Some Notes on Durable Goods Monopoly with Fixed Types

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1 Introduction

The usual treatment of the durable goods monopoly problem has the monopolist selling to a continuum of agents on an anonymous basis. The monopolist’s problem is the usual one—imperfect commitment. In this set of notes, we study the durable goods monopoly problem from a mechanism design perspective with non-anonymous buyers having fixed types over both periods.

2 Preliminaries

This time, we’ll think about the situation from the perspective of a monopoly seller of a durable good. As usual, there are two types of agent $\theta_l$ and $\theta_h$ where $Pr(\theta = \theta_l) = \lambda$. A buyer’s type should be thought of as his or her flow utility from a single unit of the product in each period. The discount rate is $\delta$. The agent should be thought of as the buyer of a good supplied by a monopoly principal at zero cost.

Full Commitment Solution

Define $T_i$ to be the NPV of the payment made by a buyer identified as type $i$. Let $Q_i = q_{i1} + \delta q_{i2}$ denote the expected “quantity” of the good consumed over the two periods. The quantity is actually the probability of consumption in a given period. Then P’s problem is

$$\max_{Q_i, T_i} \lambda T_i + (1 - \lambda) T_h$$

subject to

$$\theta_i Q_i - T_i \geq 0 \quad (IR)$$

$$\theta_i Q_i - T_i \geq \theta_j Q_j - T_j \quad (IC)$$
and

\[0 \leq Q_i \leq 1 + \delta \quad \text{(FEAS)}\]

As usual, one can show that the binding constraints are

\[\theta_t Q_t - T_t = 0\]
\[\theta_h Q_h - T_h = \theta_h Q_l - T_l\]

Substituting

\[\max_{Q_i} \lambda \theta_t Q_t + (1 - \lambda) (\theta_h (Q_h - Q_l) + \theta_l Q_l)\]

subject to the feasibility constraint.

This yields

\[\lambda \theta_t - (1 - \lambda) (\theta_h - \theta_l) - \mu_t \leq 0\]
\[(1 - \lambda) \theta_h - \mu_h \leq 0\]

The point is that there are no \(Q_i\)'s in either solution, hence we must have corner solutions. First, notice that \(Q_h > 0\) hence in any solution

\[Q_h = 1 + \delta\]

In contrast, \(Q_l\) depends on how large is \(\lambda\). When \(\lambda\) is large,

\[Q_l = 1 + \delta\]

while if \(\lambda\) is small

\[Q_l = 0\]

What is the critical value of \(\lambda\)?

Indifference between excluding and including low types

\[\theta_t (1 + \delta) = (1 - \lambda) \theta_h (1 + \delta)\]

or

\[\lambda < \lambda^* = 1 - \frac{\theta_l}{\theta_h}\]

We'll now assume that \(\lambda < \lambda^*\).

**Sales Contracts**

Now suppose that \(P\) is restricted to sales contracts and cannot commit not to sell in period 2. Suppose that if a buyer does not buy in period 1, the beliefs of \(P\) are that the buyer is low with probability \(\lambda (P_1)\). If that number is sufficiently high, then \(P\) will not exclude the low type otherwise he will. In either case, low types earn zero surplus in period 2.

Thus, a low type buyer buys in period 1 iff

\[P_1 \leq (1 + \delta) \theta_t\]
A high type buyer’s decision depends on the exclusion of the low types or not. If an \( h \) expects \( l \) types to be excluded in period, he will buy in period 1 iff

\[
P_1 \leq (1 + \delta) \theta_l
\]

If not, he’ll buy iff

\[
\theta_h (1 + \delta) - P_1 \geq \delta (\theta_h - \theta_l)
\]

or

\[
P_1 \leq \theta_h + \delta \theta_l
\]

Thus, \( P \) has three potential strategies at his disposal: (1) Sell to both types in period 1; (2) Sell to \( l \) in period 2; or (3) Exclude \( l \).

Revenues under (1)

\[
P_1 = \theta_l (1 + \delta)
\]

Hence

\[
\pi (1) = \theta_l (1 + \delta)
\]

Revenues under (2)

\[
P_1 = \theta_h + \delta \theta_l
\]

\[
P_2 = \theta_l
\]

Hence

\[
\pi (2) = (1 - \lambda) (\theta_h + \delta \theta_l) + \delta \lambda \theta_l
\]

We claim that \( \pi (2) > \pi (1) \). To see this, notice that

\[
\pi (2) - \pi (1) = (1 - \lambda) \theta_h + \delta \theta_l - \theta_l (1 + \delta) = (1 - \lambda) \theta_h - \theta_l
\]

\[
> 0
\]

since \( \lambda < \lambda^* \).

Finally, suppose that \( P_1 > \theta_h + \delta \theta_l \) and \( P_1 \leq \theta_h (1 + \delta) \). If \( P_2 = \theta_l \), then the \( h \) type buyer will not buy in the first period. If \( P_2 = \theta_h \) then the second period price is inconsistent with \( h \) types buying in the first period. Therefore, it must be that \( h \) types are mixing in the first period. The expected price in period 2 must leave an \( h \) type indifferent in period 1.

\[
\theta_h (1 + \delta) - P_1 = \delta \sigma (\theta_h - \theta_l)
\]

where \( \sigma \) is the probability that price \( P_2 = \theta_l \) will be charged. Hence

\[
\sigma = \frac{\theta_h (1 + \delta) - P_1}{\delta (\theta_h - \theta_l)}
\]
Furthermore, to induce P to undertake such a strategy, it must be that the remaining fraction of H types must leave P indifferent

\[ \lambda' = \frac{\lambda}{\lambda + (1 - \lambda)(1 - \gamma)} = 1 - \frac{\theta_l}{\theta_h} \]

where \( \gamma \) is the probability of accepting a first-period offer by a high type. Hence

\[ \gamma = 1 - \frac{\lambda}{1 - \lambda \theta_h - \theta_l} \]

P’s profit is then

\[ (1 - \lambda) \gamma P_1 + ((1 - \lambda)(1 - \gamma) + \lambda) \delta \theta_l \]

which is increasing in \( P_1 \); hence

\[ P_1 = \theta_h (1 + \delta) \]

and

\[ \pi (3) = (1 - \lambda) \gamma \theta_h (1 + \delta) + ((1 - \lambda)(1 - \gamma) + \lambda) \delta \theta_l \]

Notice that when \( \gamma \to 0 \), \( \pi (3) < \pi (2) \) whereas for \( \gamma \to 1 \), the reverse is true.

**Ratchet Effects**

Now consider the case where \( P \) can “rent” the item. This is equivalent to being able to commit to any long-term contract whatsoever. Notice that the decision in period 2 simply depends on the inference \( P \) makes following the decision in period 1. Again, there are three possibilities:

1. **Pool in period 1**: Rent at \( R_1 = \theta_l \) in period 1, then choose the optimal 1 period scheme in period 2. This earns profits equal to

\[ \pi (1) = \theta_l + \delta (1 - \lambda) \theta_h \]

2. **Separate in period 1**: In that case, no agent obtains rents in period 2. Thus, a high type agent can pretend to be low in period 1 and thereby earn rents in period 2 of \( \theta_h - \theta_l \). Hence

\[ \theta_h - R_1 \geq \delta (\theta_h - \theta_l) \]

And profits for \( P \) are then

\[ \pi (2) = (1 - \lambda) (\theta_h + \delta \theta_l) + \lambda \delta \theta_l \]

\[ = (1 - \lambda) \theta_h + \delta \theta_l \]

and this is exactly the same as a sale.

3. **Semi-separate in period 1**: In that case, one can show that things are again identical to a sale.

However, the equality is only true in two periods. With three or more periods, the sale starts to revenue dominate the rental with non-anonymous buyers (Hart and Tirole, 1988).