1. There are two players, a sender and a receiver. The state is uniformly distributed on \([0, 1]\) and preferences are quadratic. Suppose that, in contrast to the standard costless signaling model, it costs the sender an amount \(c(m, \theta)\) to send message \(m\) in state \(\theta\).

   (a) Characterize the “best” equilibrium when \(c(m) = \delta m\), where \(\delta > 0\) is a parameter.
   
   (b) What happens to this equilibrium as \(\delta \to 0\)?
   
   (c) Suppose that \(c(m) = m\) and the sender has access to a costless signal, \(\mu\), as well. Characterize the “best” equilibrium

2. A hungry parent bird must decide whether or not to give food to its offspring. There is a chance \(p\) that the offspring is truly hungry (this is known to the offspring and not to the parent). The offspring can either squawk or remain quiet. After hearing squawking or not, the parent can either keep the food or give it to the offspring. The “payoffs” to the parties are their fitness in producing offspring. Normalize the parent’s strength if it keeps the food to be 1 and let \(S < 1\) be the parent’s strength if it gives the food to the offspring. Let \(r\) be the degree of relatedness between the parent and the offspring. If the offspring is quiet and gets the food, its strength is 1. If it is not hungry and does not get the food, its strength is \(V < 1\). If it is hungry and does not get the food, its strength is zero. Squawking is costly for the offspring. Its strength is multiplied by the factor \(1 - t\) where \(t \in (0, 1)\) if it squawks. The payoff of each player is its strength plus \(r\) times the strength of the other player.

   (a) Characterize all separating and pooling equilibria in the game and the parameter conditions in which each obtains under a Bayes-Nash equilibrium.
   
   (b) Do the same exercise using the notion of sequential equilibrium to “refine” away some of the equilibria
   
   (c) Do the same exercise under the intuitive criterion.
3. A firm produces a good which is either high or low quality. Let probability \( \pi > 0 \) denote the probability that the firm produces a high quality good. The consumer values the high quality good at \( H > 0 \) and the low quality good at \( L = 0 \). It costs the firm nothing to produce the low quality good while it costs \( 0 < c_H < H \) to produce the high quality good. A firm sets a price \( p_1 \) for the good and chooses a celebrity endorser at a cost of \( E \geq 0 \) (where the level of \( E \) is under the control of the firm). The consumer observes \( p_1 \) and \( E \) and decides whether to purchase in the first period. In the second period, the quality is observable to the consumer and the firm chooses a price \( p_2 \).

(a) Find conditions on the parameters such that it is a sequential equilibrium for the firm to employ the services of a celebrity endorser only if it is a high quality good.

(b) Find conditions on the parameters where there is a pooling equilibrium (which is sequential) to the game

(c) Use the intuitive criterion to refine equilibria

(d) Suppose this is only a one period game. Characterize equilibria using the notion of sequential equilibrium.