Notes on Introduction to Contract Theory

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1 Overview of the Course

This is a readings course. The lectures are designed to distill the central ideas of the readings into models capturing the main insights leaving many of the technical details to be explained in the readings themselves. The hope is that the combination of being exposed to the motivating problems for many of these excursions in contract theory combined with the modeling exercise of distilling things down to the fundamental economic ideas will enable students wishing to pursue contract theory—either in its own right or as a “front-end” to experimental or empirical work—will have the tools and skills required to use these ideas as part of a thesis.

All materials pertaining to the course are located on the course website:
http://faculty.haas.berkeley.edu/rjmorgan/econ206
To summarize, in this course you will:

- Learn the basic tools/main ideas of contract theory
- Touch on the relationship between the main theoretical insights to come out of this literature and empirical/experimental work.
- Learn how the main ideas of contract theory can be usefully applied in a variety of economics settings including areas of finance, industrial organization, labor, and public economics.

2 Major Families of Models

The course is organized along the lines of the major families of models in contract theory. The models mainly consist of interaction between a single principal and a single agent where the principal is assumed to have all the bargaining power. That is, the principal can propose a take it or leave it contract which the agent can either accept or not. The major families are organized along two “axes”: The first axis concerns whether the primary concern of the principal is hidden information or
hidden action. When the problem is hidden information, the agent knows something economically relevant to the principal. When the problem is one of hidden action, the agent must undertake some effort to produce economic value to the principal but this effort cannot be monitored directly. The second axis concerns the timing of moves. Either the principal moves first and effectively specifies the contract (or, more generally, the game form) or the agent moves first. In the latter case, the modeler essentially determines the extensive form of the game played between principal and agent. The following table summarizes the two axes. Each of the main families of models is listed in bold and is detailed below.

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<tr>
<th>P moves:</th>
<th>Hidden: Information Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>Adverse Selection</td>
</tr>
<tr>
<td>Second</td>
<td>Moral Hazard</td>
</tr>
<tr>
<td></td>
<td>Signaling</td>
</tr>
</tbody>
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**Adverse Selection**

In adverse selection models, the agent possesses economically relevant information that the principal would like to know. The archetypal example occurs with a price discriminating monopoly. The agent knows her willingness to pay for the item. Since, by assumption, the principal has all the bargaining power, if it knew this information, it could structure a contract to extract all of the surplus from the agent. Since it doesn’t know this information, the contract proposed must be structured to induce the agent to reveal it truthfully. The key point is that, to obtain truthful reporting on the part of the agent, the principal has to pay an information rent to induce agents with different willingnesses to pay to report truthfully. It is worth noting that the space of contracts the principal might employ to achieve its ends is quite large. As we shall see later, this problem is dramatically simplified by the key tool in this section—the revelation principle. We’ll return to this in more detail later.

Key question: How to structure contracts to maximize surplus extraction
Key tool: Revelation principle.

**Signaling**

In signaling models, the central problem is that the agent wants to credibly convey his hidden information to the principal. The problem is that many “types” of agent all want to be perceived to be the “best” type. The archetypal example of this type of situation occurs with used durable goods of heterogeneous quality. All sellers want to be perceived as selling high quality goods and the question turns on how to credibly convey this information. In this setting, the use of money back guarantees reflects a sort of signaling solution.

Key distinction: We will distinguish between costly signaling, where the key economic force is that sending a given signal is differentially costly for different types and costless signaling, where credibility of a signal depends on differences in the economic outcome obtained by sending differing signals (more detail on this later in the course).
Key tool: Equilibrium refinements.

**Moral Hazard**

In moral hazard models, the central concern of the principal is to set incentives in such a way as to obtain the desired level of effort at lowest cost. Incentive contracts in industry are the archetypal example of this type of contract. The main result in this area is that, if the agent is risk-averse, then there is a trade-off between offering high-powered incentives, which motivate effort but entail substantial risk to the agent, and offering insurance. Thus, risk-aversion “blunts” the high-poweredness of incentives.

Key questions: How do incentives change with multiple agents? Multiple tasks?

Key tool: First-order approach

After covering the main families of models, we’ll talk about some extensions along three lines:

1. Long-term contracting
2. Imperfect commitment
3. Incomplete contracting

We now turn to the revelation principle:

### 3 Direct and Indirect Mechanisms

What is a contract?

On a general level, a contract is a game designed by the principal and played by the agent or agents. There may be various institutional restrictions placed on the set of feasible game forms the principal may choose from.

Thus, a contract can be thought of as a game $\Gamma$ that designates the strategies available to the agents as well as the payoffs from the realizations of the strategies. We call such a contract an *indirect mechanism*. Suppose agent $i$’s type is $\theta_i$ and her equilibrium strategy in game $\Gamma$ is $\psi_i(\theta_i)$. Let $\psi(\theta)$ denote the vector of strategies chosen by all of the agents given their type.

So we have

$$\theta$$

$$v_i(\psi(\theta)) \leftarrow \psi(\theta)$$

The mechanism is *indirect* since ultimately the vector of types $\theta$ determines the payoffs to each of the agents; however this information is transmitted indirectly through the strategy $\psi(\cdot)$.

In contrast, the principal might elect to use a *direct mechanism*. In a direct mechanism:

- Agents report their types to an neutral intermediary
The intermediary then implements the strategy $\psi_i(\cdot)$ on behalf of the agent

And the agent earns $v_i(\psi(\theta))$.

Thus, the direct mechanism looks like:

$$
\begin{array}{c}
\theta \\
\sqrt{v(\psi(\theta)), \psi(\theta)}
\end{array}
$$

Indirect Mechanisms in Practice  Now we will have you participate in an indirect mechanism. I’ll play the role of principal and you will be the agents. As principal, my objective is to maximize the amount of money I get from you. As agents, you care about your surplus. Now there are a huge number of game forms I might select from. I’ll choose this one:

I’m going to auction the following object using an auction mechanism used to sell art and wines as Sotheby’s. This mechanism is sometimes called an English or open outcry auction.

Notice how this is an indirect mechanism. Your type (your WTP for the candy bar) is transmitted only indirectly through your bidding behavior.

Direct Mechanisms in Practice  Now consider the auction format on eBay. With eBay, you submit your “type” to a bidder elf who bids on your behalf up to the value you told it. Thus, eBay is a kind of direct mechanism version of the Sotheby’s auction. Let’s try this mechanism.

The Revelation Principle  Suppose we use the solution concept of Bayes–Nash equilibrium then it must be the case that for agent $i$ of type $\theta_i$,

$$
E_{\theta_i}v_i(\psi_i(\theta_i), \psi_{-i}(\theta_{-i})) \geq E_{\theta_i}v_i(\psi_i', \psi_{-i}(\theta_{-i}))
$$

for all $\psi_i'$ in the indirect mechanism. That is, no agent should have a profitable deviation.

We might also require that the agent have an incentive to participate in the game in the first place. That is

$$
E_{\theta_i}v_i(\psi(\theta)) \geq \bar{v}
$$

for some outside option $\bar{v}$.

The point of the direct mechanism is that, through the revelation principle, it allows us to ignore analyzing all of the indirect mechanisms and focus only on the direct mechanism. Specifically,

Theorem 1 (Revelation Principle, Myerson, 1981)  Suppose that $\psi$ was a Bayes-Nash equilibrium of the indirect mechanism $\Gamma$. Then there exists a direct mechanism that is payoff-equivalent and where truthful revelation is an equilibrium.
Sketch Proof. By construction, if agents tell the truth in the direct mechanism, it is payoff equivalent to playing the $\psi$ equilibrium strategies in the indirect mechanism $\Gamma$. To see that truth-telling is an equilibrium, notice that if player $i$ with type $\theta_i$ deviates and reports his type as $\theta_i'$ then she earns $E_{\theta_{-i}} v_i(\psi_i(\theta_i'), \psi_{-i}(\theta_{-i})) = E_{\theta_{-i}} v_i(\psi_i', \psi_{-i}(\theta_{-i}))$ for some $\psi_i'$ and, from above, we know

$$E_{\theta_{-i}} v_i(\psi_i(\theta_i), \psi_{-i}(\theta_{-i})) \geq E_{\theta_{-i}} v_i(\psi_i', \psi_{-i}(\theta_{-i}))$$

Therefore, this is not profitable. QED

Mechanism Design So far, all we have done is showed that the principal could limit him or herself to eBay style auctions without any giving up anything in terms of revenues. The principal has other tools available on eBay. In particular, $P$ could set a reserve price and an opening bid level for the auction.

We’ll try that now.

Notice that people with WTP below the reserve level are excluded from the auction.

Many of these mechanisms are strategically equivalent. Specifically,

**Proposition 1** Any two eBay auctions that exclude the set of types are strategically equivalent. Further, in these auctions, the seller earns the same expected revenues.

The proof is obvious from the revelation principle.


[Explain eBay field experiments here.]

3.0.1 Main Lessons

The revelation principle should be used with care. Psychological factors can play a key role.