1 Preliminaries

This key tension in this set of topics centers around the role of commitment in long-term contracting. Previously, we assumed full commitment power on the part of the Principal in setting all aspects of a contract. In a dynamic setting, the fulfillment of the early stages of a contract provides information to the Principal about the agent’s type. Naturally, the Principal will be tempted to act on this information, and this temptation can lead to incentives to renegotiate the initial contract. Of course, the agent will anticipate these incentives as well and this will then affect the incentive constraints facing the agent at the initial contracting stage. This set of notes explores these issues.

Outline of paper:

First, we highlight the central tension in an ultra-simple application of dynamic commitment. Next, we compute the optimal dynamic contract under full commitment. Finally, we examine how the optimal contract changes when the assumption of perfect commitment is relaxed. Along the way, we introduce a slight extension of the revelation principle, the so-called renegotiation-proofness principle, to studying optimal contracts with imperfect commitment.

2 Exclusive Contracts

To highlight the stringency of the full commitment assumption in dynamic contexts, we first study how commitment in the form of exclusive dealing can forestall entry in a market.

A seller S produces a good which costs 1/2. The good is offered to a single buyer, B, who values the good at 1 and has unit demand. There is a potential entrant, E, in the market who can produce the same good at a cost $c$, which is unknown to either S or B. It is, however, commonly known that $c$ is drawn from the uniform
distribution on $[0, 1]$. The goods provided by S and E are perfect substitutes to B and they compete purely on price if both are in the market according to an English auction. Thus, if E enters, the price to the buyer will be

$$P = \max \left( c, \frac{1}{2} \right)$$

Absent a contract, then E will enter the market when $c < \frac{1}{2}$ and stay out otherwise. Obviously, the buyer will pay 1 if E does not enter. Thus, the buyer’s expected surplus absent a contract is 1/4. (50% chance of getting a price equal to 1/2)

Now suppose that B and S sign a contract prior to the entry decision by E and that $c$ is not observed ex post. A contract consists of a price, $P$, that the B pays to the seller as well as a penalty $P_0$, paid if B breaches the exclusive deal and buys from E instead.

This contract is assumed to be observable to E. If E enters the market, it will set a price equal to $P - P_0$ and B earns surplus of $1 - P$. Thus, entry will occur only if $c < P - P_0$. The probability of entry is then

$$\pi = \max \left( 0, P - P_0 \right).$$

Of course, S has to make it worth B’s while to sign the contract in the first place. Hence,

$$1 - P \geq \frac{1}{4}$$

Now, we need to derive the optimal contract. S’s profits are

$$U_s = (1 - \pi) \left( P - \frac{1}{2} \right) + \pi P_0$$

First, suppose that $P < P_0$, then

$$U_s = \left( P - \frac{1}{2} \right)$$

and hence

$$P = \frac{3}{4}$$

If $P > P_0$ then

$$U_s = (1 - (P - P_0)) \left( P - \frac{1}{2} \right) + (P - P_0) P_0$$

$$= \left( P - \frac{1}{2} \right) - (P - P_0)^2 + \frac{1}{2} (P - P_0)$$

Differentiating with respect to $P_0$, one obtains

$$2 (P - P_0) - \frac{1}{2} = 0$$

2
or

\[ P_0 = P - \frac{1}{4} \]

and again, to induce B to sign, requires \( P = \frac{3}{4} \) and yields \( \pi = \frac{1}{4} \).

Now, compare S’s surplus under the two regimes:

If \( P < P_0 \) then S earns \( U_s = \frac{1}{4} \times \frac{1}{2} + \frac{3}{4} \times \frac{1}{4} = \frac{5}{16} \).

So we have found the optimum.

Key observations:

1. Market foreclosure: S can induce B to sign an exclusive dealing contract which keeps out E even when E is more efficient.

2. Renegotiation: Notice that a more efficient (but foreclosed) entrant can pay a bribe to B and S to tear up the contract and sell to B directly.

### 3 Repeated Adverse Selection - Full Commitment

Next, we consider a dynamic version of our standard \( t, q \) model of adverse selection. Suppose that there are two periods with discount factor \( \delta \). Suppose the agent, A, has a type \( \theta \in \{ \theta_h, \theta_l \} \) which is the same in both periods. Suppose that an agent is a low type with probability \( \lambda \). Let \( q_{i,t} \) denote the quantity produced by a type \( i \) agent in period 1 and so on.

Thus, P’s problem is to choose a contract to maximize

\[
E_\theta [C(q_{i,1}) - t_{i,1} + \delta (C(q_{i,2}) - t_{i,2})]
\]

subject to individual rationality

\[
t_{i,1} - \theta_i q_{i,1} + \delta (t_{i,2} - \theta_i q_{i,2}) \geq 0
\]

for \( i \in \{ h, l \} \).

Also subject to incentive compatibility

\[
t_{i,1} - \theta_i q_{i,1} + \delta (t_{i,2} - \theta_i q_{i,2}) \geq t_{j,1} - \theta_j q_{j,1} + \delta (t_{j,2} - \theta_j q_{j,2})
\]

Note that only one message game is required for P since the agent’s type is unchanging.

Because of risk-neutrality, one can simplify the problem a bit by letting

\[
T_i = t_{i,1} + \delta t_{i,2}
\]

Thus, a contract is a triple \((T_i, q_{i,1}, q_{i,2})\) for \( i \in \{ h, l \} \).
Let $U_i$ denote the equilibrium rents to a type $i$ agent. Then we can rewrite P’s problem as

$$\max_{U_i, U_h, q_{i,1}, q_{i,2}} \lambda (C (q_{i,1}) - \theta_t q_{i,1} + \delta (C (q_{i,2}) - \theta_t q_{i,2}) - U_i)$$
$$+ (1 - \lambda) (C (q_{h,1}) - \theta_h q_{h,1} + \delta (C (q_{h,2}) - \theta_h q_{h,2}) - U_h)$$

subject to

$$U_i \geq 0$$
$$U_i \geq U_h + \Delta \theta (q_{h,1} + \delta q_{h,2})$$

and

$$U_h \geq U_i - \Delta \theta (q_{l,1} + \delta q_{l,2})$$

Using identical reasoning as the static case, one can show that the binding constraints are

$$U_h = 0$$
$$U_i = \Delta \theta (q_{h,1} + \delta q_{h,2})$$

Hence, we have the “simple” P’s problem

$$\max_{q_{i,1}, q_{i,2}} \lambda (C (q_{i,1}) - \theta_t q_{i,1} + \delta (C (q_{i,2}) - \theta_t q_{i,2}) - \Delta \theta (q_{h,1} + \delta q_{h,2}))$$
$$+ (1 - \lambda) (C (q_{h,1}) - \theta_h q_{h,1} + \delta (C (q_{h,2}) - \theta_h q_{h,2}))$$

Differentiating, we have

$$\lambda (C' (q_{l,1}) - \theta_t) = 0$$
$$\lambda (\delta (C' q_{l,2} - \theta_t)) = 0$$

and

$$(1 - \lambda) (C' (q_{h,1}) - \theta_h) = \lambda \Delta \theta$$
$$(1 - \lambda) \delta (C' (q_{h,2}) - \theta_h) = \lambda \delta \Delta \theta$$

The first pair of equations tell us that low types produce the efficient quantity in both periods.

The second pair of equations tell us that high types are distorted downward.

Finally, notice that $\delta$ doesn’t figure into the solution at all: the two period contract is just the twice repetition of the static contract.
4  Repeated Adverse Selection - Limited Commitment

Notice that, once we’ve arrived at the second period, P now knows A’s type. Thus, there is no point in distorting the output of the high cost type in the second period, as is called for in the contract. Indeed, both P and a high cost A could agree to rewrite the second period contract to make them both better off. Thus, the contract specified above is not renegotiation proof. The standard model of renegotiation-proofness goes like this:

Period 0: P and A sign long-term deal
Period 1: Message game is played and period 1 contract is executed.
End of period 1: Having observed the results of period 1, P proposes an alternative contract to A who can accept or reject. If A accepts, the proposed contract becomes the period 2 contract. Otherwise, the original contract remains in force
Period 2: Message game is played and period 2 contract is executed
And then the game ends.

Recall that the optimal contract under full commitment involved separation in period 1. How does this contract change under renegotiation? To answer this question, we need to say something about the space of indirect mechanisms. Here, we’ll use the renegotiation proofness principle which says:

\textbf{Proposition 1 (Renegotiation Proofness Principle)} Any incentive feasible allocation of some indirect mechanism with renegotiation is implementable as a direct mechanism under renegotiation-proof long-term contracts.

With this result in hand, we can restrict attention to indirect mechanisms where there are no Pareto gains possible in the second period conditional on the information revealed in the first period.

\textbf{Separating Equilibrium}

First, consider an equilibrium where the types separate in the first period. In that case, the second period renegotiation proof contract must specify efficient allocations. Hence, we know that

\[ C^* (q_{i,2}) = \theta_i \]

in any optimal renegotiation-proof contract.

Call these outputs \( q^*_i \)

As usual, we need the low type to be incentive compatible; hence

\[ U_l = U_h + \Delta \theta (q_{h,1} + \delta q^*_h) \]

likewise, the high type obtains only the IR payoff

\[ U_h = 0 \]
and we have P’s problem
\[
\max_{q_{l,1}} \lambda (C(q_{l,1}) - \theta_l q_{l,1} + \delta (C(q^*_l) - \theta_l q^*_l) - \Delta \theta (q_{h,1} + \delta q^*_h)) \\
+ (1 - \lambda) (C(q_{h,1}) - \theta_h q_{h,1} + \delta (C(q^*_h) - \theta_h q^*_h))
\]

Thus, we obtain
\[
\lambda (C'(q_{l,1}) - \theta_l) = 0
\]
and
\[
(1 - \lambda) (C'(q_{h,1}) - \theta_h) = \lambda \Delta \theta
\]
Thus, the distortion is exactly the same in the first period for the high type. But neither type’s output is distorted in the second period. However, the fact that there’s no distortion to the high type in the second period combined with the fact that that agent’s information rent is exactly the same (zero) must mean that the information rent paid to the low type of agent has to have gone up since it is now more tempting to “pretend” to be the low type. This is apparent if we revisit the incentive constraint:

Under renegotiation, a low type agent earns:
\[
U_l = \Delta \theta (q_{h,1} + \delta q^*_h)
\]
while under full commitment, that agent’s rent is
\[
U_h = \Delta \theta (q_{h,1} (1 + \delta))
\]
Since \(q_{h,1} < q^*_h\), it is obvious that this agent earns a higher information rent. Further, P must be worse off as well. To see this, notice that P could have implemented the renegotiation proof contract under full commitment; that is, it was incentive feasible. However, P chose not to implement this contract, that is, it was not incentive efficient. Hence, P is clearly worse off under renegotiation.

**Pooling Contract**

How much worse off is P? By way of comparison, suppose that P executes a full pooling contract in the first period. Clearly, in period 2, P should execute the optimal static contract hence, we have \(q_{l,2} = q^*_l\) while \(q_{h,2} = q^{FC}_{h,2}\) where \(FC\) denotes the optimum under full commitment. Thus, it remains to find an optimal pooling contract, \(q_1\), in the first period.

As usual, we have that
\[
U_h = 0
\]
and
\[
U_l = \Delta \theta (q_1 + \delta q^*_l)
\]
And P’s problem becomes
\[
\max_q \lambda (C(q_1) - \theta_l q_1 + \delta (C(q^*_l) - \theta_l q^*_l) - \Delta \theta (q_1 + \delta q^{FC}_{h,2})) \\
+ + (1 - \lambda) (C(q_1) - \theta_h q_1 + \delta (C(q^{FC}_{h,2}) - \theta_h q^{FC}_{h,2}))
\]
and solving for this yields $q_1$.

Notice that this pooling contract was also incentive-feasible but not incentive efficient.

Now, compare the two contracts. Under pooling, $P$ earns

$$V^P = V(q_1) + \delta V(q^{FC})$$

Under separating

$$V^S = V(q^{FC}) + \delta V(q_l^*, q_h^*)$$

Now, we know that

$$V(q^{FC}) \geq \max (V(q_l^*, q_h^*), V(q_1))$$

since both contracts were incentive-feasible in the static game.

Hence, we have

**Proposition 2** For large enough values of $\delta$, pooling is preferred to separating under renegotiation.