PRODUCT DIFFERENTIATION THROUGH EXCLUSIVITY:  
Is there a One-Market-Power-Rent Theorem?

15 October 2011

Benjamin E. Hermalin
and
Michael L. Katz*

* Haas School of Business
University of California
Berkeley, CA  94720
hermalin@haas.berkeley.edu
katz@haas.berkeley.edu

Two anonymous referees and a co-editor provided helpful comments that led to a substantial revision of the paper. We are grateful to seminar participants at Berkeley-Stanford IOFest 2009, Berkeley’s 9th Summer Institute in Competitive Strategy, Berkeley’s Center for Research in Telecommunications Mobile Impact Conference, IFN Stockholm, Northwestern Law School’s Third Annual Research Symposium on Antitrust Economics and Competition Policy, the University of Arizona, the University of Southern California, and Wesleyan University for helpful comments and suggestions. We also thank Nokia Corporation for supporting Berkeley’s Program in Business Model Innovation.
Abstract

In systems industries, combinations of components are consumed together to generate user benefits. Arrangements among component providers sometimes limit consumers’ ability to mix and match components, and such exclusive arrangements have been highly controversial. We examine the competitive and welfare effects of exclusive arrangements among system components in a model of relatively differentiated applications that run on relatively undifferentiated platforms. We show that there is no “One-Market-Power-Rent Theorem.” Specifically, exclusive deals with providers of differentiated applications can raise platforms’ margins without reducing applications’ margins, so that overall industry profits rise. Hence, for a given set of components and prices, exclusive arrangements can reduce consumer welfare by limiting consumer choice and raising equilibrium prices. In some cases, however, exclusivity can raise consumer welfare by increasing the equilibrium number of platforms, which leads to lower prices relative to the monopoly outcome that would prevail absent exclusivity.

Key Words

One-Monopoly-Rent Theorem, exclusive contracts, systems competition,
I. INTRODUCTION

Many products yield consumers benefits only if used together (e.g., video games and consoles). When the various components of such systems are differentiated, the ability to mix and match components can benefit consumers. In some industries, however, incompatibility among components or contractual restrictions imposed by component providers limit the set of available combinations. Examples include: video games, where some games are produced exclusively for particular brands of console; video programming distribution, where some distributors have exclusive programming arrangements; and wireless communications, where networks often enter into exclusive dealing arrangements with handset manufacturers.

The use of exclusive arrangements has been highly controversial. In the 1980s, Atari sued Nintendo for the latter’s use of exclusive contracts with game developers. More recently, there have been widespread calls for regulations requiring wireless network operators to allow consumers to utilize any access device (e.g., smart phone) or application that they wish.\(^1\) And, in the United States, the Federal Communications Commission prohibits exclusive contracts between cable operators and cable-affiliated programming networks.\(^2\)

In this paper, we examine the competitive and welfare effects of exclusive arrangements between platforms (e.g., game consoles, wireless networks, or video distributors) and applications (e.g., video games, handsets, or video programming). We examine the case of relatively undifferentiated platforms and relatively differentiated applications, and we assume that the provision of platform services is characterized by significant fixed costs.

\(^1\) See, for example, Wu (2007). One of the authors, Katz, has previously consulted for AT&T on policy issues regarding handset exclusivity. The present research was conducted after that project ended, was not funded by AT&T, and the views expressed in the present paper should not be attributed to AT&T or any other organization.
We identify three mechanisms through which exclusive arrangements can affect competition and welfare. First, and most obviously, they limit the ability of consumers to mix and match. Given a set of products and prices, this limitation is welfare reducing. Second, we show that, when otherwise undifferentiated platforms can differentiate themselves through exclusive deals with differentiated applications, equilibrium prices are higher than they otherwise would be. This softening of price competition leads to a third effect: the market equilibrium may support a larger number of platforms than when there are no exclusive arrangements. In the presence of fixed costs, platform providers must be able to charge prices greater than average variable cost in order to earn non-negative profits. When the same applications are available on multiple (undifferentiated) platforms, competition drives prices toward marginal cost. Consequently, the market equilibrium with non-exclusive applications may support only one provider. By softening price competition, exclusive arrangements can lead to (softened) platform competition instead of platform monopoly. Hence, when viewed in the context of the full process of entry and pricing, exclusive arrangements can lead to greater competition and lower prices. We show that the net effects of exclusive arrangements on consumer surplus and total surplus can be positive or negative, depending on the parameter values.

The point that product differentiation can allow the market equilibrium to support additional platforms has been made informally by Yoo (2005, § III.A.2.a) and Ford (2008, pages 5-6). However, these papers do not analyze the full set of tradeoffs involved. Critically, they do not consider the tradeoff faced by the differentiated component producers. By reaching an exclusive agreement with a platform, an application allows that platform to differentiate itself, but the application gives up potential sales to consumers who patronize rival platforms.

2 United States Court of Appeals for the District of Columbia (2010).
Moreover, there is a question of whether it can be profitable to “buy” differentiation from a party that already has it. Specifically, why couldn’t an application provider take advantage of its product’s differentiation directly by charging higher prices, which would allow it to enjoy the fruits of its market power without suffering from limited distribution? In other words, does a version of the One-Monopoly-Rent Theorem hold in the settings we consider?

The One-Monopoly-Rent Theorem concerns situations in which: (a) two goods are consumed in fixed proportions; (b) one good is supplied by a constant-returns, perfectly competitive industry; and (c) the other good is supplied by a monopolist. The theorem states that the monopolist has no incentive to engage in the tying of the two goods in order to “leverage” its monopoly from one market to the other. It is well known that the theorem does not extend to situations in which (a) or (b) are violated. When the two goods are consumed in variable proportions, bundling can be used as a vehicle for rent extraction by the monopolist (see, e.g., Burstein, 1960). When there is imperfect competition in the non-monopolized market, it can be profitable to engage in “strategic foreclosure” that weakens a duopoly rival and allows the supplier with a monopoly in one market to earn higher profits in the imperfectly competitive market (see Whinston, 1990).

The situations we examine below satisfy assumptions (a) and (b). However, we relax assumption (c) to consider situations in which the second good is supplied by imperfectly competitive duopolists offering differentiated products rather than by a monopolist. This is an important extension because there are few unregulated firms that are literally monopolists. We show that there is no “One-Market-Power-Rent Theorem.” That is, we identify conditions under which the most profitable industry configuration is one in which the good with market power
(e.g., the differentiated applications) is tied to the competitively supplied good (e.g., the undifferentiated platforms).

We show that exclusive deals raise industry profits because such deals raise the platforms’ margins without reducing the application providers’ margins. The intuition is as follows. When either undifferentiated platform can be utilized with the same set of applications, consumers will purchase the lower-priced platform. Such undifferentiated Bertrand competition between platforms leads to equilibrium margins of zero. In the presence of exclusive contracts, however, an undifferentiated platform can attract customers away from a rival platform only by inducing consumers to switch between differentiated applications. Consequently, the demand for a platform’s services is not perfectly elastic and the platform’s equilibrium margin is positive. In effect, application differentiation can be used to relax price competition twice—once for applications and once for platforms. A similar intuition applies to situations in which the platforms are differentiated but the applications are more strongly differentiated.

The increase in platform margins can raise industry profits because the duopoly applications cannot fully appropriate the benefits of their product differentiation through their own retail price setting—imperfect competition between the applications leads to equilibrium margins that are less than the cartel level. In contrast, if there were a single, monopoly application, that application could set its margin at the industry-profit-maximizing level even when the platforms price at cost.

Our paper is related to the literature exploring the competitive effects of compatibility in mix-and-match markets (e.g., Economides and Salop, 1992, and Matutes and Regibeau, 1988
and 1992). Economides and Salop examine equilibrium under a variety of market structures that differ in terms of the degree of integration, but they do not examine the effects of exclusive contracts between unintegrated suppliers. We also explore platform entry issues that are not addressed by these papers.

The Matutes and Regibeau articles focus on integrated suppliers of complementary products and examine their incentives to make their components compatible with those of rivals. In contrast, we consider settings in which platforms and applications are supplied by independent firms that choose whether to enter into exclusive agreements. Our setting raises different policy questions with respect to the institutions giving rise to exclusivity. Moreover, the effects of exclusivity are very different. Matutes and Regibeau (1988) find that incompatibility lowers equilibrium prices by blocking mix-and-match consumption: when products are incompatible, an integrated supplier knows that its components will be used together and, thus, internalizes the complementary-pricing effects across components. In contrast, we find that exclusive contracts, which block mix-and-match consumption, can lead to higher prices than under mix and match. There are two sources of the difference in our findings. First, in our model, the two components of a system are sold by separate firms even when they have exclusive contracts with one another. Hence, the firms fail to internalize the complementary-pricing effects. Second, platforms in our model inherit the greater differentiation of their exclusive applications. This effect does not arise in Matutes and Regibeau’s setting, in which platforms and applications are equally differentiated.

Besanko and Perry (1993 and 1994) also analyze the role of exclusive dealing in oligopoly. Both papers examine a vertical structure in which manufacturers sell their products

---

3 Neven (1985) shows that Hotelling duopolists with convex transportation costs can relax price competition by locating farther away from one another—the increasing marginal transportation costs reduce the price
through retailers, and only the latter set prices charged to consumers.\footnote{In this regard, these papers are closer to the bundle-pricing case that we consider briefly in Section VIII.B below.} In contrast, in our baseline model, both the platform and application providers set retail prices to consumers. Besanko and Perry (1993) consider three or more manufacturers of differentiated products, each of which is carried by many retailers. Each manufacturer chooses one of two possible regimes: exclusive dealing, in which all retailers that carry its product do so exclusively; or non-exclusive dealing, in which all retailers that carry its product are permitted to carry the products of other manufacturers who have also elected non-exclusive dealing. Besanko and Perry (1993) find that permitting exclusive dealing can lead to lower equilibrium retail prices because it eliminates an incentive for manufacturers to free ride on the promotional investments of each other. The resulting increase in promotional activity leads to lower prices. In contrast, we find that exclusive dealing raises equilibrium prices when the number of platforms is fixed at two, but can lower prices when one takes into account the effects on platform entry.

Like us, Besanko and Perry (1994) find that exclusive dealing can raise equilibrium prices. Under the exclusive dealing regime of their model, a retailer is differentiated by both its location and the manufacturer’s brand that it offers. Loosely speaking, retailer differentiation is the sum of these two components of differentiation. In contrast, in our baseline model, platforms are completely undifferentiated. Pricing affects arise from exclusive dealing because platforms inherit the product differentiation of their applications, while the degree of differentiation of applications remains unchanged. Another significant difference is that Besanko and Perry (1994) assume that the fixed cost of entering as an exclusive (single-brand) retailer is...
significantly less than that of entering as a non-exclusive (dual-brand) retailer. Hence, exclusive dealing can be welfare enhancing. In our model, platform entry costs are independent of the number applications that can be carried on the platform; the benefit of exclusive dealing arises because it partially insulates the platforms from price competition.

Lastly, there is large literature that examines other roles of exclusive contracts, particularly as a means of supporting investment. For example, an exclusive relationship might be necessary to induce a platform to invest in non-price promotion of an application because the platform would otherwise be concerned about free-riding by a rival platform.\(^5\) Exclusive arrangements can also be a means of increasing platform investment incentives by reducing the threat of hold up by an application provider.\(^6\) Finally, exclusive arrangements can be a means of overcoming certain types of contractual opportunism.\(^7\) None of these effects arise in our setting, and the papers in this literature do not examine the forces at work in our model.

The remainder of this paper is organized as follows. We introduce the model in the next section. In our baseline model, there are two undifferentiated platforms, two differentiated applications, and platform owners and application providers independently set the prices of their respective components. Sections III through VII characterize the equilibria of this baseline model and establish the results described above.

In Section VIII, we consider two extensions of the model. The first is differentiated platforms. Perhaps the most important difference in the results is that, in contrast to the case of undifferentiated platforms, there are parameter values for which both platforms enter the industry.

\(^5\) Marvel (1982) provides an early analysis of the use of exclusive relationships to prevent free riding.

\(^6\) For a general analysis of the investment incentive effects of exclusive contracts, see Segal and Whinston (2000).

\(^7\) See, for example, McAfee and Schwartz (1994) and (2004).
and consumers have full mix-and-match options. That said, as long as the degree of application
differentiation is greater than the degree of platform differentiation, there are also situations in
which a public policy that compelled full mix and match would lead to a platform monopoly,
while exclusive arrangements would lead to a platform duopoly. We also extend the model to
consider settings in which the platform owners set prices for bundles containing platform
services and applications, such as cable television operators selling bundles of programming.
We show that, here too, exclusivity can raise equilibrium industry profits.

The paper closes with a brief conclusion. Proofs not given in the text may be found in
the Appendix.

II. THE BASELINE MODEL

Consumers derive benefits from a pair of perfectly complementary components, \( X \) and \( Y \).
To realize any benefit, a consumer must consume the inputs in fixed proportion, which we
normalize to one-to-one. We also assume that an individual wants to consume at most one pair
of components. For expositional convenience, we will refer to component \( X \) as the \textit{platform}
(\textit{e.g.}, video game console, wireless network, or video programming service) and component \( Y \) as
the \textit{application} (\textit{e.g.}, video game, mobile-phone handset, or television program).

There are two potential platforms, 0 and 1, located at opposite ends of a Hotelling line of
length one. Consumers are distributed on the line and have “transportation cost” (disutility) \( s \) per
unit distance. In our baseline model, the platforms are undifferentiated: \( s = 0 \). We examine
situations in which \( s > 0 \) in Section VIII.A. There are two potential applications, 0 and 1, located
at opposite ends of their own Hotelling line of length one. Consumers have transportation costs
of \( t \) per unit distance along the application Hotelling line. A consumer located at \((x, y)\) who
consumes a platform located at \( i \) and an application located at \( j \) derives utility
\[ v = s|i - x| - t|j - y| + \theta \], where \( \theta \) is the consumption of an outside, composite good with a normalized price of 1. Consumers are uniformly located on the unit square \([0,1] \times [0,1]\).

Platforms and applications have affine cost functions, with fixed costs of creating the product and constant marginal costs of production. Because the marginal costs don’t affect the analysis, we set them to zero for notational convenience. Each platform incurs a fixed and sunk cost equal to \( F \) if it enters into production. We assume, throughout, that platform fixed costs are such that a platform guaranteed to be a monopoly would find it profitable to enter. Additionally, we assume application fixed costs are sufficiently small that the market can readily support two differentiated applications. Without further loss of generality, we set the application providers’ fixed costs to zero.

In our baseline model, unintegrated component suppliers independently set their prices. The structure of the component-pricing game is as follows:

**Public Policy Stage:** The public policy treatment of exclusive contracting is set. There are at least two important dimensions to the public policy treatment of platform-application relationships: whether a platform owner can exclude an application from operating on the platform through technological or contractual means; and whether a platform owner can pay an application provider to refrain from making its application available for use with the rival platform.

**Entry Stage:** Investment decisions are made. Potential platform providers 0 and 1 simultaneously choose whether to enter (and incur sunk cost \( F \)) or stay out. Given their zero fixed costs, both application providers can be assumed to enter the market.

**Contracting Stage:** Contracting between application providers and platform owners takes place, subject to the operative public policy. Any payments between platform and application providers are assumed to be lump-sum transfers.\(^8\)

**Pricing Stage:** First, the platform providers that have entered the market set prices. Second, those prices become common knowledge and the two application providers set their prices. Let \( p_i \) denote the price charged by platform \( i \), and let \( q_j \) denote the price charged by application \( j \).

---

\(^8\) We briefly discuss the possibility of quantity-dependent payments in Section VIII.B below.
**Consumption Stage:** Consumers choose which feasible component pairs to purchase. A consumer at location \((x,y)\) chooses from the feasible combinations the one that maximizes

\[
v - s|i - x| - t|j - y| - p \cdot q\ .
\]

If no feasible combination yields positive surplus, the consumer does not buy.

The equilibrium concept is perfect equilibrium. Hence, as usual we solve the game backward.

III. **PRICING**

Consumers behave non-strategically, so we begin our analysis with the pricing stage. We need to examine several continuation games that could arise from the entry and contracting stages. As will become evident, given the consumption benefits of application variety (i.e., \(t > 0\)) and the absence of fixed costs associated with introducing a second application, the only interesting histories are those in which each application has reached an agreement to be distributed by at least one platform:

- **Monopoly Platform.** This case arises when only one platform has entered the market or (off the equilibrium path) when both platforms have entered but one of them has exclusive contracts with both application providers. Without loss of generality, we assume platform 0 is the monopolist.

- **Mix and Match.** In this case, all four potential component combinations are available to consumers.

- **Exclusive Duopoly.** Under this outcome, one application is available exclusively for use with one platform, and the other application is available exclusively for use with the other platform. Without loss of generality, we label the components such that, under this arrangement, only systems \(Z_{00}\) and \(Z_{11}\) are available, where \(Z_i\) denotes the system comprising platform \(i\) and application \(j\).
• **Asymmetric Duopoly:** Under this outcome, one application is available exclusively on one platform and the other application is available on both platforms. We label the components so that the sole exclusive contract is between platform 0 and application 0. In this case, systems $Z_{00}$, $Z_{01}$, and $Z_{11}$ are available but system $Z_{10}$ is not.

### A. Application Pricing

It is useful to begin the analysis of application pricing by examining the exclusive duopoly continuation game. Recall $s = 0$. When platform and application prices are sufficiently high relative to a system’s benefits, some consumers do not purchase systems. Specifically, if $2v - p_0 - q_0 - p_1 - q_1 < t$, then applications 0 and 1 make sales to groups located on the intervals $[0, y_0]$ and $[1 - y_1, 1]$, respectively, where $v - p_j - q_j - y_j t = 0$, or

$$y_j = \frac{v - p_j - q_j}{t}. \quad (2)$$

An additional constraint is that the quantities be between 0 and 1. At times, we simplify the notation in ways that do not explicitly note this constraint.

In contrast, if $2v - p_0 - q_0 - p_1 - q_1 \geq t$, then prices are sufficiently low relative to benefits that all consumers purchase systems. A consumer with application-preference $y$ buys application 0 if and only if $v - yt - p_0 - q_0 \geq v - (1 - y)t - p_1 - q_1$, which yields

$$y_0 = \frac{t - p_0 - q_0 + p_1 + q_1}{2t} = 1 - y_1. \quad (3)$$

There are three cases to consider. Suppose, first, that $2v - p_0 - p_1 < 2t$ and max\{p_0, p_1\} < v. If each application ignored its rival’s existence and priced as a monopolist,
then application $j$ would choose $q_j$ to maximize $q_j y_j$, where $y_j$ is as given in equation (2). The resulting price would be

$$q_j = \frac{1}{2} (v - p_j).$$

(4)

Observe the applications will, indeed, have non-overlapping market areas:

$$y_0 + y_1 = \frac{v - p_0}{2t} + \frac{v - p_1}{2t} < 1.$$  

This is the unique continuation equilibrium.

Next, suppose that $3t < 2v - p_0 - p_1$ and $\max\{p_0, p_1\} < v$. When $y_j$ is given by equation (3), maximizing $q_j, y_j$ yields the best-response function $q_j^*(q_{-j}) = \frac{1}{2} (t - p_j + p_{-j} + q_{-j})$, where $-j$ indexes $j$’s rival. The resulting equilibrium prices and quantities are

$$q_j = t + \frac{1}{2} (p_{-j} - p_j)$$

(5)

and

$$y_j = \frac{1}{2} + \frac{p_{-j} - p_j}{6t},$$

respectively. Observe that these formulae apply only for platform prices such that $|p_0 - p_1| \leq 3t$.

Lemma A.1 in the Appendix shows that, if $p_i - p_{-i} > 3t$, then $Z_{ii} = 0$. Also observe that, when $p_0 = p_1$, consumers view the platforms as being identical to one another, and the applications differ only by their locations on the Hotelling line. In this case, the standard Hotelling equilibrium obtains in which $q_j = t$ (recall that marginal cost is equal to 0) and each application sells to half of the market and earns a profit of $\frac{1}{2} t$. 

12
Lastly, when $2v - p_0 - p_1 \in [2t, 3t]$ and $\max\{p_0, p_1\} < v$, there exist multiple equilibria.\(^9\)

To see why, suppose that the market areas “just touch” in the sense that the equilibrium values satisfy $y_i = 1 - y_0$ and

$$v - p_j - q_j - y_j t = 0 \quad . \tag{6}$$

In order for neither application to want to change its price, it must be the case that

$$2y_j - \frac{1}{t}(v - p_j) \leq 0 \quad \tag{7}$$

(or application $j$ would raise its price), and

$$\frac{1}{2} \left(3y_j - \frac{v - p_j}{t}\right) \geq 0 \quad \tag{8}$$

(or application $j$ would lower its price), where (7) and (8) make use of equation (6).

Figure 1 illustrates the solution. The vertical and horizontal dashed lines represent the boundaries of the constraint set defined by inequalities (7) and (8). The line with a slope of -1 is the constraint that $y_0 + y_i = 1$. Hence, the dark line segment represents possible equilibria.

Application $j$’s profits can be expressed in terms of its market area as

$$y_j q_j = y_j (v - p_j - y_j t) .$$

The derivative of profits with respect to $y_j$ is $v - p_j - 2y_j t$, which is non-negative by (7). Hence, each application prefers the equilibrium that yields it the greatest market share. Clearly, each platform prefers the equilibrium that yields it the greatest market share.

Summarizing this analysis, we have established:

\[^9\] Analyses of Hotelling models typically assume this case away. However, the values of the parameters that give rise to this case are endogenous in the present model, so it cannot be ruled out as unlikely a priori.
Lemma 1: Suppose the industry configuration is exclusive duopoly, platforms are undifferentiated, and $\max\{p_0, p_1\} < v$.

i. If $2v - p_0 - p_1 < 2t$, then $q_j = \frac{1}{2}(v - p_j)$ and $y_j = \frac{v - p_j}{2t}$ . Each application has a local monopoly and some consumers do not make purchases in equilibrium.

ii. If $2v - p_0 - p_1 \in [2t, 3t]$, then there exist multiple equilibria of the application-pricing continuation game. The market areas of the two applications just touch.

iii. If $3t < 2v - p_0 - p_1$ and $|p_0 - p_1| \leq 3t$, then $q_j = t + \frac{1}{3}(p_{-j} - p_j)$, $y_j = \frac{1}{2} + \frac{p_{-j} - p_j}{6t}$, and all consumers make purchases in equilibrium. If $p_i - p_{-j} > 3t$, then $Z_{ii} = 0$.

Now, consider the case of a monopoly platform. It is evident that the application-pricing continuation game is equivalent to the case of exclusive duopoly in which $p_0$ and $p_1$ are both equal to the price set by the monopoly platform.

Corollary 1: Suppose that platform 0 has a monopoly and $s = 0$. If $p_0 \leq v - t$, then $y_j = \frac{1}{2}$ in equilibrium. If $p_0 \geq v - t$, then $q_j = \frac{1}{2}(v - p_0)$ and $y_j = \frac{v - p_0}{2t}$ in equilibrium.

Under mix and match, platform $i$ can have positive sales in equilibrium if and only if $p_i \leq p_{-i}$. Hence, the mix-and-match application-pricing continuation game is equivalent to the exclusive-duopoly continuation game in which $p_0$ and $p_1$ are both equal to the lowest price set by the two platforms.
Corollary 2: Suppose that the industry configuration is mix and match and platforms are undifferentiated. If \( \min\{p_0, p_1\} \leq v - t \), then \( y_j = \frac{1}{2} \) in equilibrium. If \( \min\{p_0, p_1\} > v - t \), then \( q_j = \frac{1}{2} (v - \min\{p_0, p_1\}) \) and \( y_j < \frac{1}{2} \) equilibrium. Applications set \( q_j = t \) whenever 
\[
\min\{p_0, p_1\} \leq v - \frac{3}{2} t.
\]

Lastly, consider the case of asymmetric duopoly, where application 0 can be used solely with platform 0, while application 1 can be used with both platforms. If \( p_0 \leq p_1 \), then all consumers will utilize a platform with a price equal to \( p_0 \), and the analysis of the application-pricing continuation game is identical to that of the monopoly platform and mix-and-match cases above. If \( p_0 > p_1 \), then all consumers utilize platform 0 if and only if they are consuming application 0, and utilize platform 1 if and only if they are consuming application 1. Hence, the analysis of the application-pricing continuation game is identical to that of the exclusive-duopoly case above.

B. Platform Pricing

We now turn to platform pricing, beginning with the case of full mix and match. Given the assumption that the platforms are undifferentiated products (i.e., \( s = 0 \)), the usual logic of the Bertrand model applies and the unique equilibrium prices are \( p_j = 0 \).

Next, consider a monopoly platform. An immediate consequence of Corollary 1 is that the monopoly platform will never set a price less than \( v - t \), because doing so would result in no increase in quantity. A price of \( p_0 = v - t \) yields the platform a profit of \( v - t \). A higher platform price results in sales of \( y_0 + y_1 = (v - p_0) / t \) and profit of \( (v - p_0)^2 / t \). It follows that the monopoly platform chooses \( p_0 = v - t \) if \( v \geq 2t \) and \( p_0 = \frac{1}{2} v \) otherwise.
Now consider the exclusive duopoly case. As shown in Lemma 1, there are multiple application-pricing equilibria for some parameter values. Because our primary interest is in demonstrating the possibility of certain results, we focus our attention on values of exogenous parameters such that the knife-edge case does not exist. Specifically, we examine cases in which either: (a) consumer valuations are sufficiently low that the systems have local monopolies, or (b) consumer valuations are sufficiently high that the systems have overlapping market areas.

Suppose that \( v \leq 2t \). If each platform ignores the existence of the other and acts a monopolist by choosing \( p_i \) to maximize \( p_i y_i = p_i \frac{v - p_i}{2t} \) (where we have made use of Lemma 1), the resulting prices are \( p_i = \frac{1}{2} v \) and \( q_i = \frac{1}{4} v \), with sales \( y_i = v/(4t) \leq \frac{1}{2} \) (i.e., the two market areas do not overlap). The region in which there exist multiple application-pricing equilibria cannot arise when \( v \leq 2t \). It is also readily verified that profits would be lower at any price that induced overlapping market areas.

Next, suppose that \( v > 6t \) and each platform acts to maximize \( p_i y_i = p_i \left( \frac{1}{2} + \frac{p_{-i} - p_i}{6t} \right) \) taking \( p_{-i} \) as given. The resulting best-response functions are \( p_i^b(p_{-i}) = \frac{1}{2} (3t + p_{-i}) \), which yield the unique equilibrium values \( p_i = 3t \). By Lemma 1, \( q_j = t \) and \( y_j = \frac{1}{2} \). Observe that, when \( v > 6t \) and \( p_{-i} = 3t \), the application pricing continuation game must fall in case (iii) of Lemma 1, so that \( p_0 = 3t = p_1 \) is, in fact, an equilibrium of the full pricing continuation game.

The final industry configuration to consider is the one in which one application is available exclusively on one platform and the other application is available on both platforms. Recall our convention that platform 0 and application 0 have the exclusive arrangement. In this case, there is no pure-strategy equilibrium. The underlying intuition is as follows. Suppose the
two platforms charged different prices than one another. It would have to be the case that platform 1 was charging the lower price because otherwise it would make no sales. But then consumers would purchase only the systems $Z_{00}$ and $Z_{11}$. The two platforms would face identical pricing problems, and they could not both be choosing an optimal price. Next, suppose the two platforms charged the same price. At least one of the platforms makes sales to no more than half of the consumers purchasing application 1. For any price other than zero, that platform has an incentive to lower its price. However, if both platforms are setting prices equal to zero, platform 0 has an incentive to raise its price to earn profits from the sale of its exclusive application, 0. Although no pure-strategy equilibrium exists, Theorem 5 of Dasgupta and Maskin (1986) implies that there exists a mixed-strategy equilibrium.\(^{10}\)

Summarizing this discussion, we have shown:

**Proposition 1:** Suppose the platforms are undifferentiated (i.e., $s = 0$) and the platform and application owners independently set the prices for their respective components.\(^{11}\)

i. **Monopoly Platform:** If $v < 2t$, then the unique equilibrium prices are $p_0 = \frac{1}{2}v$ and $q_j = \frac{1}{4}v$, and some consumers do not purchase platform-application pairs. If $v \geq 2t$, then the unique equilibrium prices are $p_0 = v - t$ and $q_j = \frac{1}{2}t$, and all consumers purchase platform-application pairs.

ii. **Mix and Match:** The equilibrium platform prices are $p_i = 0$. If $v < t$, then each application has a local monopoly and the unique equilibrium prices are $q_j = \frac{1}{2}v$. If

\(^{10}\) We provide a partial characterization of that equilibrium in the proof of Proposition 2 in the Appendix.

\(^{11}\) Because of potential coordination problems among complementors, there always exists a trivial equilibrium in which all component prices are set greater than or equal to $v$ and no supplier makes any sales. We ignore equilibria of this sort.
\[ v \geq t , \text{ then there exists a unique symmetric equilibrium, under which } q_j = \min\{t, v - \frac{1}{2}t\} \]

and every consumer purchases a bundle.

iii. **Exclusive Duopoly:** If \( v < 2t \), then the unique equilibrium prices are \( p_k = \frac{1}{2}v \) and \( q_k = \frac{1}{4}v \), and the platform-application pairs have non-overlapping market areas. If \( v \geq 6t \), then the unique equilibrium prices are \( p_k = 3t \) and \( q_k = t \), and every consumer purchases a bundle.

iv. **Asymmetric Duopoly:** A mixed-strategy equilibrium exists but a pure-strategy equilibrium does not.

For the parameter values considered by Proposition 1, equilibrium prices are highest under monopoly and lowest under mix-and-match competition, with exclusive duopoly falling in between.

**IV. MORE THAN ONE MARKET POWER RENT**

In this section, we compare the different industry configurations in terms of equilibrium industry profit. We first note that, because each component supplier ignores the effects of its pricing on the sales of the complementary component supplier, equilibrium prices may be greater than the profit-maximizing prices. For example, when \( v < 2t \), the equilibrium price of a system is \( p_k + q_k = \frac{3}{4}v \). However, if platform \( k \) and application \( k \) were integrated and had a local monopoly, then they would set component prices that summed to \( \frac{1}{2}v \). For some parameter values, this double marginalization results in industry profits that are higher with mix and match than with exclusivity (e.g., when \( v < t \), full mix and match results in equilibrium prices such that \( p_k + q_k = \frac{1}{2}v \)).
There is no double-marginalization problem when the consumption value of a bundle is large relative to the degree of application differentiation; the applications’ market areas overlap under all of the industry configurations. The different configurations do, however, give rise to different prices and profits:

**Proposition 2 (More than One Market-Power Rent):** Suppose the platforms are undifferentiated (i.e., \( s = 0 \)), \( v > 6t \), and the platform and application owners independently set the prices for their respective components. Then profits gross of licensing fees are:

i. **Monopoly Platform:** \( v - t \) for the platform, and \( \frac{1}{4} t \) for each application.

ii. **Mix and Match:** 0 for each platform and \( \frac{1}{2} t \) for each application.

iii. **Exclusive Duopoly:** \( \frac{3}{2} t \) for each platform and \( \frac{1}{2} t \) for each application.

iv. **Asymmetric Duopoly:** Expected industry profits are lower under the asymmetric duopoly equilibrium than under the exclusive duopoly equilibrium.

The comparison between exclusive duopoly and mix and match is of particular interest. Under full mix and match, consumers choose their platforms and applications independently. Hence, the degree of application differentiation is irrelevant to the choice of platform, and the platforms are undifferentiated Bertrand competitors, which set price equal to marginal cost. The resulting application prices equal to \( t \). Under exclusive duopoly, platforms essentially inherit the differentiation of their applications: in order to attract a customer from platform \(-i\), platform \( i\) has to induce that customer to switch his or her application from \( i\) to \(-i\) as well. Hence, the platforms engage in differentiated-products competition with one another under exclusive
duopoly. The resulting platform prices are equal to $3t$ and application prices are equal to $t$.\footnote{Intuition might suggest that the platform’s equilibrium prices would equal $t$. However, a platform acts as a leader with respect to pricing by the two applications. Raising $p_i$ induces application $i$ to lower its price and application $-i$ to raise its price, both of which benefit platform $i$. In an earlier version of the paper, we considered simultaneous pricing by platforms and applications. In this case, the Stackelberg effects do not arise and the platforms’ equilibrium prices are equal to $s$ under mix and match and $t$ under exclusive duopoly. The analysis is available from the authors upon request.} In other words, the One-Monopoly-Rent Theorem does not extend to this case. By entering into exclusive deals, providers of differentiated applications can increase the margins earned by platforms without reducing the applications’ margins, so that industry profits rise.

Our finding the exclusive duopoly is more profitable than mix and match is driven by the fact that the differentiated applications are not monopolists. To see this fact, consider the case of a monopoly application. Industry profits (gross of fixed costs) would be maximized by having that application available on both platforms. Platforms would set their prices equal to zero, while the application monopolist would set its price equal to $\max\{\frac{1}{2}v,v-t\}$, resulting in industry-profit-maximizing system prices. In other words, the One Monopoly Rent Theorem holds in our framework. The profitability of exclusivity in our model is driven by the fact that applications are not monopolies and imperfect competition between applications prevents them from fully exercising potential market power (\textit{i.e.}, under mix and match, equilibrium application prices are equal only to $t$).

V. CONTRACTING

In the stage prior to pricing, contracting between the platforms and applications takes place subject to any public policy constraints. We will examine three policy regimes:

- \textit{Full Exclusivity}: A platform is free to exclude applications and to pay one or both application owners not to make their products available on the rival platform.
• **Limited Exclusivity**: A platform owner is free to exclude applications and can pay at most one application owner to withhold its product from the rival network.

• **Open Platforms**: A platform has to be technologically open, cannot charge applications to be on it, and cannot pay applications to be exclusive.\(^{13}\)

We assume that any payments between platforms and applications are lump sum, although these sums can be contingent on the degree of exclusivity. This assumption would be satisfied, for example, if specific quantities were unverifiable to a third-party contract enforcer (e.g., a court). Our rationale for making this assumption is to rule out the use of contracts as a form of implicit collusion.\(^{14}\)

In order to understand how the policy regime affects the outcome, and to determine the equilibrium distribution of profits among platforms and applications, it is necessary to consider an explicit model of bargaining. At present, there are no fully satisfactory non-cooperative game-theoretic models of \(2 \times 2\) bargaining. To illustrate possible outcomes, we examine a highly stylized extensive-form bargaining game. In the first round, application \(i\) is paired with platform \(i, i = 0,1\). Platform \(i\) makes a take-it-or-leave it offer to application \(i\), which the application either accepts or rejects. In the second round, application \(i\) is paired with platform \(-i\),

\(^{13}\) This corresponds to the sort of public policy sought by many advocates of so-called network neutrality. In addition to being the result of a public policy requirement, this regime could arise as the result of technological conditions (e.g., if the platforms are technologically compatible and an application developed for one platform can be run on the other without any cooperation from the platform owner).

\(^{14}\) Under some conditions, if quantity-contingent payments are feasible, then an application could sign two-part tariffs with the platforms under which the marginal price would induce the platforms to set industry-profit-maximizing prices and the fixed fee would divide the profits. Indeed, if allowed to do so by public policy, a third party with no other involvement in the industry could implement such a scheme. See Bonanno and Vickers (1988) for a demonstration of this point in a related context. An issue with such results, both here and in the Bonanno and Vickers model is that they are sensitive to whether contracts can be privately renegotiated. See Katz (1991 and 2006) for further analysis of the issues that can arise when parties seek to gain strategic advantage via the use of contracts. We also address these issues further in Section VIII.B below.
and that platform makes a take-it-or-leave-it offer. There are no additional rounds of bargaining. Observe that, a priori, bids could be positive (e.g., the platform pays the application developer for offering a complementary product) or negative (e.g., the platform demands a license fee for access to proprietary protocols needed to operate on the platform).

If there is a monopoly platform and it can exclude applications from its platform, then the platform owner will charge a fee to each application. Given that it has the bargaining power, the monopolist extracts all of the industry profits by setting its fee equal to an application’s profits gross of the license fee, \( \frac{1}{4} t \).

Next, suppose that both platforms have invested in entering the industry. Under the open platform regime, there will be no license fees and the equilibrium configuration will be mix and match. Next, consider the full exclusivity and partial exclusivity regimes. Given the structure of the bargaining game, we have:

**Lemma 2:** Suppose that two platforms have entered the market. Under the full exclusivity and partial exclusivity regimes, the equilibrium industry configuration is exclusive duopoly and each application pays a license fee of \( \frac{1}{2} t \) to the corresponding platform.

The details of the proof are given in the Appendix, but the underlying logic is the following. In the first round, each platform offers to allow the corresponding application to be on the platform in return for a payment of \( \frac{1}{2} t \) and a commitment not to become available on the other platform. Each application accepts this offer because it would earn no profits if it were to refuse the offer—in the second round, the application would face a take-it-or-leave it offer from
the other platform that would also extract all of the application’s potential profits.\textsuperscript{15} Even when full exclusivity is permitted, the parties cannot bargain their way to the industry-profit maximizing outcome, under which both applications reach exclusive agreement with a single platform. The reason is that there is no mechanism for a platform to commit to rewarding an application owner for waiting until the second bargaining round to sign an exclusive contract.

We emphasize that our objective is not to make precise predictions about the outcome of the contracting game. Indeed, the equilibrium industry structure is sensitive to our assumptions about the extensive form of the bargaining game.\textsuperscript{16} Rather we wish to demonstrate the possibility that some degree of platform exclusivity can lead to increased equilibrium platform profits.

A further limitation in our analysis is that we restrict attention to a particular set of potential contracts. An alternative approach would be to allow the platforms to make highly sophisticated offers, in which terms and outcomes are fully contingent on the behavior of the other three players. This approach allows a platform to "react" to its rival’s offer even in a simultaneous-offer game. Although a full analysis of such contracting is beyond the scope of

\textsuperscript{15} A backward induction argument demonstrates that allowing a finite number of additional rounds of bargaining would make no difference to the equilibrium outcome.

\textsuperscript{16} For example, when simultaneous offers are feasible, there is an equilibrium under which platform $i$ offers to pay each application one half of the gross monopoly platform profits in return for the application’s agreeing not to be on the rival platform, while platform $-i$ offers each application this amount minus $\frac{1}{4} t$ (the difference between gross profits per application under the exclusive duopoly and monopoly platform configurations). In equilibrium, both applications sign contracts with platform $i$. It would be unprofitable for either application to deviate or for platform $-i$ to bid more than half of the monopoly profits to either or both applications. A platform could not attract applications with a lower bid.
this paper, we note there exist equilibria similar to that of Lemma 2, in which both applications sign exclusive agreements with one of the platforms.\(^{17}\)

VI. PLATFORM ENTRY

Now consider the two platforms’ entry decisions.\(^{18}\) Rather than work through the details of the various stage-game equilibria, we offer broad observations on the structure of these equilibria. Let \(\Pi_i^m\) denote the profit gross of the entry cost that platform \(i\) would earn as a monopolist given the continuation equilibrium in the pricing and contracting stages. Similarly, let \(\Pi_i^d\) denote the profit gross of the entry cost that platform \(i\) would earn in the continuation equilibrium in competition with platform \(j\).

Observe that:

a. If \(\Pi_i^d \leq \Pi_i^m < F\), then it is a dominant strategy for platform \(i\) to stay out of the market.

b. If \(\Pi_i^d < F < \Pi_i^m\), then platform \(i\)’s best response to platform \(j\) action of entering the market or not is to do the opposite of what platform \(j\) does.

c. If \(F < \Pi_i^d \leq \Pi_i^m\), then it is a dominant strategy for platform \(i\) to enter the market.

---

\(^{17}\)Suppose the regime is full exclusivity. The following strategies support an equilibrium in which each platform signs an exclusive contract with a single application. Platform \(i\) offers contract \(C_i\), where this contract states that: (a) if platform \(-i\) offers any contract other than \(C_i\), then platform \(i\) will offer to pay half of the gross monopoly platform profits to each application to be nonexclusive, and (b) if platform \(-i\) offers contract \(C_{-i}\), then platform \(i\) will charge each application \(t/4\) for the right to be exclusively on the platform if both agree, and otherwise refuse to deal with applications. Both applications accept the offer of one of the platforms. There are no profitable deviations for either the platforms or applications. A similar equilibrium exists in the partial exclusivity regime, except each application signs a contract in which it pays \(t/2\).

\(^{18}\)Although described in terms of de novo entry, the model can also be interpreted as an existing platform’s decision to invest in additional capabilities to support a new class of applications (e.g., a mobile telephone network’s decision to invest in high-speed data transport).
The entry-game equilibria that arise under the various possible combinations of parameter values are largely self-evident. We note that, when both platforms are in case (b), there are three equilibria: two pure-strategy equilibria defined by which platform enters and which stays out, and a mixed strategy equilibrium in which each platform enters with a probability between 0 and 1. We focus on the pure-strategy equilibria.

VII. WELFARE ANALYSIS

We next examine the welfare of various public policies toward exclusive contracts. To do so, it is useful to rank the various outcomes in terms of profits, consumer surplus, and total surplus. To simplify the analysis, we assume \( v > 6t \) so that the market is covered in equilibrium under all of the industry configurations.

Proposition 2 above ranks the configurations in terms of profits gross of fixed costs. When \( v > 6t \), the monopoly platform, exclusive duopoly, and mix and match configurations all give rise to the same level of gross consumption benefits. Hence, these configurations are ranked in terms of consumer surplus in inverse order to their ranking in terms of profits gross of fixed costs. Proposition 1 and Lemmas 1 and A.3 imply that that consumer surplus is greater under mix and match than under asymmetric duopoly. Proposition 1 and Lemmas 1, A.4 and A.6 demonstrate that, from a consumer’s perspective, at worst the realization under asymmetric duopoly is equal to that under exclusive duopoly.

Turning to total surplus, the fact that the monopoly platform, exclusive duopoly, and mix and match configurations all give rise to the same level of gross consumption benefits implies that exclusive duopoly and mix and match give rise to the same level of total surplus, which is lower than the level under the monopoly platform configuration by an amount equal to the fixed
cost savings of $F$. The asymmetric case gives rise to the lowest level of total surplus of all because platform fixed costs are incurred twice and because consumer choices between the two applications are distorted when the platforms’ realized prices are unequal to one another.

Summarizing this discussion,

**Proposition 3:** Suppose platforms are undifferentiated (i.e., $s = 0$) and $v > 6t$. Then:

i. **In terms of the equilibrium level of consumer surplus, the ranking of industry configurations from highest to lowest is mix and match, asymmetric duopoly, exclusive duopoly, and monopoly platform; and**

ii. **The monopoly-platform configuration yields the highest level of equilibrium total surplus due to the non-duplication of fixed costs, while the asymmetric-duopoly configuration yields the lowest due to distortions in the relative prices of platform-application pairs. Equilibrium total surplus under both the exclusive-duopoly and mix-and-match configurations is $F$ lower than under the monopoly-platform configuration.**

We next examine the equilibrium welfare levels under the alternative public policy regimes of full exclusivity, limited exclusivity, and open platforms. Under either of the first two regimes, Proposition 1 and Lemma 2 imply that, if both platforms enter the market, then each will earn $2t - F$ in equilibrium. Hence, if $2t > F$, then both platforms will enter the market, while if $2t < F$, the only pure-strategy equilibria are those in which one platform enters the market and the other does not.

Now, consider the open-platform regime. If both platforms entered the market, then each would suffer losses because there would be no license revenues and product-market competition

---

19 Observe, too, that the ranking can be different for lower values of $v$ because at those values the higher prices associated with monopoly and exclusive duopoly can distort consumption levels.
between the two undifferentiated rivals would lead them to price at marginal cost. Hence, each firm would suffer losses equal to the fixed cost of entry, $F$. Consequently, the only pure-strategy equilibria are those in which one platform enters the market and the other does not. In other words, a public policy intended to promote competition by requiring full mix and match would, in fact, induce the monopoly platform configuration.

Summarizing the implications of this analysis:

**Proposition 4:** Suppose the platforms are undifferentiated (i.e., $s = 0$), $v > 6t$, and $2t > F$. The open-platform policy regime yields higher total surplus but lower consumer surplus and less platform competition than do either the full exclusivity or partial exclusivity regimes.

Notice that the differences in the equilibrium entry levels are not simply due to the lack of licensing revenues under the open-platform regime. Even if applications could be forced to pay all of their profits under the mix-and-match configuration to the platforms, each platform would earn only $\frac{1}{2}t$, so that the analogue of Proposition 4 would arise when $2t > F > \frac{1}{2}t$.

To be clear, Proposition 4 depends on the specific bargaining game played by the platforms and applications. As noted above, under a different bargaining game, full exclusivity would lead to a monopoly platform, whether or not fully contingent offers were feasible. However, partial exclusivity when fully contingent contract offers are feasible would lead to the same result as Proposition 4.

Lastly, we should note that, in our baseline model, exclusive duopoly always yields lower total surplus than a monopoly platform because the two industry structures lead to the same set of consumers who purchase systems but duopoly entails an additional entry expenditure of $F$. Under a different demand structure, duopoly could expand the number of consumers who purchase. Depending on parameter values, the welfare gain from more consumers’ purchasing
could outweigh the additional fixed cost of entry. The following example illustrates. Modify our baseline model so that half of the consumers have value \( v = 7t \) and the other half have value \( v = 15t \). Given these parameter values, Proposition 1 (iii) implies that exclusive duopoly leads to full market coverage. By Proposition 1(i), if the monopolist chooses to sell solely to the high-value customers, it will set a price of 14.5t. Clearly, this strategy is more profitable than setting its price low enough to attract sales from low-value customers. The resulting deadweight loss of monopoly is 3.5t. Hence, for \( F \in (0, 2t) \), allowing exclusive contracts will lead to greater total surplus than would the open-platform policy regime.

VIII. EXTENSIONS OF THE TWO-MARKET-POWER RENTS FINDING

In this section, we consider two extensions of our finding that there can be more than one market power rent.

A. Differentiated Platforms

The next proposition establishes that our central result regarding the lack of a One-Market-Power-Rent Theorem extends to the case of component pricing with differentiated platforms. Because the Hotelling model can give rise to many different cases, we simplify the extension by restricting attention to a specific range of parameter values.

**Proposition 5:** Suppose that platforms are differentiated with \( 0 < s < t \) and \( 6t < v < 3t^2 / s \). Then:

i. **Mix and Match:** There exists a unique symmetric equilibrium, under which the platform prices are \( p_i = s \), application prices are \( q_j = t \), and every consumer purchases a bundle.

ii. **Exclusive Duopoly:** There exists a unique symmetric equilibrium, under which \( p_k = 3t \), \( q_k = t \), and every consumer purchases a bundle.
Once again, the platforms inherit the application differentiation under exclusive contracting, which relaxes price competition.

It important to recognize that there are also significant differences between the cases of differentiated and undifferentiated platforms. Most notably, in the absence of platform differentiation: (a) there is no direct consumption value of platform variety, and (b) the open-platform regime cannot support two platforms for any positive fixed cost. Neither conclusion holds for differentiated platforms. When \( s > 0 \), there are parameter values for which platforms can cover the fixed costs of entry under the mix-and-match configuration even absent side payments from application providers  \( i.e., 0 < 2F < s \). Moreover, there are parameter values for which industry profits are greater under mix and match than exclusive duopoly. For example, when \( s \) and \( t \) are sufficiently close to \( v \), the platforms will find it optimal to set system prices that give rise to non-overlapping market areas. Industry profits will be greater when each platform has a greater variety of applications to offer consumers in its local monopoly areas.

**B. Bundle Pricing**

In some industries in which exclusive applications have been controversial, such as cable and satellite television, a platform obtains the right to distribute one or more applications and then the platform owner sets the price of an integrated offering, or bundle, to consumers. In this part, we show that exclusive applications can increase equilibrium prices under bundle pricing. We consider the game described in Section II above except we now assume the following form of the pricing stage:

**Pricing Stage:** Any platform providers that have entered the market simultaneously set prices for bundles comprising one platform and one application. Let \( r_{ij} \) denotes the price of a system offered by platform \( i \) that contains application \( j \).
We consider markets in which the value of a platform-application bundle is large relative to its costs, so that duopoly platforms have overlapping market areas and engage in price competition rather than acting as local monopolies.\footnote{An earlier version of the paper, which is available from the authors, provides a more complete analysis of bundle pricing.}

**Proposition 6:** Suppose that $0 \leq s < t$, $\nu \geq \frac{1}{2}s + \frac{1}{2}t$, and platform owners set the prices for platform-application bundles. If both platforms enter the market, then the unique symmetric equilibrium outcomes in the pricing continuation game are:

i. **Mix and Match:** When each platform offers both applications, the equilibrium prices are $r_{ij} = s$ and every consumer purchases a bundle.

ii. **Exclusive Duopoly:** When each platform offers a (different) single application, the prices are $r_{ii} = t$ and every consumer purchases a bundle.

Here, the underlying intuition for how exclusivity relaxes price competition is particularly clear. In order for a platform to gain sales at the margin under exclusive duopoly, it must reduce its price by an amount sufficient to induce consumers to switch applications (as well as platforms). Hence, a price cut attracts new customers at the rate $\frac{dc}{dt}$. Under mix and match, however, consumers can switch platforms without switching applications. Thus, under mix and match, a price cut can be used to attract new customers at the margin at a rate of $\frac{dc}{2\nu}$. Because demand is more elastic under mix and match, the equilibrium platform price is lower.

One question is whether this result extends to markets in which two-part tariffs or some other form of quantity-dependent licensing is used. Suppose that the industry configuration is mix and match and platforms can offer to pay two-part tariffs. Could the platforms offer...
marginal payments greater than marginal cost as a means of inducing themselves to set higher equilibrium prices under mix and match, so that exclusive duopoly would not be a more profitable configuration?

Consider an industry in which: the platforms offer two-part tariffs; only non-exclusive, non-discriminatory contracts are permitted; and a platform never observes the rival platform’s contracts. A full analysis of this setting is beyond the scope of the present paper, but consider a candidate symmetric equilibrium with each platform offer to pay each application a fixed fee of $W^*$ and a per-unit fee of $w^*$, where $w^* \geq t - s$. Given these license contracts, the equilibrium bundle prices would be $r_j = w^* + s \geq t$, suggesting that mix and match could be as profitable for the industry as exclusive duopoly. However, if platform $k$ were playing its part of the candidate equilibrium, then it is readily shown that platform $-k$ could profitably deviate by offering applications a contract with a lower per-unit fee and higher fixed fee. Intuitively, this deviation from the candidate equilibrium would be profitable because it would allow platform $-k$ to appropriate some of the margin earned by platform $k$ on its sales. This brief analysis thus suggests that allowing two-part tariffs would not overturn the conclusion that exclusive duopoly can be more profitable than mix and match for some parameter values.

One might conjecture that, if applications made the offers, then an application could induce both platforms to charge higher prices for bundles containing its product, thus raising joint profits. However, unobservable contracts raise the possibility of deceit by the application and raise difficult issues of how platforms interpret out-of-equilibrium offers. Analyses of monopoly applications suggests that such contracts are an imperfect means of offsetting
downstream competition. The central point for our purposes is that there are reasons to expect that quantity-dependent license fees are not a perfect substitute for the use of exclusive contracts.

IX. CONCLUSION

This paper has examined a straightforward model in which exclusive contracts can create product differentiation. Our central finding is that an imperfectly competitive supplier (an application provider in our model) can increase aggregate profits by tying its product to a good that would otherwise be competitively supplied. In other words, there is no One-Market-Power-Rent Theorem analogous to the One-Monopoly-Rent Theorem. This is an important limitation of the One-Monopoly-Rent Theorem because very few firms are literally monopolists.

We also find that, because they raise equilibrium prices and industry profits for a fixed number of platforms, exclusive arrangements can play a role supporting investment even when that investment is neither relationship specific nor subject to free riding by other parties. Because exclusive contracts between platforms and applications raise prices for a fixed number of platforms but can, in some circumstances, increase the equilibrium number of platforms, exclusive contracts can raise or lower equilibrium consumer and total surplus through effects on the degree of platform competition.

This analysis clearly is at too early a stage to serve as the basis for recommending for specific legal rules for the treatment of exclusive deals. The analysis does, however, have clear

---

21 For an analysis of some of these issues, see Rey and Vergé (2004). The authors examine a setting that corresponds to a monopoly application selling to two differentiated platforms. They show that, with unobservable contract offers, there is no equilibrium that supports the industry-profit-maximizing downstream price.

22 In addition, it is readily shown that, by giving rise to positive equilibrium margins for platforms, exclusive dealing can create incentives for platforms to undertake costly activities (e.g., marketing campaigns) that expand industry—as opposed to platform-specific—demand. Such incentives do not exist under the mix-and-match configuration with undifferentiated platforms.
implications for public-policy formulation. Specifically, the model demonstrates that: (a) arguments based on the One-Monopoly-Rent Theorem are not necessarily valid when applied to imperfect competitors, and (b) policy analyses should consider the full industry equilibrium, rather than assuming that critical elements of industry structure are fixed.
REFERENCES


Lemma A.1: If \( 3t < 2v - p_0 - p_1 \) and \( p_i - p_{-i} > 3t \), then \( Z_{ii} = 0 \).

Proof: Suppose, counterfactually, that the two platforms had local monopolies in equilibrium. Then by the same logic used in the application-pricing game considered previously, the equilibrium quantities would be \( y_j = \frac{v - p_j}{2t} \). Given the assumption that \( 3t < 2v - p_0 - p_1 \), we would have \( y_0 + y_1 > \frac{3}{2} \), a contradiction.

Now suppose the two market areas touch one another. Suppose, counterfactually, that the equilibrium entails \( Z_{ii} > 0 \). It would also have to be the case that \( Z_{-i-i} > 0 \), with

\[
0 < Z_{ii} = \frac{t - p_i - q_i + p_{-i} + q_{-i}}{2t} = 1 - Z_{-i-i}.
\]

This inequality implies that \( q_{-i} - q_i > 2t \). It must also be the case that \( Z_{-i-i} + q_i \frac{\partial Z_{-i-i}}{\partial q_i} \geq 0 \) or application \(-i\) would find it profitable to lower its price. But given \( \frac{\partial Z_{ik}}{\partial q_k} = -\frac{1}{2t} \), we have

\[
Z_{-i-i} + q_i \frac{\partial Z_{-i-i}}{\partial q_i} < (1 - Z_{ii}) - (q_i + 2t) \frac{1}{2t} = -Z_{ii} - q_i < 0.
\]

Again, we have a contradiction. **QED**

The proof of Propositions 1(iv) and 2 proceed via a series of lemmas. Recall our convention that, in an asymmetric duopoly, it is application 0 that is exclusive to platform 0, while application 1 is available on both platforms.

We first consider the solution to a modified platform-pricing game, where the sole modification is to restrict the platform’s prices to being chosen from the interval \([0, 3t]\). By Theorem 5 of Dasgupta and Maskin (1986), there exists a mixed-strategy equilibrium. Let \( \underline{p}_i \) and \( \bar{p}_i \) denote the inf and sup, respectively, of the prices charged by platform \( i \) with positive
probability in the mixed-strategy equilibrium that arises when the industry configuration is asymmetric duopoly. By construction, \( \bar{p}_i \leq 3t \).

**Lemma A.2:** The equilibrium of the modified game is also an equilibrium of the original game (i.e., the one in which a platform is free to choose any price in the interval \([0,v]\)).

**Proof:** Suppose that each platform expects its rival to play its equilibrium strategy from the modified game. At any \( p_1 \) greater than \( \bar{p}_0 \), platform 1 makes no sales. Hence, it could never be profitable to set \( p_1 > 3t \).

For \( p_0 \) greater than \( \bar{p}_1 \), the outcome is similar to exclusive duopoly. By Lemma 1, for any \( p_0 > 3t \), platform 0’s expected continuation profits are

\[
\int_{p_0 - 3t}^{3t} p_0 \left( \frac{3t + p_1 - p_0}{6t} \right) dG(p_1).
\]

Differentiation with respect to \( p_0 \) yields

\[
\int_{p_0 - 3t}^{3t} p_0 \left( \frac{3t + p_1 - 2p_0}{6t} \right) dG(p_1).
\]

where \( G(\cdot) \) is the distribution function characterizing platform 1's mixed strategy.

Given \( \bar{p}_1 \leq 3t \), the term in brackets is negative for any \( p_0 > 3t \). Hence, it could never be optimal to set \( p_0 > 3t \). \( \text{QED} \)

**Lemma A.3:** \( p_{\omega_0} > 0 \).

**Proof:** Even if \( p_1 = 0 \), platform 0 could guarantee itself positive profits by setting \( p_0 = \frac{1}{2} t \). Let \( \pi \) denote a positive constant less than the lower bound on platform 0’s profits. Given that platform 0’s quantity can never be greater than 1, it can never set its price below \( \pi \) in equilibrium. \( \text{QED} \)
Lemma A.4: \( \bar{p}_1 \leq \bar{p}_0 \).

**Proof:** Suppose, counterfactually, that \( \bar{p}_1 > \bar{p}_0 \). Then there exist prices that platform 1 plays with positive probability for which it makes 0 sales with probability one. Platform 1’s expected profit at these prices would be 0, which contradicts the fact that platform 1 can guarantee itself positive profits by setting a strictly positive price less than \( \bar{p}_0 \). QED

Lemma A.5: If \( v > 6t \), then the distribution of platform 0’s prices under asymmetric duopoly has an atom at \( \bar{p}_0 \).

**Proof:** There are two cases to consider.

First, suppose \( \bar{p}_0 > \bar{p}_1 \). Platform 0’s price is greater than platform 1’s price with probability one for any \( p \in (\bar{p}_1, \bar{p}_0] \). By Lemmas 1 and A.2, over the relevant range of prices, platform 0 faces the demand curve \( y_0 = \frac{3t + p_1 - p_0}{6t} \). Hence, any price in this interval played with positive probability must be a solution to

\[
\max p_0 \frac{3t + E[p_1] - p_0}{6t}.
\]

This problem has a unique solution. Hence, there must an atom at this price.

Second, suppose \( \bar{p}_0 = \bar{p}_1 \). Suppose counterfactually that there is no atom at \( \bar{p}_0 \). Then the probability that platform 1’s price is less than or equal to platform 0’s price goes to 0 as \( p_1 \uparrow \bar{p}_1 \). Hence, for all prices sufficiently close to the sup, platform 1’s expected profits would be less than the profits that the platform could guarantee itself by setting a strictly positive price less than \( \bar{p}_0 \). QED

Lemma A.6: If \( v > 6t \), then \( \bar{p}_0 < 3t \).
Proof: By expression (A.1) above,
\[ \overline{p}_0 = \frac{3t + E[p_1]}{2} < \frac{3t + \overline{p}_0}{2}. \]

The inequality follows from Lemma A.4 and the fact that platform 1’s equilibrium price distribution must be non-degenerate, which implies that the mean is less than the sup. **QED**

**Proof of Proposition 2:** Parts (i) through (iii) are immediate consequences of Proposition 1. Consider equilibrium profits under the asymmetric duopoly configuration. By Lemma 1(iii), industry revenues are less than or equal to \( t + \frac{1}{2}(p_0 + p_t) \). Hence, by Lemmas A.4 and A.6, expected industry profits in the asymmetric case are less than \( 4t \), which is the level that would be earned under exclusive duopoly. **QED**

**Proof of Lemma 2:** Suppose each platform adopts the following strategy.

- In round 1, platform \( i \) offers to let application \( i \) be on platform \( i \) in return for a payment equal to \( \frac{1}{2} t \) and a commitment not to become available on the other platform.

- In round 2, platform \( i \) makes the following offers to application \( -i \):
  - If \( -i \) has an exclusive contract with platform \( -i \), there is no offer.
  - If \( -i \) has no contract with platform \( -i \), then platform \( i \) offers to allow application \( -i \) to be on platform \( i \) in return for a payment equal to \( \frac{1}{4} t \) to the profit it could expect to earn given the resulting industry structure if it is (de facto) exclusively on platform \( i \).
  - If \( -i \) has a non-exclusive contract with platform \( -i \), then—depending on the parameter values—platform \( i \) may or may not make a non-exclusive offer to application \( -i \). If an offer is made, it will result in application \( -i \)’s earning the same profits as if no deal were reached given the ensuing industry structure.

Given these platform strategies, an application would never find it profitable to refuse the initial offer given that it expects the other application to accept in the first round. Next, consider whether a platform, say \( m \), would have incentives to deviate. There clearly is no incentive to
deviate by offering the same contract but demanding a lower payment from an application at any of the nodes above. There also clearly is no reason to change any of the forms of contracts offered at the various nodes in the second round.

It remains to consider deviations in the first round that take the form of a different type of contract offer.

- Suppose that $m$ offered no contract in the first round? Then given the equilibrium strategies, $-m$ would obtain a platform monopoly. This is an unprofitable deviation.

- Suppose that $m$ offered a non-exclusive contract in the first round.
  
  o If application $m$ accepts a non-exclusive offer, then in the next round platform $-m$ knows that application $m$ will earn $\frac{1}{2}t$ if it does not reach an agreement to be available on platform $-m$. Given that it is making a take-it-or-leave-it offer, platform $-m$ will never make an offer that yields application $m$ profits (net of the license fee to $-m$ but gross of the license fee to $m$) greater than $\frac{1}{2}t$.

  o Hence, by signing a non-exclusive agreement, the most that application can expect to earn in the overall game $\frac{1}{2}t$ minus any fee paid to platform $m$ for the non-exclusive license. Therefore, the most that platform $m$ could charge for a non-exclusive relationship is $\frac{1}{2}t$. Thus, this deviation would never increase platform $m$’s profits. It would reduce platform $m$’s profits if $-m$ reached a non-exclusive agreement with application $m$.

QED

Proof of Proposition 5: First, consider the mix-and-match configuration. Suppose that

\[
\min\{p_0, p_1\} < v - 3t \quad \text{and} \quad q_1 = t. \quad \text{By assumption,} \quad 6t < v < 3t^2 / s \quad \text{Hence, any consumer can enjoy positive surplus by purchasing a system that includes application 1. It follows that the market areas of the two applications would overlap for any} \quad q_0 \quad \text{that gives rise to positive sales.} \\
\text{Application 0’s unit sales are thus equal to} \quad (2t - q_0) / 2t, \quad \text{and its profits are maximized by setting}\]

\[
q_0 = t. \quad \text{Next, platform 0’s pricing decision given that} \quad p_1 = s. \quad \text{As just shown, the equilibrium application prices will equal} \quad t \quad \text{for any choice of} \quad p_0. \quad \text{Hence, any consumer can enjoy positive} 
\]

40
surplus by purchasing a system that includes platform 1; the market is covered. Platform 0’s unit sales are thus equal to $(2s - p_0)/2s$, and its profits are maximized by setting $p_0 = s$. We have established (i).

Now, consider the case of exclusive duopoly. It can be shown that the applications’ profit functions are continuous and strictly quasi-concave. It follows from theorems of Debreu (1952), Glicksberg (1952), and Fan (1952) that there exists a pure-strategy equilibrium of application-pricing continuation game.

There are two cases to consider. First, suppose $p_0 \in [\frac{3}{12} t, 6t - 3s], p_1 = 3t$, and $q_1 = \frac{1}{3} p_0$. Then in a neighborhood of $q_0 = 2t - \frac{1}{3} p_0$,

$$Z_{q0} = \frac{t - p_0 - q_0 + p_1 + q_1}{2t}. \quad (A.2)$$

and application 0’s profits equal $q_0 \frac{3t - 2p_0 - 3q_0 + 9t}{6t}$. Calculation shows that the derivative of profits with respect to $q_0$ is equal to 0 at $q_0 = 2t - \frac{1}{3} p_0$. By the quasiconcavity of application 0’s profit function, this value of $q_0$ is a best response. Similarly, given $q_0 = 2t - \frac{1}{3} p_0$, application 1’s profits are equal to $q_1 \frac{2p_0 - 3q_1}{6t}$ and its best response is $q_1 = \frac{1}{3} p_0$.

Next, suppose that $p_0 \approx 6t - 3s$ and $p_1 = 3t$. If, in equilibrium, the two applications’ market areas do not touch, then it must be the case that application 0’s market area is contained within triangle $A$ in Figure 2. This is so, because it is readily shown that application 1 will find it profitable to set $q_1 < v - 4t$ when its market area does not overlap with application 0’s market.

---

The proof is available from the authors upon request.
area. If the two market areas touch in area B, then the first-order conditions for the two
applications’ best-response calculations imply

\[ Z_{00} - \frac{1}{2t} q_0 = (1 - Z_{00}) - \frac{1}{2t} q_1, \tag{A.3} \]

where \( Z_{00} \) is given by (A.2) above.

Equations (A.2) and (A.3) imply \( q_1 - q_0 = \frac{2}{3} (p_0 - p_1) \). Given \( p_0 \geq 6t - 3s \) and \( p_1 = 3t \), it
follows that \( p_0 + q_0 - p_1 - q_1 \geq t - s \), with strict inequality for any \( p_0 > 6t - 3s \). Hence, the
equilibrium cannot be in B for any \( p_0 > 6t - 3s \). It is readily shown that the equilibrium market
area for application 1 can never be contained in triangle C when \( p_1 = 3t < p_0 \). Hence, it follows
that application 0’s sales region must fall within triangle A in any equilibrium of the application-
pricing continuation game when \( p_0 \geq 6t - 3s \) and \( p_1 = 3t \).

Now, consider platform 0’s best response to \( p_1 = 3t \). The analysis just completed
demonstrates that \( p_0 = 3t \) leads to \( q_0 = t = p_1 \), which would yield platform 1 profits of \( \frac{3}{2} t \) gross
of any license fees. It follows that it cannot be a best response for platform 0 to set any price
less than \( \frac{3}{2} t \). Moreover, any \( p_0 \geq 6t - 3s \) will result in unit sales less than or equal to \( s/(2t) \) and
revenues less than or equal to \( vs/(2t) < \frac{3}{2} t \). Lastly, the analysis of application price
demonstrates that, for \( p_0 \in [\frac{3}{2} t, 6t - 3s) \), platform 0’s profits are

\[ p_0 Z_{00} = p_0 \frac{6t - p_0}{6t}, \]

which is maximized at \( p_0 = 3t \). QED

**Proof of Proposition 6:**
(i) Suppose one platform, say 0, has set its prices to both be \( s \). Consider the following pseudo-problem for platform 1: For each \( y \) imagine platform 1 can set a price for its system, \( r(y) \). Assume each such system has the same application as the consumer would obtain were he to buy from platform 0. In other words, the pseudo-problem for platform 1 is to devise a best response to its rival’s price of \( s \) in a conventional Hotelling model in which the intrinsic value for the good is \( \tilde{v}(y) = v - t \min\{y,1-y\} \). Because

\[ v \geq \frac{1}{2} t + \frac{1}{2} s > \frac{1}{2} t + \frac{3}{2} s \geq t \min\{y,1-y\} + \frac{1}{2} s, \]

\( \frac{3}{2} \tilde{v}(y) > s \) for all \( y \). Per the usual analysis of such Hotelling models, it follows that \( r(y) = s \) for all \( y \). Given that platform 1 can gain no advantage trying to sell application 0 to \( y > \frac{1}{2} \) or application 1 to \( y < \frac{1}{2} \), it follows that platform 1’s best response to its actual problem is to set \( r_{10} = r_{11} = s \).

To see that no other symmetric equilibrium exists, suppose that one did in which \( r_{00} = r_{01} \neq s \). An argument similar to that just given reveals that platform 1’s best response is \( r_{10} = r_{11} \neq r_{00} = r_{01} \), a contradiction.

(ii) Without loss of generality, assume that the two systems are \( Z_{00} \) and \( Z_{11} \). There are two possible equilibrium configurations. When \( s \) and \( t \) are sufficiently low relative to \( v \), the two systems compete and have overlapping market areas in equilibrium. The border between the two customer sets its defined the condition that a consumer located at \((x,y)\) be indifferent between the two bundles:

\[ v - r_{00} - sx - ty = v - r_{11} - s(1-x) - t(1-y) \]

or

\[ y = \frac{1}{2} + \frac{1}{2t} \{r_{11} - r_{00} + s(1-2x)\} \]

Given our assumption that \( s < t \), the resulting sales are
\[ Z_{ii} = \frac{1}{2} + \frac{1}{2t} \{ r_{jj} - r_{ii} + s \} - \frac{1}{2} \int_0^1 xdx = \frac{1}{2} + \frac{1}{2t} \{ r_{jj} - r_{ii} \} \]

for values of \( r_{ii} \) corresponding to overlapping market areas. In this case, the equilibrium prices must satisfy the first-order condition \( Z_{ii} - \frac{r_{ii}}{2t} = 0 \), which implies that the unique equilibrium system prices are \( r_{ii} = t \). \(^{24}\) QED

---

\(^{24}\) It is readily shown that, if \( r_{jj} = t \), then the unique solution to this equation is \( r_{ii} = t \). Moreover, if platform \( i \) were to set \( r_{ii} \) sufficiently high that the two market areas did not touch, then its price would have to satisfy \( Z_{ii} - \frac{r_{ii}}{t} = 0 \), which has no solution with \( r_{ii} > t \).
Figure 1: A Continuum of Equilibria
The unit square represents the distribution of consumer locations, with the competing platform-application pairs located at (0,0) and (1,1).