Notes on Incomplete Contracts

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1 Introduction

Throughout the course, we have assumed that all contracts are complete. This is, of course, complete nonsense in reality but a useful starting point in theory. But why is it complete nonsense in reality? What are we missing? Many stories have been suggested. One possibility is that we can, indeed, think of all possible contingencies, but it is costly to write all of these down in a contracts (Lawyers aren’t cheap). Thus, we economize on the longshots that don’t matter much and write in the parts that do. Another possibility is that we’re simply not able to foresee all possible contingencies so that even if we wanted to write a complete contract and there were no costs in doing so, we simply couldn’t. Still a third possibility is that, while the parties engaging in the contract can observe aspects of the contract perfectly well, the courts are unable or unwilling to verify these pieces. Therefore, whether we write these outcomes down in the contract or not avails us little in the event of breach.

In the last unit on dynamics of adverse selection and moral hazard, we emphasized that renegotiation could (and sometimes did) make things worse off. The Principal would obviously like to have full commitment power. All renegotiation did was to constrain the set of outcomes that the Principal could credibly commit to, and this was bad news from his perspective. On the other hand, with incomplete contracts, there’s a new and more positive motive for renegotiation. An outcome has occurred that we simply didn’t contemplate or couldn’t contract upon ex ante and now we’d like to fix the incentive problem going forward. Thus, unlike the complete contracting literature, under incomplete contracts renegotiation now has the possibility of improving contractual outcomes rather than harming them.

Much of this literature represents attempts to build formal models to capture the insights of transaction cost economics as described by Coase, Williamson, Klein, Crawford, Alchian, and so on. The key wedge in most of the formal models is the presence of relationship specific, but non-contractible investment. This leads in turn to the famous hold-up problem. Decisions by the Principal about the boundaries of the firm and property rights more generally in response to these issues lie at the heart of many of the formalizations.
This unit is intended to be a brief survey of the main “success stories” of the incomplete contracts literature. As one might guess, each of the success stories has led to a host of caveats and limitations and we’ll explore these arguments as well. Other courses, typically those offered in Haas, offer much greater depth into the models and issues that arise. Thus, this unit should be viewed as a “primer” or “gateway” for those materials for those interested in studying these in more detail.

2 Moral Hazard and Incomplete Contracts

Consider a simple one-period moral hazard model between a risk-neutral Principal and a risk-averse Agent. Unlike the standard model, suppose that effort is perfectly and costlessly observable by P.

Full commitment with renegotiation

Suppose that P can fully commit as in the standard model. Even though the action is observable, since it is not contractible, this doesn’t change the contract. Let \( e \) be a contractible effort level and let \( w \) denote the vector of wages. Then we know that A earns expected utility of

\[
U = \sum_{i=1}^{n} p(y_i|e) u(w(y_i)) - c(e)
\]

Nowe consider the effects of allowing renegotiation. Since \( e \) is observable, P can, after having observed \( e \), propose a renegotiated schedule paying a fixed wage, \( \bar{w} \) such that

\[
\sum_{i=1}^{n} p(y_i|e) u(w(y_i)) = u(\bar{w})
\]

Since the agent is indifferent, he’s happy to substitute the one contract for the other. However, P is now better off. To see this, recall that

\[
\sum_{i=1}^{n} p(y_i|e) u(w(y_i)) \leq u \left( \sum_{i=1}^{n} p(y_i|e) w(y_i) \right)
\]

Hence

\[
u(\bar{w}) \leq u \left( \sum_{i=1}^{n} p(y_i|e) w(y_i) \right)
\]

The cost to P of implementing \( u \) utils to the agent is a function \( h = u^{-1} \). Hence

\[
h(u(\bar{w})) \leq h \left( u \left( \sum_{i=1}^{n} p(y_i|e) w(y_i) \right) \right)
\]

Hence, P’s cost of implementing effort level \( e \) has fallen with renegotiation.

Going back a step, this then implies that, under renegotiation, P’s optimal choice of effort will be higher than under full commitment and, under some conditions, P may even wish to implement the first-best contract (Hermalin and Katz, 1991).
3 Property Rights - Simple Model

We start with a simple version of the Grossman-Hart model. There are 2 parties, B and S who will trade a good. Trading occurs for sure; however, there may be an opportunity for improvement in period 2. The extra cost to the seller for the “improved” product is \( c \), which is incurred in period 2. B, however, chooses a period 1 level of investment, \( x \), that affects the chances of an improvement. If an improvement occurs, it is worth \( v \) to B otherwise it is worth zero. \( \Pr(\text{success}) = x \). We assume \( 0 < c < v < 1 \); that is, it is only worth producing the product if it is successful. The cost of the investment is \( \frac{x^2}{2} \).

The social optimum chooses \( x \) to maximize

\[
W = x(v - c) - \frac{x^2}{2}
\]

or

\[
x^* = v - c
\]

Unconstrained bargaining: Next, suppose that the parties are entirely separate and bargain ex post using the Nash bargaining solution. Then, if the improvement is successful, the buyer expects to obtain utility of

\[
U = x \left( \frac{v - c}{2} \right) - \frac{x^2}{2}
\]

and hence will choose investment level

\[
x^D = \left( \frac{v - c}{2} \right)
\]

or underinvestment.

Seller control: Under seller control, all the seller can do is threaten not to make the improvement and use the original contract terms. But, of course, this position is negotiable; hence the situation is exactly as in the case of unconstrained bargaining.

Buyer control: Under buyer control, B can insist on the improvement regardless. Of course, when \( v = 0 \), S would, of course prefer not to make the improvement and will negotiate to avoid having to do so. Thus, B chooses \( x \) to maximize

\[
U^B = xv + (1 - x) \left( \frac{c}{2} \right) - \frac{x^2}{2}
\]

or

\[
x^B = v - \frac{c}{2}
\]

which is overinvestment relative to the social optimum.
4 Property Rights

Recall that, under complete contracting, the allocation of property rights (bargaining power) affected the distribution of surplus, but had no effect on incentive feasibility or efficiency. This is consistent with the “pure” Coasian world. Of course, it seems preposterous to suggest that property rights don’t in fact, matter—indeed, were that the case, one could reasonably conclude that virtually all of the upstream and downstream integration one sees in industry represents a complete waste of time. Here, we’ll study how incomplete contracts drive integration incentives.

Consider a caricature of the situation of Intel and IBM in the early 1980s in negotiating over the 386 chip. Intel can invest in microprocessor innovations that increase willingness to pay downstream by a random amount $v$, which takes on the values $\{2, 4\}$. However, suppose that these innovations require IBM to change its production process at some random incremental cost $c$, which takes on values $\{1, 3\}$.

Suppose that, by making a specific investment, $x$, Intel can increase the probability of a “successful” innovation. That is $\Pr(v = 4|x) = x$. This costs Intel $x^2$. Likewise for IBM $\Pr(c = 1|y) = y$ at a cost of $y^2$.

There are four variables, $v, c, x, y$ all of which are observable but none of which are verifiable. Clearly, neither party can then sign a contract ex ante. Now we’ll look at how changes in the property rights affect investment outcomes. In writing down the game, it helps to think of Intel as the upstream party (U) and IBM as the downstream party (D).

The game is as follows:
1. U and D choose $x$ and $y$.
2. U and D observe $v$ and $c$.
3. U and D renegotiate to decide whether to utilize the processor innovation and, if so, how to split the surplus.

**Social Optimum**

The processor innovation will be undertaken if $v > c$, which occurs in three of four cases. Thus, the only time the innovation is not implemented occurs with probability $(1 - x)(1 - y)$. Thus, the surplus from the innovation is

$$S(x, y) = E(\max(v - c, 0)|x, y)$$

$$= 3xy + x(1 - y) + y(1 - x)$$

$$= xy + x + y$$

The social planner’s problem is to choose $x$ and $y$ to maximize:

$$S(x, y) - x^2 - y^2$$

Or

$$y + 1 - 2x = 0$$

$$x + 1 - 2y = 0$$
which has the unique solution \( x = y = 1 \). The overall welfare is

\[ W = 1 \]

**Property Rights**

First, consider the case where U and D are separate. We have to assume something about how surplus is split at the renegotiation phase. We’ll assume that the Nash bargaining solution determines this. Thus, U’s problem is to choose \( x \) to maximize

\[
\frac{S(x, y)}{2} - x^2
\]

Or

\[
\frac{y + 1}{2} - 2x = 0
\]

Likewise for D

\[
\frac{x + 1}{2} - 2y = 0
\]

This yields equilibrium investment levels of

\[ x = y = \frac{1}{3} \]

(Note that this is basically a Cournot model). The overall surplus is

\[ W = \frac{5}{9} \]

Next, suppose that U buys D after investment is undertaken and this is anticipated by both parties. Then D will clearly not bother to invest at all since this it will obtain no surplus. U on the other hand chooses \( x \) to maximize

\[ 1 - 2x = 0 \]

or \( x = \frac{1}{2} \) leading to overall welfare of

\[ W = \frac{1}{4} \]

This is an even worse situation—D underinvests and U overinvests compared to the nonintegrated case. One obtains the reverse outcome when D buys U.

This is a very simple example of the *hold-up problem*. The key insight is that contractual incompleteness combined with relationship specific investments leads to underinvestment on the part of one or both parties.
5 Solving the Hold-up Problem

Numerous “solutions” to the hold-up problem have been proposed in the literature. The key to the “problem” is the fact that the outside option in the event of a renegotiation is not specifiable at the initial contracting stage. Suppose as an alternative that the status quo is specified in the initial contract and that we make the usual asymmetric bargaining power assumption. Then Aghion-Dewatripont-Rey show that we can solve the free-rider problem.

The model: There is a buyer B and a seller S who exchange a quantity $q$ of some good at price $p$. Suppose B can make a relationship specific investment $i$ that increases its value of obtaining the good. Likewise, S can make a relationship specific investment $j$ to reduce the costs of providing the good. There are random terms $\varepsilon$ and $\eta$ associated with the surplus and cost terms as well. Thus, the buyer’s utility is

$$
U = s(q, i, \varepsilon) - p - \psi(i)
$$

$$
V = p - c(q, j, \eta) - \phi(j)
$$

We make the usual assumptions about concave benefits and convex costs.

After the realization of $i, j, \varepsilon, \eta$ efficient trade is determined by choosing $q$ to maximize $U + V$. Or

$$
\frac{\partial s(q, i, \varepsilon)}{\partial q} = \frac{\partial c(q, j, \eta)}{\partial q}
$$

Let $q^*(i, j, \varepsilon, \eta)$ be the solution to this expression. Of course, we cannot negotiate this quantity ahead of time. Instead, suppose the parties contract on $(q_0, p_0)$, which they are certainly free to do.

Suppose that at the renegotiation phase, S makes a take it or leave it proposal. In that case, we’ll obtain efficient trade and the price will be

$$
p - c(q^*, j, \eta) = p_0 - c(q_0, j, \eta) + G
$$

where $G$ is the gain from renegotiation:

$$
G = s(q^*, i, \varepsilon) - c(q^*, j, \eta) - (s(q_0, i, \varepsilon) - c(q_0, j, \eta))
$$

The buyer is, of course, made indifferent

$$
s(q^*, i, \varepsilon) - p = s(q_0, i, \varepsilon) - p_0
$$

or

$$
p = (s(q^*, i, \varepsilon) - s(q_0, i, \varepsilon)) + p_0
$$

Therefore, the seller obtains utility of

$$
V = p_0 - c(q_0, j, \eta) + s(q^*, i, \varepsilon) - c(q^*, j, \eta) - (s(q_0, i, \varepsilon) - c(q_0, j, \eta)) - \phi(j)
$$

$$
= p_0 + s(q^*, i, \varepsilon) - s(q_0, i, \varepsilon) - c(q^*, j, \eta) - \phi(j)
$$
Let \( i^* \) and \( j^* \) be the socially optimal investment levels. That is \( i^* \) and \( j^* \) maximize

\[
E \left[ s(q^*,i,\varepsilon) - c(q^*,j,\eta) - \phi(j) - \psi(i) \right]
\]

or

\[
E \left[ \frac{\partial s(q^*,i^*,\varepsilon)}{\partial i} \right] = \psi'(i^*)
\]

and

\[
E \left[ \frac{\partial c(q^*,j^*,\eta)}{\partial j} \right] = \phi'(j^*)
\]

Now, choose \( q_0 \) such that

\[
E \left[ \frac{\partial s(q_0,i^*,\varepsilon)}{\partial i} \right] = \psi'(i^*)
\]

Then, with this contract, we get first-best investment levels.

To see this, first notice that B anticipates earning utility of

\[
E[U] = E[s(q_0,i,\varepsilon) - p_0] - \psi(i)
\]

and, by construction of \( q_0 \), B can do no better than to choose \( i^* \). The seller expects to earn

\[
E[V] = E[p_0 - s(q_0,i,\varepsilon) + s(q^*,i,\varepsilon) - c(q^*,j,\eta) - \phi(j)]
\]

and, hence chooses \( j \) such that

\[
E \left[ \frac{\partial c(q^*,j^*,\eta)}{\partial j} \right] = \phi'(j^*)
\]

To close the model, choose \( p_0 \) to just satisfy B’s participation constraint.

And this yields the result.

The point is that adding enough instruments (two) to deal with the number of choice variables \( i \) and \( j \) fixes the hold-up problem.

6 Discussion

There is an odd tension about bounded rationality in this literature. On the one hand, the parties are assumed to be unable to foresee and write down all relevant conditions in their contracts. On the other, the investment decisions ex ante are made with a full view about how ex post renegotiation given various contractual outcomes will occur.

A striking illustration of this tension can be seen in a later paper by Tirole (1994 working paper version, not sure of publication date). Suppose renegotiation is impossible for some reason. Then all of the payoffs from complete contracts are attainable through complicated message games. One doesn’t really need external verifiability.