Some Lecture Notes on the Insurer as a
Monopolist

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1 Overview

This set of lecture notes studies a classic example of adverse selection—insurance markets. Unlike the more famous Rothschild-Stiglitz (1976) framework of competitive insurance markets, here we’ll study the situation from our usual Principal-Agent perspective treating the insurer as a single Principal holding bargaining power. The goal is to see which of the insights from our standard adverse selection model translate into this framework.

2 Preliminaries

A single insurer (P) is contracting with a single insured (A) over a premium ($q$) and a reimbursement ($R$) in the event of a loss of fixed size $d$. The agent has initial wealth $w$ and has private information about his type $\{\theta_L, \theta_H\}$ which determines his probability of suffering a loss $p_L < p_H$. Let $\lambda$ denote the probability that an agent is a low type.

As usual, the revelation principle allows us to restrict attention to contracts of the form $(q(\hat{\theta}), R(\hat{\theta}))$ where $\hat{\theta}$ is the agent’s report of his type. Under such a contract, an agent’s final wealth following a report $\hat{\theta}$ is

$$W_A(\hat{\theta}) = w - d(\hat{\theta}) - q + R(\hat{\theta})$$

if a loss occurs and

$$W_N(\hat{\theta}) = w - d(\hat{\theta})$$

if no loss occurs.

Thus, an agent’s of type $i$ reporting $\hat{\theta}$ earns expected utility of

$$EU_i = p_i u\left(W_A(\hat{\theta})\right) + (1 - p_i) u\left(W_N(\hat{\theta})\right)$$
where $u$ satisfies the usual risk aversion properties: $u' > 0$, $u'' < 0$.

**Individual Rationality**

The agent can always opt not to sign an insurance contract. In this case, a type $i$ agent earns expected utility of

$$E\tilde{U}_i = p_i u (w - d) + (1 - p_i) u (w)$$

(1)

The key thing to notice about equation (1) is that the outside option of an agent is **type-dependent**; thus, we have already departed from the standard model.

**Single-Crossing Property**

Since it is no longer the case that the agent’s utility is quasi-linear in money, the single-crossing condition we derived in the standard model won’t work either. Instead, let’s study the marginal rate of substitution of a type $i$ agent. Recall that

$$\frac{\partial EU_i}{\partial q} = - (p_i u' (W_A) + (1 - p_i) u' (W_N))$$

and

$$\frac{\partial EU_i}{\partial R} = p_i u' (W_A)$$

Thus

$$MRS_i = \frac{\frac{\partial EU_i}{\partial q}}{\frac{\partial EU_i}{\partial R}} = \frac{- (p_i u' (W_A) + (1 - p_i) u' (W_N))}{p_i u' (W_A)}$$

How does this vary with $i$?

$$\frac{\partial MRS_i}{\partial p_i} = \frac{- p_i u' (W_A) (u' (W_A) - u' (W_N)) - u' (W_A) (p_i u' (W_A) + (1 - p_i) u' (W_N))}{(p_i u' (W_A))^2} = \frac{u' (W_A) u' (W_N)}{(p_i u' (W_A))^2} > 0$$

And, since the MRS “tilts” with $i$, we should be in a position to separate the types.

**Insurer’s Problem**

The insurer is assumed to be risk-neutral. It seeks to maximize

$$E\pi = \lambda (q (\theta_L) - p_L R (\theta_L)) + (1 - \lambda) (q (\theta_H) - p_L R (\theta_H))$$

subject to the usual incentive compatibility and individual rationality constraints.
3 Analysis

Now we’ll try to find the solution to the insurance problem. Before proceeding, we’ll derive some properties of indifference curves of an i type agent mapping this into \((W_A, W_N)\) space. Since

\[ EU_i = p_i u(W_A) + (1 - p_i) u(W_N) \]

then

\[ \frac{\partial W_A}{\partial W_N} = -\frac{(1 - p_i) u'(W_N)}{p_i u'(W_A)} \]

Of particular note is that, under the first-best contract \(W_N = W_A\) (full insurance). Hence, the slope of the agent’s indifference curve under full insurance is simply \(-\frac{1 - p_i}{p_i}\). Likewise, the isoprofit curve for the insurer in serving an i type is

\[ \frac{\partial W_A}{\partial W_N} = -\frac{1 - p_i}{p_i} \]

so the two are tangent at first-best.

Moreover, since \(\frac{\partial MRS_i}{\partial p_i} > 0\), the slope at full insurance for the high type is flatter than that for the low type.

Now, to figure out which constraints bind, we’ll do our usual trick of considering perturbations around the first-best contract. Let \(\bar{W}_N = w\) and \(\bar{W}_A = w - d\) denote the “outside option” for the two types in \((W_A, W_N)\) space. Since \(\theta_L\) types have steeper indifference curves, the full insurance contract must offer them a higher payout in \((W_A, W_N)\) than for high types under first-best.

Notice, however, that high types can profitably deviate by pretending to be low types and enjoying the favorable rates offered to these types if the first-best contract is offered. Thus, the incentive constraint binds for high types while low types are held at the utility of their outside option. Finally, one can show that the optimal contract entails:

1. Full insurance for high types
2. Less than full insurance (i.e. deductibles) for low-types such that they are indifferent between this contract and their outside option.
3. The greater the population of high types, the larger the deductible for low types.