1 Motivation

Up until now, we’ve focused on a situation where the principal designed contracts to try to reduce the information rents of the agent while still achieving his desired economic objectives. In this section, we reverse the location of the private information—now the principal has private information about the state can convey this information either through some sort of pre-contract action or in the form of the contract offer itself.

To recap:

- Screening
  1. $P$ sets $\{x, t\}$ contract
  2. $A$ accepts/rejects
  3. $A$ learns $\theta$
  4. $A$ executes contract in profit maximizing way
  5. Payoffs

- Signaling
  1. $P$ learns $\theta$
  2. $P$ signals, sending message $m$
  3. $P$ sets contract
  4. $A$ accepts/rejects
  5. $A$ executes contract in profit maximizing way
  6. Payoffs

Whereas before we relied extensively on the revelation principle to guide us, here we’ll worry about the set of equilibria in the indirect game. Belief formation will matter a lot here.

As usual, we’ll be mainly interested in efficiency.
2 A Primer on Equilibrium and Its Refinements

Since the set of equilibria in these games turns out to hinge on the solution concept employed, we begin with a refresher course.

In this section, we present a primer on refinements liberally stolen from Banks, Camerer, and Porter (1994).

Here’s our archetypal signaling game:

Sender is one of a finite number of types: \( t \in T \).
Sender sends one of a finite number of messages: \( m \in M \).
Receiver chooses one of a finite number of actions \( a \in A \).
S’s payoff: \( u(t, m, a) \); R’s payoff \( v(t, m, a) \).
R has priors \( P(t) \) over types.
A behavioral strategy for S is a probability distribution over the set of messages given type \( t \); i.e.
\[
q(m | t)
\]
A response strategy for the receiver is \( r(m) \).
The receiver forms posterior beliefs \( P(t | m) \) on hearing message \( m \).

2.1 Bayes-Nash Equilibrium

We are now in a position to offer a solution concept.

Definition 1 A Bayes-Nash Equilibrium consists of strategies \( q, r \) and beliefs \( \mu \) such that
\[
(i) \quad q(m' | t) > 0 \quad \text{only if} \quad m' \in \arg\max_{m \in M} u(t, m, r(m)).
(ii) \quad \text{If} \quad q(m | t) > 0, \quad r(a' | m) > 0 \quad \text{only if} \quad a' \in \arg\max_{a \in A} \sum_a v(t, m, a) \mu(t | m)
(iii) \quad \text{If} \quad q(m | t') > 0 \quad \text{then} \quad \mu(t' | m) = q(m | t') P(t') / \sum_q q(m | t) P(t)
\]

S best responds given R
R best responds to equilibrium path messages by S
R uses Bayes rule to update when hearing equilibrium messages from S.

[SEE GAME 1]

2.2 Sequential Equilibrium

Standard criticism is that BNE allows too much freedom for R to do as she likes off the equilibrium path.

[SEE GAME 2]
Sequential forces R to specify some belief for every possible message and to best respond given this belief.

Definition 2 A sequential equilibrium is a BNE where (ii) is replaced by:
\[
(ii') \quad \text{For all} \quad m, \quad r(a' | m) > 0 \quad \text{only if} \quad a' \in \arg\max_{a \in A} \sum_a v(t, m, a) \mu(t | m).
\]

- Note a Perfect Bayesian equilibrium is the analog of sequential equilibrium applicable to continuous games.
2.3 The Intuitive Criterion

The criticism of sequential is that, while it requires that R best-respond to some beliefs out-of-equilibrium, these beliefs might themselves be goofy. The intuitive criterion tries to place restrictions on out of equilibrium beliefs by ascribing greater probability of defection to types that would stand to benefit more by the defection.

Formally, fix a sequential equilibrium with the associated equilibrium payoffs $u^* (t)$ for $S$. For each out of equilibrium message, define

$$T_I (m) = \left\{ t \in T : u^* (t) > \max_{r \in BR(T,m)} u(t,m,r) \right\}.$$ 

That is, this is a set of types who could not possibly benefit from sending message $m$ regardless of what beliefs it induced in $R$.

Now let

$$\Delta_I (m) = \left\{ \lambda \in \Delta_t : \lambda (t) > 0 \text{ only if } t \notin T_I (m) \right\}.$$ 

This is a set of beliefs following message $m$ that put weight only on types where $t$ is not in $T_I (m)$.

Definition 3 A sequential equilibrium satisfies the intuitive criterion if for all out of equilibrium messages $m, \mu (\cdot | m) \in \Delta_I (m)$.

- A refinements literature grew up in the 1980s to try to pick the “right” equilibrium from among the set of equilibria.
  - This literature had its problems for two reasons:
    1. Game theorists are endlessly clever at devising games where a given refinement picks out the “wrong” equilibrium (or, in some cases, no equilibrium.
    2. Dueling solution concepts—sometimes different refinements will pick out different equilibria as the “intuitive” one.
    3. Experimental results on this whole theoretical line have led to mixed results about how good are the predictions of these refinements.

3 Classic Signaling: The Spence Model

With all of the above results in hand, we are now in a position to breeze through the granddaddy of all signaling models—Spence’s piece on signaling though education.

Timing
1. $P$ learns $\theta$
2. $P$ chooses $e$
3. Contract on $w$

4. Payoffs

Dramatis Personae

The worker (Principal) is of two productivity types $r_H$ and $r_L$ where $r_H > r_L > 0$. The prior probability that the worker is a high type is $\lambda$. Workers earn wages $w$ and have a constant outside option of 0.

Workers can get education level $e$ at a cost $\theta e$ where $\theta_H < \theta_L$.

This is Berkeley hence we assume that workers have all the power and are paid wages equal to their expected productivity given the beliefs of firms.

Pooling Equilibria

In a pooling equilibrium, all workers choose education level $e$ and get paid wages $w = \lambda r_H + (1 - \lambda) r_L$.

For all education levels $e' \neq e$, wages are $r_L$.

(In a BNE we can pick whatever wage we like for education levels $e' \neq e$. In a sequential equilibrium, we’re stuck to picking wages between $r_L$ and $r_H$.)

For this to be an equilibrium, both types must want to participate, which is the same as

\[ w - \theta_L e \geq 0 \]
\[ e \leq \frac{w}{\theta_L} \]

Likewise, no one must want to deviate.

\[ w - \theta_H e \geq r_L \]
\[ e \leq \frac{w - r_L}{\theta_H} \]

And since the second condition is more stringent than the first, we have identified all pooling equilibria.

Separating equilibria

In a separating equilibrium, there must be 2 education levels $e_L$ and $e_H$ associated with wages $w_L = r_L$ and $w_H = r_H$. In a sequential equilibrium, you cannot pay a worker less than $r_L$, so this implies that $e_L = 0$.

For incentive compatibility by the $H$ types, we need

\[ r_H - \theta_H e_H \geq r_L \]

or

\[ e_H \leq \frac{r_H - r_L}{\theta_H} \]

and low types

\[ r_L \geq r_H - \theta_L e_H \]

or

\[ e_H \geq \frac{r_H - r_L}{\theta_L} \]

Thus, there are a continuum of these equilibria as well.
Intuitive Criterion

Let’s look at the pooling equilibria:

Find an \( e' \) such that

\[
  r_H - \theta_L e' = w - \theta_L e
\]

or

\[
  e' = \frac{r_H - w + \theta_L e}{\theta_L}
\]

Now, since \( \theta_H < \theta_L \), if an \( H \) type deviates just above \( e' \) it’s profitable, but there are no beliefs where it is profitable for an \( L \) type. Hence, no pooling equilibrium satisfies the intuitive criterion.

How about separating equilibria:

Suppose that \( e_H \) is such that \( \frac{e_H - r_L}{\theta_H} \geq e_H > \frac{e_H - r_L}{\theta_L} \). Then, a high type deviating to \( e' = e_H - \varepsilon \) stands to benefit by being perceived as a high type. A low type, on the other hand, derives no benefit whatsoever. Hence, the lone intuitive equilibrium is separating where

\[
e_H = \frac{r_H - r_L}{\theta_L}.
\]

4 Informed Principal

Suppose we change the timing to be:

1. \( P \) learns \( \theta \)
2. Contract on \( \{w, e\} \)
3. Payoffs

Case 1: \( \lambda r_H + (1 - \lambda) r_L \leq r_H - \theta_H \left( \frac{e_H - r_L}{\theta_L} \right) \) (i.e. the high type prefers the cheapest separating equilibrium to the cheapest pooling equilibrium)

Claim 4 In this case, the unique equilibrium is the contract mimicking the cheapest separating contract.

Intuition: Clearly once such a contract is in place, neither side has any incentive to deviate. No cheaper way for \( H \) types to separate and they prefer separating to pooling. No contract that \( L \) types can offer to induce either pooling or a higher wage at an acceptable level of education.

Case 2: \( \lambda r_H + (1 - \lambda) r_L > r_H - \theta_H \left( \frac{e_H - r_L}{\theta_L} \right) \)

In this case, the unique equilibrium is the contract mimicking the cheapest pooling outcome.

5 A Model of Takeovers

Now for another application to signaling related to the experiment we ran earlier.

We study the following situation:
Consider a situation where company A (the acquirer) is negotiating with company T (the target) over an exchange of property rights from T to A. Company A believes that combining T’s assets with its own is value enhancing, but it is not certain exactly what the net present value of T’s future cash flows combined with A are. T has a better idea about this since it knows its own operations intimately.

Fun fact: In event studies of successful takeovers, T company shares exhibit abnormal positive returns whereas A company shares exhibit abnormal negative returns.

To put this situation in our terms:
A principal has an object worth \( \theta \) (to him) where \( \theta \) is distributed uniformly on the unit interval and where \( \delta \in [0, 1] \).

The principal offers the agent the following contract:

- A bids some amount \( b \)
- If \( b \geq \delta \theta \), then the object is transferred to A at a price of \( b \).
- If \( b < \delta \theta \), then P keeps the object.

Payoffs:
- P simply earns either \( \delta \theta \) if he retains the object or \( b \) is he sells it.
- A earns 0 if she does not buy the object and \( \gamma \theta - b \) if she does where \( \gamma > 1 \) represents additional value A gets from the object compared to P.

Full information Benchmark:

- Under complete information, such a contract will lead A to bid \( b = \delta \theta \) and the object always to be transferred.
- Transfer is efficient since \( \gamma > \delta \).
- P earns surplus of \( \delta \theta \) and A earns surplus of \( (\gamma - \delta) \theta \)

## 5.1 Private Information

What happens under private information?

Since A has no information, bidding strategy cannot depend on \( \theta \). So A needs to choose a bid \( b \) to maximize

\[
E \pi = \int_0^b (\gamma \theta - b) d\theta
\]

Differentiating yields:

\[
\frac{\partial E \pi}{\partial b} = (\gamma - 2) b
\]

so the solution is bang-bang:
• If $\gamma \geq 2$, optimal bid is $b = 1$.
• If $\gamma < 2$, optimal bid is $b = 0$.

Notice that for a large enough synergy term, we achieve allocative efficiency. Compared to full-information, $P$ obtains higher surplus.

When $\gamma$ is not too large, no trade ever takes place. This is the lemons problem.

5.2 A Variation

Suppose that instead of the above contract, we are faced with this situation:

• $P$ learns $\theta$
• $P$ makes an offer $b$
• $A$ can accept/reject this offer
• Payoffs are realized.

Now $P$ has all the bargaining power

Complete information
Under complete information, $P$ offers to sell for $\gamma \theta$ and $A$ accepts
Trade is efficient, $P$ captures all the surplus.

Private information
Suppose that $P$ makes an offer $b$.
When should the agent accept?

• Let the agent’s posterior beliefs about $\theta$ be given by $\mu (\theta|b)$.

  – In a Bayesian equilibrium, posterior beliefs are obtained using Bayes’ rule wherever possible.
  – Then $A$ should accept if

$$E\pi_A (b) = \int_\theta (\gamma \theta - b) \mu (\theta|b) d\theta \geq 0$$

  and reject otherwise.

• What should $P$ do?

  – Given $\theta$, choose $b$ to maximize

$$\pi_P = bI_{E\pi_A (b) \geq 0} + \delta \theta I_{E\pi_A (b) < 0}.$$

• Separating contract:

  – Consider an equilibrium where the contract distinguishes among the $P$ types as in the first-best contract. That is where $b' (\theta) > 0$ for all $\theta$. 
– In this case, on seeing a contract $b(\theta')$, $A$ believes that with probability 1, the realization of $\theta$ is $\theta'$. Thus, $A$ will accept whenever $b(\theta) \leq \gamma \theta$.

– But this means that $P$’s optimization is simply to choose $b \in [0, \gamma]$ to maximize

$$\pi_P = b$$

and clearly this occurs at $b = \gamma$.

• In fact, we have established:

**Proposition 5** Any equilibrium contract must entail a single price at which $A$ will accept.

• Thus, $P$ is reduced to offering pooling contracts.

• In this case, $A$ does not update beliefs on seeing the equilibrium contract $b^*$ but is free to believe whatever we wish for a non-equilibrium contract $b \neq b^*$.

  – Consider out of equilibrium beliefs that $P$’s type is 0 for all $b \neq b^*$.

  – Acceptance requires that

$$\frac{\gamma}{2} - b^* \geq 0.$$ 

  – Thus, the candidate set of contracts are bounded from above by $\frac{\gamma}{2}$. Obviously, $b^* = \frac{\gamma}{2}$ is $P$’s favorite contract.

• Now for this to be optimal, it must be the case that $b^* \geq \delta$ for $P$ otherwise, by offering an outrageously expensive price, he can retain the object.

• Now we have that for $\gamma \geq 2\delta$, full efficiency is achievable otherwise, there is no pooling equilibrium.

**Key Special Case:** In many models, the outside option of $P$ is assumed to be constant. This is like $\delta = 0$. Notice that here, efficient trade is no problem.

• Finally, consider the case where $\gamma < 2\delta$. Is there a contract where some trade takes place?

  – Suppose when $b^*$ is offered, $A$ buys. Then it must be that $E\pi_A \geq 0$ conditional on types offering $b^*$.

  – We earlier established that those types consist of $\theta \leq b^*$.
Hence, 

\[ E_{\pi_A} = \frac{1}{F(b^*)} \int_{\theta \leq \frac{b^*}{2}} (\gamma \theta - b^*) \, d\theta \]

\[ = \frac{\delta}{b^*} \left( \frac{\gamma}{2} \left( \frac{b^*}{\delta} \right)^2 - \frac{1}{\delta} (b^*)^2 \right) \]

\[ = b^* \left( \frac{\gamma}{2\delta} - 1 \right) \]

where the inequality follows from the fact that \( \gamma < 2\delta \). Thus, there is no such contract and efficiency is still impossible.

6 Signaling

Now consider what happens if \( P \) can signal by having a big schmoozing party before offering a contract. Suppose that at a cost \( C(s, \theta) \), signal \( s \) can be sent. Let \( c(s, \theta) \) denote the marginal cost of sending signal \( s \) in state \( \theta \).

Assumption (Single-Crossing Property) The marginal cost of sending a given signal is decreasing in the state. That is, for all \( s, \theta' < \theta'' \), \( c(s, \theta') > c(s, \theta'') \).

We’ll assume that \( C \) takes on the ridiculously simple form \( C(s, \theta) = \frac{s}{\theta} \) so \( c(s, \theta) = \frac{1}{\theta} \) which quite obviously satisfies our favorite assumption.

Separating Equilibrium

Now consider a separating contract \( b(\theta) \) combined with signals \( s(\theta) \).

On the equilibrium path, \( A \) believes that \( P \)’s type is \( \theta = 0 \).

For this to be legit, we require

- \( b(\theta) \leq \gamma \theta \) (IR for agent)
- \( b(\theta) - C(s(\theta), \theta) \geq \theta \) (IR for principal)
- And finally incentive compatibility:

\[ b(\theta) - C(s(\theta), \theta) \geq b(\theta') - C(s(\theta'), \theta) \]

for all \( \theta' \neq \theta \).

Thus, we have the familiar maximization problem

\[ \max_{\hat{\theta}} \pi_P = b(\hat{\theta}) - C(s(\hat{\theta}), \theta) \]

which yields

\[ b' - c(s(\theta), \theta) s' = 0 \]
which is a differential equation.

Further, at the lower boundary, we know that \( b(0) = 0 \) and hence \( s(0) = 0 \).
Consider linear contracts \( b(\theta) = b\theta \). Then, the differential equation becomes

\[
\begin{align*}
\frac{b - s'}{\theta} &= 0 \\
\frac{s'}{\theta} &= b\theta
\end{align*}
\]

Hence

\[
\begin{align*}
s(\theta) &= \int_0^\theta btdt \\
&= \frac{b}{2} \theta^2
\end{align*}
\]

Finally, it remains to verify IR for \( P \).

\[
\begin{align*}
\pi_P(\theta) &= b\theta - \frac{b}{2} \theta \\
&= \frac{b}{2} \theta
\end{align*}
\]

so this condition holds only when \( b \geq 2\delta \).

Choosing the optimal \( b \) from the perspective of the principal yields \( b = \gamma \) so IR only holds when \( \gamma \geq 2\delta \). Same condition as before.

- Notice that compared to the case where there was no signaling, social surplus is actually reduced by the presence of signaling.
- Non-linear contracts are not going to help us in that the \( \gamma \theta \) contract is an upper bound on rents we can transfer to \( P \).
- Note that the details of this depended on the functional form of the signaling technology. Because the marginal cost of signaling was independent of \( s \), we are somewhat limited in what we can do.

**Pooling Equilibrium**

Now we’ll characterize the set of equilibria where a price \( b \) and a signal \( s \) is sent regardless of \( \theta \) and where there is efficient trade.

Efficient trade requires that the sale go through at the equilibrium price.

Agent IR:

\[
\frac{\gamma}{2} - b \geq 0
\]

or

\[
b \leq \frac{\gamma}{2}
\]

Principal IR

\[
b - s \geq \delta \theta
\]
for all \( \theta \). In particular, we need to satisfy this at \( \theta = 1 \) so
\[
b \geq \delta + s
\]

Finally, off equilibrium beliefs: Suppose that if \( P \) chooses anything other than \((b, s)\), \( A \) believes he is a zero type and doesn’t trade.
So we’re done.
A pooling equilibrium is any pair \((b, s)\) where \( b \leq \delta / 2 \) and \( b - s \geq \delta \)

- Obviously the best of these equilibria from the social perspective is one where \( s = 0 \). Yields the same efficiency condition as all the others.
- Just because signaling plays no informative role does not rule it out as part of an equilibrium.

7 An Application: Debt versus Equity and Signaling

The application of interest is optimal corporate financing decisions under asymmetric information.

Background:
Modigliani-Miller thm: Capital structure is irrelevant for firms’ investment decisions when there are no tax distortions, transactions costs, agency problems, or information asymmetries.

- In this world firms undertake projects iff NPV is positive.

In this application, we use signaling to show how the choice of financing—debt versus equity—affects a firm’s investment policy.

- Stylized fact: new equity issues on average lead to a drop in stock price.
  (Seems odd in that the new equity issue should indicate that a firm has found a positive NPV project and this should drive up stock prices.)

8 Model

The model is adapted from Myers-Majluf (1984).
Firm is operated by a risk-neutral manager.

- Existing assets generate a value of 1 or 0 in the future. The probability of success is \( a_i \)
- A new project is available at a cost of 0.5, which also bears either 1 or 0 in the future. The probability of success is \( b_i \).
  - \( i \) denotes the state of nature, which might be good or bad. Each state occurs with 50-50 probability. The probability of success for each project is higher in the good state.
- The manager knows the state when making the financing decision.

- A benchmark: Suppose that the manager was also the owner and had the 0.5 in cash. Then the new project would be initiated iff \( b_i > \frac{1}{2} \). This is of course efficient.

- We seek to compare this case to one where the manager needs to go to the capital markets to obtain the cash.

9 Case 1: All projects have positive NPV

Assume: \( b_G \geq b_B \geq \frac{1}{2} \).

In this case, it is always efficient to undertake a project.

**Equity Finance**

Suppose that the manager chooses to finance the project with an equity offering.

In state \( B \), the firm always wants to issue and invest since even with the most pessimistic investor beliefs, investors will give 0.5 in exchange for an equity stake \( s \) solving

\[
s (a_B + b_B) = 0.5
\]

or

\[
s = \frac{0.5}{(a_B + b_B)}
\]

(Note we are assuming a perfectly competitive capital market.)

The manager then earns an expected payoff of

\[
\pi = (1 - s) (a_B + b_B) = a_B + b_B - 0.5 > a_B.
\]

So investment is optimal in state \( B \).

Given that it is optimal to issue equity in state \( B \) regardless of investor beliefs, we turn attention to the case where the manager also issue equity in state \( G \). In this case, investors infer nothing about the state from the mere issuance of equity. Hence They are now willing to take a stake of size \( s' \) solving

\[
s' \left( \frac{1}{2} (a_B + b_B) + \frac{1}{2} (a_G + b_G) \right) = \frac{1}{2}
\]

or

\[
s' = \frac{1}{\sum_i (a_i + b_i)}
\]

Notice that since the probability of success is higher in the good stake, the stock price fraction of the company that must be sold to finance the investment
is lower here than it was when investors believed that the firm was in the bad state with certainty. Conversely, for the good state.

Now, we know that it was optimal to issue equity in the bad state. What about the good state?

\[
\pi = (1 - s') (a_G + b_G) \\
= (a_G + b_G) - \frac{(a_G + b_G)}{(a_B + b_B) + (a_G + b_G)}
\]

Now add and subtract 0.5

\[
\pi = (a_G + b_G) - 0.5 + \frac{1}{2} \frac{(a_B + b_B) + \frac{1}{2} (a_G + b_G) - (a_G + b_G)}{(a_B + b_B) + (a_G + b_G)}
\]

\[
= (a_G + b_G) - \frac{1}{2} - \frac{1}{2} \left( \frac{(a_G + b_G) - (a_B + b_B)}{(a_B + b_B) + (a_G + b_G)} \right)
\]

The last term here is the dilution of returns in the high state owing to private information.

It only pays to invest if \( \pi > a_G \). That is

\[
b_G > \frac{1}{2} \left( 1 + \frac{(a_G + b_G) - (a_B + b_B)}{(a_B + b_B) + (a_G + b_G)} \right)
\]

Notice that this exceeds \( \frac{1}{2} \), which is the threshold for efficient investing. When equation (1) holds, there is a pooling equilibrium.

Suppose that

\[
b_G < \frac{1}{2} \left( 1 + \frac{(a_G + b_G) - (a_B + b_B)}{(a_B + b_B) + (a_G + b_G)} \right)
\]

then the firm chooses to issue equity only when the project is bad.

**Key observations:**

- The value of the firm is lower than would be the case had the financing been doable internally.

- By announcing an equity offering, investors learn that the state is bad. Updating their priors this means that the stock price on the secondary market changes from \( \frac{1}{2} (a_G + a_B) \) to \( a_B + b_B - 0.5 \), which may be lower.

**Debt Finance**

Suppose that the inequality in equation (2) holds. Then projects in the good state go unfinanced. Can one use debt instead of equity?

Definitions: Describe the realized value of the firm as its value in the future \( \{0, 1, 2\} \) before any distributions to stakeholders. We contract specifies an amount to be distributed in each state. Let \( r_i \) denote the repayment when the firm has value \( i \) of some instrument.

**Straight Equity:** A pure equity contract is \( r_2 = 2r_1 \).
Debt: A debt contract with face value $D$ is such that $r_1 = \min \{D, 1\}$, $r_2 = \min \{D, 2\}$.

Notice that under a debt contract, the firm repays relatively more when its value is 1 than when it is 2. (Concave repayments characteristic of debt).

Recall that the problem in financing when things were good was that the dilution of the manager’s stake was too great to be worthwhile. If debt can reduce this dilution, then it might help.

Consider a local deviation from an equity contract. Instead of repaying $(r_1, 2r_1)$ then firm chooses the contract $(r_1 + \delta, 2r_1 - \varepsilon)$ where $\delta$ and $\varepsilon$ are chosen so that the expected repayment from the two contracts are equal.

That is

$$\frac{1}{2} \left[ \sum_i a_i (1 - b_i) + (1 - a_i) b_i \right] (r_1 + \delta) + \frac{1}{2} \left[ \sum_i a_i b_i \right] (2r_1 - \varepsilon) = k$$

for some constant $k$.

Hence

$$\frac{\partial \varepsilon}{\partial \delta} = \frac{\sum_i a_i (1 - b_i) + (1 - a_i) b_i}{\sum_i a_i b_i}$$

If the subsidy under the deviation is smaller than under equity, debt financing lowers dilution costs.

The subsidy is the difference between the true value of the repayment stream in state $G$ and the investor’s valuation of this stream.

$$S = (a_G (1 - b_G) + (1 - a_G) b_G) (r_1 + \delta) + a_G b_G (2r_1 - \varepsilon)$$

$$- \frac{1}{2} \left[ \sum_i a_i (1 - b_i) + (1 - a_i) b_i \right] (r_1 + \delta) + \frac{1}{2} \left[ \sum_i a_i b_i \right] (2r_1 - \varepsilon)$$

$$= (a_G (1 - b_G) + (1 - a_G) b_G) (r_1 + \delta) + a_G b_G (2r_1 - \varepsilon) - k$$

Now examine $\frac{\partial S}{\partial \delta}$

$$\frac{\partial S}{\partial \delta} = (a_G (1 - b_G) + (1 - a_G) b_G) - a_G b_G \frac{\partial \varepsilon}{\partial \delta}$$

$$= (a_G (1 - b_G) + (1 - a_G) b_G) - a_G b_G \frac{\sum_i a_i (1 - b_i) + (1 - a_i) b_i}{\sum_i a_i b_i}$$

And $\frac{\partial S}{\partial \delta} < 0$ provided

$$(a_G (1 - b_G) + (1 - a_G) b_G) < a_G b_G \frac{\sum_i a_i (1 - b_i) + (1 - a_i) b_i}{\sum_i a_i b_i}$$

$$= a_G b_G (a_G (1 - b_G) + (1 - a_G) b_G) + a_G b_G (a_G (1 - b_G) + (1 - a_G) b_G)$$

$$< a_G b_G (a_G (1 - b_G) + (1 - a_G) b_G) + a_G b_G (a_B (1 - b_B) + (1 - a_B) b_B)$$

Simplifying

$$a_B b_B (a_G (1 - b_G) + (1 - a_G) b_G) < a_G b_G (a_B (1 - b_B) + (1 - a_B) b_B)$$
or
\[
\frac{a_G (1 - b_G) + (1 - a_G) b_G}{a_B (1 - b_B) + (1 - a_B) b_B} > \frac{a_G b_G}{a_B b_B}
\]
which is the monotone likelihood ratio property.

Thus, we have shown that debt is better than equity.

10 Case 2: Projects Have Negative NPV in the Bad State

In this section, we assume \( b_G > 0.5 > b_B \).

Consider a separating equilibrium. The firm is supposed to finance only in the good state. Thus, the stake required to finance the project is

\[
s = \frac{0.5}{a_G + b_G}
\]

Now suppose the manager deviates and issues equity in the bad state. Then, his expected payoff is

\[
\pi = \left(1 - \frac{0.5}{a_G + b_G}\right) (a_B + b_B)
\]

\[
= a_B + b_B - 0.5 + \frac{1}{2} \left( \frac{a_G + b_G - (a_B + b_B)}{a_G + b_G} \right)
\]

So if

\[
b_B + \frac{1}{2} \left( \frac{a_G + b_G - (a_B + b_B)}{a_G + b_G} \right) > \frac{1}{2}
\]

it pays to deviate and this wrecks efficiency.

As before, self-financing gets it right and debt financing does better than equity financing.