On Accounting-Based Valuation Formulae*

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Abstract. This paper considers accounting-based valuation formulae. Its initial focus is on two problems related to residual income valuation (RIV). First, insofar valuation depends on the present value of expected dividends per share, applying RIV requires clean surplus accounting on a per share basis. Awkwardly, equity transactions that change the number of shares outstanding generally imply $\text{eps} \neq \frac{\text{bvps}}{\text{dps}}$. A clean surplus equality holds only if one “re-conceptualizes” either end-of-period bvps or eps as a forced “plug”. Second, one cannot circumvent the per share issue by evaluating RIV on a total dollar value basis unless one introduces relatively subtle MM-type restrictions. In light of RIV’s unsatisfactory aspects, the paper proposes an alternative to RIV. This new approach maintains a strict eps-focus. It derives by replacing $\text{bvps}_t$ in RIV with $\text{eps}_{t+1}$ capitalized (i.e. divided by $r$). One obtains a formula such that the current market price equals next-period expected earnings capitalized plus the present value of expected abnormal earnings growth, referred to as AEG. A number of propositions then demonstrate the advantages of the AEG approach as compared to RIV. These results follow because $\text{eps}_{t+1}$ capitalized generally approximates market price better than $\text{bvps}_t$.

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Introduction and Summary

During the past decade, Residual Income Valuation (RIV) has become prominent in the accounting literature.\(^1, 2\) The main reason for RIV’s widespread acceptance rests on its apparent ability to procure a constructive role for accounting data in equity valuation. Traditional equity valuation, with its emphasis on future cash flows, in contrast suggests a general irrelevance of future earnings and other accounting data. Accounting-oriented textbooks like to counter this impression, of course. They demonstrate how RIV derives from PVED combined with clean surplus accounting: Current book value acts as a “valuation-anchor”, and the PV of expected future residual earnings reconciles the difference between intrinsic and book values. While this accounting-based valuation formula seems simple enough, the literature also recognizes that GAAP’s earnings construct violates clean surplus accounting. It is then generally argued that the dirty surplus items have approximately zero expected values, which essentially eliminates the problem. Overall, the literature does not view such

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practical issues as detrimental to RIV, and it positions RIV as a robust and sensible way of placing accounting data at the center of equity valuation. A commentary by Bernard (1995) appears to have been especially influential in popularizing RIV as the approach to equity valuation.

This paper takes a critical look at RIV and its conceptual underpinnings. Most of the analysis deals with problems associated with capital transactions which change the number of shares outstanding. Such transactions undermine RIV’s standard foundation by opening up issues related to the relevance and validity of clean surplus on a per share basis. Moreover, expected changes in shares outstanding touches the PVED concept itself because it can be applied on either a per share basis, or on a total (dollar) equity basis. Capital transactions thus introduce intertwined accounting and finance theory issues. But this analysis of capital transactions and RIV brings out a broader question: To what extent do theoretical constructs bear on the appeal of RIV as compared to alternative, accounting-based, valuation formulae? The paper addresses the question by introducing a framework that allows for a comparison of alternative (but PVED-equivalent) valuation formulae. One of these alternatives usefully captures the spirit of practical equity valuation. That is, the formula works such that the growth in eps explains the price to forward-earnings ratio. And one cannot reasonably claim that the underpinning concepts of RIV connect to the earnings growth principle observed in practice. Perhaps more than anything else, this aspect of RIV “not conforming to observed practice” leaves a question mark as to what ought to be the formula’s status for purposes of equity valuation.

Major points of the paper are:

- The concept that value equals the present value of expected dividends derives from a per share perspective, and where dividends are regular cash dividends. RIV and PVED equivalence therefore directs attention to clean surplus on a per share basis. Total dollar value clean surplus does not guarantee the same on a per share basis because potential capital transactions change the number of shares outstanding. “Regular”, GAAP-type, accounting with respect to eps and bvps thus leads to the possibility Δ bvps ≠ eps – dps. Only by introducing unorthodox, contrived, accounting can one achieve clean surplus accounting on a per share basis. Such accounting forces either eps or end-of-period bvps to act as a pure “plug”.

- A potential way of finessing the per share problem starts from a D in PVED that represents the total dollar amount of dividends net of all capital contributions. This comprehensive PVED approach, however, yields the correct equity value for the current shareholders only if the (expected) equity transactions are neutral/irrelevant to prospective “future new” shareholders.

- A general analytical machinery demonstrates that RIV is but one specific accounting-based valuation formula within a broad class. Using this machinery, the paper develops a competing valuation formula with the same core mathe-
matical structure as RIV. In contrast to RIV, the alternative formula makes no reference to book values; it focuses solely on future (expected) eps. Next-period expected eps capitalized—$\frac{\text{eps}_{t+1}}{r}$—replaces current book value as the starting point. And $\frac{\text{eps}_{t+1}}{r}$ replaces $\frac{\text{bvps}_t}{1+r}$ as it appears in the book value representation of residual earnings, $\frac{\text{bvps}_t + \text{dps}_t - (1 + r) \cdot \text{bvps}_{t-1}}{1+r}$.

The resulting formula can be interpreted as one of abnormal earnings growth (AEG) valuation.

• Though AEG and RIV both equal PVED, the 2 formulae can be compared in terms of their truncation errors. It is shown that, under mild conditions, the AEG approach builds in smaller truncation—errors than the RIV approach. In other words, if one estimates value by assessing the “near future” only, then a finite-terms AEG works better than a finite-term RIV. This dominance result captures the idea that the expected earnings capitalized generally approximates the expected market value closer than the expected book value.

• A final result pertains to the text-book model which relates the price-to-book ratio to the next period’s residual earnings (Penman (2004) chapter 14, for example). It is shown that this model is a special case of the Ohlson and Jeuttner (2004) model which relates the price-to-forward earnings to the change in residual earnings. As a consequence, one can again claim that an earnings and growth in earnings perspective is more general than a book value and growth in book value perspective.

1. Basics The PVED Formula

Researchers and practitioners interest in RIV depends on its equivalence to PVED. The latter formula is virtually always taken as fundamental in valuation theory. Nevertheless, the ‘D’ in PVED can refer to two distinct definitions. One possibility defines ‘D’ in terms of dividends per share (dps), in which case the left hand side of the PVED formula targets price per share. A second possibility defines ‘D’ as total dividends net of capital contributions, in which case PVED targets the total (dollar) value of the equity. These two approaches differ potentially; the matter does not reduce to scaling for the number of shares outstanding at the date of evaluation. The acronym PVED therefore raises the following question: Should it be expressed on a per share basis, a total-equity dollar basis, or do both approaches generally work?

The details of any answer depends on the assumptions, of course. Under weak assumptions consistent with the so-called “random walk hypotheses for stock prices”, it turns out that the per share approach provides the starting point.

To appreciate why the ‘D’ in PVED should equal dps, consider an investor who buys a share (ex dividend) at date $t$, paying $P_t$. At the subsequent date the total payoff equals $P_{t+1} + \text{dps}_{t+1}$. Capital contributions do not influence the payoff because the investor can refuse any such transactions. Nor can pre-existing owners
always partake in an offering (retirement) of shares. Owning a share therefore guarantees only the “regular” cash dividend.

Suppose further that the investment is a fair game. This equilibrium requirement means that an investor cannot improve on his/her forecast of returns by expanding the information set. Specifically, an investor who buys one share of a stock at date $t$ can expect a return of $E_t[(P_{t+1} + dPS_{t+1})/P_t] = R$, regardless of the date $t$ information. It is implicitly assumed that risk and risk-free rates are held constant over time so that the expected equity return, $R$, requires no subscript ‘t’. Recursive substitution now implies,

$$P_t = \sum_{s=1}^{\infty} R^{-s} E_t[dPS_{t+s}]$$

and where, to be sure, both sides of the equation are on a date $t$ per share basis. Conversely, (*) implies $E_t[(P_{t+1} + dPS_{t+1})/P_t] = R$. PVED on a per share basis accordingly corresponds to the traditional dividend-adjusted “random walk hypotheses”. Neglecting issues related to changes in risk and interest rates, theoretical finance relies on this modeling when investors’ expectations are homogeneous.

Consider next the validity of PVED as a representation of a firm’s total equity, i.e., the ‘D’ now stands for total (dollar) dividends net of capital contributions. PVED on a total equity basis is of course as valid as (*) if the shares outstanding never change in the future. If, on the other hand, shares outstanding can change then one must recognize that the issue of dividends per share versus total dividends net of capital contributions arises because capital contributions—as opposed regular cash dividends—perturbs the ownership structure by introducing “new” owners (or “eliminate” owners). A firm that operates as a proprietorship simply “taxes” the (single) owner if the firm requires a capital infusion; the number of shares outstanding need not change. With these observations in place it becomes clear that PVED on a total basis may differ from the market value because PVED can include (expected) wealth created for individuals who are not currently shareholders. Dealing with this issue requires the articulation of MM-conditions so that (*) reconciles with PVED on a total equity basis. A subsequent section formalizes such analysis. Before undertaking this task, it helps to identify conditions on the accounting such that RIV meshes with PVED on a per share, or (*), basis.

2. Per Share Accounting and Clean Surplus

It goes almost without saying that the change in book value per share ($\Delta bvps$) will generally differ from earnings per share (eps) minus dividends per share (dps). This claim does not rest on dirty surplus items, or on complexities associated with eps due to potentially dilutive securities (like convertible bonds). Ruling out these possibilities, but allowing for (regular) capital transactions, still leads to $\Delta bvps \neq eps - dps$, unless (i) number of shares outstanding does not change, or,
(ii), issue price per share equals bvps at the transaction date. Neither exception is of practical interest. With respect to (i), firms sometimes announce plans to engage in acquisitions and capital structure changes or, as another example, maintain share repurchase programs to support the stock price. And GAAP makes no attempt to satisfy condition (ii)—which corresponds to mark-to-market accounting—even as an approximation.

Absent clean surplus on a per share basis, the usual analysis that leads to equivalence in RIV and PVED falls apart. Hence, if one defines

$$\text{reps}_t \equiv \text{eps}_t - r \cdot \text{bvps}_{t-1},$$

where $r = R - 1$; one can then not rule out

$$\text{PVED} \neq \text{bvps}_t + \sum_{t=1}^{\infty} R^{-t} E_t[\text{reps}_{t+1}],$$

where ‘ED’ refers to $E_t[\text{dps}_{t+1}], E_t[\text{dps}_{t+2}], \ldots$. This inequality, to be sure, can hold even if capital transactions occur only at the end of periods so that there are no ambiguities associated with eps, and bvps.

Per share clean surplus does not hold if a firm issues/buys shares insofar the transaction changes bvps. An end-of-period issuance of shares increases bvps if $P_t$ exceeds the bvps prior to the issuance. But the transaction leaves eps, and dps, unaffected, and it follows that $\Delta \text{bvps}_t > \text{eps}_t - \text{dps}_t$ (assuming no other capital transactions).

Clean surplus on a per share basis can be achieved by “reconceptualizing” either of the two accounting variables, of course. The two reference schemes are as follows:

(i) Define

$$\text{eps}_t \equiv \Delta \text{bvps}_t + \text{dps}_t.$$  

Hence, eps, will not generally equal “regular” eps, even if clean surplus holds on a total dollar basis. As a further complication, bvps, may be ambiguous if there are dilutive securities outstanding (GAAP does not prescribe how one determines bvps, in sharp contrast to eps). But this issue is subordinated and separate. Approach (i) stems from the construct that within a per share context eps derives as a “plug” from the change in bvps adjusted for dividends per share. Because changes in shares outstanding have a direct influence on bvps, this eps construct includes “realized gains/losses due to capital transactions”. The difference, $\text{eps}_t - \text{eps}_t$ determines the net of this “expense” item, and its sign depends on the sign of $P - \text{bvps}$ and whether shares are issued or purchased. Put mildly, this kind of accounting is unorthodox.

(ii) Recursively, define

$$\text{bvps}_t \equiv \text{bvps}_{t-1} + \text{eps}_t - \text{dps}_t$$

with the initialization $\text{bvps}_0 \equiv \text{bvps}_0$.

In contrast to (i), approach (ii) specifies book value per share instead of earnings per share as the “plug”. Empirical papers applying RIV on a per share basis rely on the scheme. Forecasted eps thereby take on a central role, though there also has to be
some assumption about forecasted dps. Accounting gains/losses allocated to current shareholders because of (forecasted) capital transactions are now no longer part of the eps construct. This feature perhaps suggests that (ii) is preferable to (i). Still, to claim that the approach leads to an appealing residual earnings construct would seem to be farfetched.

Both approaches, (i) and (ii), seem awkward in their reliance on mechanical plugs. A requirement of clean surplus earnings on a per share basis makes the RIV model look contrived rather than intuitive. Because the per share basis causes the smudge, one may instead consider a RIV that targets a firm’s total (dollar) equity value. The accounting then becomes more palatable by removing per share issues. Nevertheless, the next section shows how potential future capital contributions still mar the analysis.

3. PVED on a Total Equity Basis

Consider PVED when the ‘D’ stands for total dividends net of all capital contributions. Without referring to any concepts of equity valuation, it is of course true that PVED equals RIV given the clean surplus relation. To justify a total dollar basis RIV therefore requires an analysis of conditions such that PVED equals the market value of the total equity at date zero. RIV lacks proper grounding if PVED on a total basis differs from an evaluation (*) of the expected dps—sequence. In analytical terms, the issue arises whether potential changes in the anticipated number of shares outstanding allows for

\[ n_0 \sum_{t=1}^{\infty} \frac{R^{-t}E_t[dps_t]}{C_t} \neq \text{PVED} \]

where ‘D’ = all dividends net of capital contributions and \( n_0 \) denotes the number of shares outstanding at the date of evaluation \( (t = 0) \).

As an example below shows, the two expressions equate if, and only if, the issuance/buy-back of shares is value-neutral from the point of view of expected "new future" shareholders. Accordingly, the condition represents an "outsider" capital transaction irrelevancy concept in the spirit of MM.

To illustrate the subtleties associated with a total-equity PVED approach, consider the following:

(1) Regular cash dividends are declared and paid on a per share basis every 12/31. Let \( \text{dps}_t \geq 0 \) denote these dividends.

(2) The current date is 1/1 and \( t = 0 \). Only one share is outstanding. The firm plans to issue \( k \) new shares at a price of \( p^* \) one year later, where \( k \) and \( p^* \) depend on the circumstances at that date. These two variables are therefore random from a date \( t = 0 \) perspective. Without substantive loss of generality, assume further that shares outstanding will not change beyond date \( t = 1 \). Total dividends net of capital contributions equal
One can next evaluate PVED on a total equity basis. Define it as $V_0$:

$$V_0 = R^{-t_1} \left( E_0 [dps_1] - E_0 \left[ \tilde{k} \cdot \tilde{p}^* \right] \right) + \sum_{i=2}^{\infty} R^{-t_i} E_0 \left[ (1 + \tilde{k}) \cdot dps_i \right]$$

$$= \sum_{i=1}^{\infty} R^{-t_i} E_0 [dps_1] + \left\{ \sum_{i=2}^{\infty} R^{-t_i} E_0 \left[ \tilde{k} \cdot \tilde{p}^* \right] - R^{-t_1} E_0 \left[ \tilde{k} \cdot \tilde{p}^* \right] \right\}.$$ 

Because the PVED-formula on a per share basis, equation (*), equals $P_0$, it follows that $V_0 = P_0$ if and only if the expression inside $\{}$ equals zero. The latter restriction means that the “new future” shareholders view the issuing of shares as neutral ex ante: They are indifferent to the anticipated transaction. MM applies in this confining sense.

For the “current” shareholders to potentially benefit it suffices if the anticipated transaction affects the sequence of expected dividends per share positively. Thus the condition $P_0 = V_0$ allows for economically relevant capital transactions. But the allocation of welfare across the two groups of shareholders has to be extreme in that only the current shareholders can benefit. This is of little practical interest unless neither group benefits. No benefits whatsoever can be thought of as capital-irrelevance in a traditional MM sense. Treasury stock transactions fit neatly into this category. Acquisitions of business do not, since the “new” shareholders typically require a premium to surrender their business assets. (In this case $k; p^*$ corresponds to the market value of shares issued prior to the announcement of the transaction.) In sum, PVED on a total equity basis, $V_0$, identifies value created to all shareholder groups, including those that in the future leave or enter as owners on less than fair terms. Contrast this approach to PVED on a per share basis, (*), which identifies the value accruing to a current owner who does not ever change his ownership status, except on fair-value terms.

A final point about capital transactions relates to the requirement that all capital transactions (and dividends) subsumed by ‘D’ must be measured by their market values. GAAP violates this market value condition for some capital transactions, such as convertible bonds when converted and (compensation) options when exercised. Further, expected values of the GAAP-measures of the capital contributions will most likely be materially less than the related expected market values given GAAP’s inherent downward biases.

Observations in this and the prior section suggest that neither of the two RIV models, on a per share basis or on a total equity basis, satisfactorily circumvent problems caused by capital transactions. The per share model seems particularly awkward with its contrived schemes to attain clean surplus. PVED on a total equity basis, on the other hand, depends on a relatively reasonable condition of fair capital transactions. This condition can in some ways be weakened, a point developed by Christensen and Feltham (2003). They discuss a full range of capital transactions and
how these can be accounted for outside GAAP to reflect wealth-redistributions across ownership interests, all within a clean surplus framework. Whether these refined accounting-schemes would aid practical equity valuation can be debated. The issue actually deflects by suggesting that accounting-based valuation ought to hone in on some version of RIV.

RIV’s primary problem concerns the need to juggle two accounting measures, earnings and book values. A model that relies on 2 value-attributes runs at obvious cross-purposes with how investors in the real world look at the future. Practical investment analysis emphasizes (expected) earnings (or eps) as a core value attribute; expected book values or residual earnings have no such role (or at least rarely so). These aspects make it troublesome to assign RIV the elevated status of being the accounting-based valuation formula.

Subsequent sections develop a reasonable alternative to RIV. This new formula maintains an unequivocal eps-focus. Further, the analytical machinery on which RIV depends remains essentially the same.

4. A Machinery for Accounting-Based Valuations

RIV’s fragile underpinnings suggests that there is need for alternative, accounting-based, valuation formulae. A powerful approach to the broad problem of identifying valuation formulae consistent with PVED on a per share basis should thus embed RIV merely as a special case. To develop the analytics that equates PVED and RIV, it helps if one first articulates a class of valuation schemes consistent with PVED and thereafter introduces book values, earnings and clean surplus. This approach will usefully demonstrate the arbitrariness of RIV as an accounting-based valuation formula. An alternative to RIV, which focuses on future eps without any reference to byps, will become apparent.

Equating RIV to PVED depends only on a simple scheme that requires no reference to economic or accounting concepts. Consider the following mathematical equality: Let \( \{y_t\}_{t=0}^\infty \) be any sequence of numbers that satisfies \( R^{-t} \cdot y_t \to 0 \) as \( t \to \infty \); then

\[
0 = y_0 + R^{-1}(y_1 - R \cdot y_0) + R^{-2}(y_2 - R \cdot y_1) + \cdots
\]

Adding the last equation and (*) implies that

\[
P_0 = y_0 + \sum_{t=1}^{\infty} R^{-t} \cdot z_t
\]

where

\[
z_t \equiv y_t + \hat{d}p_t - R \cdot y_{t-1}
\]

and \( \hat{d}p_t \equiv E_0 \left[ \hat{d}p_t \right] \). Though the left hand side of (**) does not depend on the \( y_t \)-sequence, that is, (**) always equals (*)—the \( z_t \) sequence does. In what follows, \( z_t \) should always be viewed as dependent on some specification of \( (y_t, y_{t-1}) \).
Armed with the expression (**), nothing stops us from putting \( y_0 = \text{bvps}_0 \) and \( y_t = \text{bvps}_t, t \geq 1 \), so that \( z_t = \frac{\Delta \text{bvps}_t + \text{dps}_t}{\text{bvps}_{t-1}} - R \cdot \text{bvps}_{t-1} \). This approach demonstrates the centrality of book values in RIV. Earnings, by contrast, enter the analysis via the “back-door”—\( \text{eps}_t = \Delta \text{bvps}_t + \text{dps}_t \)—and this incremental step is hard to justify unless one assigns a “conceptual label” to \( z_t \), namely, residual earnings (per share). Given the dubious nature of clean surplus on a per share basis, it would probably be more accurate to say define \( \tilde{\text{eps}}_t = \text{bvps}_t + \text{dps}_t \), and define \( \tilde{\text{reps}}_t \equiv \tilde{\text{eps}}_t - r \cdot \text{bvps}_{t-1} \), and without attaching any labels to \( \text{eps}_t \) and \( \text{reps}_t \). RIV on a per share basis applies, but the letter ‘I’ in the acronym RIV misleads insofar the formula is more of a book value than an income approach to equity valuation.

The above steps bring us back to the clean surplus implication (i) in Section III with its unorthodox \( \text{eps}_t \)-concept. An alternative to (i) relies on the construct (ii), which treats \( \text{bvps} \) as the plug. In terms of the Section III notation, put \( y_t = E_0[\text{bvps}_t] \) and re-define per share residual income, \( \tilde{\text{reps}}_t \equiv \tilde{\text{eps}}_t - r \cdot \text{bvps}_{t-1} \). This unrestricted choice of a \( \text{bvps} \) -construct reflects the saying that “accounting measurement/valuation principles do not influence the validity of RIV”. Both constructs of \( \text{bvps} \) clearly “work”. But the utter lack of any conceptual restrictions on what determines the \( \text{bvps} \)-sequence inevitably points toward a more poignant truth: The mathematical machinery by itself provides no compelling reasons why the \( y \)-sequence should correspond to a book value-sequence.

One can perhaps argue that a \( \text{bvps} \) -specification of \( y \) makes sense because \( \text{bvps}_0 \) supplies a “natural starting point” in equity valuation. Such a claim, however, immediately raises the question whether one can identify “starting point” measures other than current book value. Casual observation, if not theory, points toward capitalized next period expected earnings as a much better first-cut indicator of equity value. In other words, investment practice with its focus on forward \( \text{eps} \) suggests the pick \( y_0 = E_0[\text{eps} \text{-next year}] / r \). The difference between \( P_0 \) and \( E_0[\text{eps} \text{-next year}] / r \) should then relate to the expected increments in subsequent expected \( \text{eps} \). It seems reasonable, therefore, that one should entertain the sequence

\[
y_t = E_0[\text{eps}_{t+1}] / r \equiv \frac{\text{eps}_{t+1}}{r} \quad t = 0, 1, 2, \ldots
\]

For \( t = 1, 2, \ldots \), substituting into the definition of \( z_t \) results in

\[
z_t = \frac{1}{r} \left[ \text{eps}_{t+1} + r \cdot \text{dps}_t - R \cdot \text{eps}_t \right] = \frac{1}{r} \cdot \left[ \Delta \text{eps}_{t+1} - r \cdot (\text{eps}_t - \text{dps}_t) \right]
\]

The analytics is definitional; it applies regardless of the accounting rules which influence the \( \text{eps}_t \)-sequence. One interprets \( rz_t \) as the expected \( \text{eps} \)-increment in excess of what should be expected due to earnings retained in the prior period. Positive \( z_t \) result in a positive premium, \( P_0 > \text{eps}_1 / r \). Alternatively, the price-to-forward-earnings ratio, \( P_0 / \text{eps}_1 \), exceeds the benchmark \( 1/r \) in case of above-normal future expected earnings increments.

Every specification of \( \{y_t\}_{t=0}^\infty \) maintains PVED, but conceptual issues arise if one wants to approximate PVED by cutting (***) short in terms of the number of years that enter the evaluation. Finite horizons appeal in a world of bounded rationality.\(^6\)
In this spirit, one can potentially judge the usefulness of accounting constructs in terms of the errors

\[ \text{PVED} - \left( y_0 + \sum_{i=1}^{T} R^{-i}z_i \right) = \sum_{i=T+1}^{\infty} R^{-i}z_i = \text{Err}(T; y) \]

where \( y = (y_0, y_1, \ldots) \) specifies the accounting measurements and their expected values. As a boundary case, for \( T = 0 \) define \( \text{Err}(0; y) \equiv P_0 - y_0 \).

While \( \text{Err}(T; y) \to 0 \) as \( T \to \infty \), finite \( T \) generally leads to errors. And the smaller the errors across various horizons (\( T \)), the better. In what follows the analysis considers

\[ y' \equiv (\text{bps}_1/r, \text{bps}_2/r, \text{bps}_3/r, \ldots) \]

and

\[ y'' \equiv (\text{bpvps}_0, \text{bpvps}_1, \text{bpvps}_2, \ldots). \]

There are no reasons why \( \text{Err}(0; y) = 0 \) should hold as a practical matter. But the outcome is of theoretical interest for both \( b_{0v} \) and \( \text{eps}_1/r \). To avoid strange knife-edge cases, one may consider more generally the possibility of \( \text{Err}(T; y) = 0 \) for all \( T \geq 0 \). This “no-error” outcome implies that some hypothetical accounting measurements have succeeded in satisfying \( z_t = 0 \) and \( \text{PVED} - y_0 = P_0 - y_0 = 0 \). Addressing what it takes to achieve the stringent no truncation-error regardless of the horizon \( T \) provides a useful baseline when one compares book values and earnings (capitalized) as value attributes.

5. The Specification of \( \{y_t\}_{t=0}^\infty \): Book Value vis-à-vis Earnings

This section compares the implications of specifying the \( y \)-sequence as either earnings or book values on the truncation-error \( \text{Err}(T; y) \). The sign of

\[ |\text{Err}(T; \text{bpvps}_0, \ldots)| - |\text{Err}(T; \text{bps}_1/r, \ldots)| \]

for various \( T \geq 0 \) thus evaluates the relative performance.

The initial setup disallows equity transactions, other than cash dividends, to keep matters simple. In other words, the firm will effectively operate as a proprietorship with only 1 share outstanding. The assumption makes the ‘D’ in PVED unambiguous. Given this simplification the analysis can be tied to the prior literature on valuation, and thus it helps to use more “standard” notation. Let \( x_t \) denote earnings where thus \( x_t = \text{eps}_t \) and \( d_t \) denotes dividends where \( d_t = \text{dps}_t = D \). Further, let \( x_t^a = x_t - r b_t \) where \( b_t = \text{bpvps}_t \) and \( x_t^a = \text{reps}_t \). In what follows the notation \( x_t, d_t, b_t, x_t^a \) will be used only if the firm operates as a proprietorship (one share is outstanding at all dates, but \( d_t \) may be negative).

With these preliminaries in place the analysis next addresses under what conditions
\[ z'_t' \equiv (\bar{x}_{t+1} + r \cdot \bar{d}_t - R \cdot \bar{b}_t)/r \]

or

\[ z''_t \equiv (\bar{b}_t + \bar{d}_t - R \cdot \bar{b}_{t-1}) \]
equals zero. Which of the two seems the more likely candidate to satisfy this zero-condition?

It goes almost without saying that both \( z'_t \) and \( z''_t \) equal zero if the firm corresponds to a "saving-account". Next-period earnings will be certain, but subsequent earnings can be uncertain because next-period dividends may be uncertain. In spite of this uncertainty, one readily shows that for all future dates \( t \)

\[ z'_t \equiv r^{-1} \cdot (\bar{x}_{t+1} + r \cdot \bar{d}_t - R \cdot \bar{x}_t) = 0 \]

and

\[ z''_t \equiv \bar{b}_t + \bar{d}_t - R \cdot \bar{b}_{t-1} = 0. \]

Alternatively, \( P_t = \bar{x}_{t+1}/r = (R/r)\bar{x}_t - \bar{d}_t = \bar{b}_t = R^{-1} \cdot (\bar{b}_{t+1} + \bar{d}_{t+1}) \) and, for both of the two \( y \)-specifications, \( \text{Err}(T; \cdot) = 0 \) all \( T \geq 0. \)

The savings account serves as a useful starting point because it shows that the right accounting constructs can yield a clean result. Thus one may next examine what happens in case of mark-to-market accounting, which is somewhat less stringent than a savings account.

A powerful analytical observation handles a more general version of mark-to-market accounting. Note that

\[ \Delta x_{t+1} = (\bar{b}_{t+1} + \bar{d}_{t+1} - R \cdot \bar{b}_t) = (\bar{b}_t + \bar{d}_t - R \cdot \bar{b}_{t-1}) \]

\[ = \bar{x}_{t+1} - R \cdot \Delta \bar{b}_t - \bar{d}_t = \bar{x}_{t+1} + r \cdot \bar{d}_t - R \cdot \bar{x}_t = r \cdot z'_t \]

To interpret the relation in terms of its underlying accounting constructs, recall that \( z'_t = x'_t \) and thus \( r \cdot z'_t = \Delta x'_t \). Thus, one moves from a focus on book values, \( z'_t \) or \( x'_t \), to one on earnings, \( z''_t \) or \( \Delta x''_{t+1} \), by taking the first difference in \( z''_t \) or \( x''_t \). It follows immediately that \( z''_t = 0 \) for all \( t \) implies \( z'_t = 0 \). But the opposite is obviously false. Applying these observations to \( \text{Err}(\cdot; \cdot) \) yields the result below.

**Proposition 1** Assume PVED and clean surplus accounting. Consider the truncation-error definition \( \text{Err}(T; y) \) where \( y \) corresponds to either \( y' \equiv (\bar{x}_1/r, \bar{x}_2/r, \ldots) \) or \( y'' \equiv (b_{t0}, b_{t1}, \ldots) \). If one further assumes that \( \text{Err}(T; y'') = 0 \) for all \( T \geq T^* \), then \( \text{Err}(T; y') = 0 \) for all \( T \geq T^* \) as well. The converse, however, is false.

Traditional accounting concepts supply an intuitive and direct interpretation of the above proposition. Perfect balance sheets, which equate a firm’s book value to its economic value, result in perfect earnings measures if one uses clean surplus accounting. Reversing this claim does not work, however, because of the “canceling error” concept. That is, if one deducts a constant from all perfect balance sheets then the earnings measure does not change and thus it remains perfect. One cannot, therefore, infer perfect balance sheets from perfect measures of earnings (and
dividends). These ideas, which go to the heart of accounting, impel the proposition: There should be no surprise why earnings, as opposed to book values, take on such dominant role in fundamental analysis of equity value. RIV, as noted, runs at cross-purposes with this core idea.

Does the proposition depend on no capital transactions and the absence of potential new/former shareholders? The answer is no. If a firm’s market value always equals its book value, then the issuance of shares leads to no implicit or explicit gains/losses to the new/former shareholders; clean surplus on a per share basis therefore applies if it does so on a dollar basis. With respect to perfect (expected) earnings—Err(Ti′; y′) = 0 but Err(Ti′; y′′) ≠ 0, T ≥ T′ > 1—one can of course argue that capital transactions make the scenario implausible. But this matter of plausibility goes beyond the scope; the formal analysis goes through.

6. Illustrations of a Book Value vis-à-vis Earnings Focus

Via two examples, this section illustrates dynamic underpinnings of an earnings-focus vis-à-vis a book-value-focus. The first example shows how a book-value, or RIV, focus with Err(T; y′′) = 0 can be thought of as mark-to-market in expectation. The second example shows how an earnings focus with Err(T; y′) = 0 but possibly Err(T; y′′) ≠ 0 can be thought of as permanent earnings in expectation. Both examples rely on the information dynamic found in Ohlson (1995), slightly extended.

Example 1 Mark-to-Market in Expectation

Consider the dynamic

\[ x_{t+1} = v_{1t} + \tilde{e}_{1t+1} \]

\[ \tilde{v}_{1t+1} = v_{2t} + \tilde{e}_{2t+1} \]

\[ \tilde{v}_{2t+1} = +\tilde{e}_{3t+1} \]

where \( E_t[\tilde{e}_{kt+1}] = 0 \), all \( t \geq 1 \). Using RIV yields the valuation function:

\[ P_t = b_t + R^{-1} \cdot v_{1t} + R^{-2} \cdot v_{2t} \]

Hence, for \( t \geq 2 \),

\[ E_t[P_{t+1} - \tilde{b}_{t+2}] = 0, \]

and where \( t \geq 2 \) is also necessary for the equality. It follows that Err(T; y′′) = 0, all \( T \geq 2 \), and, due to the proposition, Err(T; y′) = 0 as well.

To see more concretely why Err(T; y′) = 0 for T ≥ 2, note that \( E_t[v_{kt+1}] = 0 \) for \( t \geq 2 \). Next, since \( E_t[\tilde{x}_{kt+1}] = r \cdot E_t[\tilde{b}_{kt+1}] \) one has \( E_t[P_{t+1}] = r^{-1} \cdot E_t[\tilde{x}_{t+1}] \). The equilibrium condition \( E_t[P_{t+1} + \tilde{d}_{t+1}] = R \cdot E_t[P_{t+1}] \) (all \( t \geq 2 \)) combined with the last relation implies \( r^{-1} \cdot E_t[\tilde{x}_{t+1}] + E_t[\tilde{d}_{t+1}] = R \cdot r^{-1} E_t[\tilde{x}_{t+1}] \), which indeed corresponds to \( \tilde{z}_t = 0 \) for all \( t \geq 2 \) and Err(T; y′) = 0, \( T \geq 2 \).

The labeling of this setting, mark-to-market in expectation, appeals because \( P_{t+1} = \tilde{b}_{t+2} \) beyond the next year (\( t \geq 2 \)). The special case \( v_{1t} = v_{2t} = \tilde{e}_{1t} = \tilde{e}_{2t} = 0 \) all
\( t \), defines strict mark-to-market accounting: \( P_t = b_t \) and \( \text{Err}(0; y^\prime) = 0 \). More generally, one can extend the information dynamic so that \( P_{t+\tau} = b_{t+\tau}, \tau \geq T^* \), for any preselected \( T^* \).

**Example 2** Permanent Earnings in Expectation

Consider the dynamic

\[
\begin{align*}
\hat{v}_{t+1}^\prime & = v_{t+1} + \tilde{e}_{t+1} \\
v_{t+1} & = v_t + v_{2t} + \tilde{e}_{2t+1} \\
\tilde{e}_{2t+1} & = \tilde{e}_{3t+1}
\end{align*}
\]

where \( E_t[\tilde{e}_{3t+1}] = 0 \) all, \( \tau \geq 1 \). Using RIV and clean surplus it follows that

\[ P_t = b_t + r^{-1} \cdot v_t + (R \cdot r)^{-1} \cdot v_{2t} = r^{-1} \cdot E_t[\tilde{e}_{t+1}] + (R \cdot r)^{-1} v_{2t}. \]

As to the last equation, note that \( E_t[\tilde{e}_{t+1}] = v_t \). Next, because \( E_t[\tilde{e}_{2t+1}] = 0 \) for all \( \tau \geq 1 \), it follows that \( E_t[\tilde{e}_{2t+1}] = r^{-1} \cdot E_t[\tilde{e}_{t+2}] \), \( \tau \geq 1 \). The equilibrium condition \( E_t[\tilde{e}_{t+1} + \tilde{e}_{t+2}] = R \cdot E_t[\tilde{e}_{t+1}] \), \( \tau \geq 0 \) now implies \( r^{-1} \cdot E_t[\tilde{e}_{t+2}] + E_t[\tilde{e}_{t+1}] = R \cdot r^{-1} \cdot E_t[\tilde{e}_{t+1}] \). One sees that \( z_t = \{E_0[\tilde{e}_{t+1}] + r \cdot E_0[\tilde{d}_t] - R \cdot E_0[\tilde{x}_t]/r = 0 \), for all \( t \geq 1 \), so that \( \text{Err}(T; y^\prime) = 0 \) if \( T \geq 1 \). On the other hand, \( \text{Err}(T; y^\prime^\tau) \neq 0 \) since \( E_t[\tilde{e}_{t+1}] = E_t[\tilde{h}_{t+1}] + r^{-1} v_t \) for all \( \tau \geq 1 \), which violates mark-to-market in expectation.

As to the terminology, its motivation stems from the permanent earnings concept. The earnings dynamic now satisfies \( E_t[\tilde{e}_{t+1}] = R \cdot x_t - r \cdot d_t \), which is more demanding than \( E_t[\tilde{e}_{t+1}] = R \cdot E_t[\tilde{e}_{t+1}] - r \cdot E_t[\tilde{d}_{t+1}] \), \( \tau \geq 2 \) but not necessarily \( \tau = 1 \). Permanent earnings therefore suffice for permanent earnings in expectations, but the converse is false. Finally, note that the dynamic generalizes so that \( \text{Err}(T; y^\prime^\tau) = 0 \) for \( T \geq T^* \), and where one can pick any number \( T^* \) (not just \( T^* = 1 \)).

The proposition combined with the examples suggests the following summary. Mark-to-market in expectation corresponds to \( P_t = b_t \) and \( \text{Err}(t; y^\prime^\tau) = 0 \), \( t \geq T^* \). This model contrasts with permanent earnings in expectations which corresponds to \( P_t = r^{-1} \cdot \tilde{x}_t + (R/r) \cdot \tilde{x}_t - d_t \), for all \( t \geq T^* \) and \( \text{Err}(t; y^\prime^\tau) = 0 \). Moreover—with this point is indeed central—whereas mark-to-market in expectation implies permanent earnings in expectation, the converse is false. On the basis of this observation, at least, a focus on earnings clearly dominates a focus on book values.

**7. Book Values vis-à-vis Earnings when the Accounting is Conservative**

So far the analysis has stated assumption that imply no truncation-error beyond some (future) date. This setup build in the benchmarking conditions \( P_T = b_T \) or \( P_T = \tilde{x}_{T+1}/r \). These exclude the more realistic case when both capitalized earnings and book values have a consistent downward bias relative to market values. Biases of this kind can be thought of as manifestations of conservative accounting. Feltham and Ohlson (1995), Ozair (2003), Zhang (2000), and others, formalize the concept of conservative accounting and relate it to such biases. Equivalently, conservative
accounting implies that a firm’s long run expected earnings can continue to drift upwards even if the (net) dividend payout is fixed at 100%. In this sense growth in expected earnings and conservative accounting are two sides of the same coin. Proposition I, to be sure, excludes conservative accounting because of the property $E_t[\Delta x_{t+1}] = 0$ as $t \to \infty$ if $x_t = d_t$.

Relying on Zhang (2000), Ohlson (2002) outlines how capitalized earnings dominate book values as a value attribute when one evaluates truncation-errors in settings with conservative accounting. Again the traditional partially “canceling errors” concept makes its presence felt. In analytical terms, the difference $P_t - \bar{x}_{t+1}/r$ is smaller than $\bar{P}_t - \bar{b}_t$ as $t \to \infty$.

Granting the centrality of the asymptotic inequalities $P_t > \bar{x}_{t+1}/r > b_t$, one can usefully show how a partial “canceling error” concept applies in a setting with (a) long-run expected growth, (b) clean surplus, (c) conservative accounting. Define the (expected) “errors” in book values as

$$\bar{e}_t \equiv \bar{P}_t - b_t,$$

where $t = 0$ is today’s date and $t = 1, 2,\ldots$. For large $t$, $\bar{e}_t$ is positive due to conservative accounting. Further, assume that $\bar{e}_{t+1}/\bar{e}_t, P_{t+1}/P_t, b_{t+1}/b_t > 1 + g$, i.e., the 3 variables in question have the same asymptotic growth rate, $g$. Convergence in PVED requires $g < r$, which is another mild regularity condition. Next, combining clean surplus with the equilibrium condition, $P_{t+1} + \bar{d}_{t+1} = r \bar{P}_t$, implies

$$P_t = \bar{x}_{t+1}/r + (\bar{e}_{t+1} - \bar{e}_t)/r.$$

Due to growth, $\bar{e}_{t+1} > \bar{e}_t$ so that $\bar{P}_t > \bar{x}_{t+1}/r$: this proves the first inequality. The second inequality states that $\bar{x}_{t+1}/r > b_t$, which is equivalent to $(\bar{e}_{t+1} - \bar{e}_t)/\bar{e}_t$. In other words, do the balance sheet errors cancel each other sufficiently such that the difference capitalized is less than the error itself? The answer is ‘yes’ simply because $(\bar{e}_{t+1} - \bar{e}_t)/\bar{e}_t > 0$.

The preceding paragraph motivates the following proposition.

**Proposition II** Assume PVED and clean surplus accounting. Further assume that the expected dividend sequence satisfies the regularity condition $\lim_{t \to \infty} d_t/d_{t-1} = g + 1$ were $0 < g < r$. Similarly, assume $\lim_{t \to \infty} b_t/b_{t-1} = 1 + g$.

Conservative accounting in the sense of

$$\lim_{t \to \infty} (P_t/\bar{b}_t) > 1$$

then implies there exists some $T^*$ such that for all $T \geq T^*$

$$\text{Err}(T; y') > \text{Err}(T; y') > 0.$$

Conversely, the accounting is conservative if the last statement is true.

**Proof** One readily shows that $\text{Err}(T; y) = R^{-T}(\bar{P}_T - \bar{y}_T)$ Hence $R^T \cdot |\text{Err}(T; y)|$.

$-\text{Err}(T; y') = \bar{x}_{T+1}/r - b_T$. Zhang (2000), in his Proposition I, has shown that there exists some $T^*$ such that for all $T > T^*(\bar{x}_{T+1}/b_T) > r$. Sufficiency follows.
Necessity is straightforward because Zhang (2000), also in his Proposition I, shows that \( \left( x_{t+1} \right) / x_T > r \) implies conservative accounting.

Because the assumptions are relatively weak, it would seem that RIV as an (intrinsic) valuation formula lacks appeal as compared to the AEG formula which focuses on expected earnings and their subsequent expected increments. A complete evaluation of the result’s robustness nevertheless requires that one considers capital transactions. In its current version the Proposition’s PVED assumption implicitly requires no expected changes in shares outstanding, or the expected capital transactions have to be fair. If neither condition is met, then, of course PVED must be evaluated on a per share basis. And for this setting clean surplus will not generally apply given “regular” accounting for \( \text{eps}_t \) and \( \text{bvps}_t \).

A relative minor modification of the proposition handles the per share problem and the absence of a “regular” clean surplus. Referring back to Section III and the definition of \( \text{bvps}_t \), \( \text{bvps}_t = \text{bvps}_{t-1} + \text{eps}_t - \text{dps}_t \), consider the case when both “regular” \( \text{bvps}_t \) and \( \text{bvps}_t \) are conservative, i.e., \( \lim \{ P_t / \max \{ \text{bvps}_t, \text{bvps}_t \} \} > 1 \).

Additional mild conditions ensure that \( \text{eps}_t + \text{eps}_{t+1} > \max \{ \text{bvps}_t, \text{bvps}_t \} \) and accordingly the proposition generalizes to admit changes in shares outstanding.

8. The Modeling of Earnings and their Growth vs. Book Values and their Growth

In comparing earnings and book values in equity valuation one may usefully consider implications of parametric models. Via assumptions on the sequence of future expected residual earnings, such modeling instructs by expressing values in simple terms. This section argues that the traditional focus on the market-to-book ratio should be complemented, if not replaced, by a model that focuses on the price-to-(expected) forward earning ratio.

The next paragraph reviews the popular, well-known, model that explains market value when anchored to (current) book value. With these derivations in place the rest of the section can show why it makes sense to replace book value with next-period expected earnings capitalized. Certain aspects of the analyses will be striking. Whether one anchors value to book values or to earnings, the assumptions and the math will be much the same. Yet the analysis builds in unambiguous conclusions as to why one wants to focus on earnings.8

Following Penman (2004) and other textbooks, consider the most straightforward parameterization of the sequence of future expected residual earnings:

\[
\hat{x}_t = \gamma \cdot \hat{x}_{t-1} \quad t = 2, 3, \ldots
\]

and, as always, \( \hat{x}_t \equiv \hat{x}_t - r \cdot \hat{b}_{t-1} \). The parameter \( \gamma \) satisfies \( \gamma < R \), and it determines the growth in the sequence of anticipated residual earnings. Given clean surplus, RIV applies so that
\[ P_0 = \text{PVED}_0 = b_0 + \sum_{t=1}^{\infty} R^{-t} \bar{x}^u_t = b_0 + (R - \gamma)^{-1} \cdot \bar{x}^u_t = b_0 \cdot \left[ \left( \frac{\text{RoE}_1 - g}{r - g} \right) \right], \]

where \( \text{RoE}_1 \equiv \bar{x}_1/b_0 \) and \( g = \gamma - 1 \). Hence the valuation equation explains the market-to-book ratio in terms of the forthcoming accounting rate-of-return, discount-rate, and growth parameter \( g \). In this form the model provides well-known insights about the market-to-book ratio. As \( \text{RoR} \) increases, so does the ratio. Further, \( (P_0/b_0) > 1 \) if and only if \( \text{RoE}_1 > r \); \( P_0 = b_0 \) corresponds to \( \text{RoE} = r \). These observations have theoretical appeal, but it is noteworthy that book value rather than capitalized earnings acts as the anchor to value.

It turns out that the assumptions of the model \( (P_0 = \text{PVED}, \text{clean surplus and the recursive residual earnings equation}) \) also lead to an entirely different valuation expression. This alternative expression explains value in terms of forward earnings and the subsequent growth in forward earnings. It makes no reference to book values, explicitly or implicitly, even though the assumptions remain the same. Understanding how the derivations yield the conclusion will be critical. There are three steps.

First, taking the first difference of the recursive residual earnings equation yields

\[ \Delta \bar{x}^u_{t+1} = \gamma \cdot \Delta \bar{x}^u_t, \quad t = 2, 3, \ldots \]

Second, due to the clean surplus relations, as shown earlier

\[ \Delta \bar{x}^u_{t+1} = r \cdot \bar{z}^u_t, \quad t = 2, 3, \ldots \]

where, as before,

\[ \bar{z}^u_t = r^{-1} \cdot \left[ \bar{x}_{t+1} + r \cdot \bar{d}_t - R \cdot \bar{x}_1 \right]. \]

Thus one can write

\[ \bar{z}^u_{t+1} = \gamma \cdot \bar{z}^u_t, \quad t = 2, 3, \ldots \]

Third, combining the AEG-model,

\[ P_0 = \bar{x}_1/r + \sum_{t=1}^{\infty} R^{-t} \cdot \bar{z}^u_t, \]

with the last recursive equation yields

\[ P_0 = \bar{x}_1/r + (R - \gamma)^{-1} \cdot \bar{z}^u_1 = \bar{x}_1/r + (R - \gamma)^{-1} \cdot \frac{1}{r} \cdot \Delta \bar{x}^u_2 \]

\[ = \bar{x}_1 \cdot \frac{\left[ g_2 - g \right]}{r - g} \]

where \( g = \gamma - 1 \), as before, and

\[ g_2 = \frac{\Delta \bar{x}_2 + r \cdot \bar{d}_1}{\bar{x}_1}. \]
One conceptualizes \( g_2 \) as the growth in expected earnings, year 2 vs. year 1, and where \( r \cdot d_1 \) corrects year 2 earnings for the earnings foregone due to year 1 dividends. This model, developed in Ohlson and Juettner (2004), thus explains value as a function of \( g_2 \) (short-term growth in earnings), \( r \) (the discount factor) and \( g \) which will, under reasonable conditions on the dividend policy, correspond to long-term growth in expected earnings, \( g = \frac{\Delta x_t}{\Delta x_{t-1}} \) as \( t \to \infty \). Book value is nowhere to be found, and, to be sure, it cannot be inferred from \((\bar{x}_2, \bar{x}_1, d_1)\) (the accounting data that suffices for \( P_0 \)).

Looking at the two valuation expressions reveals their different accounting variables yet similar structures. As noted, the first should be thought of as a book-value model. Because \( \text{RoE}_1 = \frac{(b_1 + d_1 - b_0)}{b_0} \), one naturally writes \( P_0 = f(b_0, b_1, d_1) \). With respect to the second valuation expression, \( P_0 = f(\bar{x}_1/r, \bar{x}_2/r, d) \equiv h(\bar{x}_1, \bar{x}_2, d_1) \).

In words, the second valuation expression should be thought of as an earnings model of equity value. From a “structural” point of view, the second expression is the same as the first in that \( \text{RoE} \equiv \frac{b_0}{b_1} \), one can read \( \text{RoE} \) as “growth in book value adjusted for dividends” and \( g_2 \) as “growth in earnings adjusted for dividends”.

It is readily shown that the book value model cannot be valid unless \( \bar{x}_{t+1} = \gamma \cdot \bar{x}_t \), and the earnings model cannot be valid unless \( \Delta x_{t+1} = \gamma \cdot \Delta x_t \). Moreover—and this point is critical indeed—whereas \( \bar{x}_{t+1} = \gamma \cdot \bar{x}_t \) implies \( \Delta x_{t+1} = \gamma \cdot \Delta x_t \), the converse does not hold. (For a proof, simply note that \( \bar{x}_{t+1} = \gamma \cdot \bar{x}_t + k, k \neq 0 \), also implies \( \Delta x_{t+1} = \gamma \cdot \Delta x_t \)).

By way of summary, the following has been proved.

**Proposition III** Assume \( P_0 = \text{PVED} \) and clean surplus accounting. Consider the following two models

\[
\begin{align*}
\text{A} & : \quad x_{t+1}^{\text{a}} = \gamma \cdot x_t^{\text{a}}, \quad t \geq 1 \\
\text{B} & : \quad \Delta x_{t+1}^{\text{a}} = \gamma \cdot \Delta x_t^{\text{a}}, \quad t \geq 2
\end{align*}
\]

which implies, and is implied by,

\[
\begin{align*}
P_0 = b_0 + \frac{x_1^{\text{a}}}{R - \gamma} \\
\text{B} : \quad \Delta x_{t+1}^{\text{a}} = \gamma \cdot \Delta x_t^{\text{a}}, \quad t \geq 2
\end{align*}
\]

which implies, and is implied by,

\[
\begin{align*}
P_0 = \frac{x_1}{r} + \frac{1}{r} \cdot \frac{\Delta x_1^{\text{a}}}{R - \gamma} \\
\text{B} : \quad \Delta x_{t+1}^{\text{a}} = \gamma \cdot \Delta x_t^{\text{a}}, \quad t \geq 2
\end{align*}
\]

Model A then implies model B, but the converse is false.

Disregarding the mathematical aspects of the two models, the following statement captures the Proposition’s essence. If the valuation expression \( P_0 = f(b_0, b_1, d_1) \) applies, then so does \( P_0 = h(\bar{x}_1, \bar{x}_2, d_1) \). The converse does not apply, however. The model focusing on earnings is therefore distinctly more robust than the one focusing on book-value.
No apparent reasons suggest why one should hone in on the book value model as a practical matter. This model is arguably of interest only because it becomes part of a demonstration why earnings rather than book values provide the focus in equity valuation.

Model B is without question an earnings model: Its accounting data input—earnings, dividends or change in residual earnings—do not suffice to infer any book value. If one writes \(\Delta x_{t+1}^a\) as a function of \(b_{t+1}, b_t, b_{t-1}\), and suppresses \(d_{t+1}, d_t\), i.e., \(\Delta x_{t+1}^a = \Delta x^e(b_{t+1}, b_t, b_{t-1})\), then \(\Delta x^e(b_{t+1} + k, b_t + k, b_{t-1} + k)\) does not depend on \(k\). Again, the “canceling errors” concept manifests itself. In sharp contrast, \(x_{t+1}^a\), by itself and without the delta-sign, satisfies no such property. Nor will any “canceling error” concept apply to the book-value valuation expression. Given \(P_0 = f(b_0, \hat{b}_1, \hat{d}_1)\), the function \(f(b_0 + k, \hat{b}_1 + k, \hat{d}_1)\) depends on \(k\). (An exception pertains to the singularity when \(P_0 = \hat{x}_1/r\) and \(\hat{x}_1^a = \Delta \hat{x}_1^e = 0\).

Allowing for capital transactions in the analysis has the usual implications. It causes problems in a context of clean surplus accounting on a per share basis. Specifically, putting Model A on a per share basis leads back to the issues raised in Section III. The recursive residual earnings equation requires, at least in principle, a resolution to the awkward question how clean surplus applies. Does bvsp or eps serve as a plug in the clean surplus relation? This question cannot be avoided unless one dismisses concepts of earnings. That is, one has to assume there exists some appropriate bvsp construct such that

\[
(bvps_{t+1} + dps_{t+1} - R \cdot bvps_t) = \gamma \cdot (bvps_t + dps_t - R \cdot bvps_{t-1})
\]

for \(t = 1, 2, \ldots\). Model A so modified avoids capital transaction problems, but at the cost of being devoid of any eps-concept except as a forced definition, \(\text{eps}_t \equiv \Delta \text{bvps}_t + \text{dps}_t\).

Model B avoids the capital transactions and clean surplus issue because it requires no bvsp-construct. Consistent with the real world, the model centers around eps and its future increments, without any reference to how it may reconcile with any bvsp-construct. With respect to the underlying sequence of expected outcomes, it actually misleads to refer to the equation \(\Delta x_{t+1}^e = \gamma \cdot \Delta x_t^e\) insofar it suggests a role for clean surplus and residual earnings. Given the generic irrelevance of book values, the more general, Ohlson-Juettner, version of Model B eliminates clean surplus. Thus, it reduces to the following, which will be referred to as Model C:

\[
z_{t+1}^i = \gamma \cdot z_t^i,
\]

where \(z_t^i \equiv \text{eps}_{t+1} + r \cdot \text{dps}_t - R \cdot \text{eps}_t\) implies, and is implied by, the valuation expression

\[
P_0 = \frac{\text{eps}_1}{r} \cdot \left[\frac{g^2 - g}{r - g}\right].
\]

Model C raises legitimate questions about the extent to which GAAP’s measure of eps aligns with the model’s dynamic requirements. These questions are not easily resolved, especially in light of the complexities associated with the accounting for capital trans-
actions. It is also unclear how theoretical concepts, such as matching or the role of accruals, fit into Model C’s restrictions. Nevertheless, in spite of these ambiguities, the broader point needs to be underscored.\textsuperscript{9} Put succinctly,

Model A ⇒ Model B ⇒ Model C

whereas the converses do not hold. Because all 3 models have the same single degree of freedom—growth (or \( \gamma \))—only Model C is of interest as a practical matter. And this model makes no reference to the clean surplus relation or restricts how book values (adjusted for dividends) are expected to evolve over time.\textsuperscript{10, 11}

9. Concluding Remarks

This paper has examined RIV in a somewhat critical light, but these observations should not overshadow that the RIV model has contributed to accounting theory. Two points deserve to be mentioned.

First, one can use RIV to identify residual earnings as a measure of a firm’s value creation. By taking the present value of the expected residual earnings, the difference between market to book values comes into focus. Many students of accounting undoubtedly have found the underlying analyses insightful. Among other things, these have shown that the market-to-book ratio relates directly to a firm’s expected profitability, and that a firm’s profitability in turn relates to conservative accounting. Second, RIV captures settings with dividend policy irrelevancy effectively. Under reasonable conditions, residual earnings do not depend on dividends. This independence property usefully shows a close connection between accounting constructs and dividend policy irrelevancy. Moreover, RIV streamlines the analysis of formal models because there is a simple way of evaluating PVED that does not require an explicit sequence of expected dividends. Ohlson (2001) emphasizes this point. But it also follows that core insights of such models in no way hinge on RIV. Nor do these analyses implicitly suggest that RIV is a “natural” or “preferable” way of looking at equity valuation. Though analytical expediency can potentially lead to conceptual and practical implications, one need to keep in mind RIV may not be the only convenient scheme available to evaluate PVED.

RIV’s most deficient aspect pertains to the elevated status it assigns to the current and expected book values. It would be more accurate to relabel RIV as ABG, where the new acronym stands for abnormal book values growth. That is, the model takes the present value of above/below benchmark increments in expected book values, adjusted for dividends, to explain the market minus book value premium. There is no need to refer to earnings or residual earnings. In fact, earnings enter the model via the clean surplus relation in a somewhat underhanded way. And capital transactions illustrate the contrived nature of relying on clean surplus to introduce earnings. Taken in its totality, it seems difficult to argue that RIV appeals conceptually or that investors ought to rely on it as a premier, practical, valuation tool.
The analytical framework that leads to accounting-based valuation formulae shows that RIV provides but one out of many possibilities. Within the available class, one can avoid all of RIV’s disadvantages. One simply replaces RIV’s expected book value with the subsequent period’s expected earnings capitalized. This valuation approach, AEG, anchors valuation to expected next-period earnings capitalized, and then it expresses this premium in terms of subsequent increments in expected earnings, adjusted for dividends. The Appendix summarizes all the major properties of the model.

Compared to RIV, AEG embeds three distinct advantages:

First, the model demands no book value construct. Nor does the model rely on clean surplus. The relatively weak assumptions mean that eps is as easy to work with as total dollar earnings. And the possibility of changes in shares outstanding has no adverse implications per se.

Second, a focus on earnings can never be worse than a focus on book value, but the converse is false. As the three propositions of this paper suggest, the power of the partial “canceling error” concept seems undeniable. It leads to the important idea that, \textit{ex ante}, earnings capitalized approximate market value more closely than book value.

Finally, investment practice revolves around earnings and their subsequent growth, not book values and their subsequent growth. AEG, not RIV, builds in this central organizing principle of equity valuation. RIV has arguably been “oversold” by text-books as a practical tool in investment analysis; a switch to AEG seems warranted. All the theoretical arguments support this contention.

\section*{Appendix}

The Abnormal Earnings Growth (AEG) Model Summary

\subsection*{Formula}

\[ P_0 = \frac{\text{eps}_1}{r} + \sum_{t=1}^{\infty} R^t z_t \]

where

\[ z_t \equiv \left[ \frac{\text{eps}_{t+1}}{r} + r \cdot \text{dps}_t - R \cdot \text{eps}_t \right] / r = \frac{\Delta \text{eps}_{t+1} - r \cdot (\text{eps}_t - \text{dps}_t)}{r} \]

The term \( r \cdot (\text{eps}_t - \text{dps}_t) \) modifies \( \Delta \text{eps}_{t+1} \) for “the expected increment of period \( t+1 \) earnings due to earnings retained during the prior period, \( r \)”. Hence the special case \( z_t = \Delta \text{eps}_{t+1} / r \) applies only if the per share payout is 100%. The payout does not, however, affect \( z_t \) under suitable assumptions; see IV below.

\subsection*{Properties}

(a) \( z_t = 0 \) if

(i) the “firm” is a savings-account
(ii) the firm uses mark-to-market accounting (in expectation)

(iii) the firm uses permanent earnings accounting (in expectation) Note that (i) is a special case of (ii), which in turn is a special case of (iii).

(b) \( P_0 > \frac{\bar{\text{eps}}_1}{r} \) if \( z_t > 0 \)

That is, positive expected abnormal earnings growth implies the price-to-forward-earnings ratio exceeds the benchmark, \( 1/r \).

**Baseline Models**

(a) Suppose that for all \( t \geq 1 \)

\[ z_{t+1} = \gamma \cdot z_t, \quad 1 \geq \gamma. \]

Then

\[ P_0 = \frac{\text{eps}_1}{r} \cdot \left[ \frac{g_2 - g}{r - g} \right] \]

where

\[ g_2 = \frac{\Delta \text{eps}_2 + r \cdot \bar{\text{dps}}_1}{\text{eps}_1} \] (short-term growth)

\[ g = \gamma - 1 = \lim_{t \to \infty} \frac{\Delta \text{eps}_{t+1}}{\text{eps}_t} \] (long-term growth)

Solving for \( r \) leads to the cost-of-capital formula

\[ r = A + \sqrt{A^2 + \frac{\text{eps}_1}{P_0} \cdot \left( \frac{\Delta \text{eps}_2}{\text{eps}_1} - g \right)} \]

where

\[ A \equiv \frac{1}{2} \left( g + \frac{\text{dps}_1}{P_0} \right) \]

As a first special case, if \( g = 0 \) then \( r = \sqrt{\text{PEG}^{-1}} \)

where

\[ \text{PEG} \equiv \frac{(P_0/\text{eps}_1)}{g_2} \]

As a second special case, if \( \frac{\Delta \text{eps}_2}{\text{eps}_1} = g \) then the constant growth model applies and \( r = g + \frac{\text{dps}_1}{P_0} \).

(b) An assumption of Clean Surplus added to \( z_{t+1} \equiv \gamma z_t \). If \( \bar{x}_t' \equiv \text{residual earnings} \)
then clean surplus and the definition of $z_t$ imply

$$z_t = \Delta x^{\alpha}_{t+1} / r$$

It also follows that

$$P_0 = \frac{\text{eps}_1}{r} + \frac{1}{r} \cdot \frac{\Delta x^{\alpha}_n}{r-g}$$

Note: This model, (b), is strictly less general than (a).

(c) $x^{\alpha}_{t+1} = \gamma \cdot \ddot{x}_t$ in lieu of $\Delta x^{\alpha}_{t+1} = \gamma \cdot \dot{x}_t$. As one—but only one—of the solutions to $\Delta x^{\alpha}_{t+1} = \gamma \Delta x^{\alpha}_t$, consider

$$\ddot{x}_{t+1} = \gamma \cdot \ddot{x}_t.$$

In that case

$$P_0 = b_0 \cdot \left[ \frac{R\delta_1 - g}{r-g} \right] = b_0 + \frac{\ddot{x}_1}{r-g},$$

where $b_0 = \text{date 0 book-value (per share)}$.

Solving for $r$ leads to $r = g \cdot [(P_0 - b_0)/P_0] + \ddot{x}_1/r$; that is $r$ is linear in the book-to-market premium and the forward-earnings yield. (This model does not seem to be known by finance researchers who suggest (somewhat vaguely) that the market-to-book ratio should relate positively to risk.)

Note: The model above, (c), is strictly less general than (b).

**MM-considerations**

In I, div-policy irrelevancy applies if (i) $\partial \text{eps}_1 / \partial \text{dps}_0 = -r$ and (ii) $\partial z_t / \partial d_{t-1} = 0$. These conditions correspond to the idea that the payment of dividends on the margin has the effects of lowering next-period eps with $-r$ for each dollar of dividends.

In III (a), dividend policy irrelevancy applies in the following sense: Consider the dynamic

$$\text{eps}_{t+1} = R \cdot \text{eps}_t - r \cdot \text{dps}_t + r \cdot z'_t$$

$$z'_{t+1} = \gamma \cdot z'_t$$

$$\text{dps}_{t+1} = \delta_1 \cdot \text{eps}_t + \delta_2 \cdot \text{dps}_t + \delta_3 \cdot z'_t.$$

Then PVED does not depend on $\delta_1, \delta_2, \delta_3$.

The above analysis does not depend on whether one assume clean surplus or not. Thus, it applies to model III (b) as well.

With respect to III (c), consider

$$(\ddot{b} + \ddot{d})_{t+1} = R \cdot (\ddot{b} + \ddot{d})_t - R \cdot \dot{d}_t + z''_t$$

$$z''_{t+1} = \gamma \cdot z''_t.$$
\[ \overline{dps}_{t+1} = \delta_1 \cdot b_t + \delta_2 \cdot d^2ps_t + \delta_3 \cdot z''_t \]

Then PVED does not depend on $\delta_1$, $\delta_2$, $\delta_3$.

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Notes

1. This paper depends significantly on some pre-existing papers of mine, in particular, Ohlson (1999) and (2002), and Ohlson and Juettner (2004). Penman (2004) discusses many of the issues associated with residual income valuation. Also, an earlier version of the current paper exists, “Residual Income Valuation: The Problems.”


3. The expression defines residual earnings if one eliminates earnings via the clean surplus equation.

4. Another way of thinking about AEG recognizes that RIV “applies for all accounting principles”. Hence, one can “measure” bvps in terms of $ep_{1-t}/r$.


7. This permanent earnings concept is due to Ryan (1988).

8. Reference to this model can also be found in Brief and Lawson (1992), Easton (2004), Fairfield (1994), Gjesal (1999) and Hutton (2000). All of these papers use too restrictive assumption. Either they assume $x_{t+1} = \gamma x_t^t$ for $t = 0$ as well as $t \geq 1$ or they put restrictions on the dividend policy.

9. One can potentially deal with this issue using the research methodology found in Ohlson and Zhang (1988).


11. Ozair (2003) identifies information dynamics that sustain the Ohlson and Juettner model.

References


