Reconciling the Return Predictability Evidence

Martin Lettau
Columbia University, New York University, CEPR, NBER

Stijn Van Nieuwerburgh
New York University and NBER

Evidence of stock-return predictability by financial ratios is still controversial, as documented by inconsistent results for in-sample and out-of-sample regressions and by substantial parameter instability. This article shows that these seemingly incompatible results can be reconciled if the assumption of a fixed steady state mean of the economy is relaxed. We find strong empirical evidence in support of shifts in the steady state and propose simple methods to adjust financial ratios for such shifts. The in-sample forecasting relationship of adjusted price ratios and future returns is statistically significant and stable over time. In real time, however, changes in the steady state make the in-sample return forecastability hard to exploit out-of-sample. The uncertainty of estimating the size of steady-state shifts rather than the estimation of their dates is responsible for the difficulty of forecasting stock returns in real time. Our conclusions hold for a variety of financial ratios and are robust to changes in the econometric technique used to estimate shifts in the steady state. (JEL 12, 14)

1. Introduction

The question of whether stock returns are predictable has received an enormous amount of attention. This is not surprising because the existence of return predictability is not only of interest to practitioners but also has important implications for financial models of risk and return. One branch of the literature asserts that expected returns contain a time-varying component that implies predictability of future returns. Due to its persistence, the predictive component is stronger over longer horizons than over short horizons. Classic predictive variables are financial ratios, such as the dividend-price ratio, the earnings-price ratio, and the book-to-market ratio (Rozeff, 1984; Fama and French, 1988; Campbell and Shiller, 1988; Cochrane, 1991; Goetzmann and Jorion, 1993; Hodrick, 1992; Lewellen, 2004, and others), but other variables have also been found to be powerful predictors of long-horizon returns (e.g., Lettau and...
Ludvigson, 2001; Lustig and Van Nieuwerburgh, 2005a; Menzly, Santos, and Veronesi, 2004; Piazzesi, Schneider, and Tuzel, 2006). Moreover, these studies conclude that growth rates of fundamentals, such as dividends or earnings, are much less forecastable than returns, suggesting that most of the variation of financial ratios is due to variations in expected returns.

These conclusions are controversial because the forecasting relationship of financial ratios and future stock returns exhibits a number of disconcerting features. First, correct inference is problematic because financial ratios are extremely persistent; in fact, standard tests leave the possibility of unit roots open. Nelson and Kim (1993); Stambaugh (1999); Ang and Bekaert (2006); Ferson, Sarkissian, and Simin (2003); and Valkanov (2003) conclude that the statistical evidence of forecastability is weaker once tests are adjusted for high persistence. Second, financial ratios have poor out-of-sample forecasting power, as shown in Bossaerts and Hillion (1999) and Goyal and Welch (2003, 2004), but see Campbell and Thompson (2007) for a different interpretation of the out-of-sample evidence.¹ Third, and related to the poor out-of-sample evidence, the forecasting relationship of returns and financial ratios exhibits significant instability over time. For example, in rolling 30-year regressions of annual log CRSP value-weighted returns on lagged log dividend-price ratios, the ordinary least squares (OLS) regression coefficient varies between zero and 0.5 and the associated $R^2$ ranges from close to zero to 30%, depending on the subsample. Not surprisingly, the hypothesis of a constant regression coefficient is routinely rejected (Viceira, 1996; Paye and Timmermann, 2005).

In addition to concerns that return forecastability might be spurious, the benchmark model of time-varying expected returns faces additional challenges. The extreme persistence of price ratios implies that expected returns have to be extremely persistent as well. However, if shocks to expected returns have a half-life of many years or even decades, as implied by the high persistence of financial ratios, they are unlikely to be linked to many plausible economic risk factors, such as those linked to business cycles. Instead, researchers have to identify slow-moving factors that are primary determinants of equity risk. In addition, the extraordinary valuation ratios in the late 1990s represent a significant challenge for the benchmark model. Given the historical record of returns, fundamentals, and prices, it is exceedingly unlikely that persistent stationary shocks to expected returns are capable of explaining price multiples like those seen in 1999 or 2000.

In summary, the return predictability literature has yet to provide convincing answers to the following four questions: What is the source of parameter instability? Why is the out-of-sample evidence so much weaker than the in-sample evidence? Why has even the in-sample evidence disappeared in the late 1990s?

¹ There is some ambiguity about the use of the term “forecast” in this literature. Most papers use “forecast” to refer to in-sample regressions using the entire sample. In contrast, predictions using only currently available data are referred to as “out-of-sample forecasts.” We follow this convention.
Why are price ratios extremely persistent? In this paper, we show that these puzzling empirical patterns can be explained if the steady-state mean of financial ratios has changed over the course of the sample period. Such changes could be due to changes in the steady-state growth rate of economic fundamentals resulting from permanent technological innovations and/or changes in the expected return of equity caused by, for example, improved risk sharing, changes in stock market participation, changes in the tax code, or lower macroeconomic volatility.

Using standard econometric techniques, we show that the hypothesis of permanent changes in the mean of various price ratios is supported by the data. We then ask how such changes affect the forecasting relationship of returns and lagged price ratios. Standard econometric techniques that assume that the regressor is stationary will lead to biased estimates and incorrect inference. However, since deviations of price ratios from their steady-state values are stationary, it is straightforward to correct for the nonstationarity if the timing and magnitudes of shifts in steady states can be estimated. We conduct tests that incorporate such adjustments from the perspective of an econometrician with access to the entire historical sample (in-sample tests), as well as from the perspective of an investor who forecasts returns in real time (out-of-sample tests).

Our in-sample results conclude that “adjusted” price ratios have favorable properties compared to unadjusted price ratios. In the full sample, the slope coefficient in regressions of annual log returns on the lagged log dividend-price ratio increases from 0.094 for the unadjusted ratio to 0.235 and 0.455 for the adjusted ratio with one and two steady-state shifts, respectively. While the statistical significance of the coefficient on the unadjusted dividend-price ratio is marginal, coefficients on the adjusted dividend-price ratios are strongly significant. Finally, the regression coefficients using adjusted price ratios as regressors are more stable over time. We find similar differences for other price ratios, such as the earnings-price ratio and the book-to-market ratio.

In real time, however, the changes in the steady state are not only difficult to detect but also estimated with significant uncertainty, making the in-sample return forecastability hard to exploit. Results for out-of-sample forecasting tests reflect this difficulty. While adjusted price ratios have superior out-of-sample forecasting power relative to their unadjusted counterparts, they do not outperform the benchmark random walk model. Why does the real-time prediction fail to beat the random walk model? In real time an investor faces two challenges. First, she has to estimate the timing of a break. Second, if she detects a new break, she has to estimate the new mean after the break occurs. If the new break occurred toward the end of the sample that the investor has access to, the new mean has to be estimated using a small number of observations and is subject to significant uncertainty. We perform additional tests to evaluate the relative difficulty of estimating the break dates versus estimating the means relative to the pure out-of-sample forecasts and the ex post adjusted dividend-price ratio. We find that (i) the estimation of the break dates
in real time is not crucial and the resulting prediction errors are smaller than for the random walk model, and (ii) that the estimation of the magnitude of the break in the mean dividend-price ratio entails substantial uncertainty, and is ultimately responsible for the failure of the real-time out-of-sample predictions to beat the random walk. These findings can explain the lack of out-of-sample predictability documented by Goyal and Welch (2004).

Several papers have explored the impact of structural breaks on return predictability. For example, Viceira (1996) and Paye and Timmermann (2005) reported evidence in favor of breaks in the OLS coefficient in the forecasting regression of returns on the lagged dividend-price ratio. Our focus is instead on shifts in the mean of financial ratios, which, in turn, render the forecasting relationship unstable if such shifts are not taken into account. In other words, in contrast to Viceira (1996) and Paye and Timmermann (2005), we focus on the behavior of the mean of price ratios instead of the behavior of the slope coefficient. Pastor and Stambaugh (2001) use a Bayesian framework to estimate breaks in the equity premium. They find several shifts in the equity premium since 1834 and identify the sharpest drop in the 1990s, which is consistent with the timing of the shift in price ratios identified in this paper. This paper is also related to the recent literature on inference in forecasting regressions with persistent regressors (e.g., Amihud and Hurwich, 2004; Ang and Bekaert, 2006; Campbell and Yogo, 2002; Lewellen, 2004; Torous, Volkanov, and Yan, 2004; Eliasz, 2005). In these papers, asymptotic distributions for OLS regressions are derived under the assumption that the forecasting variable is a close-to unit, yet stationary, root process. In contrast, we allow for the presence of a small but statistically important nonstationary component in forecasting variables.

The rest of the paper is organized as follows. In Section 2, we establish that the standard dividend-price ratio does not significantly forecast stock returns or dividend growth. We find much stronger evidence for return predictability in various subsamples. The slope coefficient in the return equation is much smaller in the full sample than in any of the constituent subsamples, which confirms the instability of the forecasting relationship over time. In Sections 3 and 4, we show how changes in the steady-state affect the dividend-price ratio and other price ratios. For the log dividend-price ratio, we find evidence for either one break in the early 1990s or two breaks around 1954 and 1994. Other valuation ratios, such as the earnings-price ratio and the book-to-market value ratio, exhibit similar breaks. We show that filtering out this nonstationary component yields adjusted price ratios that have strong and stable in-sample return predictability. In Section 5, we study out-of-sample predictability. We use a recursive Perron procedure that estimates both the break dates and the means of the regimes in real time. We show that using the break-adjusted dividend-price series produces superior one-step-ahead return forecasts compared to using the unadjusted dividend-price series, but does slightly worse than the naive random walk model. Using a Hamilton (1989) regime-switching model, we show that if the investor did not have to estimate regime means in real
time, but only the regime-switching dates, her out-of-sample forecast would improve substantially, and beat the random walk. The Hamilton procedure leads to slightly later break dates but predictability results that are virtually as good as those when the (ex post) break dates were known and used. In sum, the hardest part of real-time out-of-sample prediction in the presence of regimes is the estimation uncertainty about the mean of the new regime. In Section 6, we consider a vector error correction model that includes the return and dividend growth predictability equations and imposes a joint present value restriction on the slope parameters from both equations. We find that this restriction is satisfied when we use the adjusted dividend-price ratio as an independent variable, but not when we use the unadjusted series. We use this framework to estimate long-horizon regressions. Finally, in Section 7, we find that our simple model serves as a plausible data generating process. It is able to replicate both the findings of no predictability when the unadjusted dividend-price ratio is used and the findings of in-sample and out-of-sample predictability when the adjusted series is used.

2. Instability of Forecasting Relationships

In this section, we document the instability of the forecasting relationship of returns, dividend growth, and the lagged dividend-price ratio. The forecasting relationship of returns and other financial ratios (such as the earnings-price ratio and the book-to-market ratio) and alternative measures of dividends (such as accounting for repurchases or considering only dividend-paying firms) are similar and will be presented later. The data are based on annual CRSP value-weighted returns from 1927 to 2004 and are described in Appendix A. The top panel of Figure 1 shows the estimation results for the forecasting regression of demeaned returns on the demeaned lagged dividend-price ratio using 30-year rolling windows:

\[ r_{t+1} - \bar{r} = \kappa_r (d_{t} - \bar{d}p) + \tau_{t+1}, \]  

(1)

where \( r_t \) denotes the log return, \( d_{t} \) denotes the log dividend-price ratio \( d_t - p_t \), and \( \bar{r} \) and \( \bar{d}p \) denote the sample means of returns and the log dividend-price ratio in each of the subsamples, respectively. The top panel plots the slope coefficient \( \kappa_r \) along with two standard error bands. The instability of the forecasting relationship is strikingly illustrated by the variation of the return predictability coefficient over time. The estimates of \( \kappa_r \) are around 0.5 in the subsamples ending in the late 1950s and in the samples ending in the early 1980s to the mid-1990s. In contrast, \( \kappa_r \) is much smaller for the samples ending in the mid-1960s and is close to zero and statistically insignificant in samples ending in the late 1990s and early 2000s. Similarly, the \( R^2 \) of the forecasting regression displays instability with values ranging from 34% in 1982 to 0% at the end of the 1990s (not shown). This evidence has led some researchers to conclude that the dividend-price ratio
does not forecast stock returns, or at least not robustly so. Not surprisingly, the hypothesis of a constant regression coefficient is routinely rejected.

We also estimate a predictability regression for demeaned dividend growth rates:

\[ \Delta d_{t+1} - \bar{d} = \kappa_d (dp_t - \bar{dp}) + \tau_{d,t+1}, \]  

where \( d_t \) denotes log dividends and \( \bar{d} \) denotes the sample mean of dividend growth. Dividend growth rates are even less forecastable than returns. For most of the sample, the point estimate is not statistically significantly different from
zero, and the regression $R^2$ never exceeds 16% (not shown). Interestingly, the dividend-price ratio at the end of the 1990s seems to forecast neither stock returns nor dividend growth. This is a conundrum from the perspective of any present value model (see Section 3.1), as also pointed out by Cochrane (2007) and Bainsbergen and Koijen (2007).

The left two columns of Table 1, denoted “No Break,” report the coefficients $\kappa_r$ and $\kappa_d$ from equations (1) and (2) and their asymptotic standard errors for the entire 1927–2004 sample, as well as for various subsamples. The first row shows that the dividend-price ratio marginally predicts stock returns (first column); the coefficient is significant at the 5% level if asymptotic standard errors are used for inference. However, small-sample standard errors computed from a Bootstrap simulation suggest that the coefficient $\kappa_r$ is not statistically different from zero for the entire sample.$^2$ The dividend-price ratio does not forecast dividend growth at conventional significance levels (third column). Thus, we cannot reject the hypothesis that the dividend-price ratio forecasts neither dividend growth nor returns.

Rows 2 and 3 report the results for two nonoverlapping samples that span the entire period: 1927–1991 and 1992–2004. We will justify this particular choice of subsamples in Section 3. The estimates of $\kappa_r$ display a remarkable pattern across subsamples: in both subsamples, $\kappa_r$ is much larger than its estimate in the whole sample. In fact, the estimates are almost identical in the two subsamples: .2353 in the 1927–1991 subsample compared to .2351 in the later 1992–2004 subsample. Yet, when we join the two subsamples, the point estimate drops to .094. In addition, $\kappa_r$ is strongly statistically significant in both subsamples but only marginally significant in the whole sample. Confirming the instability of $\kappa_r$ estimates, row 4 reports the results of a Chow test, which rejects the null hypothesis of no structural break in 1991 at the 4% level. Finally, the dividend growth forecasting relationship displays less instability, and the coefficient remains insignificant in both subsamples.

The pattern of $\kappa_r$ is not unique to the specific subsamples chosen. We obtain very similar results when we use three nonoverlapping subsamples: 1927–1954, 1955–1994, and 1995–2004 (bottom half of Table 1). Again, we find that the return predictability coefficient $\kappa_r$ is estimated to be much higher in each of the three subsamples than in the entire subsample. In row 5, the predictability coefficient is .09, whereas it is .51, .38, and .53 in rows 6–8, respectively.

$^2$ Asymptotic standard errors may be a poor indicator of the estimation uncertainty in small samples, and the $p$ values for the null of no predictability may be inaccurate. The asymptotic corrections advocated by Hansen and Hodrick (1980) have poor small-sample properties. Ang and Bekaert (2006) find that use of those standard errors leads to overrejection of the no-predictability null. The Bootstrap exercise imposes the null of no predictability and asks how likely it is to observe the estimated $\kappa_r$ coefficients reported in the first column of Table 1. We find that the small-sample $p$ value for $\kappa_r$ is 6.8% compared to an asymptotic $p$ value of 4.1%. We also conduct a second Bootstrap exercise to find the small-sample bias in the return coefficient. Consistent with Stambaugh (1999), we find an upward bias. If the true value is .094, the Bootstrap exercise estimates a coefficient of .115. Detailed results are available upon request. The empirical size of tests-based asymptotic and bootstrapped standard errors tends to be larger than their nominal size if the regressor is highly persistent (e.g., Amihud, Hurvich, and Wang, 2005). Alternative tests with better size properties weaken the evidence for forecastability with the dividend-price ratio further.
Table 1
Forecasting returns and dividend growth with the dividend-price ratio

<table>
<thead>
<tr>
<th>Sample</th>
<th>No break</th>
<th>One break</th>
<th>No break</th>
<th>One break</th>
<th>No break</th>
<th>One break</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Returns</td>
<td>Dividend growth</td>
<td>Excess returns</td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1927–2004</td>
<td>.094</td>
<td>.005</td>
<td>.113</td>
<td>.235</td>
<td>.019</td>
<td>.282</td>
</tr>
<tr>
<td></td>
<td>(.046)</td>
<td>(.037)</td>
<td>(.049)</td>
<td>(.058)</td>
<td>(.047)</td>
<td>(.059)</td>
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<td></td>
<td></td>
<td>[.038]</td>
<td>[.000]</td>
<td></td>
<td>[.014]</td>
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<tr>
<td></td>
<td>(.065)</td>
<td>(.053)</td>
<td>(.071)</td>
<td>(.065)</td>
<td>(.053)</td>
<td>(.071)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[.100]</td>
<td>[.006]</td>
<td></td>
<td>[.006]</td>
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</tr>
<tr>
<td></td>
<td>(.134)</td>
<td>(.103)</td>
<td>(.139)</td>
<td>(.134)</td>
<td>(.103)</td>
<td>(.139)</td>
</tr>
<tr>
<td>Chow F stat</td>
<td>3.408</td>
<td>.295</td>
<td>4.383</td>
<td>.235</td>
<td>.295</td>
<td>.235</td>
</tr>
</tbody>
</table>

This table reports estimation results for the equations \( r_{t+1} - \bar{r} = \kappa_r (dp_t - \bar{dp}) + \tau_{r_{t+1}} \) and \( \Delta d_{t+1} - \bar{d} = \kappa_d (dp_t - \bar{dp}) + \tau_{d_{t+1}} \). The first two columns report the equation for returns. The next two columns report the predictability equation for dividend growth. The last two columns are for excess returns instead of gross returns. The table reports point estimates and standard errors in parentheses of \( \kappa_r \) and \( \kappa_d \), as well as regression \( R^2 \) in square brackets. The parameters \( (\bar{r}, \bar{d}, \bar{dp}) \) are the sample means of log returns \( r \) (log excess returns in the last two columns), log dividend growth \( \Delta d \) and the log dividend-price ratio \( dp \). The top panel compares the case of no break in the log dividend-price ratio (\( dp \) is fixed) with the case where there is a break in the log dividend-price ratio: \( dp_1 \) is the sample mean log dividend-price ratio for 1927–1991 and \( dp_2 \) is the mean for 1992–2004. The estimation is by GMM, where the moments are the OLS normal conditions. Standard errors are by Newey-West with four lags. Row 1 reports results for the full sample; rows 2 and 3 report results for two subsamples. Row 4 reports the \( F \)-statistic and associated \( p \)-value from a Chow test with null hypothesis of no structural break in 1991 in the forecasting equations. The bottom panel compares the case of no break in the log dividend-price ratio (\( dp \) is fixed) with the case where there are two breaks in the log dividend-price ratio: \( dp_1 \) is the sample mean log dividend-price ratio for 1927–1991 (row 6), \( dp_2 \) is the mean for 1955–1994 (row 7), and \( dp_3 \) is the mean for 1995–2004 (row 8). Row 9 reports the \( F \)-statistic and associated \( p \)-value from a Chow test with null hypothesis of no structural breaks in 1954 and 1994 in the forecasting equations.

Moreover, it is statistically significant in each subsample. Row 9 shows that we strongly reject the joint null hypothesis of parameter stability in 1954 and 1994. For dividend growth, the evidence is more mixed. We fail to reject the same null hypothesis of no breaks in 1954 and 1994, but the \( \kappa_d \) coefficient is marginally statistically different from zero in rows 7 and 8.
Finally, the last two columns repeat the analysis using returns in excess of a 90-day treasury-bill rate instead of gross returns. The exact same findings hold for excess returns. In the rest of the analysis, we proceed with gross returns only.

We conclude that the forecasting relationship between returns and the dividend-price ratio is unstable over time. Coefficient estimates of $\kappa_r$ are almost identical in nonoverlapping subsamples, but the point estimate for the whole sample is much lower than it is in each of the subsamples. Next, we investigate what might explain this intriguing pattern of the regression coefficients that links returns to past dividend-price ratios.

3. Steady-State Shifts and Forecasting

The macroeconomics literature has recently turned to models with persistent changes in fundamentals to explain the dramatic change in valuation ratios in the bull market of the 1990s. Most such models imply a persistent decline in expected returns or an increase in the steady-state growth rate of the economy. Lettau, Ludvigson, and Wachter (2004) argue that a persistent decline in the volatility of aggregate consumption growth leads to a decline in the equity premium. Another class of models argues for persistent improvements in the degree of risk sharing among households or regions, either due to developments in the market for unsecured debt or the market for housing-collateralized debt (Krueger and Perri, 2005; Lustig and Van Nieuwerburgh, 2006). In the model of Lustig and Van Nieuwerburgh (2007), the improvement in risk sharing implies a persistent decline in the equity premium. McGrattan and Prescott (2005) argue that persistent changes in the tax code can explain the persistent decline in the equity premium. Lastly, models of limited stock market participation argue that the gradual entry of new participants has persistently depressed equity premia (Vissing-Jorgensen, 2002; Calvet, Gonzalez-Eiras, and Sadini, 2003; Guvenen, 2003). Other models argue that there was a persistent increase in the long-run growth rate of the economy in the 1990s (Quadrini and Jermann, 2003; Jovanovic and Rousseau, 2003). The first set of models lowers the long-run required return of equity ($\bar{r}$); the last set of models raises the long-term growth rate of the economy ($\bar{d}$). Intuition based on the Gordon growth model implies that either effect lowers the steady-state level of the dividend-price ratio $\bar{d}/\bar{p}$. In this section, we augment the Campbell-Shiller framework for such changes in $\bar{d}/\bar{p}$, we estimate these shifts in the data, and explore their implications for return predictability.

3.1 Changes in the mean of price ratios

The standard specification of stock returns and forecasting variables assumes that all processes are stationary around a constant mean. For example, Stambaugh (1986, 1999), Mankiw and Shapiro (1986), Nelson and Kim
(1993), and Lewellen (1999) considered the following model:

\[ r_{t+1} = \bar{r} + \kappa r_t + \tau'_{t+1} \]  
\[ y_t = \bar{y} + v_t \]

The mean of the forecasting variable \( y_t, \bar{y} \), is constant and the stochastic component \( v_t \) is assumed to be stationary, often specified as an AR(1) process. Means of financial ratios are determined by properties of the steady state of the economy. For example, the mean of the log dividend-price ratio \( \bar{dp} \) is a function of the growth rate \( \bar{d} \) of log dividends and expected log return \( \bar{r} \) in steady state

\[ \bar{dp} = \log(\exp(\bar{r}) - \exp(\bar{d})) - \bar{d}, \]

whereas the stochastic component depends on expected future deviations of returns and dividend growth from their steady-state values (Campbell and Shiller, 1988):

\[ dp_t = \bar{dp} + E_t \sum_{j=1}^{\infty} \rho^{j-1} [(r_{t+j} - \bar{r}) - (\Delta d_{t+j} - \bar{d})], \]

where \( \rho = (1 + \exp(\bar{dp}))^{-1} \) is a constant. Similar equations can be derived for other financial ratios (e.g., Vuolteenaho, 2000). Berk, Green, and Naik (1999) show how stock returns and book-to-market ratios are related in a general equilibrium model.

A crucial assumption is that the steady state of the economy is constant over time: The average long-run growth rate of the economy as well as the average long-run return of equity are fixed and not allowed to change. However, if either the steady-state growth rate or expected return were to change, the effects on financial ratios and their stochastic relationships with returns would be profound. Even relatively small changes in long-run growth and/or expected return have large effects on the mean of the dividend-price ratio, as can be seen from Equation (5). The effects of steady-state shifts on other valuation ratios, such as the earnings-price ratio and the book-to-market ratio, are similar. In this paper, we entertain the possibility that the steady state of the US economy has indeed changed since 1926, and we study the effect of these changes on the forecasting relationship of returns and price ratios.

A steady state is characterized by long-run growth and expected return. Any short-term deviation from steady state is expected to be only temporary and the economy is expected to return to its steady state eventually. Thus, steady-state growth and expected return must be constant in expectations, but the steady state might shift unexpectedly. Correspondingly, we assume that

\[ E_t \bar{r}_{t+j} = \bar{r}_t, \quad E_t \bar{d}_{t+j} = \bar{d}_t, \quad E_t \bar{dp}_{t+j} = \bar{dp}_t. \]

3 Although the log dividend-price ratio is a nonlinear function of steady-state returns and growth, we assume that the steady-state log dividend-price ratio is also (approximately) a martingale: \( E_t \bar{dp}_{t+j} = \bar{dp}_t \). This assumption
The log linear framework introduced earlier illustrates the effect of time-varying steady states, though none of our results depend on the accuracy of the approximation. Just as in the case with constant steady state, the log dividend-price ratio is the sum of the steady-state dividend-price ratio and the discounted sum of expected returns minus expected dividend growth in excess of steady-state growth and returns:

$$dp_t = \overline{dp}_t + E_t \sum_{j=1}^{\infty} \rho_t^{j-1}[(r_{t+j} - \overline{r}_t) - (\Delta d_{t+j} - \overline{d}_t)].$$

(7)

where $\rho_t = (1 + \exp(dp_t))^{-1}$. The important difference of Equation (7) compared to Equation (6) is that the mean of the log dividend-price ratio is no longer constant. In fact, it not only varies over time but it is nonstationary. If, for example, the steady-state growth rate increases permanently, the steady-state dividend-price ratio decreases and the current log dividend-price ratio declines permanently. While the log dividend-price ratio contains a nonstationary component, it is important to note that deviations of $dp_t$ from steady states are stationary as long as deviations of dividend growth and returns from their respective steady states are stationary, an assumption we maintain throughout the paper. In other words, the dividend-price ratio $dp_t$ itself contains a nonstationary component $\overline{dp}_t$ but the appropriately demeaned dividend-price ratio $dp_t - \overline{dp}_t$ is stationary. The implications for forecasting regressions with the dividend-price ratio are immediate. First, in the presence of steady-state shifts, a nonstationary dividend-price ratio is not a well-defined predictor and this nonstationarity could cause the empirical patterns described in the previous section. Second, the dividend-price ratio must be adjusted to remove the nonstationary component $\overline{dp}_t$ to render a stationary process.

While we emphasized the effect of steady-state shifts on the dividend-price ratio, the intuition carries through to other financial ratios. Changes in the steady state have similar effects on the earnings-price ratio and the book-to-market ratio. However, other permanent changes in the economy, such as changes in

is justified for the specific processes for steady-state returns and growth that we will consider next. Appendix C spells out a simple asset pricing model where the price-dividend ratio in levels follows a (bounded) martingale. It shows that $\overline{dp}$ and $\overline{d}$ are approximate (bounded) martingales.

4 Appendix B presents a detailed derivation. Under our assumption, the log approximation in a model with time-varying steady states is as accurate as the approximation for the corresponding model with constant steady state. In fact, the ex ante expressions of the approximate log dividend-price ratio (6) and (7) are exactly the same. Only their ex post values are different in periods when the steady state shifts.

5 Of course, in a finite sample, it is impossible to conclusively distinguish a truly permanent change from an extremely persistent one. Thus, our insistence of nonstationarity might seem misguided. However, the important insight is that the dividend-price ratio is not only a function of (less) persistent changes in expected growth rates and expected returns that could potentially have cyclical sources but is also affected by either extremely persistent or permanent structural changes in the economy. This distinction turns out to be very useful, as we will show in the remainder of the article. In this sense, our assumption of true nonstationarity can be regarded to include “extremely persistent but stationary.” In a finite sample, the conclusions will be the same in either setting. The distinction of “permanent” versus “extremely persistent” is important, however, for structural asset pricing models because permanent shocks might have a much larger impact on prices than very persistent ones.
payout policies, could affect different ratios differently. In the following section, we provide evidence that steady-state shifts have occurred in our sample and propose simple methods to adjust financial ratios for such shifts.

3.2 Steady-state shifts in the dividend-price ratio

Has the steady-state relationship of growth rates and expected returns shifted since the beginning of our sample in 1926? If so, have these shifts affected the stochastic relationship between returns and price ratios? In this section, we use econometric techniques that exploit the entire sample to detect changes in the steady state. In Section 5, we study how investors in real time might have assessed the possibility of shifts in the steady states without the benefit of knowing the whole sample. In both cases, there is strong empirical evidence in favor of changes in the steady state and we find that such changes have dramatic effects on the forecasting relationship of returns and price ratios. We suggest a simple adjustment to the dividend-price ratio and revisit the forecasting equations from Section 2. We first study shifts in the dividend-price ratios in detail and consider alternative ratios in Section 4.

Our econometric specification is directly motivated by the framework that allows for changes in the steady state laid out in the previous section. Equation (7) implies that the log dividend-price ratio is the sum of a nonstationary component and a stationary component. In this section, we model the nonstationary component as a constant that is subject to rare structural breaks as in Bai and Perron (1998).

The full line in each of the panels of Figure 2 shows the log dividend-price ratio from 1927 to 2004. Visually, the series displays evidence of nonstationarity. In particular, the bull market of the 1990s is hard to reconcile with a stationary model. The dividend-price ratio has risen since, but at the end of our sample in 2004, prices would have to fall an additional 46% for the dividend-price ratio to return to its historical mean. One possible explanation is that the bull market of the 1990s represents a sequence of extreme realizations from a stationary distribution.

The solid line in Figure 3 shows the smoothed empirical distribution of the log dividend-price ratio \( dp_t \). This distribution has a fat left tail, mainly due to the observations in the last 15 years. To investigate whether this is a typical plot from a stationary distribution, we conduct two exercises. Following Campbell, Lo, and MacKinlay (1997); Stambaugh (1999); Campbell and Yogo (2002); Ang and Bekaert (2006); and many others we estimate an AR(1) process for the log dividend-price ratio. First, in a Bootstrap exercise, we draw from the empirical distribution with replacement. The smoothed Bootstrap distribution is the dash-dotted line in the figure. Second, we compute the density of \( dp_t \) using Monte Carlo simulations from an estimated AR(1) model with normal

---

6 As an alternative, we have also studied in-sample predictability in a Hamilton regime-switching model. Because the estimated regimes are so persistent, the predictability coefficients are very close to the ones we report here. In Section 5, we revisit the Hamilton model in the context of out-of-sample predictability.
innovation. This density is plotted as the dashed line. The graph shows that neither the Bootstrap nor the Monte Carlo can replicate the fat left tail that we observe in the data. Interestingly, the stationary model also cannot generate the right tail of the empirical distribution. In summary, it is unlikely that the $dp_t$ data sample from 1927 to 2004 was generated by a stationary distribution.

An alternative explanation is that the long-run mean of the log dividend-price ratio is subject to structural breaks. To investigate this possibility, we test the null hypothesis of no break against the alternative hypotheses of one or two breaks with unknown break dates. Table 2 reports sup-$F$-test statistics suggested by Bai and Perron (1998). The null hypothesis of no break is strongly rejected (the $p$-value is less than 1%) in favor of a break in 1991 or two breaks in 1954 and 1994. While the evidence against no breaks is very strong, the question of whether the dividend-price ratio is subject to one or two breaks does not have a clear answer. The sup-$F$-test of the null of a single break against the alternative of two breaks is rejected at the 10% level but not at
The empirical distribution of the dividend-price ratio

The figure plots the smoothed empirical distribution of the log dividend-price ratio $dp$ (solid line), alongside the smoothed density obtained from drawing from the empirical distribution with replacement (Bootstrap, dash-dotted line), and the smoothed density from a Monte Carlo exercise (dashed line).

The table also reports the estimated change in the log dividend-price ratio before and after the break. In the one-break case, the change in $\bar{dp}$ is $-0.86$, whereas in the two-break case, the first change in 1954 is $-0.37$ and the second change is $-0.78$. The two plots in the left column of Figure 2 overlay the long-run mean $\bar{dp}$ on the raw dp series. For now, we are agnostic as to whether the break(s) is (are) due to a change in the long-run mean of dividend growth or expected returns, or a combination of the two. We return to this question later. It is worth emphasizing, however, that the date(s) of the shift in the dividend-price ratio is (are) consistent with the breaks in the equity premium identified by Pastor and Stambaugh (2001).

This result motivates us to construct two adjusted dividend-price series, one for the one-break case and one for the two-break case. For each, we simply
Reconciling the Return Predictability Evidence

Table 2
Tests for change in mean of log dividend-price ratio

<table>
<thead>
<tr>
<th># of Breaks</th>
<th>Date(s)</th>
<th>$\Delta \overline{dp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1991</td>
<td>-.86</td>
</tr>
<tr>
<td>2</td>
<td>1954, 1994</td>
<td>-.37, -.78</td>
</tr>
</tbody>
</table>

Test ($H_0$, $H_1$) | Statistic   |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>sup-$F$ (0,1)</td>
<td>13.7***</td>
</tr>
<tr>
<td>sup-$F$ (0,2)</td>
<td>23.9***</td>
</tr>
<tr>
<td>sup-$F$ (1,2)</td>
<td>9.64*</td>
</tr>
</tbody>
</table>

Information criterion | # of Breaks |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LWZ</td>
<td>2</td>
</tr>
<tr>
<td>BIC</td>
<td>2</td>
</tr>
</tbody>
</table>

Persistence properties of adjusted dividend-price ratio

<table>
<thead>
<tr>
<th></th>
<th>AC(1)</th>
<th>AC(2)</th>
<th>ADF Test</th>
<th>p val</th>
<th>s.d.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$dp$, unadjusted</td>
<td>.91</td>
<td>.81</td>
<td>-.1383</td>
<td>.586</td>
<td>.42</td>
</tr>
<tr>
<td>$dp$, adjusted, one break</td>
<td>.77</td>
<td>.55</td>
<td>-.3016</td>
<td>.038</td>
<td>.26</td>
</tr>
<tr>
<td>$dp$, adjusted, two breaks</td>
<td>.61</td>
<td>.23</td>
<td>-.4731</td>
<td>.010</td>
<td>.20</td>
</tr>
</tbody>
</table>

The first panel reports dates of structural breaks in the mean of the log dividend-price ratio estimated by the Perron procedure as well as the changes in the mean before and after the breaks. The second panel reports sup-$F(i, j)$ statistics where $i$ is the number of breaks under the null hypothesis and $j$ is the number of breaks under the alternative. ‘*’, ‘**’, ‘***’ denote significance at the 1%, 5%, and 10% level, respectively. The third panel reports the number of breaks chosen according to the Bayesian Information Criterion (BIC) and the modified Schwartz criterion proposed by Liu, Wu, and Zidek (1997) (LWZ). The tests allow for autocorrelation in the residuals and the trimming value is set to 5% of the sample. The bottom panel reports first- and second-order autocorrelation coefficients, an Augmented Dickey-Fuller test, testing the null hypothesis of a unit root (and associated $p$-value), and the time-series standard deviation for the unadjusted log dividend-price ratio, the log price ratio adjusted for a change in its mean in 1991, and the log dividend-price ratio adjusted for a change in its mean in 1954 and 1994.

subtract the mean in the relative subsample(s). In the one-break case with break date $\tau$, the adjusted ratio is defined as

$$\tilde{dp}_t = \begin{cases} 
  dp_t - \overline{dp}_1 & \text{for } t = 1, \ldots, \tau \\
  dp_t - \overline{dp}_2 & \text{for } t = \tau + 1, \ldots, T,
\end{cases}$$

where $\overline{dp}_1$ is the sample mean for 1927–1991 and $\overline{dp}_2$ is the sample mean for 1992–2004. The adjusted $dp$ ratio in the two-break case is defined analogously. The right column of Figure 2 illustrates this procedure graphically.

The bottom half of Table 2 compares the autocorrelation properties of the unadjusted and adjusted dp series. As is well known, the raw $dp$ series is very persistent. The first- and second-order autocorrelations are .91 and .81. The null hypothesis of a unit root cannot be rejected, according to an Augmented Dickey Fuller (ADF) test (third column). In contrast, the two adjusted $\tilde{dp}$ series are much less persistent; the first-order autocorrelation drops to .77 and .61, respectively. The null of a unit root in the adjusted series is rejected at the 4% and 1% levels. Interestingly, the volatility of the adjusted series is only half as
large as for the adjusted series (last column). This substantially alleviates
the burden on standard asset pricing models to match the volatility of the price-
dividend ratio, once the nonstationary nature of the mean dp ratio has been
taken into account.

3.3 Forecasting with the adjusted dividend-price ratio
We now revisit the return and dividend growth predictability Equations (1)
and (2), but use the adjusted dividend-price ratios instead of the raw series
as predictor variable. The second and fourth columns of Table 1 show the
estimation results of the return and dividend growth predictability regressions
using \( \tilde{d}p \), respectively. Rows 1–4 are for the one-break case; rows 5–9 are for the
two-break case. Starting with the one-break case because the adjusted dividend-
price ratio is the same as the raw series with each subsample, the results in rows 2
and 3 are unchanged. But now in row 1, we find that the adjusted dividend-price
ratio significantly predicts stock returns. The coefficient for the entire sample
is .235, which is almost identical to the estimates in the two subsamples. Thus,
the low point estimate for \( \kappa_r \) in the first column was due to averaging across
regimes. Not taking the nonstationarity of the \( dp \) ratio into account severely
biases the point estimate for \( \kappa_r \) downward. Furthermore, row 4 shows that
the evidence for a break in the forecasting relationship between returns and the
dividend-price ratio has disappeared. The null hypothesis of parameter stability
can no longer be rejected when using \( \tilde{d}p \). The full-sample regression \( R^2 \) is 10%,
more than twice the value of the first column. The results for dividend growth
predictability remain largely unchanged. This is not surprising given that we
did not detect much instability in the relationship between \( \Delta d_{t+1} \) and \( dp_t \) to
begin with.

The rolling window estimates confirm this result.\(^7\) The middle panel of
Figure 1 shows that the coefficient \( \kappa_r \) is much more stable in the one-break case
than in the no-break case (top panel). In particular, its value in the 1990s hovers
around .3, compared to zero without the adjustment. Likewise, the regression
\( R^2 \) is also more stable and does not drop off in the 1990s. The same exercise
shows that the dividend growth relationship is stable and that \( \kappa_d \) never moves
far from zero (not shown). The evidence for dividend growth predictability is
weak at best.\(^8\)

The bottom panel of Table 1 uses \( \tilde{d}p \), adjusted for breaks in 1954 and 1994.
The full-sample estimate for \( \kappa_r \) is now .455 (row 5) and highly significant.\(^9\)

\(^7\) In the rolling window estimation, we assume that the break in \( \tilde{d}p \) is caused by a break in mean expected returns \( \bar{r} \).
The alternative assumption that the break is in the long-run growth rate of the economy \( \bar{g} \) gives identical results.

\(^8\) The lack of predictive power of the dividend-price ratio for dividend growth does not imply that dividend growth
is not forecastable, because any correlated movement in expected returns and expected dividend growth cancels
in \( d - p \), as shown in Lettau and Ludvigson (2005).

\(^9\) A Bootstrap analysis confirms that the small-sample \( p \)-value (asymptotic \( p \)-value) is 1.11% (0.00%) in the one-
break case and 0.00% (0.00%) in the two-break case. A second Bootstrap exercise shows that the small-sample
bias in the coefficients is small relative to their magnitude. In the one-break case, the bias is .019 (we estimate
The full-sample regression $R^2$ is 22%. In contrast, dividend growth is not predictable. The bottom panel of Figure 1 shows that the rolling estimates for $\kappa_r$ are very stable when we use $\tilde{dp}$ adjusted for two breaks. The point estimate hovers around .4 and the return regression $R^2$ goes up as high as 40%. Moreover, the Chow test in row 9 finds no evidence for instability in either forecasting equation.

We conclude that taking changes in the long-run mean of the dividend-price ratio into account is crucial for forecasts of stock returns. Forecasting with the unadjusted dividend-price ratio series results in coefficient instability in the forecasting regression and unreliable inference (insignificance in small samples, and results depending on the subsample). These disconcerting properties are due to a nonstationary component that shifts the mean of the dividend-price ratio. In Section 3.1, we extend the model to allow for such nonstationarity in $\tilde{dp}$. In this section, we examined a simple form of nonstationarity, a structural break. Appropriately adjusting the dividend-price ratio for the structural break strengthens the evidence for return predictability, but not dividend growth predictability. The predictability coefficient is stable over time and least-squares coefficient estimates are highly significant. Finally, the in-sample return predictability evidence stands up to the usual problem of persistent regressor bias (Nelson and Kim, 1993; Stambaugh, 1999; Ang and Bekaert, 2006; Valkanov, 2003) because the adjusted dividend-price ratio is much less persistent.

4. Other Financial Ratios

While the dividend-price ratio has been the classic prediction variable, it is useful to investigate to what extent our results are robust to a different measure of payouts. Lamont (1998) finds that the log earnings-price ratio $ep$ forecasts returns. We find very much the same patterns for the earnings-price ratio as for the dividend-price ratio. The earnings data start in 1946 and are described in Appendix A. The book-to-market ratio is computed from the same earnings and dividend data using the clean-surplus method (Vuolteenaho, 2000).

Table 3 shows that the null hypothesis of no structural break in the $ep$ ratio is strongly rejected in favor of one or two breaks (first row). The Perron test estimates a 1990 break date in the one-break case and 1953 and 1994 break dates in the two-break case. These line up almost perfectly with the $dp$ break dates in Table 2. One other often-used valuation ratio, the log book-to-market ratio (bm) also displays strong evidence of two breaks with similar break dates in 1953 and 1990 (row 2). Clearly, there is evidence for a permanent or strongly persistent component in all valuation ratios.

Some researchers have argued that there were persistent changes in firms’ payout policies in the 1990s and have argued to adjust dividend-price ratios for .254 when the true coefficient is .235). In the two-break case, the bias is .013 (we estimate .468 when the true value is .455). Detailed results are available upon request.
Table 3
Tests for change in mean of financial ratios

<table>
<thead>
<tr>
<th>Structural break tests</th>
<th>$H_0$</th>
<th>$H_1$</th>
<th>$sup-F$-test</th>
<th>p-value</th>
<th>Date(s)</th>
<th>$\Delta$ mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>ep</td>
<td>zero break</td>
<td>one break</td>
<td>15.5</td>
<td>&lt; 1%</td>
<td>1990</td>
<td>−.67</td>
</tr>
<tr>
<td></td>
<td>zero break</td>
<td>two breaks</td>
<td>18.0</td>
<td>&lt; 1%</td>
<td>1953, 1994</td>
<td>−.50, −.62</td>
</tr>
<tr>
<td>bm</td>
<td>zero break</td>
<td>one break</td>
<td>9.3</td>
<td>&lt; 10%</td>
<td>1953</td>
<td>−.80</td>
</tr>
<tr>
<td></td>
<td>zero break</td>
<td>two breaks</td>
<td>17.9</td>
<td>&lt; 1%</td>
<td>1953, 1990</td>
<td>−.71, −.33</td>
</tr>
<tr>
<td>de</td>
<td>zero break</td>
<td>one break</td>
<td>5.6</td>
<td>&gt; 10%</td>
<td>1993</td>
<td>−.24</td>
</tr>
<tr>
<td></td>
<td>zero break</td>
<td>two breaks</td>
<td>3.3</td>
<td>&gt; 10%</td>
<td>1990, 1993</td>
<td>+.27, −.46</td>
</tr>
<tr>
<td>dp$^{nas}$</td>
<td>zero break</td>
<td>one break</td>
<td>10.2</td>
<td>&lt; 5%</td>
<td>1992</td>
<td>−.75</td>
</tr>
<tr>
<td></td>
<td>zero break</td>
<td>two breaks</td>
<td>18.1</td>
<td>&lt; 1%</td>
<td>1954, 1995</td>
<td>−.35, −.70</td>
</tr>
<tr>
<td>dp$^{rep}$</td>
<td>zero break</td>
<td>one break</td>
<td>3.7</td>
<td>&gt; 10%</td>
<td>1990</td>
<td>−.43</td>
</tr>
<tr>
<td></td>
<td>zero break</td>
<td>two breaks</td>
<td>4.5</td>
<td>&gt; 10%</td>
<td>1954, 1991</td>
<td>−.23, −.34</td>
</tr>
<tr>
<td></td>
<td>zero break</td>
<td>three breaks</td>
<td>20.1</td>
<td>&lt; 1%</td>
<td>1957, 1973, 1990</td>
<td>−.46, +.50, −.57</td>
</tr>
</tbody>
</table>

The top half of the table reports sup-$F$ Perron tests of the null hypothesis of no break against the alternative hypothesis of one (first row) or two (second row) breaks with unknown break date. It reports the p-value of the test statistic, as well as the resulting break date. The last column reports the estimated change in means before and after the break(s). These tests are performed for the log earnings-price ratio $ep = e - p$, the log book-to-market value of equity ratio $bm = b - m$, the log dividend-earnings ratio $de = d - e$, the log book-to-market value of equity ratio $bm = b - m$, the log dividend-earnings ratio $de = d - e$, the log dividend-price ratio adjusted for repurchases $dp^{rep}$, and the log dividend-price ratio of the universe of CRSP firms that excludes the NASDAQ firms $dp^{nas}$.

repurchases (Fama and French, 2001; Grullon and Michaely, 2002; Boudoukh et al., 2004). First, we find no evidence for a break in the payout ratio $de = d - e$ at the 10% level (row 3 of Table 2). This is consistent with the view that both $dp$ and $\overline{ep}$ contain structural breaks. Second, even if there was a break in the $de$ ratio and we took the point estimate for $de$ in the subsamples, we would find that more than three-fourths of the change in the mean dividend-price ratio comes from a change in the mean earnings-price ratio and less than one-fourth from a change in $de (\overline{dp} = \overline{ep} + \overline{de})$. In particular, for our S&P 500 sample from 1946 to 2004 with a 1991 break date, we find a change in $\overline{dp}$ of −.81, a change in $\overline{ep}$ of −.63, and a change in $\overline{de}$ of −.18.

To further investigate the role of repurchases and the role of a changing composition in CRSP, we consider two additional valuation ratios. First, we consider the CRSP universe without NASDAQ stocks. Arguably, removing NASDAQ stocks goes a long way toward eliminating new economy and non-dividend-aying companies that became more prevalent in the 1990s.10 We compute the dividend-price ratio, $dp^{nas}$, and the dividend growth for this group. This time series has properties very similar to those of the series with the NASDAQ. The Perron tests in row 5 of Table 3 show a break of −75% in 1992, close to the −86% change in the full-sample series in 1991. For the two-break

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10 Fama and French (2001) document that the fraction of nonfinancial, nonutility firms that paid dividends declined by almost 45% between 1978 and 1999. However, most of that decline is attributable to new firms and to small firms. They write: “The characteristics of dividend payers (large profitable firms) do not change much after 1978.” We take this group to be the value-weighted CRSP index without NASDAQ stocks. This series starts to deviate from the full-sample series in 1973. We verified that the dividend growth rate of this set of firms did not change in the 1990s. Average dividend growth from 1927 to 1991 was 5.45%. Average dividend growth from 1992 to 2004 was 5.46%. 1624
Table 4
Forecasting returns with other financial ratios

<table>
<thead>
<tr>
<th>Predictor y</th>
<th>No break</th>
<th>One break</th>
<th>Two breaks</th>
<th>Three breaks</th>
</tr>
</thead>
<tbody>
<tr>
<td>ep</td>
<td>.119</td>
<td>.214</td>
<td>.216</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.030)</td>
<td>(.039)</td>
<td>(.045)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[.104]</td>
<td>[.190]</td>
<td>[.185]</td>
<td></td>
</tr>
<tr>
<td>bm</td>
<td>.070</td>
<td>.255</td>
<td>.308</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.036)</td>
<td>(.063)</td>
<td>(.064)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[.030]</td>
<td>[.154]</td>
<td>[.188]</td>
<td></td>
</tr>
<tr>
<td>dp^{nas}</td>
<td>.110</td>
<td>.250</td>
<td>.417</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.048)</td>
<td>(.056)</td>
<td>(.090)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[.043]</td>
<td>[.105]</td>
<td>[.182]</td>
<td></td>
</tr>
<tr>
<td>dp^{rep}</td>
<td>.191</td>
<td>.282</td>
<td>.361</td>
<td>.576</td>
</tr>
<tr>
<td></td>
<td>(.054)</td>
<td>(.065)</td>
<td>(.084)</td>
<td>(.097)</td>
</tr>
</tbody>
</table>

This table reports estimation results for the equation \( r_{t+1} - \bar{r} = \kappa_y (y_t - \bar{y}) + \tau r_{t+1} \), where \( y \) is the log earnings-price ratio \( ep = e - p \) in the first row, the log book-to-market value ratio \( bm = b - m \) in the second row, the log dividend-price ratio without the NASDAQ firms \( dp^{nas} \) in the third row, and the repurchase adjusted log dividend-price ratio \( dp^{rep} \) in the fourth row. The table reports point estimates and standard errors in parentheses of \( \kappa_y \), and the regression \( R^2 \) in brackets. The regressor is the unadjusted valuation ratio in the first column, the one-break adjusted valuation ratio in the second column, and the two-break adjusted valuation ratio in the third column. For the predictor \( dp^{rep} \), we report the three-break case as well. The break dates for all regressors are reported in Table 3. The sample is 1946–2004 in row 1, and 1927–2004 in all other rows.

case, the break dates and magnitudes are also very similar: 1954 and 1995 and −35% and −70%.

Second, we use the Boudoukh et al. (2004) repurchase yield data, available from 1971 onward, and construct a corrected dividend-price ratio and dividend growth rate series. We label this repurchase-adjusted dividend price series \( dp^{rep} \). The case favored by the data is a three-break case with break dates in 1957, 1973, and 1990 (see last row of Table 3). We show next that these two adjustments do not materially affect our predictability results with the standard dividend-price ratio presented earlier. This leads us to conclude that structural changes in payout policies and/or the composition of firms in the 1990s can only explain a small part of the change in the dividend-price ratio.

We first turn to the forecasting regressions with the earnings-price ratio. When we use the earnings-price ratio as a return predictor, we obtain similar results to what we reported for the dividend-price ratio in Table 1. The first row of Table 4 shows that when the unadjusted earnings-price ratio is the independent variable, the slope coefficient is .119. Just as in the dividend-price ratio regressions, this coefficient displays parameter instability among subsamples: the full-sample point estimate is lower than the estimates in all subsamples, and the Chow test of no break has a \( p \)-value of only .15 (not shown). The next two columns show that this bias is due to averaging over

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11 We note that this is just one possible adjustment. The correct adjustment depends on the investor under consideration. Here, an investor’s cash flows are adjusted for aggregate repurchases, but not for seasoned equity offerings nor initial public offerings.
subsamples. Once we use the adjusted $\tilde{ep}$ ratio, the instability disappears and the full-sample point estimate increases to .215 in both the one-break and two-break cases. These coefficients are twice the size of the ones obtained with the unadjusted $ep$ ratio and are measured precisely. The regression $R^2$ almost doubles. The slope coefficient is very similar to the one we found in the first panel of Table 1: $\kappa_r = .235$. One difference from the results in Table 1 is that the adjusted earnings-price ratio also significantly forecasts earnings growth, with a negative sign (not shown).

For the unadjusted lagged log book-to-market ratio $bm = b - m$, we find that the predictability coefficient $\kappa_r$ is only marginally significant. The point estimate is .07, lower than the point estimates in the subsamples 1927–1952 (.26), 1953–1990 (.44), and 1991–2004 (.72), all of which are strongly significant. Again, this downward bias is due to averaging over the break(s). The full-sample point estimate increases to .255 with the one-break adjusted $\tilde{bm}$ series as regressor and to .308 with the two-break adjusted $\tilde{bm}$ series. The regression $R^2$ increases from 3% in the first column to 19% in the third column.

The return predictability findings for $dp^{nas}$ and $dp^{rep}$ are also similar to the benchmark $dp$ results. First, using $dp^{nas}$ without break adjustment, we find a point estimate for $\kappa_r$ of .11. This point estimate is lower than in either subsample (.24 in 1927–1992 and .30 in 1993–2004). Once we use the break-adjusted series, the point estimate more than doubles to .250. Just as for the standard $dp$ ratio, the downward bias comes from averaging over the break. The break-adjusted point estimate is close to the .235 we found for the sample that includes the NASDAQ. We obtain further increases in the point estimate and the $R^2$ in the two-break case. Second, using $dp^{rep}$, the full-sample return predictability coefficient is .19, higher than the .09 for the standard $dp$ series, but again lower than in either subsample (.25 in 1927–1990 and .53 in 1991–2004). Clearly, adjusting for repurchases improves the forecasting power of the $dp$ ratio. However, adjusting for the breaks is important and further strengthens the case for predictability. In the preferred case of three breaks, the predictability coefficient is .58, three times its unadjusted value. The regression $R^2$ is also three times higher.

We conclude that the other financial ratios indicate a predictability pattern similar to that of the dividend-price ratio. Without an adjustment for the change in their long-run mean, the relationship between 1-year ahead returns and financial ratios is unstable over time. However, once we filter out the nonstationary component, we find a stable forecasting relationship and a large predictability coefficient. The fact that the results are so similar for earnings and dividend data suggests that an explanation that exclusively rests on changing payout policies misses the most important structural changes in the economy: changes in long-run growth rate or long-run expected returns.
5. Out-of-Sample Predictability

The in-sample predictability results presented so far used a break adjustment constructed using the entire data sample. In this section, we investigate how an investor who forms an adjusted dividend-price ratio in real time fares in predicting out-of-sample returns. We compare the out-of-sample forecasting properties of adjusted dividend-price ratios to the unadjusted series and a random walk model. We find that a real-time dividend-price ratio adjustment yields uniformly smaller prediction errors compared to the unadjusted series but slightly larger forecast errors than the random walk model.

Why does real-time prediction fail to beat the random walk model? In real time, an investor faces two challenges. First, she has to estimate the timing of a break. Second, if she detects a new break, she has to estimate the new mean after the break occurs. If the new break occurred toward the end of the sample that the investor has access to, the new mean can only be estimated using a small number of observations and is subject to significant estimation uncertainty. To investigate which issue is responsible for the deterioration of the out-of-sample forecasting power, we consider two additional exercises. In the first exercise, the investor predicts out-of-sample using the ex post adjusted dividend-price ratio series considered in Section 3.1. In other words, we endow the investor with information about estimated break dates and means from the entire sample—thus, this case is not a pure out-of-sample test. However, it sets an informative benchmark for the analysis of the pure out-of-sample forecasts. In a second exercise, we consider a Hamilton (1989) regime-switching model for the mean of the dividend-price ratio. In this model, the regime means are estimated using data from the entire data sample. The Hamilton model computes two estimates for the break dates (regime-switching probabilities): (i) “unsmoothed” probabilities estimated using only currently available data, and (ii) “smoothed” probabilities based on the entire sample. While this case is not a pure out-of-sample forecast, it allows us to study the relative difficulty of estimating the break dates versus estimating the means relative to the pure out-of-sample forecasts and the ex post adjusted dividend-price ratio.

The conclusions from these two exercises are that (i) the estimation of the break dates in real time is not crucial and the resulting prediction errors are smaller than for the random walk model, and (ii) the estimation of the magnitude of the break in the mean dividend-price ratio entails substantial uncertainty, and is ultimately responsible for the failure of the real-time out-of-sample predictions to beat the random walk. These findings can explain the lack of out-of-sample predictability documented by Goyal and Welch (2004).

5.1 Real-time dividend price adjustments

Before presenting the estimation results, we confirm our earlier conclusion (based on ex post data; see Figure 3) that it is extremely unlikely that the dividend-price ratio sample is drawn from a stationary distribution, based on
Recursive Bootstrapped Quantiles of d-p

---

**Figure 4**
Recursive estimation of the empirical distribution of the dividend-price ratio

We recursively estimate an AR(1) for the log dividend-price ratio $d_p$, $d_{p,t+1} = c + \phi d_{p,t} + e_{d_{p,t}}$ using data up to time $t + 1$, and Bootstrap percentiles of the empirical distribution by drawing with replacement from the residuals $\{e_{d_{p,t}}, \ldots, e_{d_{p,t+1}}\}$. The dashed lines represent the 2.5, 5, 95, and 97.5 percentiles of the Bootstrapped distribution. The initial sample is 1927–1951. Each successive exercise adds one year of data. The solid line represents the observed log-dividend-price ratio in deviation from its recursive sample mean.

---

real-time data only. Figure 4 shows a recursive (i.e., real time) estimation of the empirical distribution of the log dividend-price ratio. In each year, the investor estimates an AR(1) model for $d_p$, using data up to the current year. She then Bootstraps from the available sample to compute the empirical distribution of the log dividend-price ratio. Each year she recomputes the 2.5, 5, 95, and 97.5 percentiles of the Bootstrapped distribution (dashed and dotted lines in the figure). The figure also plots the realized dividend-price ratio, in deviation from its recursive sample mean (full line). By 1958, the investor is quite confident that the realized dividend-price ratio is far below the mean; it hits the 2.5 percentile of the empirical distribution. Likewise, in 1994, the observed dividend-price ratio falls in the 2.5% tail of the distribution. Between 1995 and 1999, the investor is almost certain that the observed dividend-price ratio has not been drawn from a stationary distribution. Interestingly, these “crossing” dates are almost identical to the break dates estimated in the previous sections. This shows that the permanent changes in the dividend-price ratio that were identified by the ex post break tests do not rely on having the benefit of the entire sample through 2004. Even an investor in real time would have concluded that extreme
observations of dividend-price ratios are unlikely to be generated by a stationary process with constant parameters.

Next we construct a real-time adjustment of the dividend-price ratio that can be used in out-of-sample forecasting tests. In each year $T' \leq T$, the investor estimates the Perron structural break test using data available up to year $T'$ using one of the three tests: the sequential sup-$F$-test with a 10% critical value, the BIC criterion, and the LWZ criterion. Given the break dates and corresponding means, the real-time adjustment of the dividend-price ratio is analogous to the adjustment using the entire sample in Equation (8), with the exception that only data up to date $T'$ instead of $T$ are used in the estimation. Denote this corrected ratio $\hat{dp}_{t,T'}$, $t = 1, \ldots, T'$. The out-of-sample return forecast for period $T' + 1$ is then computed from a regression of returns $r_{t+1}$ on $\hat{dp}_{t,T'}$ for $t = 1, \ldots, T' - 1$ and the currently observed adjusted dividend-price ratio $\hat{dp}_{t,T'}$.\footnote{This procedure implies that there are no breaks between $T'$ and $T' + 1$, consistent with our assumption that the break is unpredictable.}

To gauge the relative difficulties of estimating the timing of a break versus the estimation of the means, we also use a “pseudo” real-time adjustment based on a Hamilton (1989) regime-switching model in real time. The dividend-price ratio is assumed to follow an AR(1) with different means in either two regimes (one-break case) or three regimes (two-break case). The top panel of Figure 5 shows the smoothed (based on the entire sample) and unsmoothed (real-time) estimates of the probability that the dividend-price ratio is drawn from the low regime, when two regimes are considered. Using real-time data, the investor puts nonzero probability on a shift to the low $d - p$ regime starting in early 1990. By 1995, she is more than 50% certain that the shift occurred. The difference between smoothed probabilities and unsmoothed probabilities is that the investor is assigning a higher likelihood to the new, low dividend-price regime earlier because she has access to the entire data set. Note that the $dp$ estimates for the two regimes coincide with our ex post estimates because they use the entire data sample. When she considers three regimes instead, the investor increases the real-time probability of a switch from the high to the middle $dp$ regime in 1954. By 1960, she is more than 50% certain that the first shift occurred (see middle panel of Figure 5). In 1990, she starts to attribute probability mass to the low-$p$ regime, and by 1996, she is more than 50% confident that the economy left the middle-$dp$ regime for the low-$dp$ regime. As in the two-regime case, the smoothed probabilities indicate that knowing the entire sample yields a faster transition to the new regime.

Not surprisingly, the regimes in the Hamilton model are estimated to be very persistent. The probability of remaining in the current regime is above 0.9 in all cases. In other words, the regimes identified by the Hamilton estimation are close to permanent structural breaks as specified in the Perron model.
Figure 5
Regime-switching model—state probabilities
This figure reports estimation results from a Hamilton regime-switching model for the log dividend-price ratio \( d - p \). The top panel is for the two-regime (one-break) case; the bottom panel is for the three-regime (two-break) case. The solid lines represent unsmoothed probabilities, based on real-time information. The dashed lines represent smoothed probabilities. The top panel represents the probability of being in the low \( dp \) regime; the bottom panel plots the probabilities of being in the low and in the middle \( d - p \) regimes. The latter are denoted by squares and diamonds.

5.2 Out-of-sample forecast errors
We follow the approach taken by Goyal and Welch (2003) and predict 1-year ahead returns with the lagged dividend-price ratio. The first forecasting regression uses 20 years of data, so that the first forecasted return is the one in 1946. For all future years, we use expanding windows and compare three different forecasters. The first is the current sample mean return implied by the “naive” random walk model. The second is the standard unadjusted dividend-price ratio \( dp \). The third is the break-adjusted dividend-price ratio. To explain the results, we use several alternatives for the break-adjusted series.

Panel 1 of Table 5 reports the mean absolute forecast error (MAE) and the root mean-squared forecasting error (RMSE). Comparing the second to the first row, we confirm the result of Goyal and Welch (2003): the random walk model
Table 5
Out-of-sample predictability

<table>
<thead>
<tr>
<th>Benchmarks</th>
<th>Mean absolute error</th>
<th>Root mean-squared error</th>
</tr>
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<tr>
<td>Random walk</td>
<td>.1338</td>
<td>.1605</td>
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<tr>
<td>Unadjusted $dp$</td>
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<td>.1685</td>
</tr>
<tr>
<td>Pure OOS</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>.1661</td>
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<tr>
<td>$\hat{dp}_p$—Perron LWZ criterion</td>
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<td>.1680</td>
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<tr>
<td>$\hat{dp}_p$—Perron BIC criterion</td>
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<td>.1646</td>
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<tr>
<td>Pseudo OOS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{dp}$—Perron, ex post, one break</td>
<td>.1309</td>
<td>.1558</td>
</tr>
<tr>
<td>$\tilde{dp}$—Perron, ex post, two breaks</td>
<td>.1158</td>
<td>.1421</td>
</tr>
<tr>
<td>$\tilde{dp}_H$—Hamilton, one break, smoothed</td>
<td>.1286</td>
<td>.1543</td>
</tr>
<tr>
<td>$\tilde{dp}_H$—Hamilton, two breaks, smoothed</td>
<td>.1100</td>
<td>.1340</td>
</tr>
<tr>
<td>$\tilde{dp}_H$—Hamilton, one break, unsmoothed</td>
<td>.1330</td>
<td>.1590</td>
</tr>
<tr>
<td>$\tilde{dp}_H$—Hamilton, two breaks, unsmoothed</td>
<td>.1243</td>
<td>.1512</td>
</tr>
</tbody>
</table>

Panel 1: Data

Panel 2: Monte Carlo—one break

| Random walk                 | .1212               | .1525                   |
| Unadjusted $dp$             | .1210               | .1555                   |
| $\tilde{dp}$—ex post, one break | .1058               | .1334                   |

Panel 3: Monte Carlo—two breaks

| Random walk                 | .1222               | .1533                   |
| Unadjusted $dp$             | .1203               | .1516                   |
| $\tilde{dp}$—ex post, one break | .1165               | .1493                   |
| $\tilde{dp}$—ex post, two breaks | .1064               | .1336                   |

The table reports one-period ahead return forecast errors based on the Random Walk model (row 1) and based on the forecasting equation $r_{t+1} = k_r (dp_t - \bar{dp}) + \epsilon_{t+1}$ with fixed $\bar{dp}$ (row 2). Rows 3 through 5 use the real-time Perron procedure to estimate $\hat{dp}_p$. We report results for three different methods of selecting the number of break: the sequential sup-$F$-test with 10% critical value, and the LWZ and BIC information criteria. Rows 6 and 7 use the ex post break-adjusted dividend price ratios with a change in $\tilde{dp}$ in 1991 (row 6), and two changes in the mean $\tilde{dp}$ in 1954 and 1994 (row 7). Rows 8–11 use the Hamilton approach to construct adjusted dividend-price ratios $\tilde{dp}_H$. We consider the cases of two regimes and three regimes. All numbers denote returns per annum.

The third through fifth rows report the real-time adjusted dividend-price ratio $\tilde{dp}_p$ obtained from the Perron procedure. For all three criteria for dating the breaks, sup-$F$, BIC, and LWZ, the prediction errors of the adjusted series are lower than those obtained from using the unadjusted series (row 2) but the out-of-sample prediction errors are larger than for the random walk model. The sequential sup-$F$-test has the lowest mean absolute error while the BIC criterion delivers the lowest root mean-squared error.

The table reports one-period ahead return forecast errors based on the Random Walk model (row 1) and based on the forecasting equation $r_{t+1} = k_r (dp_t - \bar{dp}) + \epsilon_{t+1}$ with fixed $\bar{dp}$ (row 2). Rows 3 through 5 use the real-time Perron procedure to estimate $\hat{dp}_p$. We report results for three different methods of selecting the number of break: the sequential sup-$F$-test with 10% critical value, and the LWZ and BIC information criteria. Rows 6 and 7 use the ex post break-adjusted dividend price ratios with a change in $\tilde{dp}$ in 1991 (row 6), and two changes in the mean $\tilde{dp}$ in 1954 and 1994 (row 7). Rows 8–11 use the Hamilton approach to construct adjusted dividend-price ratios $\tilde{dp}_H$. We consider the cases of two regimes and three regimes. All numbers denote returns per annum.

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has superior out-of-sample properties compared to the standard dividend-price ratio specification. The latter’s prediction errors are almost 1% per year higher.

The third through fifth rows report the real-time adjusted dividend-price ratio $\tilde{dp}_p$ obtained from the Perron procedure. For all three criteria for dating the breaks, sup-$F$, BIC, and LWZ, the prediction errors of the adjusted series are lower than those obtained from using the unadjusted series (row 2) but the out-of-sample prediction errors are larger than for the random walk model. The sequential sup-$F$-test has the lowest mean absolute error while the BIC criterion delivers the lowest root mean-squared error.
Figure 6
Regime-switching model—mean dividend-price ratio
This figure reports the mean dividend-price ratio based on the estimation results from a Hamilton regime-switching model for the log dividend-price ratio $d - p$. The top panel is for the two-regime (one-break) case; the bottom panel is for the three-regime (two-break) case. The solid line is the weighted average of the regime means, where the weights are the unsmoothed (real-time) state probabilities. The dashed line is a weighted average of the same regime means, but the weights are the smoothed state probabilities (based on the entire sample).

We contrast these results with three “pseudo” out-of-sample cases to investigate why the out-of-sample forecast errors are higher than those for the random walk model. First, we use the dividend-price ratio adjustment based on the Perron break test estimated in the entire sample $\tilde{dp}$. Second, we construct adjusted dividend-price ratios based on the Hamilton model $\hat{dp}_t^H$ by subtracting the average of the means of each regime weighted by the time-$t$ probability of each regime from $dp_t$. Figure 6 plots the mean dividend-price ratios that result from probability-weighting the regime mean estimates. The weights are either the unsmoothed (solid line) or the smoothed probabilities (dashed line). It also plots the ex post mean (dotted line). Naturally, the mean dividend-price ratio based on smoothed probabilities tracks the ex post breaks somewhat faster than the unsmoothed mean.
Rows 6 and 7 in Panel 1 of Table 5 show that the use of the ex post adjusted dividend-price ratios $\tilde{dp}$ substantially reduces the forecasting error. The MAE and RMSE of the dividend-price ratio adjusted for a single break in 1991 are lower than those for the random walk model. The out-of-sample forecasting power of the dividend-price ratio adjusted for two breaks in 1954 and 1994 is dramatically improved compared to the unadjusted ratio and to the random walk model. The RMSE and MAE are reduced by 12–15% compared to the random walk model.

The pseudo out-of-sample forecast errors based on the Hamilton correction with smoothed probabilities are similar but slightly lower than those in the ex post Perron correction. Since both cases use the entire sample to estimate the regime means and probabilities, it is not surprising that the out-of-sample results are comparable. The Hamilton model allows for a smoother adjustment to a new regime compared to the Perron model, which might account for the lower forecast error.

For the Hamilton regime-switching model, we also compute pseudo out-of-sample forecast errors based on unsmoothed probabilities. Recall that these probabilities are estimated using only currently available information, but estimates of regime means are based on information of the entire sample. Thus, this case enables us to assess the relative difficulty of real-time estimation of the timing of breaks versus real-time estimation of the level of regimes. Interestingly, the out-of-sample prediction errors of the case with unsmoothed probabilities are only slightly higher than those with ex post Perron adjustment and smoothed probabilities. In the one-break example, the RMSE for the unsmoothed Hamilton adjustment is 15.90% compared to 15.58% for the ex post Perron adjustment and 15.43% for the smoothed Hamilton adjustment. In contrast, the forecast errors for the pure OOS adjustments are significantly higher; between 16.46 and 16.80%, depending on the selection criterion.

Where does the difference in forecast errors come from? In the Hamilton adjustment with unsmoothed probabilities, the investor estimates the probability of each regime in real time but has access to the entire time series to estimate the mean $dp$ ratio in the different regimes. In the ex post Perron and smoothed Hamilton procedures, the investor estimates not only break dates in real time, but also the long-run mean in the current regime. Our results suggest that the poor real-time out-of-sample forecasts are to a larger extent due to uncertainty about the magnitudes of breaks rather than uncertainty about the exact timing of a break. Estimating a new long-run mean based on a few data points incurs a lot of measurement error. For example, if the Perron investor in 2001 detects a break in 1995, she has only six data points to estimate the new

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A satisfactory resolution that avoids any look-ahead bias in the Hamilton model would be to conduct a full Bayesian analysis in which the investor estimates the number of regimes, the regime-switching dates, and their associated long-run means based on real-time available data and prior information. Pettenuzzo and Timmermann (2005) have worked out such an estimation in the context of an asset allocation problem.
\( \bar{dp} \). The difficulty in estimating this mean is what accounts for the increase in prediction errors.

6. Long-Horizon Predictability

An important component of the empirical work on return predictability uses long-horizon regressions. In this section we provide a framework for analyzing long-horizon predictability. We use the ex post dividend-price ratio as the predictor and derive theoretical restrictions that link return and dividend growth to lagged dividend-price ratios at different horizons. Rather than estimating regressions for various horizons separately, the advantage of this approach is that fewer parameters have to be estimated. Moreover, the restrictions guarantee that the estimates across horizons are consistent with each other.

Recall the return and dividend growth predictability Equations (1) and (2)

\[
\tilde{r}_{t+1} = \kappa_r \tilde{dp}_t + \tau_{r,t+1},
\]

\[
\Delta \tilde{d}_{t+1} = \kappa_d \tilde{dp}_t + \tau_{d,t+1},
\]

where variables with a tilde are appropriately demeaned and stationary. Subtracting Equation (10) from Equation (9) and using the log linear approximation for log returns \( \tilde{r}_{t+1} = \tilde{dp}_t - \rho \tilde{dp}_{t+1} + \Delta \tilde{d}_{t+1} \) yields the implied AR(1) process for the dividend-price ratio in (11):

\[
\tilde{dp}_{t+1} = \phi \tilde{dp}_t + \tau_{dp,t+1},
\]

where the innovations are linked by \( \rho \tau_{dp,t+1} = \tau_{d,t+1} - \tau_{r,t+1} \). The model imposes a nonlinear present value restriction (12) on the predictability coefficients \( \kappa_r \) and \( \kappa_d \). Because \( \rho < 1 \) and stationarity implies \( |\phi| < 1 \), \( \kappa_r - \kappa_d \) must be positive. This is another way of saying that either returns \( (\kappa_r \neq 0) \) or dividend growth \( (\kappa_d \neq 0) \) have to be forecastable (or both). Most researchers work with Equations (9) and (11); we work with (9) and (10) instead because long-horizon restrictions are more easily derived in this case. Iterating forward on Equations (9) and (10), we obtain the (annualized) \( H \)-period dividend growth and return forecasting equations

\[
\frac{1}{H} \sum_{j=1}^{H} \tilde{r}_{t+j} = \kappa_r(H) \tilde{dp}_t + \tau_{r,t+H},
\]

\[
\frac{1}{H} \sum_{j=1}^{H} \Delta \tilde{d}_{t+j} = \kappa_d(H) \tilde{dp}_t + \tau_{d,t+H},
\]
where

\[ \kappa_d(H) = \kappa_d \frac{1}{H} \left( \frac{1 - \phi^H}{1 - \phi} \right) \]  \hspace{1cm} (15) 

\[ \kappa_r(H) = \kappa_r \frac{1}{H} \left( \frac{1 - \phi^H}{1 - \phi} \right) \]  \hspace{1cm} (16)

Let \( N \) be the number of horizons \( H > 1 \). Then, the joint system of 1-year ahead and \( H \)-year ahead predictability regressions for returns and dividend growth contains \( 2 + 2N \) equations but only two free parameters (\( \kappa_d, \kappa_r \)). The parameter \( \phi \) is implied by the present value constraint (12). The typical approach in the literature is to estimate univariate long-horizon return predictability equations without imposing these restrictions. Instead, we estimate the entire system of long-horizon return and dividend growth regressions jointly. This estimation procedure not only takes the high correlation of return regression coefficients at different horizons, pointed out by Boudoukh, Richardson, and Whitelaw (2005), explicitly into account but also takes the present value relationship (12) explicitly into account.

The estimation routine we describe next finds parameters to match the entire “term structure” of univariate predictability coefficients. Figure 7 illustrates this procedure for \( H = \{1, 3, 5, 7, 10\} \). The top panel plots the univariate predictability coefficients obtained from standard OLS regressions of \( H \)-year ahead returns and dividend growth on the unadjusted dividend-price ratio \( dp \), as well as the predictability coefficients (\( \kappa_d(H), \kappa_r(H) \)) implied by the joint estimation of the system of equations. To match the pattern of the 10 OLS predictability coefficients, the optimization routine chooses a value for \( \kappa_r \) above the value from the 1-year ahead univariate regression.

We start by estimating the one-period ahead equations for returns and dividend growth (9)–(10). These are the same equations we estimated in Section 3.3, but the additional restriction provides two new insights. First, the estimation delivers a value for the autocorrelation coefficient of the dividend-price ratio \( \phi \) because we impose the present value constraint (12). Second, we use the break-adjusted series \( \tilde{dp} \) from Section 3 in the estimation.

Because return and dividend growth series were demeaned by their sample averages, the previous sections implicitly assumed a break in \( \tilde{dp} \) without associated break in \( \bar{r} \) or \( \bar{d} \). The model tells us that a break in \( \tilde{dp} \) must be associated with a break in either \( \bar{r} \) or \( \bar{d} \), or both. First, we assume that \( \bar{d} \) is constant and focus on changes in the expected returns, consistent with the evidence on breaks in the equity premium in Pastor and Stambaugh (2001). The change in \( \bar{r} \) implied by the change in \( \tilde{dp} \) can be inferred from \( \bar{r}_t = (1 + \bar{d}) \exp(\tilde{dp}_t) + \bar{d} \). The top panel of Table 6 shows how large the change in \( \bar{r} \) is (row 2) corresponding to the change in the mean dividend-price ratio in the data (row 1). The left panel is for the one-break case, the right panel for the two-break case. The observed change in \( \tilde{dp} \) implies a decline in mean expected returns of 2.6%
Figure 7
Estimation with long-horizon moments
This figure compares the univariate OLS long-horizon regression coefficients, $\kappa_d(H) - OLS$ and $\kappa_r(H) - OLS$, to the GMM estimates that impose the present-value restriction (12), $\kappa_d(H) - GMM$ and $\kappa_r(H) - GMM$. The system contains 10 equations, 5 return and 5 dividend growth equations. The horizons (in years) are $H \in 1, 3, 5, 7, 10$. The top panel uses the unadjusted $dp$ ratio as predictor, the middle panel uses the $\tilde{dp}$ ratio adjusted for one break in 1991, and the bottom panel uses the $\tilde{dp}$ ratio adjusted for two breaks in 1954 and 1994.
### Table 6
Implied changes in steady-state expected returns and dividend growth

<table>
<thead>
<tr>
<th>dividend-price ratio</th>
<th>27–91</th>
<th>92–04</th>
<th>Change</th>
<th>27–54</th>
<th>55–94</th>
<th>95–04</th>
<th>Changes</th>
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<tbody>
<tr>
<td>$\ddp$</td>
<td>−3.133</td>
<td>−3.968</td>
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<td>$\dfrac{\ddp}{\dfrac{r}{d}}$, $\dfrac{d}{\dfrac{r}{d}}$ constant</td>
<td>8.86%</td>
<td>6.29%</td>
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<td>9.83%</td>
<td>8.16%</td>
<td>6.07%</td>
<td>−1.67%, −2.09%</td>
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<tr>
<td>$\dfrac{\ddp}{\dfrac{r}{d}}$, $\dfrac{r}{d}$ constant</td>
<td>5.00%</td>
<td>7.54%</td>
<td>2.54%</td>
<td>4.07%</td>
<td>5.68%</td>
<td>7.76%</td>
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<table>
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<tr>
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<th>91–04</th>
<th>Change</th>
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<th>54–94</th>
<th>95–04</th>
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<tr>
<td>$\dfrac{ep}{d}$</td>
<td>−2.540</td>
<td>−3.202</td>
<td>−0.662</td>
<td>−2.217</td>
<td>−2.652</td>
<td>−3.267</td>
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<tr>
<td>$\dfrac{d}{\dfrac{r}{d}}$, $\dfrac{d}{\dfrac{r}{d}}$ constant</td>
<td>11.72%</td>
<td>9.24%</td>
<td>−2.47%</td>
<td>13.70%</td>
<td>11.25%</td>
<td>8.87%</td>
</tr>
<tr>
<td>$\dfrac{ep}{d}$, $\dfrac{d}{\dfrac{d}{r}}$ constant</td>
<td>6.38%</td>
<td>8.79%</td>
<td>2.41%</td>
<td>4.52%</td>
<td>6.83%</td>
<td>9.16%</td>
</tr>
</tbody>
</table>

The top panel (bottom panel) of the table reports the mean log dividend-price ratio (log earnings-price ratio) in the subsamples, as well as the difference between the two. The second row reports the mean return $r$ in the subsamples if all of the changes in $\ddp$ ($\dfrac{ep}{d}$) in the first row were attributable to changes in mean returns. The third row reports mean dividend growth rates $d$ in the subsamples if all of the changes in $\ddp$ ($\dfrac{ep}{d}$) in the first row were attributable to changes in mean dividend growth. The left panel reports the case of one break; the right panel the case of two breaks. For the log dividend-price ratio the break date is estimated to be 1991 for the one break case (1954 and 1994 for the two break case). For the log earnings price ratio, the break is estimated to be 1990 for the one break case (1953 and 1994 for the two break case). In the bottom panel, we also report the change in the payout rate, the log dividend-earnings ratio $\dfrac{ed}{d}$. The sample for the top panel is 1927–2004; the sample for the bottom panel is 1946–2004 (see Appendix A).

in 1991 or a dual decline of 1.7% in 1954 and 2% in 1994, assuming long-run dividend growth did not change. Alternatively, it can stem from an increase in long-run dividend growth of 2.5% in 1991 or a dual increase of 1.6% in 1954 and 2% in 1994, when mean expected returns are held constant. In the results reported next, we choose to correct $\dfrac{r}{d}$, but this choice turns out to be unimportant for the point estimates. The second panel reports the change in the mean earnings-price ratio in the various subsamples, as well as the implied change in the long-run mean return or long-run mean dividend growth rate. Appendix A describes how the latter two are computed. We find that the change in the mean return of $-2.47\%$ that accounts for the change in the earnings-price ratio $\dfrac{ep}{d}$ (bottom panel) is very similar to the $-2.60\%$ change that accounted for the change in the dividend-price ratio $\ddp$ (top panel). The same is true when the long-run growth rate does all the adjustment.

Panel A of Table 7 reports the estimation of the 1-year ahead system. Row 1 uses the unadjusted $dp$ series, whereas rows 2 and 3 use the adjusted series $\ddp$ for the one-break and two-break case, respectively. The GMM estimation uses the OLS normal conditions to estimate $\kappa_d$ and $\kappa_r$. Therefore, the point estimates are identical to the ones reported in Table 1. Three differences are worth pointing out. First, the adjustment in $\dfrac{r}{d}$ delivers slightly lower standard errors for $\kappa_r$ in rows 2 and 3. Second, as foreshadowed by Table 2, the point estimates for $\phi$ are substantially lower when we use the adjusted dividend-price

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14 The reason is that returns and dividend growth are very volatile, compared to the change in their mean implied by the change in $\ddp$. For the same reason, the results reported below are virtually identical if we assume that the break takes place in $d$ instead. This validates the results in Section 3.3.
Table 7
Estimation with long-horizon moments

<table>
<thead>
<tr>
<th></th>
<th>( \kappa_d )</th>
<th>( \kappa_r )</th>
<th>( \phi )</th>
<th>PV violation</th>
<th>Moment violation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: No long-horizon moments ( H = {1} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No break</td>
<td>.005</td>
<td>.094</td>
<td>.945</td>
<td>-.046</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>(.037)</td>
<td>(.046)</td>
<td>(.052)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>One break (’91)</td>
<td>.019</td>
<td>.235</td>
<td>.813</td>
<td>.004</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>(.047)</td>
<td>(.055)</td>
<td>(.052)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two breaks (’54, ’94)</td>
<td>.124</td>
<td>.455</td>
<td>.694</td>
<td>-.001</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>(.073)</td>
<td>(.079)</td>
<td>(.070)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel B: Long-horizon moments ( H = {1, 3, 5} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No break</td>
<td>.021</td>
<td>.068</td>
<td>.990</td>
<td>.189</td>
<td>.205</td>
</tr>
<tr>
<td></td>
<td>(.018)</td>
<td>(.038)</td>
<td>(.032)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>One break (’91)</td>
<td>.019</td>
<td>.210</td>
<td>.834</td>
<td>.076</td>
<td>.085</td>
</tr>
<tr>
<td></td>
<td>(.019)</td>
<td>(.043)</td>
<td>(.042)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two breaks (’54, ’94)</td>
<td>.080</td>
<td>.409</td>
<td>.697</td>
<td>.100</td>
<td>.144</td>
</tr>
<tr>
<td></td>
<td>(.065)</td>
<td>(.078)</td>
<td>(.060)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table reports GMM estimates for the parameters \( (\kappa_d, \kappa_r, \phi) \), their asymptotic standard errors, and \( p \)-values. The results in panel A are for the system with 1-year ahead equations for dividend growth and returns \( (H = 1, N = 0) \). The results in panel B are for the system with 1-year, 3-year, and 5-year ahead equations for dividend growth and returns \( (H = \{1, 3, 5\}, N = 2) \). The estimation is by GMM. The first-stage weighting matrix is the identity matrix. The asymptotic standard errors and \( p \)-values are computed using the Newey-West HAC procedure (second-stage weighting matrix) with four lags in panel A and \( H = 5 \) lags in panel B. The first number in the last column denotes the present-value constraint violation of the univariate OLS slope estimators: \( (1 - \rho \phi^{ols})^{-1}(\kappa_{d,ols} - \kappa_{r,ols}) \). It is expressed in the same units as \( \kappa_d \) and \( \kappa_r \). In panel B, this number is the average violation of the three constraints, one constraint at each horizon. The second number in the last column reports the average moment violation. In panel A, that number is not available (N/A) because the system is exactly identified. The dividend-price ratio in rows 1 and 4 is the unadjusted one. In rows 2 and 3, the dividend-price ratio is adjusted for one break in 1991 (see Equation (8), and in rows 3 and 6, it is the series adjusted for two breaks in 1954 and 1994. All estimation results are for the full sample 1927–2004.
standard error indicate that we cannot reject the null hypothesis of a unit root in the dividend-price ratio.15

Results using adjusted series reported in rows 5 and 6 are quite different. Once we use the adjusted ratio \( \tilde{dp} \), we find strong evidence for return predictability. The point estimates remain large: .210 in the one-break case and .409 in the two-break case. Moreover, the asymptotic standard errors on \( \kappa_r \) are reduced. The reason is that we use restrictions of the term structure of predictability coefficients that cannot be uncovered by estimating the long-horizon moments in isolation; imposing these constraints improves the inference on \( \kappa_r \) and \( \kappa_d \). Put differently, the univariate OLS long-horizon coefficients violate the present value constraint. The first number in the last column reports the average violation across the three constraints (the RMSE); it is 19% in row 1. This violation is lower in rows 5 and 6. The second number in the last column reports the average moment violation (RMSE) and measures the degree to which the imposed restrictions are satisfied. In row 5, the average moment violation is only 8.5%, less than half as big as in row 4.

The middle and bottom panels of Figure 7 graphically illustrate the long-horizon estimation results with the adjusted dividend-price ratio for the larger \{1, 3, 5, 7, 10\}-year system. First, the one-period coefficients are very similar to the ones we reported for the \{1, 3, 5\}-year system. Second, the two panels show that when the adjusted dividend-price ratio is used, the pattern of GMM long-horizon regression slopes generated by our model lines up almost perfectly with the univariate OLS regression slopes. Put differently, the normal conditions for the long-horizon moments are satisfied at the model-implied long-horizon predictability coefficients. This contrasts with the first panel, which is for the unadjusted \( dp \) ratio. There, the OLS coefficients are quite different from the GMM estimates. The correction supports both our specification and our main argument.

The results for the \{1, 3, 5\}-year and \{1, 3, 5, 10\}-year systems are representative of the results we found for different numbers of long-horizon moments and choices of horizons. Imposing long-horizon information confirms the results of the earlier sections: returns are predictable by the dividend-price ratio, once its nonstationary component is removed.

7. Monte Carlo Simulations

In this section, we provide further evidence that the model in Equations (9)–(11) captures the moments of the data well. We use a Monte Carlo exercise to show (i) that this model replicates the failures that are found in the in-sample and out-of-sample predictability literature using the unadjusted \( dp \) ratio, and (ii) that it matches the moments once the dividend-price ratio is properly adjusted for and after taking into account small-sample inference issues.

15 The standard error of \( \phi \) is implied by the estimates for \( \kappa_d \) and \( \kappa_r \) through Equation (12) and computed using the delta method.
As Campbell, Lo, and MacKinlay (1997) point out, there is no closed-form solution for the long-horizon \( R^2 \). We approximate it by simulating the structural model for 100,000 periods.

### Table 8

**Long-horizon predictability in data and Monte Carlo exercise**

<table>
<thead>
<tr>
<th>Horizon</th>
<th>( k_e )</th>
<th>s.e.</th>
<th>( R^2 ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>True values in sim.</td>
<td>.222</td>
<td>.187</td>
<td>.159</td>
</tr>
<tr>
<td>Sim., ( dp ) unadj.</td>
<td>.115</td>
<td>.092</td>
<td>.075</td>
</tr>
<tr>
<td>Data, ( dp ) unadj.</td>
<td>.087</td>
<td>.095</td>
<td>.071</td>
</tr>
<tr>
<td>Sim., ( dp ) adj.</td>
<td>.264</td>
<td>.214</td>
<td>.175</td>
</tr>
<tr>
<td>Data, ( dp ) adj.</td>
<td>.220</td>
<td>.200</td>
<td>.150</td>
</tr>
<tr>
<td>True values in sim.</td>
<td>.456</td>
<td>.330</td>
<td>.249</td>
</tr>
<tr>
<td>Sim., ( dp ) unadj.</td>
<td>.128</td>
<td>.089</td>
<td>.064</td>
</tr>
<tr>
<td>Data, ( dp ) unadj.</td>
<td>.087</td>
<td>.095</td>
<td>.071</td>
</tr>
<tr>
<td>Sim., ( dp ) adj.</td>
<td>.483</td>
<td>.340</td>
<td>.249</td>
</tr>
<tr>
<td>Data, ( dp ) adj.</td>
<td>.442</td>
<td>.355</td>
<td>.231</td>
</tr>
</tbody>
</table>

The table reports results from univariate regressions of cumulative long-horizon returns on the log dividend-price ratio. The left columns denote slope coefficients for 1-year, 3-year, and 5-year horizon regressions; the middle columns report standard errors for the slope coefficients; the right columns report the corresponding regression \( R^2 \). The rows labeled “Data, \( dp \) unadj.” denote regressions run with real data using the unadjusted log dividend-price ratio as independent variable. The rows labeled “Data, \( dp \) adj.” denote regressions run with real data using the log dividend-price ratio, adjusted for a break in the mean \( dp \). In the top panel there is one break in this mean in 1991; in the bottom panel, there are two breaks in 1954 and 1994. The results for the data are contrasted with the results from a Monte Carlo exercise. The return and dividend growth system is estimated until 1991 in the top panel (1954 in the bottom panel) on real data. The estimated parameters imply “true” structural parameters. The theoretical long-horizon slope coefficients and regression \( R^2 \) are reported in the row with label “True values in sim.” For these parameters, the structural model is then simulated 10,000 times for 78 periods. The row “Sim., \( dp \) unadj.” denotes the Monte Carlo average slope coefficient and \( R^2 \) statistic using the unadjusted log dividend-price ratio as independent variable. The row “Sim., \( dp \) adj.” also reports regression coefficients and statistics of regressions on artificial data, but now the independent variable comes from a model where the mean \( r \) is adjusted to equal the change in \( dp \) in the data.

For the Monte Carlo exercise, we specify a structural model for the joint behavior of expected returns, expected dividend growth, and dividend growth innovations. In Appendix D, we derive the regression residuals \( \tau = (\tau^d, \tau^e, \tau^{dp}) \) as functions of the structural innovations and show how to identify the structural parameters from the parameters of the vector error correction model (VECM) in Equations (9)–(11). We back out the structural parameters from the previously reported estimates of the VECM parameters and simulate the structural model generating 10,000 time series for returns, dividend growth, and the dividend-price ratio of length \( T = 78 \), the same length as the data. We then run univariate predictability equations on the model-generated data and compare the parameter estimates to the true predictability coefficients and to the data. The Monte Carlo exercise also serves as a way to investigate the small-sample properties of the regression coefficients. Appendix E describes the algorithm in detail.

We start by studying the properties of univariate return predictability regressions. The first row of Table 8 reports the “true” predictability coefficients for the 1-year, 3-year, and 5-year horizon equations, as well as the theoretical regression \( R^2 \). In the top panel, the true parameters come from an estimation of \( \tau \).
the VECM that assumes a break in $dp$ in 1991; in the bottom panel we specify two breaks, in 1954 and 1994.

The second and fourth rows report the same coefficients and $R^2$ estimates on simulated data. This simulation ignores the presence of the break(s). Row 2 uses the unadjusted $dp$ ratio, and row 4 the adjusted $\tilde{dp}$ to predict returns. The second row shows that the model with the unadjusted $dp$ ratio fails to detect the return predictability that is present in the data. The simulation-based estimate of $\kappa_r$ and the regression $R^2$ are too small. The regression $R^2$ does not increase enough with the horizon. The 1-year ahead coefficient is not significant when we use the small-sample standard error (fourth column). Moreover, this failure of the model matches the failure in the data. Row 3 shows the results of the same univariate regressions in the data using the unadjusted $dp$ ratio. The slope coefficients, standard errors, and $R^2$ line up closely with the results from the Monte Carlo simulation without break adjustment.

In row 4, we adjust each Monte Carlo series for a break in 1991 (top panel) or two breaks in 1954 and 1994 (bottom panel). The predictability coefficients at all horizons now line up closely with their true values. The model with adjusted $dp$ ratio recovers the true predictability pattern of row 1. Moreover, the predictability coefficients, their standard errors, and the regression $R^2$ from the model in row 4 match the ones from regressions of observed returns on observed adjusted dividend-price ratios (row 5). Results for dividend growth regressions are not reported but the simulations recover the lack of predictability in the true $\kappa_d$ coefficients implied by the VECM.

Comparing row 4 to row 1, we notice that there is some small-sample bias. In line with the findings of Stambaugh (1999), the estimate for $\kappa_r$ is upward biased. In the first panel, the true value of the slope coefficient in the 1-year ahead return regression is .222 versus .264 in simulation, a bias of .042. At the 5-year horizon, the upward bias is only .016. Likewise, the $R^2$ of the regression is slightly upward biased in the simulation: 10.5% versus 8.9% at 1-year horizon and 28% versus 25% at the 5-year horizon. In the two-break case reported in the second panel, the bias is smaller. The 1-year ahead coefficient estimate is .483 versus the true value of .456. The upward bias disappears at the 5-year horizon. Overall, the bias is small relative to the magnitude of the coefficients, and therefore does not affect our conclusions. Finally, the small-sample standard errors, averaged across Monte Carlo simulations of the same length of the data, are very similar to the asymptotic standard errors from the data (middle columns).

Panels 2 and 3 of Table 5 report the out-of-sample prediction errors from a second Monte Carlo exercise. We simulate the structural model under the null hypothesis that the data generating process has one break in 1991 (panel 2) or has two breaks in 1954 and 1994 (panel 3). We compare the same three out-of-sample forecasting exercises as in the data (panel 1). In addition, we look at the forecast errors when we only correct for the second break and not the first one. The “true” parameters in panels 2 and 3 are the same and were obtained from
the VECM parameters estimated under the assumption of two breaks in 1954 and 1994 for the period 1947–2004, the same forecasting period as in panel 1. The details are shown in Appendix E. The Monte Carlo exercise regenerates the pattern we found in the data. When the unadjusted dividend-price ratio is used as a forecasting variable, the out-of-sample prediction errors are large and close to the random walk errors. On the other hand, when we implement the one-break or two-break adjustment in the model, the simulated data generate substantially lower prediction errors, mimicking the improvement in the data.

The simple model (9)–(11) replicates the patterns of univariate 1-year ahead and long-horizon regression results in-sample as well as 1-year ahead out-of-sample prediction errors found in the data. In particular, it regenerates (i) the failures of using the unadjusted $dp$ ratio as a predictor, and (ii) the successes of using the adjusted $\tilde{dp}$ ratio.

8. Conclusion

The macroeconomics literature has recently turned to models with persistent changes in fundamentals to explain the dramatic change in valuation ratios in the bull market of the 1990s. Most such models imply a persistent decline in expected returns or a persistent increase in the dividend growth rate. In this paper, we argue that either of such changes leads to a persistent decline in the mean of financial ratios. Such changes in the mean of valuation ratios have important effects on estimation and inference of return forecasting regressions. We consider various econometric techniques to detect shifts in the mean of price ratios and suggest a simple procedure to extract their stationary component. The adjusted price ratios robustly forecast returns in sample. At the same time, we show that shifts in the steady-state expected returns and growth rate of fundamentals are responsible for the instability of the return forecasting relation. Out-of-sample return predictions based on the adjusted price ratios improve relative to those based on the unadjusted ratios. However, estimation uncertainty about the magnitude of the break, rather than the break date, is ultimately what prevents a real-time investor to profit.

Appendix

A. Data description

We use annual end-of-year data from 1926 to 2004 from CRSP on the value-weighted market return (NASDAQ, NYSE, AMEX), with and without dividend capitalization. Net return including dividends are denoted $R_t$ and returns excluding dividends are $R_{\text{ex}}$. The dividend yield $D_{t+1}/P_t$ is the difference of capitalized and uncapitalized series; the dividend level is the product of the dividend yield $D_{t+1}/P_t$ and the price $P_t$ of a portfolio that does not reinvest dividends. The dividend-price ratio is defined as $D_{t+1}/P_{t+1}$ and dividend
growth as the change in the dividend level \((D_{t+1} - D_t)/D_t\). We then define log returns \(r_{t+1} = \log(1 + R_{t+1})\), log dividend growth \(\Delta d_{t+1} = \log(D_{t+1}/D_t)\), and the log dividend-price ratio \(d_{P_t} \equiv d_t - p_t = \log(D_t/P_t)\).


The second panel of Table 6 reports the subsample and full-sample means of the log earnings-price ratio \((\bar{eP})\), dividend-earnings ratio (payout rate), and the implied mean changes in long-run expected returns \(\bar{r}\) or expected dividend growth \(\bar{d}\). The implied change in \(\bar{r}\), holding \(\bar{d} = 0.0709\) fixed, is computed as \(\bar{r} = \bar{d} + (1 + \bar{d})(D/E)(E/P)\). Likewise, the implied change in \(\bar{d}\), holding \(\bar{r} = .1098\) fixed, is computed as \(\bar{d} = (\bar{r} - (D/E)(E/P))/(1 + (D/E)(E/P))\).

### B. A modified log-approximation

In this section, we extend the Campbell-Shiller log linear approximation to allow for time-varying steady-state growth rates and returns. This framework is a useful organizing principle for the empirical analysis in the main text, but most of our specifications do not impose the approximation. Our results do not depend on the accuracy of the approximation, but the framework helps to understand the intuition and implications.

The gross return of an asset is defined as

\[
R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{D_{t+1} 1 + P_{t+1}/D_{t+1}}{D_t 1 + P_t/D_t}.
\]

As of period \(t\), the steady-state (gross) growth rate of dividends is \(\bar{D}_t\) and steady-state expected (gross) returns are \(\bar{R}_t\), implying a steady state for the price-dividend ratio in levels \((\bar{P}D_t)\) and logs \((\bar{p}d_t)\)

\[
\bar{P}D_t = \frac{\bar{D}_t}{\bar{R}_t - \bar{D}_t} \quad \Rightarrow \quad \bar{p}d_t = \bar{d}_t - \log(\exp(\bar{r}_t) - \exp(\bar{d}_t)).
\]

Instead of presuming that the steady-state growth rates and expected returns are constant, we allow for the possibility that the steady state may change over time. The only requirement that we impose on the steady state log returns and log growth rates is that they are martingales; \(E_t[\bar{r}_{t+j}] = \bar{r}_t\), \(E_t[\bar{d}_{t+j}] = \bar{d}_t\). In other words, the steady state is constant in expectation only. We also assume that the steady-state log \(P/D\) ratio is a martingale; \(E_t[\bar{p}d_{t+j}] = \bar{p}d_t\). Although, in general, these assumptions can be inconsistent with each other since the log \(P/D\) ratio is a nonlinear function of \(\bar{r}_t\) and \(\bar{d}_t\), we show in Appendix C that the martingale assumptions are satisfied to a very good approximation for reasonable break processes. Log linearizing (B1) around the steady state in
\[pd_t \rightarrow \overline{pd_t} = (\Delta d_{t+1} - \overline{d_t}) - (r_{t+1} - \overline{r_t}) + \rho_{t+1}(pd_{t+1} - \overline{pd}_{t+1}) + \Delta \overline{pd}_{t+1} + \Delta \overline{r}_{t+1} - \Delta \overline{d}_{t+1}, \]

(B3)

where \(pd_t = p_t - d_t\) and \(\rho_{t+1} = \frac{\exp(pd_t)}{1 + \exp(pd_t)}\). The last two assumptions state that \(\rho_t\) is a martingale, and that deviations from the mean price-dividend ratio are uncorrelated with \(\rho\); \(E_t[\rho_{t+j}] = \rho_t\) and \(E_t[\rho_{t+j}(pd_{t+j} - \overline{pd}_t)] = 0\). Given our assumptions, we can take conditional expectations and solve the expectational difference equation for \(pd_t\):

\[pd_t \rightarrow \overline{pd_t} = E_t[\Delta d_{t+1} - \overline{d_t}] - E_t[r_{t+1} - \overline{r_t}] + \rho_t E_t[pd_{t+1} - \overline{pd}_{t+1}]. \quad (B4)\]

\[= \sum_{j=1}^{\infty} \rho_t^{-1-j} E_t[\Delta d_{t+j} - \overline{d_t}] - E_t[r_{t+j} - \overline{r_t}] . \quad (B5)\]

The log price-dividend ratio is the sum of the steady-state price-dividend ratio and the discounted sum of expected dividend growth minus expected returns in excess of steady-state growth and returns

\[pd_t = \overline{pd_t} + \sum_{j=1}^{\infty} \rho_t^{-1-j} E_t[\Delta \overline{d}_{t+j}] - E_t[\overline{r}_{t+j}], \quad (B6)\]

where \(\Delta \overline{d}_{t+j} = \Delta d_{t+j} - \overline{d_t}\) and \(\overline{r}_{t+j} = r_{t+j} - \overline{r_t}\). The expression in the main text for \(dp_t\) follows from (B6) and \(dp_t = -pd_t\).

C. An asset pricing model with steady-state shifts

This appendix solves a fully specified asset pricing model where the steady-state log dividend-price ratio follows an approximate, bounded martingale. Breaks in the steady-state dividend-price ratio originate in breaks in the steady-state mean dividend growth rate. Evidence for such breaks in US data is provided in Timmermann (2001).

Log dividend growth is \(\Delta d_{t+1} = g + q_t + \varepsilon_{t+1}\). The unconditional average consumption growth rate is \(g\), \(q_t\) is the stochastic component of the long-run mean, and \(\varepsilon_{t+1}\) is a temporary consumption growth shock, \(\varepsilon_{t+1} \sim \mathcal{N}(0, \sigma^2)\).

Let \(Q_t = (1 - A \exp(q_t))^{-1}\) for a constant \(A\) given next. We assume that \(Q_t\) is a martingale and show below that this implies that log dividend growth is also a martingale.

\[Q_{t+1} = Q_t + \eta_{t+1}\]
Reconciling the Return Predictability Evidence

The innovation to the random walk is independent from $\varepsilon_{t+1}$ and has the following distribution:

$$\eta_{t+1} = \begin{cases} 0, & \text{w.p. } \pi \\ w_{t+1}, & \text{w.p. } 1 - \pi \text{ and } w_{t+1} \sim \mathcal{N}(0, \sigma^2_{w,t+1}) \end{cases}.$$  

With probability $\pi$, the innovation $w_{t+1}$ is 0, where $\pi$ is close to 1 to capture that a break is a rare occurrence. With the complementary probability $1 - \pi$, there is a break with normally distributed break size. The innovation standard deviation of $w_{t+1}$ is time varying. It takes on the following form:

$$\sigma_{w,t+1} = \bar{\sigma}_w (1 - \exp(\kappa (1 - Q_t))). \quad (C1)$$

This implies that for $Q_t \gg 1$, $\sigma_{w,t+1} \approx \bar{\sigma}_w$ and as $Q_t \to 1$, $\sigma_{w,t+1} \to 0$. The parameter $\kappa$ governs how fast the standard deviation of $w$ shrinks as $Q_t$ approaches 1 from above. This specification guarantees that $Q_t > 1$ (at least in continuous time). In other words, $Q_t$ is a bounded martingale.

If investors are risk neutral with a time discount factor of $\beta$, asset returns have to satisfy the Euler equation

$$1 = E_t \left[ \beta \frac{D_{t+1}}{D_t} 1 + \frac{P_{t+1}}{D_{t+1}} \frac{P_t}{D_t} \right].$$

Conjecture that the price-dividend ratio takes the following form:

$$\frac{P_t}{D_t} = Q_t - 1. \quad (\text{C2})$$

Using this conjecture, the Euler equation becomes

$$1 = E_t \left[ \beta e^{\delta + \varepsilon_{t+1}} A^{-1} \frac{Q_{t+1}}{Q_t} \right].$$

Because the innovation to $Q_{t+1}$ is independent from the innovation $\varepsilon_{t+1}$, the conjecture is verified for

$$A = \beta e^{\delta + \sigma^2_t/2} E_t \left[ \frac{Q_{t+1}}{Q_t} \right] = \beta e^{\delta + \sigma^2_t/2},$$

because $Q_t$ is a martingale. The log dividend-price ratio is also an (approximate) martingale. Consider a Taylor approximation of $d_{t+1} - p_{t+1}$ around $Q_t$:

$$d_{t+1} - p_{t+1} = d_t - p_t + \frac{1}{1 - Q_t} (Q_{t+1} - Q_t) + O_{t+1}. \quad (\text{C3})$$

Because $Q_t$ is a martingale, $E_t[d_{t+1} - p_{t+1}] = d_t - p_t + E_t[O_{t+1}]$. As long as the conditional mean of the higher order terms $O_{t+1}$ is small, the log dividend-price ratio is approximately a martingale. For reasonable break processes the
higher order terms turn out to be negligible. For example, if $\pi = .99$, $\bar{\sigma}_w = 5$, and $\kappa = \ldots$, the conditional mean of the higher-order terms is 0.00006 for a price-dividend ratio level of 40, 0.00017 for a price-dividend ratio level of 20, and 0.00048 for a price-dividend ratio of 2, which is tiny compared to the value of $d - p$, which is $-3.10$ on average in a simulation with the same parameters.

By the same token, $E_t[q_{t+1}] \approx q_t$. The difference between the conditional expectation of $q_{t+1}$ and $q_t$ is $-0.0003114$ for a price-dividend ratio of 2.

Again, this is small relative to the simulation average for $q$ of $-0.0094$.

The aforementioned specification imposes that $Q_t > 1$, or equivalently $q_t > -\infty$. It would be easy to impose tighter restrictions on the steady-state log dividend growth rate process $q_t$, based on an priori “reasonable range” for log dividend growth by modifying (C1). For example, one could impose that the long-run dividend growth rate should not be smaller than $-5\%$ per year. This amounts to price-dividend ratios in excess of 11.5 ($\hat{Q} > 12.5$). The same bounded martingale approach would work with such a restriction.

D. Structural model

We propose a model for log dividend growth $\Delta d$ and log returns $r$, where expected dividend growth $z$ and expected returns $x$ follow an AR(1) with autoregressive coefficient $\phi$

$$\Delta d_{t+1} - \bar{d} = z_t + \epsilon_{t+1}$$
$$z_{t+1} = \phi z_t + \zeta_{t+1}$$
(D1)

$$r_{t+1} - \bar{r} = x_t + \eta_{t+1}$$
$$x_{t+1} = \phi x_t + \xi_{t+1},$$
(D2)

where $\bar{d}$ is the long-run mean log dividend growth and $\bar{r}$ is the long-run mean return. The model has three fundamental shocks: a dividend innovation $\epsilon_{t+1}$, an innovation in expected dividends $\zeta_{t+1}$, and an innovation in expected returns $\xi_{t+1}$. The return decomposition in Campbell (1991) implies that

$$\eta_{t+1} = -\frac{\rho}{1 - \rho\phi} \xi_{t+1} + \frac{\rho}{1 - \rho\phi} \zeta_{t+1} + \epsilon_{t+1}.$$  
(D3)

We assume that all three errors are serially uncorrelated and have zero cross covariance at all leads and lags: $\text{Cov}(\epsilon_{t+1}, \zeta_{t+j}) = 0$, $\forall j \neq 1$, and $\text{Cov}(\epsilon_{t+1}, \xi_{t+j}) = 0$, $\forall j \neq 1$, except $\text{Cov}(\zeta_t, \xi_t) = \chi$ and $\text{Cov}(\zeta_t, \epsilon_t) = \lambda$.

In steady-state, the log dividend-price ratio is $\bar{dp} = \log(\bar{r} - \bar{d})$, hence $\bar{dp} \approx \bar{r} - \bar{d}$ and $\rho \approx \frac{1 + \bar{d}}{1 + \bar{r}}$. The log dividend-price ratio can be written as

$$dp_t = d_t - p_t = \bar{dp} + \frac{x_t - z_t}{1 - \rho\phi}.$$  
(D4)

This equation clearly shows that the demeaned dividend-price ratio is an imperfect forecaster of returns. Returns are predicted by $x_t$ (D2), which con-
tains not only the demeaned \( dp \) ratio, but also expected dividend growth \( z_t \):

\[
x_t = (1 - \rho \phi)(dp_t - \overline{dp}) + z_t.
\]

This structural model implies a reduced form model that recovers the two predictability equations from Section 2:

\[
\begin{align*}
(\Delta d_{t+1} - \overline{d}) &= \kappa_d(dp_t - \overline{dp}) + \tau_{d_{t+1}}^d \quad (D5) \\
(r_{t+1} - \overline{r}) &= \kappa_r(dp_t - \overline{dp}) + \tau_{r_{t+1}}^r \quad (D6) \\
(dp_{t+1} - \overline{dp}) &= \phi(dp_t - \overline{dp}) + \tau_{dp_{t+1}}^d. \quad (D7)
\end{align*}
\]

The third equation of this Vector Error Correction Model is an AR(1) process for the dividend-price ratio. Because of Equation (6), the dividend-price ratio is the difference of two AR(1) processes with the same root \( \phi \), which is again an AR(1) process. We also considered a model where expected dividend growth has a different autoregressive coefficient \( \psi \neq \phi \):

\[
z_{t+1} = \psi z_t + \zeta_{t+1}. \quad (D8)
\]

Since the \( dp \) ratio is well described by an AR(1) model in the data, we set \( \phi = \psi \).

The slope coefficients are related to the structural parameters

\[
\begin{align*}
\kappa_d &= \frac{\text{Cov}(\Delta d_{t+1}, d_t - p_t)}{\text{Var}(d_t - p_t)} = \frac{-(1 - \rho \phi)(\sigma_z^2 - \chi)}{\sigma_z^2 + \sigma_z^2 - 2\chi}. \quad (D8) \\
\kappa_r &= \frac{\text{Cov}(r_{t+1}, d_t - p_t)}{\text{Var}(d_t - p_t)} = \frac{(1 - \rho \phi)(\sigma_z^2 - \chi)}{\sigma_z^2 + \sigma_z^2 - 2\chi}. \quad (D9)
\end{align*}
\]

The innovations to the VECM, \( \tau = (\tau^d, \tau^r, \tau^{dp}) \), are given by

\[
\begin{align*}
\tau_{t+1}^d &= \epsilon_{t+1} + x_t \left( \frac{-\kappa_d}{1 - \rho \phi} \right) + z_t \left( \frac{\kappa_r}{1 - \rho \phi} \right) \quad (D10) \\
\tau_{t+1}^r &= \epsilon_{t+1} + x_t \left( \frac{-\kappa_d}{1 - \rho \phi} \right) + z_t \left( \frac{\kappa_r}{1 - \rho \phi} \right) - \rho \frac{\xi_{t+1} - \xi_{t+1}}{1 - \rho \phi} \quad (D11) \\
\tau_{t+1}^{dp} &= \frac{\xi_{t+1} - \xi_{t+1}}{1 - \rho \phi}. \quad (D12)
\end{align*}
\]

The structural model imposes a restriction on the innovation vector: \( \rho \tau_{t+1}^{dp} = (\tau_{t+1}^d - \tau_{t+1}^r) \). Another way to write this restriction is as a restriction on a weighted sum of \( \kappa_r \) and \( \kappa_d \):

\[
\kappa_r - \kappa_d = 1 - \rho \phi.
\]

We call this restriction the present value constraint.

Leaving aside the mean parameters (\( \overline{d}, \overline{r}, \overline{dp} \)), which play no role in the demeaned system, the structural parameter vector is \( \Theta = (\phi, \sigma_r, \sigma_z, \sigma_e, \chi, \lambda) \).

The variance \( \sigma^2_\eta \) is implied by (D3). The vector of parameters from the re-
duced form model (VECM) is \( b = (\kappa_d, \kappa_r, \phi, \Sigma_\tau) \), where \( \Sigma_\tau \) is the variance-covariance matrix of \( \tau \). There are six unique elements in this covariance matrix and the present value constraint imposes three restrictions on these six elements. In addition, the present value constraint imposes a restriction on one of the elements in \( (\kappa_r, \kappa_d, \phi) \). Hence, there are five unique elements in \( b = (\kappa_r, \kappa_d, \sigma_{\tau'}, \sigma_{\tau'}, \sigma_{\tau'}, \sigma_{\tau'}) \), where \( \sigma_{\tau'} \) and \( \sigma_{\tau'} \) are the standard deviations of \( \tau' \) and \( \tau_d \), and \( \sigma_{\tau', \tau'} \) is the covariance between the two. When we estimate the nonsingular system of Equations (D5) and (D6), we can use these five coefficients to identify five out of six structural parameters in \( \Theta_1 \). Therefore, whenever we back out structural parameters from the reduced form estimates, we find it convenient to tabulate results for a range of values for \( V_z \), which measures the contribution of expected dividend growth to the variance of the dividend-price ratio:

\[
V_z = \frac{\sigma^2_z}{(1 - \rho\phi)^2 \sigma^2_{dp}} = \frac{\sigma^2_\xi}{(\sigma^2_\xi + \sigma^2_\tau - 2\chi)}. \tag{D13}
\]

E. Monte Carlo simulations

The Monte Carlo exercise simulates the model under the null hypothesis of one break or two breaks in the dividend-price ratio. It then asks what the univariate long-horizon slope coefficients and \( R^2 \) are in a small sample (of the same length as the data) when we use the unadjusted \( dp \) as regressor versus the adjusted \( \tilde{dp} \). For the one-break case reported in the top panel of Table 8, we use the following algorithm:

step 1 To find the true parameters, we estimate the (1, 3, 5)-year VECM under the assumption of one break in 1991 for the full sample. This delivers reduced form estimates: \( b = (.0121, .2098, .1410, .1868, .0176) \). Throughout, \( \rho = .9616 \), the value implied by the mean price-dividend ratio in the sample.

step 2 We invert these reduced form parameters to obtain structural parameters based on the identification scheme described earlier. We need to take a stance on the fraction of the variance in the dividend-price ratio attributable to expected dividend growth (D13). We set \( V_z = 0.3 \), but the results are not sensitive to this choice. The implied structural parameters are \( \Theta = (.8344, .0182, .0355, .1371, .0004, -.0005) \).

step 3 In each Monte Carlo iteration, we draw a new 178 × 3 vector of i.i.d. standard normal variables. The structural shocks \( (\varepsilon_t, \xi_t, \xi_t) \) are these standard normal variables premultiplied by \( \Sigma^2_u \), where \( \Sigma_u \) is the covariance matrix of the structural innovations:

\[
\Sigma_u = \begin{bmatrix}
\sigma^2_\varepsilon & \lambda & 0 \\
\lambda & \sigma^2_\xi & \chi \\
0 & \chi & \sigma^2_\xi
\end{bmatrix}.
\]
step 4 We recursively build up time series for $x, r - \bar{r}, z, \text{ and } \Delta d - \bar{d}$ according to (D1) and (D2). We form a time series for the demeaned dividend-price ratio from (D4).

step 5a Under the null hypothesis of one break in 1991, the break-adjusted series for returns, dividend growth, and dividend-price ratio are obtained by adding constant long-run means $\bar{r}, \bar{d}, \text{ and } \bar{dp}$ to the demeaned series. i.e., the adjusted $dp$ series is the demeaned series plus the mean over the entire sample 1927–2004, which is $\bar{dp} = -3.272$. Likewise, the adjusted dividend growth series $\Delta d$ is formed by adding to the demeaned series $\Delta d - \bar{d}$ the sample mean $\bar{d} = 0.0432$. We then back out $\bar{r} = \bar{d} + (1 + \bar{d}) \exp(\bar{dp}) = 0.0827$, and form the adjusted return series as the demeaned series plus this $\bar{r}$.

step 5b On the other hand, the unadjusted series still displays a break in 1991. To obtain the unadjusted series, we need to add in a different mean before and after 1991. As before, we assume $\bar{d}$ did not change, so that the change in $\bar{dp}$ entirely comes from a change in $\bar{r}$. The unadjusted $dp$ series is obtained by adding in the 1927–1991 mean ($-3.133$) before 1991 and the 1992–2004 mean ($-3.968$) after 1991. Likewise, for returns, we add in $0.0886$ before 1991 and $0.0629$ after 1991. These are the subsample means $\bar{dp}$ and $\bar{r}$ that were reported in lines 1 and 2 of Table 6 (left panel).

step 6 We form annualized, cumulative long-horizon returns and dividend growth rates $1 \sum_{j=1}^{H} \Delta r_{t+j}$ and $1 \sum_{j=1}^{H} \Delta d_{t+j}$ in the same way. There is one set of unadjusted series and one set of adjusted series, corresponding to each horizon $H$.

step 7 After the formation, we discard the first 100 observations (burn-in), and are left with the same number of observations as in the data: 78—longest horizon +1. For the 1-, 3-, and 5-year system we report on, the longest horizon is 5, so all statistics are computed with 74 observations.

step 8 We then run univariate 1-, 3-, and 5-year ahead predictability regressions of adjusted returns and dividend growth on both unadjusted and adjusted dividend-price ratios. We keep track of the predictability coefficients and regression $R^2$.

step 9 We repeat this procedure 10,000 times and report the average predictability coefficients and $R^2$ across Monte Carlo iterations.

The Monte Carlo exercise in the two-break case is exactly analogous (bottom panel of Table 8). In step 1, we estimate the 1-, 3-, and 5-year ahead VECM system under the null of two breaks in 1954 and 1994. The VECM coefficient vector is $b = (0.0795, 0.4089, 0.1391, 0.1743, 0.0157)$. The implied structural parameter vector for $V_z = 0.3$ is $\Theta = (0.6974, 0.0293, 0.0635, 0.1339, 0.0014, -0.0012)$. To construct the unadjusted $dp$ and return series we use the means reported in the right panel of Table 6.

We conduct a separate Monte Carlo exercise for the out-of-sample predictability (panels 2 and 3 in Table 5). We simulate the model under the null hypothesis of two breaks in 1954 and 1994. For simplicity, we use the same true
parameters in both panels 2 and 3. They are obtained from estimating the 1-year ahead VECM under the assumption of two breaks in 1954 and 1994, but for the period 1947–2004, which is the same sample period as we use in the data (panel 1 of Table 5). We then construct adjusted and unadjusted $dp$ ratios as in the previous algorithm, and run the same out-of-sample predictions as in the data.

References


Reconciling the Return Predictability Evidence


