Part 1  Background

1.a  Two-Sided Markets

- Definition: Economic platforms having two distinct user groups that provide each other with network benefits.

- Examples: Credit card markets, online auction platforms, offline auction entities (e.g., Sotheby’s, Christie’s), online retail platforms (e.g., Amazon), standards battles (e.g., VHS v. Beta).

- Observation: These markets tend toward concentration. Why?

1.b  Assumptions

- A seller prefers the site with the highest expected price.

- A buyer prefers the site that provide the highest expected consumer surplus (i.e. lower prices).

1.c  Countervailing forces

- **Scale (or efficiency) effect**: Larger markets typically provide greater expected surplus per participant.
  \[ \Rightarrow \text{Pushes the market toward concentration.} \]

- **Market impact effect**: When a seller contemplates switching markets, she considers that joining a market will increase the seller-buyer ratio there and thus lower the expected price.
  \[ \Rightarrow \text{Pushes the market toward interior equilibrium multiplicity. That is, a plurality of markets may coexist in equilibrium.} \]
  - Example: "If the decrease in price in Market 2 that results from a seller switching from Market 1 to Market 2 is large enough to offset Market 2’s initial price advantage, then sellers in Market 1 may be happy to stay in Market 1 while those in Market 2 are happy to stay in Market 2."
  - Note: These markets may differ significantly in size.

- Relative impact of effects
  - Both effects become smaller as the markets grow.
  - Both effects shrink at the same rate.
Part 2  Theoretical Analysis: The EFM Model Part I

Ellison, Möbius & Fudenberg, *Competing Auctions*

2.a  The Model

- Two Markets
  - $B$ ex-ante identical buyers, each with unit demand
    - Buyers’ values $v$ are drawn (iid) from cdf $F$.
    - $F$ has density function $f$ that is positive on support $[0, \bar{b}]$, with $\bar{b}$ allowed to be infinite.
    - $E[v]$ is assumed finite.
  - $S$ sellers, each endowed with a single unit of the good, each with reservation value of zero.
    - $S + 1 < B \Rightarrow$ The market price is strictly positive.

2.b  The Game

- Stage 1: Buyers and sellers simultaneously choose whether to attend Market 1 or Market 2.
- Stage 2: Buyers learn their valuations and a uniform price auction occurs in each market.
  - Explanation: Buyers may need to go to the auction site and inspect the good to learn their valuation. Or, buyers may be dealers who participate in many auctions, and may have time-varying idiosyncratic valuations due to inventory or other factors, and may choose a single auction site for all purchases because they can save on transaction costs or build a better reputation.

2.c  The Outcomes

- Price: $p$ or $v^{S+1:B}$ (If a market with $S$ sellers and $B$ buyers does not have excess supply, the price is the $S + 1$st highest of the $B$ buyer values.)
- $v^{k:n}$: the $k$th highest order statistics of a draw of $n$ values.
- $f^{k:n}$: the density of $v^{k:n}$.
- $u_s(S,B) = E(v^{S+1:B}) \equiv \bar{p}(S,B)$: A seller’s expected utility, which is the seller’s expected price in the chosen market.
- $u_b(S,B) = E(v - v^{S+1:B} | v \geq v^{S:B})PR(v \geq v^{S:B})$: A buyer’s expected utility in a market of $S$ sellers and $B$ buyers.
- $E(v|v \geq v^{S:B}) = E(v|v > v^{S+1:B}) = w(S,B)$: The surplus per seller.
  - Note: $w(S,B) = \int_0^{\bar{b}} (\int_x^b v f(v|x)dv) f^{S+1:B}(x)dx$
2.d Pure Strategy Quasi-Equilibrium Constraints

Quasi-Equilibrium: \( S_1, S_2, B_1, B_2 \) need only be nonnegative real numbers, not integers.

- \( u_s(S_1, B_1) \geq u_s(S_2 + 1, B_2) \)
- \( u_s(S_2, B_2) \geq u_s(S_1 + 1, B_1) \)
- \( u_b(S_1, B_1) \geq u_b(S_2, B_2 + 1) \)
- \( u_b(S_2, B_2) \geq u_b(S_1, B_1 + 1) \)

2.e Proposition 1. A vector of nonnegative real numbers \((S_1, S_2, B_1, B_2)\) with \( S_1 + S_2 = S \) and \( B_1 + B_2 = B \) is a quasi-equilibrium if and only if it satisfies the following four constraints:

- \((B1')\) \( u_b(S_2, B_2) - u_b(S_2, B_2 + 1) \geq u_b(S_2, B_2) - u_b(S_1, B_1) \)
- \((S1')\) \( u_s(S_2, B_2) - u_s(S_2 + 1, B_2) \geq u_s(S_2, B_2) - u_s(S_1, B_1) \)
- \((B2')\) \( u_b(S_1, B_1) - u_b(S_1, B_1 + 1) \geq u_b(S_1, B_1) - u_b(S_2, B_2) \)
- \((S2')\) \( u_s(S_1, B_1) - u_s(S_1 + 1, B_1) \geq u_s(S_1, B_1) - u_s(S_2, B_2) \)

Part 3 Theoretical Analysis: The EFM Model Part II

3.a Now Assume Uniformly-Distributed Buyer Valuations

- Specifically, assume that buyer valuations are uniform on \([0, 1]\).
- Result 1: Markets of different sizes can coexist.
- Result 2: An auction site needs a critical mass of participants to be viable.

3.b Proposition 2. When buyer valuations are uniform on \([0, 1]\), the utility functions are:

- \( u_s(S, B) = \frac{B - S}{B + 1} \)
- \( u_b(S, B) = \frac{S(1 + S)}{2B(B + 1)} \)

3.c Proposition 3: Assume buyer valuations are uniform on \([0, 1]\), and assume the same seller/buyer ratio \( \gamma \) on both sites. Then:

- (a) The per-seller scale advantage of the larger market is:
  \[ w(\gamma B_2, B_2) - w(\gamma B_1, B_1) = \frac{(1 - \gamma)(B_2 - B_1)}{2(B_2 + 1)(B_1 + 1)} \]
  The payoff advantage/disadvantage of the larger market for the sellers/buyers is:
  \[ u_s(\gamma B_2, B_2) - u_s(\gamma B_1, B_1) = (1 - \gamma)\frac{(B_2 - B_1)}{(B_2 + 1)(B_1 + 1)} \]
  \[ u_b(\gamma B_2, B_2) - u_b(\gamma B_1, B_1) = -\frac{\gamma(1 - \gamma)(B_2 - B_1)}{2(B_2 + 1)(B_1 + 1)} \]

- (b) The market impact effects for both \( j = 1 \) and \( j = 2 \) are given by:
  \[ u_s(S_j, B_j) - u_s(S_j + 1, B_j) = \frac{1}{B_j + 1} \]
  \[ u_b(S_j, B_j) - u_b(S_j, B_j + 1) = \frac{S_j(S_j + 1)}{2B_j(B_j + 1)(B_j + 2)} \]
3.d Proposition 4:

Fix $B$ and $S$ with $B > S + 2$. When buyer values have the uniform distribution, there is a unique $B_1 \in [0, B/2]$ for which there is an $S_1$ such that (S1) and (B1) both hold with equality at $(S_1, S - S_1, B_1, B - B_1)$. There exists an $S_1$ such that $(S_1, S - S_1, B_1, B - B_1)$ is a quasi-equilibrium if and only if $B_1 \in [B_1, B - B_1]$. Moreover, $-\frac{5}{4B} < \frac{B}{B_1} - \frac{1}{4}(1 - \frac{S}{B}) < \frac{3}{4B} + \frac{1}{8}$.

3.e Lemma A1

Fix $S$ and $B$ with $S + 1 < B$. Consider a general model in which $S$ sellers and $B$ buyers simultaneously choose between two locations, and receive payoffs of $u_s(S_i, B_i)$ and $u_b(S_i, B_i)$ if they choose Market $i$ and Market $i$ attracts $S_i$ sellers and $B_i$ buyers. Assume $B_1 \leq B/2$ and that the utility functions satisfy three conditions:

- (A1-Boundary) $u_s(S, B) > 0$ if $B > S$ and $u_s(S, B) = 0$ otherwise.
  $u_b(S, B) > 0$ if $S > 0$ and $u_b(S, B) = 0$ otherwise.
- (A2-Monotonicity) If $B > S$, then $\frac{\partial u_s}{\partial S} < 0$, $\frac{\partial u_s}{\partial B} > 0$, $\frac{\partial u_b}{\partial S} > 0$, and $\frac{\partial u_b}{\partial B} < 0$.
- (A3-Large Market Efficiency) If $B_1 < B_2$, then $u_s(S_1, B_1) < u_s(S_2, B_2)$ or $u_b(S_1, B_1) < u_b(S_2, B_2)$.

Then, there exists an $S_1$ such that $(S_1, S - S_1, B_1, B - B_1)$ is a quasi-equilibrium if and only if there exists and $S_1$ such that $(S_1, S - S_1, B_1, B - B_1)$ satisfies the (B1) and (S1) constraints. (Proof in Ellison and Fudenberg (2002)).

3.f Lemma A2

Suppose buyers’ values have the uniform distribution. Fix $B$ and $S$ with $B > S + 2$. For every partition $(B_1, B_2)$ with $\frac{B_2}{B_1} \in \left[\frac{1}{2} - \frac{S}{2B}, \frac{1}{2} + \frac{S}{2B}\right]$, there is a quasi-equilibrium $(S_1, S_2, B_1, B_2)$ with $u_s(S_1, B_1) = u_s(S_2, B_2)$. Specifically, choosing $S_1$ and $S_2$ with $\frac{S_1 + 1}{B_1 + 1} = \frac{S_2 + 1}{B_2 + 1}$ gives such a quasi-equilibrium.

Part 4 Empirical Analysis

Jennifer Brown & John Morgan, How much is a Dollar Worth? Tipping versus Equilibrium Coexistence on Competing Online Auction Sites

4.a Model

- Ellison, Fudenberg, and Möbius (2004) generates two testable hypotheses:
  (1) The "law of one price" should hold across competing sites.
  (2) Coexisting sites should have similar ratios of buyers and sellers.
- Both expected to hold in a two-platform equilibrium.
- Neither need hold when a market is in the process of tipping.
- Are Yahoo and eBay coexisting in equilibrium??
4.b Field Experiment
- Compared auction markets in coins on eBay and Yahoo.
- Varied reserve price through opening bid amount, not secret reserve option.
- Varied the ending rule (exact time v. soft-close) at Yahoo.

Ockenfels & Roth (2006) suggest soft-close may generate more revenue.

4.c Results and Conclusions
- Setting positive reserve price increases revenue. → Consistent with auction theory.
- Ending rule did not affect revenue. → Little support for Roth & Ockenfels (2002).
- Higher revenues (29.6%) and numbers of bidders (58.3%) on eBay. → Hypotheses not supported if platforms in equilibrium.
- Arbitrage: Could buy on Yahoo and sell on eBay at a profit.

4.d Policy Concerns
- Market tipped to eBay, which raised prices up to 67%. Monopoly pricing?
- Antitrust danger. Is it possible to prevent monopoly in a market so inclined to tipping?
- Why have we not observed tipping in other two-sided markets?
- Possible answer: Some frictions may be more salient in these markets than the on-line auction market (e.g., platform differentiation, switching costs, trustworthiness, liquidity).

Part 5 Imitation Dynamics: A Refinement of the EFM Model
5.a Types
- Two types of buyers and sellers (eBay and Yahoo).
- A system state \((s, b)\) at any point in time that identifies the number of sellers and buyers who are eBay types. The remaining \(S - s\) sellers and \(B - b\) buyers are Yahoo types.
- Buyers and sellers in state \((s, b)\) have expected payoffs of \(\pi_a\) and \(u_a\) for \(a \in \{e, y\}\).
- Types evolve in proportion to the payoffs from their strategy relative to the average payoffs in the population.

5.b Equilibria
- States \((0, 0)\) and \((S, B)\) are fixed points. ⇒ Once the market has tipped, it remains tipped.
- Imitation dynamics yields a unique interior fixed point: \((S/2, B/2)\).
- This prediction contrasts with the EFM model, which typically generates a continuum of interior equilibria.
5.c Proposition 3: Under imitation dynamics, equilibrium coexistence only occurs when both platforms enjoy equal market shares.

- In the EFM model, agents are fully rational. "Small price differences between platforms do not induce sellers and buyers to switch because increased competition would wipe out any possible gains."

- In the Imitation Dynamics model, agents are boundedly rational. Because agents simply imitate more successful strategies, the market impact effect does not sustain equilibrium coexistence.

5.d Proposition 4: The interior fixed point, \((s_1 = S/2, b_1 = B/2)\), is a saddle point while the tipped fixed points, \((s = 0, b = 0)\) and \((s = S, b = B)\), are attractors.

- Thus, for almost all initial states, the system will eventually tip.

- For example, suppose prices and buyer-seller ratios are higher on eBay. Then buyers head to Yahoo for the higher prices until buyers are indifferent between the two platforms. But sellers continue to switch to eBay for the lower prices. Then buyers start switching back to eBay. eBay’s market share grows monotonically until complete tipping occurs.

- Tipping is relatively slow, based on use of empirical data to simulate the tipping process. As the number of Yahoo agents decreases, fewer agents decide to switch each period. What might speed up the process?