PRODUCT DIFFERENTIATION THROUGH EXCLUSIVITY:
Is there a One-Market-Power-Rent Theorem?

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Abstract

In systems industries, combinations of components are consumed together to generate user benefits. Arrangements among component providers sometimes limit consumers’ ability to mix and match components, and such exclusive arrangements have been highly controversial. We examine the competitive and welfare effects of exclusive arrangements among system components in a model of relatively differentiated applications that run on relatively undifferentiated platforms. For a given set of components and prices, exclusive arrangements reduce consumer welfare by limiting consumer choice and raising equilibrium prices. In some cases, however, exclusivity raises consumer welfare by increasing the equilibrium number of platforms, which leads to lower prices relative to the monopoly outcome that would prevail absent exclusivity. We also show that there is no “One-Market-Power-Rent Theorem.” That is to say, exclusive deals with providers of differentiated applications can raise platforms’ margins without reducing application margins, so that overall industry profits rise.

Key Words

Exclusive contracts, systems competition, One-Monopoly-Rent Theorem.
I. INTRODUCTION

Many products are systems products: end users consume combinations of components in order to derive benefits. In the presence of component differentiation, end users can benefit from mix-and-match consumption. In some industries, end users can combine components as they wish. In others, technological differences among components or contractual arrangements among component providers limit the set of available combinations. Examples of industries in which combinations are limited by contractual arrangements include: video programming distribution, where some distributors have exclusive programming arrangements; video games, where game producers sometimes agree to make their games available exclusively for particular brands of console; and wireless communications, where networks often enter in exclusive dealing arrangements with handsets manufacturers. The use of exclusive arrangements has been highly controversial. For example, Apple’s agreement with AT&T, under which the iPhone is available in the U.S. exclusively for use on AT&T’s wireless network, has drawn the attention of Congress and the Federal Communications Commission.\(^1\) Similar arrangements in other nations have been challenged by competition authorities.

In this paper, we examine the competitive and welfare effects of exclusive arrangements between platforms (e.g., video distributors, game consoles, or wireless networks) and applications (e.g., video programming, video games, or handsets). We examine the case of relatively undifferentiated platforms and relatively differentiated applications, and we assume that the provision of platform services is characterized by significant fixed costs.

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We identify three mechanisms through which exclusive arrangements between platforms and applications can affect competition and welfare. First, and most obviously, exclusive arrangements limit the ability of consumers to mix and match components. For a given set of products and prices, this limitation reduces welfare. Second, we show that, when otherwise undifferentiated platforms can differentiate themselves through exclusive deals with differentiated applications, equilibrium prices under platform competition are higher than they otherwise would be. The softening of price competition leads to a third effect: the market equilibrium may support a larger number of platforms than when there are no exclusive arrangements. In the presence of fixed costs, platform providers must be able to charge prices greater than average variable cost in order to earn non-negative profits. When the same applications are available on multiple (undifferentiated) platforms, competition drives prices toward marginal cost. Consequently, the market equilibrium with non-exclusive applications may support only one provider. By softening price competition, exclusive arrangements can lead to (softened) platform competition instead of platform monopoly. Hence, when viewed in the context of the full process of entry and pricing, exclusive arrangements can lead to greater competition and lower prices. We show that the net effects of exclusive arrangements on consumer surplus and total surplus can be positive or negative, depending on the parameter values.

The point that product differentiation can allow the market equilibrium to support additional platforms has been made informally by Yoo (2005, § III.A.2.a) and Ford (2008, pages 5-6). However, these papers do not analyze the full set of tradeoffs involved, specifically the tradeoff faced by the differentiated component producers. By reaching an exclusive agreement with a platform, an application allows that platform to differentiate itself, but the application
gives up potential sales to users who patronize rival platforms. Moreover, there is a question of whether it can be profitable to “buy” differentiation from a party that already has it. Specifically, why couldn’t an application take advantage of its differentiation directly by charging higher prices, which would allow an application provider to enjoy the fruits of its market power without suffering from limited distribution? In other words, does a version of the One-Monopoly-Rent Theorem hold in the settings we consider?

The One-Monopoly-Rent Theorem concerns situations in which: (a) two goods are consumed in fixed proportions; (b) one good is supplied by a constant-returns, perfectly competitive industry; and (c) the other good is supplied by a monopolist. The theorem states that the monopolist has no incentive to engage in the tying of the two goods in order to “leverage” its monopoly from one market to the other. It is well known that the theorem does not extend to situations in which (a) or (b) are violated. When the two goods are consumed in variable proportions, bundling can be used as a vehicle for rent extraction by the monopolist (see, e.g., Burstein, 1960). When there is imperfect competition in the non-monopolized market, it can be profitable to engage in “strategic foreclosure” that weakens a duopoly rival and allows the supplier with a monopoly in one market to earn higher profits in the imperfectly competitive market (see Whinston, 1990).

The situations we examine satisfy assumptions (a) and (b). However, we relax assumption (c) to consider situations in which the second good is supplied by imperfectly competitive duopolists offering differentiated products rather than by a monopolist. We establish conditions under which there is not a One-Market-Power-Rent Theorem paralleling the One-Monopoly-Rent Theorem. In other words, the most profitable industry configuration can be one
in which the purchase of the good with market power (e.g., the differentiated applications) are tied with the competitively supplied good (e.g., the undifferentiated platforms).

Our paper is also related to the extensive literature exploring the competitive effects of compatibility in mix-and-match markets (e.g., Economides and Salop, 1992, and Matutes and Regibeau, 1988 and 1992). Economides and Salop examine equilibrium under a variety of market structures that differ in terms of the degree of integration, but they do not examine the effects of exclusive contracts between unintegrated suppliers. The Matutes and Regibeau articles focus on integrated suppliers of complementary products and examine their incentives to make their components compatible with those of rivals. In contrast, we consider settings in which platforms and applications are supplied by independent firms that choose whether to enter into exclusive agreements. Our setting raises different policy questions with respect to the institutions giving rise to exclusivity. Moreover, the effects of exclusivity are very different. Matutes and Regibeau (1988) find that incompatibility lowers equilibrium prices by blocking mix-and-match consumption: when products are incompatible an integrated supplier knows that its components will be used together and, thus, internalizes the complementary pricing effects across components. In contrast, we find that exclusive contracts, which block mix-and-match consumption, lead to higher prices than under mix and match because—in the presence of exclusive contracts—an undifferentiated platform can attract new customers only by inducing end users to switch between differentiated applications. Moreover, we explore platform entry issues that are not addressed by these papers.

Lastly, there is large literature that examines other roles of exclusive contracts, particularly as a means of supporting investment. For example, an exclusive relationship might be necessary to induce a platform to invest in promoting an application because the platform
would otherwise be concerned about free-riding by a rival platform.\textsuperscript{2} Exclusive arrangements can also be a means of increasing platform investment incentives by reducing the threat of hold up by an application provider.\textsuperscript{3} Finally, exclusive arrangements can be a means of overcoming certain types of contractual opportunism.\textsuperscript{4} None of these effects arise in our setting, and the papers in this literature do not examine the forces at work in our model.

The remainder of this paper is organized as follows. We introduce the model in the next section. In our baseline model, there are two undifferentiated platforms, two differentiated applications, and the platform owners set prices for bundles containing platform services and applications. Sections III through VI characterize the equilibria of this baseline model and establish the results described above. In Section VII, we extend the analysis to consider differentiated platforms. Perhaps the most important difference in the results is that, in contrast to the case of undifferentiated platforms, there are parameter values for which both platforms enter the industry and consumers have full mix-and-match options. That said, as long as the degree of application differentiation is greater than the degree of platform differentiation, there are also situations in which a public policy that compelled full mix and match would lead to a platform monopoly, while exclusive arrangements would lead to a platform duopoly.

We also extend the model to consider settings in which platform owners and application providers independently set the prices of their respective components. This case is of interest both because it better fits some industries (e.g., the sale of video games and the sale of mobile phones in many countries outside of the U.S.) and because it provides a setting more readily

\textsuperscript{2} Marvel (1982) provides an early analysis of the use of exclusive relationships to prevent free riding.

\textsuperscript{3} For a general analysis of the investment incentive effects of exclusive contracts, see Segal and Whinston (2000).
comparable to those typically considered in applications of the One-Monopoly-Rent Theorem. We show that there is no “One-Market-Power-Rent Theorem”: exclusive deals raise the platforms’ margins without reducing the application providers’ margins, so that overall industry profits rise. The paper closes with a brief conclusion. Proofs not given in the text may be found in the Appendix.

II. THE BASELINE MODEL

End users consume a pair of perfectly complementary components, $X$ and $Y$. To realize any benefit, a consumer must consume the inputs in fixed proportion, which we normalize to one-to-one. We also assume that an end user wants to consume at most one pair of components. For expositional convenience, we will refer to component $X$ as the platform (e.g., video programming distributor, video game console, or wireless network) and component $Y$ as the application (e.g., television program, video game, or mobile phone handset).

There are two potential platforms, 0 and 1, located at opposite ends of a Hotelling line of length one. Consumers are distributed on the line and have “transportation cost” (disutility) $s$ per unit distance. There are two potential applications, 0, and 1, located at opposite ends of their own Hotelling line of length one. Consumers have transportation costs of $t$ per unit distance along the application Hotelling line. A consumer located at $(x, y)$ who consumes a platform located at $i$ and an application located at $j$ derives utility $v - s|i - x| - t|j - y| + \theta$, where $\theta$ is the consumption of an outside, composite good with a normalized price of 1. Consumers are uniformly located on the unit square $[0, 1] \times [0, 1]$. In our baseline model, the platforms are undifferentiated: $s = 0$. In later sections, we examine extensions in which $s > 0$.

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See, for example, McAfee and Schwartz (1994) and (2004).
Platforms and applications have affine cost functions, with fixed costs of creating the product and constant marginal costs. Because the marginal costs don’t affect the analysis, we set them to zero for notational simplicity. Each platform incurs a fixed and sunk cost equal to $F$ if it enters into production. We assume throughout that the platform fixed costs are sufficiently small that a monopoly platform would earn revenues greater than $F$. Typically there are fixed costs associated with the creation of applications as well. Our maintained assumption is that these fixed costs are sufficiently small that the market can readily support two differentiated applications. There is, then, no further loss in generality in assuming that the application providers’ fixed costs are zero.

In our baseline model, a platform that has entered the market reaches agreements with one or more application providers that allow the platform provider to sell applications as part of the platform provider’s integrated offering to consumers. The structure of the game is as follows:

**Stage 0:** The public policy treatment of exclusive contracting is set. There are several possible public policy regimes, including: (a) no public policy constraints on private contracting; (b) each application provider is permitted to sign an exclusive contract with at most one platform owner; (c) exclusive contracts between applications and platforms are banned; and (d) platform and application providers are compelled to allow consumers to engage in full mixing and matching.

**Stage 1:** Investment decisions are made. Potential platform providers 0 and 1 each choose whether to enter (and incur sunk cost $F$) or stay out. Both application providers are assumed always to enter the market.

**Stage 2:** Contracting between application providers and platform owners takes place, subject to the public policy chosen in Stage 0. Any payments between platform and application providers are assumed to be lump-sum transfers. We make this assumption to avoid issues that can arise when firms use per-unit license fees as a means of supporting higher equilibrium retail prices.

**Stage 3:** Any platform provider that has entered the market set prices. Let $r_{ij}$ denote the price of a system that contains variant $i$ of component $X$ and variant $j$ of component $Y$. If there are two platforms, they set their system prices simultaneously.
Stage 4: Consumers choose which components to purchase subject to any exclusive contracts in place. A consumer at location \((x,y)\) chooses among the feasible combinations that maximize

\[ v - s|\bar{i} - x| - t|\bar{j} - y| - r_{ij}. \]  

(1)

If no combination yields positive surplus, the household makes no purchase.

The equilibrium concept is perfect equilibrium. Hence, as usual we solve the game backward.

III. PRICING

Given that atomistic end users behave non-strategically, we begin our analysis with the stage in which the platform providers simultaneously set prices for platform-application bundles. In a slight abuse of notation, we use \(Z_{ij}\) to denote both the product comprising platform \(i\) and application \(j\) and the units sales of that product. We have to examine several continuation games that could arise from the entry and contracting stages. As will become evident, given the consumption benefits of application variety \((i.e., t > 0)\) and the absence of fixed costs associated with introducing a second application, the only interesting histories are those in which each application has reached an agreement to be distributed by at least one platform:

- There is a platform offering services to consumers and that platform distributes both the applications. This case arises when only one platform has entered the market or (off the equilibrium path) if both platforms have entered but one of them has exclusive contracts with both application providers. Without loss of generality, we assume platform 0 is the monopolist.

- Both platforms distribute both applications. In this case, all four potential component combinations are available to consumers.
• Each platform reaches an exclusive distribution agreement with an application provider. We label the components such that, under this arrangement, only systems $Z_{00}$ and $Z_{11}$ are available.

• One application is available exclusively on one platform and the other application is available on both platforms. We label the components so that the sole exclusive contract is between platform 0 and application 0. In this case, only systems $Z_{00}$, $Z_{01}$, and $Z_{11}$ are available.

First, consider the case in which there is a monopoly platform, say platform 0. It is readily shown that the monopolist finds it optimal to charge identical prices for its two bundles (intuitively, if the firm charges different prices, more than half of its sales are at the lower price). The monopolist’s unit sales are equal to $\min\{\frac{2(\nu - p)}{t}, 1\}$ in our baseline model, with $s = 0$.

Straightforward calculations show that, when $\nu < t$, the monopolist sets $r_{0i} = \frac{1}{2} \nu$ and consumers located in a neighborhood of $y = \frac{1}{2}$ do not consume systems; the standard monopoly output restriction arises. When $\nu > t$, the monopolist sets its price so that all consumers purchase bundles but consumers located at $y = \frac{1}{2}$ are indifferent between purchasing bundles and not: $r_{0i} = \nu - \frac{1}{2} t$.

Next, consider the case in which both platforms have invested in providing service and both applications are available on both platforms. Given the assumption that $s = 0$, the bundles $Z_{0j}$ and $Z_{1j}$ are undifferentiated products. Given price competition, the usual logic of the Bertrand model applies and the unique equilibrium prices are $r_{ij} = 0$. All households purchase a bundle if $\nu \geq \frac{1}{2} t$. Otherwise, sales equal $2\nu / t$. 
Now suppose that platforms and applications form two pairs, each with an exclusive arrangement. Recall, in this case, we take the two systems to be $Z_{00}$ and $Z_{11}$. The platforms are differentiated by the applications that work with them. There are two possible equilibrium configurations. When $v < t$, the two platforms serve non-overlapping market areas, and the outcome is equivalent to the monopoly outcome above. For $v > t$, the two platforms compete and have overlapping market areas in equilibrium. In equilibrium, the border between the two areas is defined by the condition that any consumer located at $y = \frac{1}{2}$ be indifferent between the two bundles: $v - r_{00} - ty = v - r_{11} - t(1 - y)$. The resulting unit sales for the two bundles are $Z_{ii} = \frac{1}{2} + \frac{1}{2} (r_{ji} - r_{ii})$ for $|r_{ji} - r_{ii}| \leq t$. The pricing subgame is equivalent to a standard, one-dimensional Hotelling pricing game, and the unique symmetric equilibrium entails bundle prices $r_{ii} = \min\{t, v - \frac{1}{2}t\}$.

Lastly, suppose that one application is available exclusively on one platform and the other application is available on both platforms. Recall our convention that platform 0 and application 0 are the ones with the exclusive arrangement. Observe that, if $r_{01} = 0 = r_{11}$, then neither firm can increase its profits by unilaterally raising $r_{ii}$ because bundles $Z_{i1}$ and $Z_{j1}$ are perfect substitutes for one another. Clearly there are no other equilibrium prices because any such price would be undercut by the usual undifferentiated Bertrand logic. Given $r_{01} = 0 = r_{11}$, Platform 1 chooses $r_{00} = \arg\max_r r \times \min\{\frac{v - t}{t}, \frac{t - r}{2t}\}$. Straightforward calculations reveal that $r_{00}$ is equal to $\frac{1}{2}v$ if $v \leq \frac{3}{4}t$, $2v - t$ if $\frac{3}{4}t \leq v \leq \frac{3}{2}t$, and $\frac{1}{2}t$ if $\frac{3}{2}t \leq v$.

\[5\] It is readily shown that, if $t < v < 3t/2$, then a continuum of asymmetric equilibria also exist due to the kink in the residual demand curves faced by the two platforms.
Summarizing this discussion, we have shown:

**Proposition 1:** Suppose that platforms are undifferentiated (i.e., \( s=0 \)) and the platform owners set the prices for platform-application bundles. The unique symmetric equilibrium outcomes in the pricing continuation game are:

i. **Monopoly Platform:** A monopoly platform provider offers both applications and sets prices \( r_{ij} = \max\{\frac{1}{2}v, v - \frac{1}{2}t\} \). The monopolist serves all end users in equilibrium if and only if \( v \geq t \).

ii. **Mix and Match:** If both platforms enter the market and each one offers both applications, then the unique symmetric equilibrium prices are \( r_{ij} = 0 \). Every consumer purchases a bundle if \( v \geq \frac{1}{2}t \). Otherwise, sales equal \( 2v/t \).

iii. **Exclusive Duopoly:** Suppose each platform offers one application. If \( v < t \), then each platform has a local monopoly for its bundle and the unique equilibrium prices are \( r_{ii} = \frac{1}{2}v \). If \( v \geq t \), then there exists a unique symmetric equilibrium, under which \( r_{ii} = \min\{t, v - \frac{1}{2}t\} \) and every consumer purchases a bundle.

iv. **Asymmetric Duopoly:** If one application is available exclusively on platform 0 and the other application is available on both platforms, then there exists a pure-strategy equilibrium in which \( r_{01} = 0 = r_{11} \) and \( r_{00} \) is equal to \( \frac{1}{2}v \) if \( v \leq \frac{3}{7}t \), \( 2v - t \) if \( \frac{3}{7}t \leq v \leq \frac{1}{7}t \), and \( \frac{1}{4}t \) if \( \frac{3}{7}t \leq v \).

As expected, equilibrium prices are highest under monopoly, and lowest under mix-and-match competition, with exclusive duopoly falling in between. Exclusive duopoly leads to an outcome equivalent to platform monopoly when the value of a bundle, \( v \), is low relative to the
degree of application differentiation, $t$. When the consumption value of a bundle is large relative
to the degree of application differentiation, the platforms engage in differentiated-products
competition with one another under exclusive duopoly: In order for a platform to gain additional
sales, it must reduce its price by an amount sufficient to induce consumers to switch applications
(as well as platforms). Under full mix and match, however, consumers can switch platforms
without switching applications. Hence, the degree of application differentiation does not affect
the prices that platforms set.

Letting $\pi^r_i$ denote platform $i$'s profits gross of licensing fees and fixed costs in regime $r$,
straightforward calculations based on Proposition 1 demonstrate that

$$0 = \pi^a_0 + \pi^a_1 < \pi^e_0 + \pi^e_1 < \pi^e_0 + \pi^e_1 \leq \pi^m,$$

where $r = n, a, e,$ and $m$ correspond to the non-exclusive duopoly, asymmetric duopoly, exclusive
duopoly, and monopoly regimes, respectively. In addition, $\pi^a_1 < \pi^e_0 < \pi^e_1 = \pi^e_i$. Lastly, note that
the industry incurs platform fixed costs of $F$ under monopoly but $2F$ under all of the duopoly
configurations.

IV. APPLICATION CONTRACTING

In the stage prior to pricing, contracting between the platforms and applications takes
place subject to any public policy constraints. At present, there are no fully satisfactory non-
cooperative game-theoretic models of $2 \times 2$ bargaining. Instead, we consider a highly stylized
extensive-form bargaining game in which the platforms make simultaneous take-it-or-leave-it
offers to the two application providers. In order to avoid the non-existence of equilibria due
infinitesimal undercutting, we assume that bids must be made in increments of a small, positive
amount, $\lambda$. Each platform’s bid has four components. $L^i_{ij}$ denotes platform $i$’s bid for an
exclusive relationship with application $j$, while $L_{ij}^e$ denotes the platform’s bid for a non-exclusive relationship. An application chooses to accept those bids that maximize its profits given the actions of the other firms.

First consider the case of a monopoly platform. Because the platform can make take-it-or-leave it offers, it will offer $\lambda$ for any application that it wishes to distribute. Because there are of application variety due to $t > 0$, the monopolist will reach agreements to carry both applications when $\lambda$ is sufficiently small (Lemma A.1 in the Appendix).

Now, suppose that both platforms have invested in entering the industry. As discussed above, Proposition 1 implies that industry profits gross of licensing fees and entry costs are greatest when there is a monopoly platform and smallest when both applications are available on both platforms. It also implies that industry profits are greater when each platform offers an exclusive application than when one application is available exclusively on one platform and the other application available on both platforms. This pattern of profitability has strong implications for equilibrium contracting. Straightforward calculations establish the following result:

**Proposition 2:** Suppose the platforms are undifferentiated (i.e., $s=0$), set bundled prices, and both have entered the industry.

i. If there are no limits on the use of exclusive contracts, then one platform (say platform 0) will reach exclusive agreements with both applications. The equilibrium bids satisfy

$$\pi^m - \lambda \leq L_{00}^e + L_{01}^e \leq \pi^m \quad \text{and} \quad \min\{L_{00}^e, L_{01}^e\} \geq \pi_1^e.$$ 

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6 We assume that the applications (credibly) announce that they will reject bids of 0 except in cases in which public policy compels acceptance.
ii. If each platform is allowed to have at most one exclusive contract, then each platform will reach an exclusive contract with a single application and the resulting outcome will be exclusive duopoly. The equilibrium bids are $L_{iy}^e = \lambda$, $L_{ii}^e = 2\lambda$, and $L_{iy}^e = 0$, $i = 0, 1$, $i \neq j$.  

iii. If exclusive contracts are banned (but dealing is not compelled), then each platform will bid for a non-exclusive contract with a single application and the resulting outcome will be exclusive duopoly. The equilibrium bids are $L_{ii}^e = \lambda$ and $L_{iy}^e = 0$, $i = 0, 1$, $i \neq j$.

Case (iii) above demonstrates that, in our model, banning exclusive contracts is insufficient to prevent *de facto* exclusive contracting. It is in the interest of both platforms to avoid reaching deals with the same application. In the light of this result, policy makers might adopt a rule that compels full mix and match. Such a policy could take various forms, which would have different implications for the equilibrium values of licensing fees. One form would require each platform to make the same offers to both applications and compel an application to accept an offer from platform $i$ whenever the application accepts an offer from platform $j$ of equal or lesser value. If both platforms have entered the market, then in the licensing equilibrium, each platform will bid $\lambda$ for an application license and both application providers will accept the offer.

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7 $L_{iy}^e = 0$, $L_{ii}^e = \lambda$, and $L_{iy}^e = 0$ is not an equilibrium because a platform $i$ could offer $L_{iy}^e = 2\lambda$ and become a de facto monopolist.

8 As we will see, this policy is more natural in the case of component pricing, where it can be implemented by prohibiting an application provider from limiting the platforms with which it customers can use its application.
V. PLATFORM ENTRY

We next consider the two platforms’ entry decisions. Rather than work through the
details of the various stage-game equilibria, we offer several broad observations on the structure
of these equilibria. Let $\Pi_i^m$ denote the revenues that platform $i$ would earn as a monopolist given
the application licensing contracts that it holds. Similarly, let $\Pi_i^d$ denote the revenues that
platform $i$ would earn in equilibrium in competition with platform $j$ given the application
licensing contracts that each platform holds.

Observe that:

a. If $\Pi_i^d \leq \Pi_i^m < F + L_i$, then it is a dominant strategy for platform $i$ to stay out of
   the market.

b. If $\Pi_i^d < F + L_i < \Pi_i^m$, then platform $i$’s best response to platform $j$ action of
   entering the market or not is to do the opposite of what platform $j$ does.

c. If $F + L_i < \Pi_i^d \leq \Pi_i^m$, then it is a dominant strategy for platform $i$ to enter the
   market.

The entry-game equilibria that arise under the various possible combinations of parameter
values are largely self-evident. We note that when both platforms are in case (b), there are three
equilibria: one in which platform 0 enters and platform 1 stays out; one in which platform 1
enters and platform 0 stays out; and a mixed strategy equilibrium in which each platform enters
with a probability between 0 and 1. We focus on the pure-strategy equilibria.

VI. WELFARE ANALYSIS

We are now ready to examine the welfare of various public policies toward exclusive
contracts.
We begin with consumer surplus. Proposition 1 demonstrates that consumer surplus is strictly greater in the mix-and-match case than under any other industry configuration: consumers have the full ranges of platform-application pairs available and the equilibrium prices are the lowest of any market configuration. Proposition 1 also demonstrates that the equilibrium prices for platform-application bundles are weakly higher when there is a monopoly platform than when there are duopoly platforms with exclusive applications, and strictly higher when \( v > t \). When demand is sufficiently strong that duopoly platforms with exclusive applications would compete with one another, consumers benefit from platform competition in comparison with platform monopoly.

Summarizing,

**Proposition 3:** Under the bundled-pricing regime: (i) equilibrium consumer surplus is strictly greater under mix-and-match than any other industry configuration, and (ii) equilibrium consumer surplus is weakly greater with duopoly platforms offering exclusive applications than with a monopoly platform, and it is strictly greater for \( v > t \).

Next, consider total surplus. In the mix-and-match case, the equilibrium price of each bundle is equal to its marginal cost and users consume efficient amounts of the platform-application pairs. When \( v \geq t \) and there is a monopoly platform, all users consume bundles in equilibrium and the first-best outcome is attained. For \( v < t \), the monopoly platform sets prices that induce inefficiently little consumption and result in deadweight loss

\[
\int_{\min\left\{ \frac{v}{t}, \frac{t}{v} \right\}}^{\min\left\{ \frac{v}{t}, \frac{t}{v} \right\}} \{v - yt\}dy.
\]

Hence, total surplus may be higher under mix and match or monopoly, depending on the size of the deadweight loss (which depends on the values of \( v \) and \( t \)) and the fixed costs associated with a second platform \((F)\).
When \( s = 0 \), a monopoly platform always yields higher total surplus than do two platforms each having with exclusive applications. This is so because, when \( s = 0 \), the monopolist always sells systems to exactly the same set of end users as do the exclusive duopolists, albeit at different prices in some cases. Consequently, gross consumption benefits are the same under monopoly and duopoly, but there is the expenditure of an additional \( F \) in the latter.

We have shown:

**Proposition 4:** Suppose the platforms are undifferentiated and engage in bundled pricing. When \( v \geq t \), total surplus is maximized by having a monopoly platform. When \( v < t \), equilibrium total surplus may be higher or lower with mix-and-match duopoly than with a monopoly platform. Equilibrium total surplus under the exclusive-duopoly configuration: (i) is \( F \) lower than total surplus under the monopoly platform configuration, and (ii) is less than or equal to total surplus under the mix-and-match configuration, with equality if and only if \( v \geq t \).

In contrast to consumers, a total-surplus-maximizing policy maker would never prefer exclusive duopoly to monopoly.

We next consider three policy regimes: (a) mandatory mix and match; (b) a platform can engage in exclusive contracting with no more than one application; and (c) any degree of exclusive contracting is permitted.\(^9\)

Suppose public policy mandates mix and match. If platforms entered the market, each would earn 0 revenues and would suffer losses. Hence, there is a pair of pure-strategy equilibria,

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\(^9\) Recall from Proposition 2 that banning exclusive contracts gives rise to the same industry configuration (i.e., exclusive duopoly) as does allowing each platform to engage in exclusive contracting with at most one application.
each of which entails one of the platform’s entering the market and the other not. There are no pure-strategy equilibria in which both platforms enter the market. In other words, a public policy intended to promote competition by requiring full mix and match would induce the monopoly configuration.

Next, suppose that there are no limitations on exclusive contracts. Industry profits are greatest when there is a monopoly platform and, by Proposition 2(i), bargaining between platform owners and application providers leads to a situation in which one platform obtains the exclusive rights to both applications. Because platform owners will rationally anticipate this outcome when making their earlier entry decisions, only one platform will enter the market in the equilibrium of the full game. A public policy that allowed unlimited use of exclusive contracts would induce the monopoly configuration.

Lastly, consider a public policy that would allow a platform to reach an exclusive arrangement with one, but not both, of the applications. If both platforms enter the industry, then by Proposition 2(ii), each will reach a contractual agreement to have a single exclusive application. Together, Propositions 2(ii) and 1(iii) imply that: (a) if $v < t$, then each platform earns $\frac{v^2}{4t} - F - 2\lambda$, and (b) if $v \geq t$, then each platform earns $\frac{1}{2} \min \{t, v - \frac{1}{2} t\} - F - 2\lambda$. Hence, if either: (a) $v < t$ and $\frac{v^2}{4t} > F + 2\lambda$, or (b) $v \geq t$ and $\frac{1}{2} \min \{t, v - \frac{1}{2} t\} > F + 2\lambda$, then a policy that allows limited exclusive contracting will lead to the configuration in which both platforms enter the market and each has an exclusive application. If neither of these conditions is satisfied, then only one platform will enter the market in equilibrium.
**Proposition 5:** Suppose undifferentiated platforms engage in bundled pricing. A public policy regime that compels full mix and match or allows a platform to sign exclusive contracts with both application providers yields higher total surplus than does a regime that allows each platform to reach an exclusive arrangement with at most one application. If \( v > t \) and 
\[
\frac{1}{2} \min \{ t, v - \frac{1}{2} t \} > F + 2 \lambda,
\]
then a public policy regime that allows each platform to reach an exclusive arrangement with at most one application yields greater consumer surplus than does a regime that compels mix and match or allows a platform to sign exclusive contracts with both application providers.

In our baseline model, the configuration with duopoly platforms each having an exclusive applications always yields lower total surplus than does having a monopoly platform. The reason is that platforms in the two regimes always serve the same set of users. This fact is due to the specifics of the demand system used in our baseline model. To see this point, consider the following example, in which the structure of demand is similar to that of our baseline model except that consumers now differ in terms of their valuations of a bundle gross of transportation costs. Specifically, suppose there is a unit mass of consumers with values \( v_L \) and a unit mass with values \( v_H \), where \( \frac{3}{2} t < v_L < \frac{11}{5} v_H \). Given these parameter values, exclusive duopoly leads to full market coverage. The monopolist, however, will choose a price equal to \( v_H - \frac{1}{2} t > 2v_L \) and serve all type-H users but no type-L users. The deadweight loss of monopoly exceeds \( v_L - \frac{1}{2} t > 2t \). Hence, for \( F \in (0, 2t) \), allowing partial exclusive contracts will lead to greater total surplus.
VII. DIFFERENTIATED PLATFORMS

In our baseline model, the two platforms are undifferentiated (i.e., $s = 0$). The central finding of this model is that exclusive contracts can relax price competition. The next result demonstrates that this finding extends to settings with differentiated platforms when the value of a platform-application bundle is large relative to its costs, so that duopoly platforms have overlapping market areas and engage in price competition rather than acting as local monopolies:

**Proposition 6:** Suppose that $0 < s < t$, $v \geq \frac{1}{2} s + \frac{3}{2} t$, and platform owners set the prices for platform-application bundles. If both platforms enter the market, then the unique symmetric equilibrium outcomes in the pricing continuation game are:

i. **Mix and Match:** When each platform offers both applications, the equilibrium prices are $r_{ij} = s$ and every consumer purchases a bundle.

ii. **Exclusive Duopoly:** When each platform offers a (different) single application, the prices are $r_{ii} = t$ and every consumer purchases a bundle.

As in the case of undifferentiated platforms, exclusive applications allow competing platforms to inherit the differentiation of their applications. Under mix and match, a price cut can be used to induce end users to switch platforms (but not applications) at a rate of $\frac{dr}{dt}$. However, with exclusive applications and overlapping markets, a platform can attract new sales only by inducing end users to switch applications, which occurs at rate $\frac{dr}{2t}$.

---

10 It is not possible to derive an analytic solution for the pure-strategy equilibrium of the asymmetric duopoly game, in which one application is exclusive to one of the platforms while the other application is available on both platforms. We can, however, calculate the equilibrium prices numerically. In the numerous examples we have calculated, we find that each platform earns lower profit under asymmetric duopoly than it would under exclusive duopoly. The Mathematica program used to construct these equilibria is available from the authors upon request.
With undifferentiated platforms, the fact that exclusivity relaxes platform competition can increase the equilibrium number of platforms. This relationship also holds for differentiated platforms when \( v > \frac{1}{2} s + \frac{1}{2} t \): industry profits equal \( s - 2F \) when both applications are available on both platforms and \( t - 2F \) when each platform offers an exclusive application. Hence, when \( s < 2F < t \), exclusive arrangements can lead to duopoly platforms, while mandatory mix and match would lead to a monopoly platform in the full game.

It is important to recognize that there are also significant differences between the cases of differentiated and undifferentiated platforms. Most notably, in the absence of platform differentiation there is no direct consumption value of platform variety and the mix-and-match regime cannot support two platforms for any positive fixed cost. Neither conclusion holds for differentiated platforms. When \( s > 0 \), there are parameter values for which mandatory mix and match is compatible with the existence of a duopoly in equilibrium (i.e., \( 0 < 2F < s < t \)). Moreover, there are parameter values for which industry profits are greater under mix and match than exclusive duopoly. For example, when \( s \) and \( t \) are sufficiently close to \( v \), the platforms will find it optimal to set system prices that give rise to non-overlapping market areas. Industry profits will be greater when each platform has a greater variety of applications to offer users in its local monopoly areas.

For those cases in which exclusivity facilitates platform entry, it is useful to ask whether consumers benefit from the additional competition. In any equilibrium with a monopoly platform, the platform always sets its price sufficiently high that consumers at least one location are indifferent between purchasing a platform-application pair and not. Hence, the level of aggregate consumer surplus is always less than the level that would obtain if prices were set to yield a consumer located at \((1,\frac{1}{2})\) surplus of 0, or \( \frac{1}{2} s + \frac{1}{4} t \). In contrast, the equilibrium prices for
platform-application pairs when there are duopoly platforms with exclusive applications are bounded from above by $t$. Hence, equilibrium aggregate consumer surplus increases without bound as $v$ gets large. In other words, consumer surplus is weakly greater under the equilibrium with duopoly platforms offering exclusive applications than under the equilibrium with a monopoly platform, and strictly greater for sufficiently large values of $v$.

Turning to total surplus, a question of particular interest is whether there are cases in which, in comparison with a monopoly platform, the benefits of competition under exclusive duopoly outweigh the additional fixed cost associated with entry of the second platform. Tedious calculations (available from the authors on request) demonstrate that there is no case in which both: (a) exclusive contracts are necessary to support entry of the second platform, and (b) the benefits of the second platform are greater than the fixed costs. Hence, at least within the confines of a Hotelling model in which consumers differ only in terms of their location, the policy rationale for allowing exclusive contracts arises under a consumer-surplus standard but not under a total-surplus standard. Of course, as we showed in Section VI above, exclusivity can raise total surplus when consumers also differ in terms of their overall willingness to pay for bundles.

VIII. COMPONENT PRICING AND ONE MARKET-POWER RENT?

In this section, we analyze settings in which the prices of the different components of a system are independently set by unintegrated component suppliers. The structure of the component-pricing game is identical to that of the bundle-pricing game described in Section II above except for the pricing stage, which is now:

**Stage 3’**: The prices of all components are simultaneously chosen. Each supplier chooses the price of its component. Let $p_i$ denote the price of component $X$ produced by
firm $i$, and let $q_j$ denote the price of component $Y$ produced by firm $j$. The system price is denoted $r_{ij} = p_i + q_j$.

The case of component pricing case is of interest because it better fits some industries (e.g., sale of mobile phones in many countries outside of the U.S.) and because it provides a setting more readily comparable to those typically considered in applications of the One-Monopoly-Rent Theorem. In particular, this structure raises the important question of whether the application providers can exercise their market power fully without entering into exclusive agreements with the platforms.

As in the bundled pricing game, there are several different market structures to consider. To simplify the analysis, we return to the assumption that platforms are undifferentiated.

**Proposition 7:** Suppose the platforms are undifferentiated (i.e., $s = 0$) and the platform and application owners independently set the prices for their respective components.$^{11}$

i. **Monopoly Platform:** Suppose there is a monopoly platform provider, $0$. If $v < \frac{1}{2} \ell$, then $p_0 = \frac{1}{3} v = q_j$ and some consumers do not purchase platform-application pairs. If $v \geq \frac{1}{3} \ell$, then $p_0 + q_j = v - \frac{1}{2} \ell$, and all consumers purchase platform-application pairs.

ii. **Mix and Match:** If both platforms enter the market and each one offers both applications, then the equilibrium platform prices are $p_i = 0$. If $v < \ell$, then each application has a local monopoly and the unique equilibrium prices are $q_j = \frac{1}{2} v$. If $v \geq \frac{1}{3} \ell$,

$^{11}$ Because of potential coordination problems among complementsors, there always exists a trivial equilibrium in which all component prices are set greater than or equal to $v$ and no supplier makes any sales. We ignore equilibria of this sort.
\( v \geq t \), then there exists a unique symmetric equilibrium, under which \( q_j = \min \{ t, v - \frac{1}{2} t \} \) and every consumer purchases a bundle.

iii. **Exclusive Duopoly:** Suppose each platform offers one application. If \( v < \frac{3}{2} t \), then there exists an equilibrium outcome in which \( p_k = \frac{1}{3} v = q_k \) for all \( k = 0,1 \) and the platform-application pairs have non-overlapping market areas. If \( v \geq \frac{3}{2} t \), then \( p_k = \min \{ t, \frac{1}{2} v - \frac{1}{4} t \} = q_k \) and all households purchase a bundle.

iv. **Asymmetric Duopoly:** When one application is available exclusively on one platform and the other application is available on both platforms, no pure-strategy equilibrium exists.

A significant difference from the bundled-pricing case is that there can be double marginalization: each component supplier ignores the effects of its pricing on the sales of the complementary component supplier. Consequently, equilibrium prices may be greater than the profit-maximizing prices. Consider \( v < \frac{3}{2} t \), for example. Under component pricing, the equilibrium component prices for a bundle sum to \( \frac{2}{3} v \). However, if platform 0 and application 0 were integrated and had a local monopoly, they would set component prices that summed to \( \frac{1}{2} v \). In other cases, this double marginalization can result in industry profits that are higher with competition than with exclusivity. When \( v < t \), for example, full mix and match results in equilibrium prices such that \( p_k + q_k = \frac{1}{2} v \), which is the bundle price that maximizes industry profits, while exclusive duopoly results in \( p_k + q_k = \frac{1}{3} v \). Note that, although mix and match maximizes industry profits, it minimizes platform providers’ profits.

Turning to the effects of exclusivity on price competition, we see that the component-pricing equilibria are similar to the bundled-pricing equilibria. Specifically, parts (ii) and (iii) of
Proposition 7 parallel the corresponding parts of Proposition 1: under exclusivity, platforms essentially inherit the differentiation of their applications. When \( v > \frac{s}{2t} \), for example, the equilibrium platform prices are 0 (the value of the platform differentiation parameter) under mix and match and \( t \) (the value of the application differentiation parameter) under exclusive duopoly. The equilibrium prices for a platform-application pair under component pricing are thus \( t \) with mix and match and \( 2t \) with exclusive duopoly. In other words, the One-Monopoly-Rent Theorem does not extend to this case. By entering into exclusive deals, providers of differentiated applications can increase the margins earned by platforms without reducing the applications’ margins.

Proposition 7 demonstrates that consumer surplus is strictly greater in the mix-and-match case than under either monopoly or exclusive duopoly: consumers have the full range of platform-application pairs available and the lowest equilibrium prices are realized in this market configuration. Proposition 7 also demonstrates that the equilibrium prices for platform-application pairs are weakly higher when there is a monopoly platform than when there are duopoly platforms with exclusive applications, with strictly inequality when \( v > \frac{s}{2t} \). Hence, consumers prefer mix and match to exclusive duopoly, and they prefer both to monopoly. Observe, however, that exclusive duopoly always leads to lower total surplus than do either mix and match or a monopoly platform: mix and match leads to greater variety and more efficient consumption levels at the same cost, while monopoly gives rise to equivalent consumption patterns to exclusive duopoly but at a cost that is lower by \( F \).

Summarizing,

**Proposition 8:** *Suppose \( s = 0 \) and there is component pricing. Then:*
i. equilibrium consumer surplus under the exclusive-duopoly configuration is weakly less than than under the mix-and-match configuration and weakly greater than under the monopoly-platform configuration (with strict inequality for \( v > \frac{s}{2}t \)); and

ii. equilibrium total surplus under the exclusive-duopoly configuration is \( F \) lower than total surplus under the monopoly platform configuration and is weakly lower than under the mix-and-match configuration, with strict inequality for all \( v \neq \frac{1}{2}t \).

Our final proposition establishes that our central result regarding the lack of a One-Market-Power-Rent Theorem extends to the case of component pricing with differentiated platforms when end users place a sufficiently high value on the consumption of platforms and applications that there is full market coverage in equilibrium:

**Proposition 9:** Suppose there is component pricing, \( 0 \leq s < t \) and \( v > \frac{1}{2}s + \frac{s}{2}t \). Then:

i. **Mix and Match:** If both platforms enter the market and each one offers both applications, then there exists a unique symmetric equilibrium, under which the platform prices are \( p_i = s \), application prices are \( q_j = t \), and every consumer purchases a bundle.

ii. **Exclusive Duopoly:** Suppose each platform offers one application. There exists a unique symmetric equilibrium, under which \( p_k = t = q_k \) and all households purchase a bundle.

Once again, the platforms inherit the application differentiation under exclusive contracting. Industry profits are \( s + t \) under the mix-and-match regime and \( 2t \) under the exclusive duopoly regime.
IX. CONCLUSION

This paper has examined a straightforward model of the use of exclusive contracts to create product differentiation. A central finding is that exclusive contracts raise equilibrium prices for a fixed number of platforms. In this way, exclusive arrangements can support investment even when that investment is neither relationship specific nor subject to free riding by other parties. Because exclusive contracts between platforms and applications raise prices for a fixed number of platforms but can, in some circumstances, increase the equilibrium number of platforms, exclusive contracts can raise or lower equilibrium consumer surplus through their effects on the degree of platform competition.

The analysis (particularly that of the component pricing model) also demonstrates that there is no One-Market-Power-Rent Theorem analogous to the One-Monopoly-Rent Theorem. That is, an imperfectly competitive supplier (applications in our model) can increase its profits by tying its product to a good that would otherwise be competitively supplied. This is an important limitation of the One-Monopoly-Rent Theorem because most firms are not literal monopolies.

This analysis clearly is at too early a stage to serve as the basis for recommending for specific legal rules for the treatment of exclusive deals. The analysis does, however, have clear implications for public-policy formulation. Specifically, the model demonstrates that: (a) policy analyses should consider the full industry equilibrium, rather than assuming that critical elements of industry structure are fixed; and (b) arguments based on the One-Monopoly-Rent Theorem may not be valid when applied to imperfect competitors.
REFERENCES


APPENDIX

Lemma A.1: Suppose only one platform has entered the market.

i. If $v \leq t$, then if and only if $\frac{v^2}{4t} > \lambda$, the monopoly platform will offer $\lambda$ to each application and its offers will be accepted.\(^{12}\)

ii. If $t < v < 2t$, the monopolist will offer $\lambda$ to at least one application if and only if $v^2 \geq 4t\lambda$ and will offer $\lambda$ to both platforms if and only if $v - \frac{t}{2} - \frac{v^2}{4t} \geq \lambda$. The offers will be accepted.

If $v > 2t$, then the monopolist will offer $\lambda$ to at least one application if and only if $v - t \geq \lambda$, and will offer $\lambda$ to both platforms if and only if $\frac{t}{2} \geq \lambda$. The offers will be accepted.

Proof of Lemma A.1: The result follows from direct calculation. QED

Proof of Proposition 6:

(i) Suppose one platform has set its prices to both be $s$. Without loss of generality, assume it is platform 0 that has done this. Consider the following pseudo-problem for platform 1: For each $y$ imagine platform 1 can set a price for its system, $r(y)$. Assume each such system has the same application as the consumer would obtain were he to buy from platform 0. In other words, the pseudo-problem for platform 1 is to devise a best response to its rival’s price of $s$ in a conventional Hotelling model in which the intrinsic value for the good is

\[ \bar{v}(y) \equiv v - t \min \{y, 1-y\} \].

Because

\[ \bar{v}(y) \equiv v - t \min \{y, 1-y\} \].

\[^{12}\] We assume that the applications (credibly) announce that they will reject bids of 0 except in cases in which public policy compels acceptance.
\[ v \geq \frac{3}{2} t + \frac{1}{2} s > \frac{1}{2} t + \frac{1}{2} s \geq t \min\{y, 1-y\} + \frac{3}{2} s, \]

\[ \frac{3}{2} \tilde{v}(y) > s \text{ for all } y. \] Per the usual analysis of such Hotelling models, it follows that \( r(y) = s \) for all \( y \). Given that platform 1 can gain no advantage trying to sell application 0 to \( y > \frac{1}{2} \) or application 1 to \( y < \frac{1}{2} \), it follows that platform 1’s best response to its actual problem is to set \( r_{10} = r_{11} = s \).

To see that no other symmetric equilibrium exists, suppose that one did in which \( r_{00} = r_{01} \neq s \). An argument similar to that just given reveals that platform 1’s best response is \( r_{10} = r_{11} \neq r_{00} = r_{01} \), a contradiction.

**(ii)** Without loss of generality, assume that the two systems are \( Z_{00} \) and \( Z_{11} \). In this case, platforms are differentiated by the applications that work with them. There are two possible equilibrium configurations. When \( s \) and \( t \) are sufficiently low relative to \( v \), the two systems compete and have overlapping market areas in equilibrium. The border between the two customer sets its defined the condition that a consumer located at \((x, y)\) be indifferent between the two bundles:

\[ v - r_{00} - sx - ty = v - r_{11} - s(1-x) - t(1-y) \]

or

\[ y = \frac{1}{2} + \frac{1}{2t} \{r_{11} - r_{00} + s(1-2x)\}. \]

Given our assumption that \( s < t \), the resulting sales are

\[ Z_i = \frac{1}{2} + \frac{1}{2t} \{r_{ji} - r_{ii} + s\} - \frac{1}{t} \int_0^1 xd\alpha = \frac{1}{2} + \frac{1}{2t} \{r_{ji} - r_{ii}\} \]
for values of $r_i$ corresponding to overlapping market areas. In this case, the equilibrium prices must satisfy the first-order condition $Z_{ii} - \frac{r_{ii}}{2t} = 0$, which implies that the unique equilibrium system prices are $r_i = t$.\footnote{It is readily shown that, if $r_{ii} = t$, then the unique solution to this equation is $r_i = t$. Moreover, if platform $i$ were to set $r_{ii}$ sufficiently high that the two market areas did not touch, then its price would have to satisfy $Z_{ii} - \frac{r_{ii}}{t} = 0$, which has no solution with $r_{ii} > t$.} \textbf{QED}

\textbf{Proof of Proposition 7:}

(i) Suppose that only platform 0 has entered the market or else has exclusive contracts with both application providers. If the market areas of different systems do not overlap, then $Z_{0j} = \frac{v - p_0 - q_j}{t}$. Taking the platform price as given, application provider $j$ chooses its price to maximize $q_j Z_{0j}$, which yields $q_j = \frac{1}{2} (v - p_0)$. Taking the application prices as given, the monopoly platform provider maximizes $p_0 (Z_{00} + Z_{01})$, which has the solution $p_0 = \frac{1}{2} (v - \bar{q})$, where $\bar{q}$ equals the mean of the application prices. The unique equilibrium prices are $p_0 = \frac{1}{3} v = q_j$. The two systems thus will have non-overlapping market areas if and only if $v - p_0 - q_j - \frac{1}{2} t < 0$, or $v < \frac{3}{2} t$.

Next, suppose that $v > \frac{1}{2} t$, so that the market areas touch one another. Given that there is a monopoly platform, we know that consumers at the boundary between the two market areas must earn zero surplus: $p_0 + q_j = v - ty$. This means we have to be at the knife edge. Lower $q$ leads to competition, higher $q$ leads to local monopoly. Hence, a range of prices support this...
outcome. Focusing on symmetric equilibria, we will take \( y^* = \frac{1}{2} \), which determines the system prices, if not the individual component prices.

(ii) Because consumers can mix and match, the two platforms are differentiated solely by their inherent characteristics, not by the applications bundled with them. The unique equilibrium platform prices are \( p_i = 0 \). Given these prices and the fact that \( s = 0 \), the pricing problem faced by application providers is identical to that of the two platforms in our earlier analysis of bundled pricing when platform has an exclusive agreement with one of the applications.

(iii) A consumer purchases platform \( i \) if and only if he or she purchases application \( i \). Hence, sales of both components are equal to \( Z_{ii} \). As above, there are two configurations to consider: either the market areas touch or they do not. When the market areas do not touch,

\[
Z_{ii} = \frac{v - p_i - q_i}{t}
\]

and the component producers have reaction functions \( p_i = \frac{1}{2}(v - q_i) \) and \( q_i = \frac{1}{2}(v - p_i) \). As above, the unique equilibrium prices are \( p_i = \frac{1}{3}v = q_i \) and the two systems will have non-overlapping market areas if and only if \( v < \frac{1}{3}t \).

Next suppose that the two market areas overlap and the consumers at the boundary earn strictly positive surplus. Then \( Z_{ii} = \frac{t - r_{ii} + r_{ij}}{2t} \) and \( \frac{\partial Z_{ii}}{\partial p_i} = \frac{-1}{2t} = \frac{\partial Z_{ii}}{\partial q_i} \). Platform \( i \) chooses its price to maximize \( p_i Z_{ii} \), which has the solution \( p_i = \frac{1}{2}(t + r_{ij} - q_i) \). Similarly,

\[
q_i = \frac{1}{2}(t + r_{ij} - p_i)
\]

Conditional on the price of the competing system, the unique equilibrium prices are \( p_i = \frac{1}{3}(t + r_{ij}) = q_i \). We can use this fact to express the price of one system as a function of the price of the other: \( r_{ii} = \frac{2}{3}(t + r_{ij}) \), which yields equilibrium prices \( r_{kk} = 2t \), or
\( p_k = t = q_k \). Note that consumers at \( y = \frac{1}{2} \) earn strictly positive surplus if and only if

\[ v - r_{kk} - \frac{1}{2} t > 0 \quad \text{or} \quad v > \frac{5}{2} t. \]

The remaining case to consider is \( \frac{1}{2} t < v < \frac{5}{2} t \). In this case, there exists a symmetric equilibrium in which the two market areas just touch and consumers at the boundary earn zero surplus:

\[ p_k = \frac{1}{4} v - \frac{1}{4} t = q_k. \]  

**(iv)** We assume that platform 0 cannot engage in price discrimination based on whether the consumer is using application 0 or application 1 in conjunction with platform 0.  There is no pure-strategy equilibrium. To see why, suppose to the contrary that a pure-strategy equilibrium exists. Suppose \( 0 < p_0 < p_1 \). Then platform 1 would make no sales and would have incentives to undercut the price of platform 0. Next suppose that \( 0 < p_1 < p_0 \). Then platform 0 would make sales only in conjunction with application 0 and platform 1 would make sales only in conjunction with application 1. It can be shown that at least one of the platforms would not be setting an optimal price. Lastly, suppose \( p_0 = p_1 \). By the usual homogeneous good, Bertrand undercutting argument, the only candidate for the common price would be 0. But, because \( t > 0 \) and platform 0 is the exclusive complement for application 0, platform 0 can always guarantee itself positive profits as long as \( q_0 < v \). Hence, it must be the case that \( p_0 \) is strictly positive.

**QED**

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14 There also exists a continuum of asymmetric equilibria, all of which entail full market coverage.

15 When discrimination is feasible, a pure-strategy equilibrium exists in which: (a) both platforms charge 0 for their use with application 1, and (b) platform 0 charges a positive price for its use with application 0, which is equal to the price charged by application 0. Platform 0 and application 0 engage in double marginalization, while application 1 benefits from platform competition.

16 Recall that we ignore the degenerate equilibrium in which \( p_0 = v = q_0 \). This would not survive a trembling hand or weak-dominance refinement.
Proof of Proposition 9: Because \( v > \frac{1}{2}s + \frac{1}{2}t \), we have \( v > \frac{3}{2}(s + t) \) given \( t > s \).

As will be shown, given these assumptions, there is an equilibrium under mix and match in which the market is covered; that is, there is no consumer who, in equilibrium, couldn’t enjoy non-negative surplus from at least one system. In a covered market, it is readily shown that the demands of platforms and applications are, respectively,

\[
X(p_i, p_j) = \frac{s + p_j - p_i}{2s} \quad \text{and} \quad Y(q_k, q_l) = \frac{t + q_l - q_k}{2t}
\]

in the neighborhood of the equilibrium prices. Given these demands, it is readily seen that mutual best responses are: \( p_0 = p_1 = s \) and \( q_0 = q_1 = t \). A necessary condition for this to be an equilibrium is that all consumers earn non-zero surplus; that is,

\[
v - \frac{1}{2}s - \frac{1}{2}t - s - t \geq 0;
\]

or, equivalently, that \( v \geq \frac{3}{2}(s + t) \). As noted, this condition holds by assumption. Profits are, correspondingly, \( s/2 \) each for platforms and \( t/2 \) each for applications.

The demands used above are valid only if firms are not pricing too high. Hence, we need verify that no firm wishes to deviate to a particularly high price. Consider a platform (the analysis for an application is similar and, thus, omitted). Consider \( p > s \). There are two cases depending on whether

\[
v - t/2 - s|x - i| - t - p \geq 0 \quad \forall x \text{ such that } 0 \leq v - t/2 - s(1 - |x - i|) - t - s.
\]

If that condition holds, then

\[
\frac{d}{dp} pX(p,s,t,t) = \frac{s - p}{s} < 0
\]

if \( p > s \); hence, the platform will not wish to deviate to a higher price in this circumstance. If that condition does not hold, then
\[ X_i(p,s,t,t) \leq \tilde{X}(p,s,t,t) = \frac{2v - 3t - 2p}{2s} \]

and \( p \geq 2v - 3t - 2s = \tilde{p} \). It is readily shown that

\[ \tilde{X}(p,s,t,t) = \frac{2s - \tilde{p}}{2s}. \]

Observe

\[ \frac{d}{dp} p\tilde{X}(p,s,t,t) = \frac{2v - 3t - 4p}{2s} \leq \frac{8s + 9t - 6v}{2s} < 0, \]

where the inequality follows given the maintained assumptions. We therefore have the series of inequalities:

\[ \frac{s}{2} \geq \tilde{p}\tilde{X}(\tilde{p},s,t,t) \geq p\tilde{X}(p,s,t,t) \geq pX_i(p,s,t,t) \]

for \( p \geq \tilde{p} \). Having exhausted all possibilities, we can conclude \( p_i = s \) is indeed a best response to \( p_j = s \) and \( q = t \). A similar analysis can be conducted for the application providers.

Now consider exclusives. Without loss of generality, assume application \( i \) is exclusive to platform \( i \). Again, we focus on overlap (a covered market). Hence, the consumers indifferent between the two systems lie on the line defined by

\[ v - sx - ty - p_0 - q_0 = v - s(1-x) - t(1-y) - p_1 - q_1; \]

or, equivalently,

\[ y = \frac{s + t + p_1 + q_1 - p_0 - q_0}{2t} \cdot \frac{s}{t} x. \]

Because \( s < t \),

\[ Z_{00} = \int_0^t \left( \frac{s + t + p_1 + q_1 - p_0 - q_0}{2t} - \frac{s}{t} x \right) dx = \frac{t + p_1 + q_1 - p_0 - q_0}{2t}. \]

It follows that
\[ Z_{11} = 1 - Z_{00} = \frac{t + p_0 + q_0 - p_1 - q_1}{2t}. \]

The first-order condition for maximizing \( r_i Z_{zi} \) (\( r = p \) or \( r = \bar{p} \)) is equivalent to

\[ t + p_j + q_j - \bar{r}_i - 2r_i = 0, \]

where \( \bar{r}_i \) is the price of the other component. Solving, in a symmetric equilibrium it must be that \( p_0 = p_1 = q_0 = q_1 = t \). Given \( \nu > \frac{1}{2}s + \frac{5}{2}t \), it is readily verified that these are, indeed, equilibrium prices and the market is covered. QED