equilibrium where each advertiser wins the more relevant keyword. Furthermore, the later case Pareto dominates the former from the advertisers' point of view. Thus, we rule out the 'reverse' equilibrium where each advertiser bids for and wins the less relevant keyword.

Let $\Pi_{duopoly}$ denote the profits when each firm advertises through its more relevant keyword, and the expression is obtained from Proposition 1. It is easy to show that the profit of a firm that advertises through both keywords is

$$\Pi_{monopoly} = \lambda \frac{1+\alpha_p}{2}$$

because all the consumers who have a positive valuation for this firm's product will buy from this firm. We prove the following:

- When $\Pi_{duopoly} \geq \Pi_{monopoly} - \Pi_{duopoly}$, the envy free equilibrium consists of (1) the following allocation: each firm wins one keyword; and (2) the following $p()$ function $\Pi_{monopoly} - \Pi_{duopoly} \leq p^*(\{i\}) \leq \Pi_{duopoly}$, $i = 1, 2$.

  First we consider the given allocation and verify that the no deviation conditions lead to the above $p^*()$ function. Neither advertiser has any incentive to acquire the other advertiser's keyword, if and only if $\Pi_{monopoly} - p^*(\{1, 2\}) \leq \Pi_{duopoly} - p^*(\{i\})$, $i = 1, 2$. Furthermore, each advertiser has no incentives to give up her own keyword if and only if $\Pi_{duopoly} - p^*(\{i\}) > 0$, $i = 1, 2$. These two conditions lead to the restriction on $p^*(\{i\})$. The minimal envy free equilibrium consists of $p^*(\{1, 2\}) = 2p^*(\{1\}) = 2p^*(\{2\}) = 2(\Pi_{monopoly} - \Pi_{duopoly})$.

  Next we verify that this type of equilibrium is unique. For the allocation that firm 1 wins both keyword, there does not exist a $p()$ function that satisfies the no deviation condition. To see this, consider the condition that firm 1 does not want to give up firm 2's keyword $\Pi_{monopoly} - p^*(\{1,2\}) \geq \Pi_{duopoly} - p^*(\{1\})$. For this condition to be satisfied, we have $p^*(\{2\}) \leq \Pi_{monopoly} - \Pi_{duopoly}$. However at these prices, firm 2 will get his own keyword. A contradiction.

- When $\Pi_{duopoly} < \Pi_{monopoly} - \Pi_{duopoly}$, the envy free equilibrium consists of (1) one firm wins both keywords; and (2) $p^*(\{1,2\}) = \Pi_{monopoly}$ and $p^*(\{i\}) > \Pi_{duopoly}$, $i = 1,2$.

  Suppose firm 1 wins both keywords. The no deviation conditions imply the the following: firm 1 doesn't give up both keywords if and only if $\Pi_{monopoly} \geq p^*(\{1,2\})$. Firm 2 does not obtain both keywords if and only if $\Pi_{monopoly} \leq p^*(\{1,2\})$. Thus $p^*(\{1,2\}) = \Pi_m$. Moreover, $p^*(\{i\}) > \Pi_{duopoly}$, $i = \{1,2\}$ is a necessary condition such that firm 2 does not want to get keyword 2. In particular, $p^*(\{1\}) = p^*(\{2\}) = \frac{\Pi_{monopoly}}{2}$ is a pair of equilibrium.

  Next we verify one firm winning both keywords is the only type of equilibrium. For the allocation that each firm wins one keyword, there does not exist a $p()$ function that satisfies the no deviation condition. This can be easily seen by observing that $\Pi_{monopoly} - \Pi_{duopoly} \leq p^*(\{i\}) \leq \Pi_{duopoly}$ is an empty set.

Next, we show that the equilibrium allocation of the advertising slots is the same under price setting. This can be easily shown by observing the optimal pricing behavior of the intermediary. Under the assumption that the intermediary is a monopolist, it always sets prices equal to the advertisers' willingness to pay. When $\Pi_{monopoly} \leq 2\Pi_{duopoly}$, the firm should set the price for each keyword to $\Pi_{duopoly}$. In the unique equilibrium, each advertiser will advertise through the more relevant keyword. No advertiser will purchase both keywords since the total price will exceed the total revenue, $\Pi_{monopoly}$. Any price higher than $\Pi_{duopoly}$ will make the advertisers not purchase the keywords. Any price lower than $\Pi_{duopoly}$ will still sell both keywords, leading to lower profit for the intermediary.

When $\Pi_{monopoly} \geq 2\Pi_{duopoly}$, the firm should optimally set the price for each keyword at $\frac{\Pi_{monopoly}}{2}$. In equilibrium, one advertiser will purchase both keywords. It is not an equilibrium that each advertiser purchases her more relevant keyword since her willingness to pay $\Pi_{duopoly}$ is lower than the price. Any total price lower than $\Pi_{monopoly}$ will still sell both keywords, leading to lower profit for the intermediary. If the intermediary sets a total price higher than $\Pi_{monopoly}$, the advertiser will purchase at most one keyword. Since the maximal price for one keyword cannot exceed $\Pi_{monopoly}$, the intermediary makes lower profit.

We now determine the parameter values under which $2\Pi_{duopoly} \geq \Pi_{monopoly}$. Plugging in the expressions for the revenues yields

$$2\Pi_{duopoly} - \Pi_{monopoly} = \lambda \frac{1-3\alpha_p}{2}\alpha_c + \lambda\rho\left(\frac{1-3\alpha_p}{2}(1-\alpha_c) + \max(\alpha_p - \alpha_c, 0)\right)$$

When $\alpha_p \leq \alpha_c$ this is clearly positive iff $\alpha_p < 1/3$. When $\alpha_p \geq \alpha_c$, we have to solve

$$\frac{1-3\alpha_p}{2}\alpha_c + \lambda\rho\left(\frac{1-3\alpha_c}{2}(1-\alpha_p)\right) > 0,$$

which does not hold unless $\alpha_c < 1/3$. Finally, when $\alpha_p > 1/3$ and $\alpha_c < 1/3$, solving the inequality for α_p yields

$$\alpha_p \leq \frac{\alpha_c(1-\rho) + (1-2\alpha_c)\rho}{3\alpha_c(1-\rho) + \rho}.$$

The right hand hand side of the above is always higher than $1/3$ as long as $\alpha_c < 1/3$, completing the proof. □

PROOF OF PROPOSITION 3: When the prices are determined by the auction mechanism, the profit of the intermediary is the same as the auctioneer profit a la Varian (2007), defined by $p^*(1,2)$ in this case. From Proposition 2, the equilibrium profit of the intermediary is $2(\Pi_{monopoly} - \Pi_{duopoly})$ if $\Pi_{duopoly} \geq \Pi_{monopoly} - \Pi_{duopoly}$ and $\Pi_{monopoly}$ otherwise.

When the prices are determined through price setting, the intermediary is always able to extract all the surplus. This equals to $2\Pi_{duopoly}$ when the intermediary sells each keyword to one firm and $\Pi_{monopoly}$ when the intermediary sells both keywords to one firm. Substituting the formulas for $\Pi_{monopoly}, \Pi_{duopoly}$ yields the results stated in Proposition 3. □

Proof of Proposition 4: First we consider the case of a second price auction. From Propositions 2 and 3, the intermediary profit is

$$R^{auction} = \begin{cases} 2(\Pi_{monopoly} - \Pi_{duopoly}) & \text{when } \Pi_{monopoly} - \Pi_{duopoly} < \Pi_{duopoly} \\ \Pi_{monopoly} & \text{when } \Pi_{monopoly} - \Pi_{duopoly} > \Pi_{duopoly} \end{cases}$$

This can be written as $\mathbb{R}^{auction} = \min(2(\Pi_{monopoly} - \Pi_{duopoly}), \Pi_{monopoly})$. Since $\Pi_{monopoly}$ does not depend on the choice variables α_c and ρ, $argmax_{\alpha_c,\rho}\mathbb{R}^{auction} = argmax_{\alpha_c,\rho}(-\Pi_{duopoly})$.

Utilizing the results from Proposition 1, we have $\frac{\partial \Pi_{duopoly}}{\partial \rho} > 0$ when $\alpha_p < \frac{1}{3}$ and $\frac{\partial \Pi_{duopoly}}{\partial \rho} < 0$ when $\min(\alpha_p, \alpha_c) > \frac{1}{3}$. Thus, $\rho^* = 0$ is an optimal strategy when $\alpha_p < \frac{1}{3}$. Since $\frac{\partial \Pi_{duopoly}}{\partial \alpha_c} < 0$ when $\alpha_p > \frac{1}{3}$, $\alpha_c^* = 1$ is an optimal strategy when $\alpha_p > \frac{1}{3}$.

When $\alpha_p > \frac{1}{3}$, the sign of $\frac{\partial \Pi_{duopoly}}{\partial \rho}$ depends on the value of α_c. Since $\alpha_c^* = 1$, we have $\frac{\partial \Pi_{duopoly}}{\partial \rho}|_{\alpha_c=1} < 0$. Thus, $\rho^* = 1$ is an optimal solution when $\alpha_p < \frac{1}{3}$. When $\alpha_p < \frac{1}{3}$, the sign of $\frac{\partial \Pi_{duopoly}}{\partial \alpha_c}$ depends on the value of ρ. Since $\rho^* = 0$, it is easy to show that $\frac{\partial \Pi_{duopoly}}{\partial \alpha_c}|_{\rho=0} > 0$, therefore, $\alpha_c^* = 0$.

Finally, observe that in the case of price setting,

$$R^{pricing} = \begin{cases} 2\Pi_{duopoly} & \Pi_{monopoly} - \Pi_{duopoly} < \Pi_{duopoly} \\ \Pi_{monopoly} & \Pi_{monopoly} - \Pi_{duopoly} > \Pi_{duopoly} \end{cases}$$

Thus, $R^{pricing} = 2\Pi_{monopoly} - R^{auction}$ and the optimal of $R^{pricing}$ is achieved at $\alpha_c^* = \rho^* = 0$ when $\alpha_p > \frac{1}{3}$ and $\alpha_c^* = \rho^* = 1$ when $\alpha_p < \frac{1}{3}$. □

Proof of Proposition 5: When quality score is not implemented, bidders are ranked according to their bid-per-click while the click-through rates only determines the total payment of the winning firm. Under our assumption that a consumer will click on an ad if and only if she is interested in the advertised product, we have $CTR^s = (1-\rho)\frac{1+\alpha_p}{2} + \rho \min(1, \frac{1+\alpha_p}{1+\alpha_c})$ and $CTR^d = (1-\rho)\frac{1+\alpha_p}{2} + \rho(\frac{\alpha_c+\alpha_p}{1+\alpha_c})$.

The envy-free equilibrium in this case consists of a keyword allocation and a price vector, where the prices correspond to pay-per-click[11]. Following the arguments presented

[11]This is in contrast to the baseline model where quality score is implemented. In that case, the total

in Proposition 2 each firm advertises through the more relevant keyword in equilibrium if $\frac{(\Pi_{monopoly}-\Pi_{duopoly})}{CTR^d} < \frac{\Pi_{duopoly}}{CTR^s}$. To prove this, consider the price vector $p^*(\{1\}) = p^*(\{2\}) = \frac{\Pi_{monopoly}-\Pi_{duopoly}}{CTR^d}$. It can be verified that $\Pi_{duopoly} - p^*(\{1\})CTR^s \geq \Pi_{monopoly} - p^*(\{1\})CTR^s - p^*(\{2\})CTR^d$ and $\Pi_{duopoly} - p^*(\{2\})CTR^s \geq 0$ when $\frac{(\Pi_{monopoly}-\Pi_{duopoly})}{CTR^d} < \frac{\Pi_{duopoly}}{CTR^s}$. Thus, the allocation is envy-free for both firms under the current prices.

The keyword shelving outcome is an equilibrium if $\frac{(\Pi_{monopoly}-\Pi_{duopoly})}{CTR^d} > \frac{\Pi_{duopoly}}{CTR^s}$. Suppose firm 1 wins both keyword without loss of generality, we look for a price vector that is (1) envy-free and (2) leads to the lowest profit for the intermediary. The envy-free conditions imply:

$$\Pi_{monopoly} - p^*(\{1\})CTR^d - p^*(\{2\})CTR^s \leq 0 \tag{5}$$

$$\Pi_{monopoly} - p^*(\{1\})CTR^s - p^*(\{2\})CTR^d \geq \Pi_{duopoly} - p^*(\{1\})CTR^s \tag{6}$$

$$\Pi_{monopoly} - p^*(\{1\})CTR^s - p^*(\{2\})CTR^d \geq \Pi^r_{duopoly} - p^*(\{2\})CTR^d \tag{7}$$

$$\Pi_{monopoly} - p^*(\{1\})CTR^s - p^*(\{2\})CTR^d \geq 0 \tag{8}$$

$$\Pi_{duopoly} - p^*(\{2\})CTR^s \leq 0 \tag{9}$$

$$\Pi^r_{duopoly} - p^*(\{1\})CTR^d \leq 0 \tag{10}$$

Conditions (5) and (8) imply that the equilibrium prices have to satisfy $p^*(\{1\}) > p^*(\{2\})$. Thus, conditions (9) and (10) yield that (6) and (7) have to hold. Thus, the above set of conditions can be reduced to (5), (8), (9), and (10). The four conditions define four half-spaces and the intersection is the set of envy-free equilibrium prices. $\frac{(\Pi_{monopoly}-\Pi_{duopoly})}{CTR^d} > \frac{\Pi_{duopoly}}{CTR^s}$ is the condition that the set is non-empty. Observe that $p^*(\{1\})CTR^s + p^*(\{2\})CTR^d$ is minimized when inequalities (10) and (5) are binding. The equilibrium prices are therefore $p^*(\{1\}) = \frac{\Pi^r_{duopoly}}{CRT^d}$ and $p^*(\{2\}) = \frac{\Pi_{monopoly}-\Pi^r_{duopoly}}{CRT^s}$.

Next, we compare the equilibrium profits in the case when quality scores are imple-

willingness-to-pay for an ad position determines the winner in an auction and the price we consider in an envy-free equilibrium also corresponds to the total willingness-to-pay.

mented with the case in which they are not. When $\frac{\Pi_{monopoly}-\Pi_{duopoly}}{CTR^d} < \frac{\Pi_{duopoly}}{CTR^s}$, the equilibrium profit is $2(\Pi_{monopoly} - \Pi_{duopoly})$ with quality scores and $2\frac{CTR^r}{CTR^s}(\Pi_{monopoly} - \Pi_{duopoly})$ without. When $\Pi_{monopoly} - \Pi_{duopoly} > \Pi_{duopoly}$, the equilibrium profit is $\Pi_{monopoly}$ with quality scores and $\frac{\Pi^r_{duopoly}}{CRT^r}CTR^s + \frac{\Pi_m - \Pi^r_{duopoly}}{CRT^s}CTR^d$ without. The profit is therefore higher when quality scores are implemented. When $\frac{\Pi_{duopoly}}{CTR^s} < \frac{\Pi_{monopoly}-\Pi_{duopoly}}{CTR^d} < \frac{\Pi_{duopoly}}{CTR^d}$, the equilibrium profit is $2(\Pi_{monopoly} - \Pi_{duopoly})$ with quality scores and $\frac{\Pi^r_{duopoly}}{CRT^r}CTR^s + \frac{\Pi_{monopoly} - \Pi^r_{duopoly}}{CRT^s}CTR^d$ without. The profit in this case can be higher in either case depending on the specific parameter values. $\square$

Proof of Proposition 6: The proof has a similar structure to that of Proposition 2. When two firms bid for two ad slots, both firms will always be able to advertise in equilibrium. We consider two sub games: (1) one firm wins the first ad positions for both keywords and (2) each firm wins the first position for the more relevant keyword, respectively. In the first case, the winning firm will advertise through the first positions for both keywords and the other firm will advertise through the second positions for both keywords. In the second case, each firm advertises in the first position for the more relevant keyword and the second position for the less relevant keyword.

When each firm advertises through the more relevant keyword, the game is symmetric and the equilibrium revenues are determined by the number of consumers who will consider only one product. These consumers are composed of two groups: the consumers who only click on the first ad position and the consumers who click on both. The size of the first group is $S_1 = \left((1-\rho)\frac{1-3\alpha_p\alpha_c+\alpha_p+\alpha_c}{4} + \lambda\rho\max\left(\frac{1-\alpha_p}{2}, \frac{1-\alpha_c}{2}\right)\right)$ as calculated in Proposition 1. The second group of consumers always learn about both products. Among them, a $S_2 = \frac{1-\alpha_p}{2}$ fraction will only consider one product. Thus, the total number of consumers who will only consider one product is $\theta S_1 + (1-\theta)S_2$.

When firm 1 advertises through the first positions at both keywords, the pricing subgame is asymmetric. There is a larger segment of consumers who will only consider firm 1's product. This segment includes all the consumers who have only viewed the first ad position and are interested in firm 1's product (this segment is of size $\theta\frac{1+\alpha_p}{2}$). In addition, among all the consumers who have viewed both ad positions, firm 1's loyal customers will only consider firm 1's product. The size of this segment is $(1-\theta)\frac{1-\alpha_p}{2}$. Thus, the number of consumers who will only consider firm 1's product is $L_1 = \theta\frac{1+\alpha_p}{2} + (1-\theta)\frac{1-\alpha_p}{2}$. The number of consumers who will only consider firm 2's product is $L_2 = (1-\theta)\frac{1-\alpha_p}{2}$ and the number of consumers who will consider both firms' products is $S = (1-\theta)\alpha_p$. Firm 2's revenue can be calculated as $\Pi_2 = \frac{L_2+S}{L_1+S}\Pi_1$ according to Narasimhan (1988).

The equilibrium conditions can be determined according to the advertiser profits. When $\Pi_1 - \Pi_d > \Pi_d - \Pi_2$, one firm acquires the first positions at both keywords in equilibrium. Suppose the opposite, namely that each firm acquire the first ad position at the more relevant keyword. The price of each keyword has to be smaller than $\Pi_d - \Pi_2$, such that firms do not give up the first position at the more relevant keyword. Given this price level, each firm finds it more attractive to purchase the first position at the less relevant keyword as well, since the incremental benefit $\Pi_1 - \Pi_d$ is higher than the price. The allocation is thus not envy free: a contradiction.

When $\Pi_1 - \Pi_d < \Pi_d - \Pi_2$, each firm wins the first ad position at the more relevant keyword. The equilibrium keyword price is between $\Pi_d - \Pi_2$ and $\Pi_1 - \Pi_d$, such that both firms don't have the incentives to give up the second position at the less relevant keyword or the first position at the more relevant keyword. $\square$

Proof of Proposition 7: We first determine the advertiser revenues in each subgame. There are three possibilities as follows:

- When one firm gets all three keywords, the firm's ad covers a $\lambda_1 + \lambda_2 - \lambda_1\lambda_2$ fraction of the entire market. Among all the consumers who have received the ad, a $\frac{1+\alpha_p}{2}$ fraction will consider the firm's product. Thus, the firm's revenue equals $(\lambda_1 + \lambda_2 - \lambda_1\lambda_2)\frac{1+\alpha_p}{2}$.

- When firm 1 advertises through keywords $0, 1$ and the firm 2 advertises through keyword 2, firm 1's ad covers a larger fraction of the market, and its profit is determined by the number of consumers who will only consider product 1. At the same time firm 2's profit is $\frac{L_2+S}{L_1+S}\Pi_l$. We first calculate Π_l. The consumer segment who will only consider product 1 is composed of three parts. First, an s fraction of the consumers who receive ads from either keyword 1 or 2 will only consider firm 1's product. The value of s can be derived as explained in Proposition 1. The total size of this group of consumers is $\lambda_1(1 - \lambda_2)s$. Second, if a consumer only receives ads through keyword 0, she will consider product 1 as long as she is interested in the product. The size of this segment is $\lambda_2(1 - \lambda_1)\frac{1+\alpha_p}{2}$. Finally, if a consumer reads ads from both keyword 0 and keyword 1/2 (a $\lambda_1\lambda_2$ fraction), she will only consider product 1 if she is loyal to firm 1 (a $\frac{1-\alpha_p}{2}$ fraction) or if she is interested in both products but is not informed of product 2. The size of the last segment equals $\alpha_p - (\rho(s^A_{2b} + s^A_{bb}) + (1 - \rho)(s^I_{2b} + s^I_{bb}))$, where s^A_{1b} and s^I_{1b} are calculated as in Proposition 2. Summing the sizes of these segments, we obtain the expression Π_l in Proposition 7.

  To obtain Π_s, we need to calculate the size of the consumer segment who will consider both products as well as the number of consumers who will consider only firm 2's product. The number of consumers who will only consider firm 2's product is $\lambda_1(1 - \lambda_2)s$. As in Proposition 1, the consumers who will consider both products are those consumers who are interested in both products and are informed of both products. This segment includes a $(1 - 2s)$ fraction of those who receive ad only from keyword 1/2: $\lambda_1(1-\lambda_2)(1-2s)$. Among the consumers who have received ad from both keyword 1/2 and keyword 0, all of them are informed of firm 1's product. A $(\rho(s^A_{2b} + s^A_{bb}) + (1 - \rho)(s^I_{2b} + s^I_{bb}))$ fraction will learn about both products. Plugging these terms into the formula for $\frac{L_2+S}{L_1+S}\Pi_l$, we obtain the expression given in the proposition.

- When firm 1 advertises through keywords 1,2 and firm 2 advertises through keyword 0: a $\lambda_1\lambda_2$ fraction of consumers will learn about both products. $\lambda_1(1-\lambda_2)$ and $\lambda_2(1-\lambda_1)$ fractions of consumers will only learn about product 1 and 2, respectively. Among those who learn about both products, a α_p fraction of consumers will consider both products. Among those who only learn about one product, a $\frac{1+\alpha_p}{2}$ will consider this product. In summary, a $\lambda_1\lambda_2\alpha_p$ fraction of consumers will consider both products. A fraction $\lambda_i(1-\lambda_j)\frac{1+\alpha_p}{2}+\lambda_i\lambda_j\frac{1-\alpha_p}{2}$ will consider only product i.

  The firm revenues are determined according to the same formulas in (Narasimhan 1988). Depending on whether λ_1 or λ_2 is larger, we obtain two possible sets of expressions for revenues.

$\square$