Problem Set #1

Doing these problems is optional. The solutions to these questions will be posted on Thursday, September 3rd and discussed in section the next day. The educational value of these exercises will be maximized if you attempt to answer these questions before you look at the answers.

Sometimes students find a question in these problem sets frustrating. Since these are not graded, you are free to stop working on a problem whenever you feel the gain from further effort is not worth the cost of further frustration and time. What is important is that you have thought seriously about the problems, not that you have necessarily gotten the correct answer to every one of them.

Question 1

Imagine that you work for the World Bank, a development bank that provides loans and policy advice to governments around the world. The World Bank is trying to encourage the Russian government to privatize an industry, and you have been asked to help the Bank determine the market price and quantity that would prevail in the Russian market if competitive forces were allowed to equilibrate the market. The best estimates of the market demand and supply for the Russian good (in U.S. dollar equivalent prices) are given by $Q_D = 10 - 2P$ and $Q_s = 2 + 2P$, respectively. Both $Q_D$ and $Q_s$ are measured in billions of units of the good.

(a) Determine the competitive equilibrium price and quantity.

The competitive equilibrium is determined by the intersection of the market demand and supply curves. Algebraically, this simply means that $Q_D = Q_s$. Equating demand and supply in this case yields:

$10 - 2P = 2 + 2P$

$8 = 4P$

$P^* = 2$

Plugging this $P$ back into either the supply or the demand curve yields $Q^* = 6$.

(b) Based on your analysis, a Russian minister raises the concern that the free market price might be too high for the typical Russian citizen to pay.
Accordingly, he asks you to explain what would happen if the Russian government privatized the market, but then set a ceiling price at the Russian equivalent of $1.50. How do you answer?

Since the price ceiling is below the equilibrium price of $2, a shortage will result. Specifically, when the price ceiling is set at $1.50, quantity demanded is:

\[ Q_D = 10 - 2(1.50) = 7 \]

and quantity supplied is:

\[ Q_S = 2 + 2(1.50) = 5 \]

**Question 2**

Rex Maxprof is trying to decide whether to attempt a certain hostile takeover. One of his advisors, Bobby Surefire thinks that Maxprof Industries should attempt the takeover of Minprof Associates. If successful, the takeover would increase the net value of Maxprof Industries by $1 million. If the attempt failed, the net value Maxprof industries would decline by $200,000. Surefire says that the probability of success is 0.5.

Maxprof's other advisor, Rick Steady, says that a new defense against takeovers makes the probability of successfully taking over Minprof Associates only 0.1. Surefire disagrees, saying that this defense is not legal. Rex trusts each of his advisors equally and has no other information, so he thinks that they are equally likely to be correct about the legality of this defense (i.e., a .5 probability that each is correct).

There is currently a lawsuit underway in another takeover attempt that will determine whether the new takeover defense is legal. Rex could either pursue the takeover now or wait until the lawsuit is decided. If he waits, however, a successful takeover will be worth only $900,000 since Minprof will have squandered more assets in the mean time, while a failed attempt will still cost $200,000.

(a) Draw the decision tree that Rex faces.

*See Figure 1.*

(b) If Surefire is right and Rex proceeds with the takeover now, what is the expected value of the attempt?
If Surefire is right and the takeover defense is illegal, then the probability of success is 0.5, and the expected profits from proceeding with the takeover now are:

\[0.5 \times 1,000,000 + 0.5 \times -200,000 = $400,000\]

(c) If Steady is right and Rex proceeds with the takeover now, what is the expected value of the attempt?

If Steady is right and the takeover defense is legal, then the probability of success is 0.1, and the expected profits from proceeding with the takeover now are:

\[0.1 \times 1,000,000 + 0.9 \times -200,000 = -$80,000\]

(d) If Rex proceeds with the takeover now, what is the expected value of the attempt?

Rex believes that the outcomes in (b) and (c) are equally likely, so the expected value of proceeding with the takeover now is 0.5x$400,000 + 0.5x-$80,000 = $160,000. If there were no other information that Rex could obtain before making the decision (and if he were not too risk averse), then Rex would want to proceed with the takeover now. However, Rex can get more information by waiting to see how the ongoing lawsuit is decided.

(e) If Rex waits and the current lawsuit shows that Surefire is right, what is the expected value of the attempt after the lawsuit is decided?

If after the legal decision Rex knows that Surefire is right, then he knows that the probability of success is 0.5. The expected value of attempting the takeover is 0.5x$900,000 + 0.5x-$200,000 = $350,000.

(f) If Rex waits and the current lawsuit shows that Steady is right, what is the expected value of the attempt after the lawsuit is decided?

If after the legal decision Rex knows that Steady is right, then he knows that the probability of success is 0.1. The expected value of attempting the takeover is 0.1x$900,000 + 0.9x-$200,000 = -$90,000. In that case, Rex would not attempt the takeover at all, which has an expected value of zero.

(g) What should Rex do?

If Rex waits, there is a 0.5 probability that he will get the outcome in (e) and a 0.5 probability that he will get the outcome in (f). The expected
value is then \(0.5 \times \$350,000 + 0.5 \times \$0 = \$175,000\). This is greater than the value in (d). Rex should wait to see how the lawsuit is decided.

(h) What is the value of the information that Rex would learn by waiting until after the lawsuit to decide on whether to attempt the takeover?

The value of the information is \(\$15,000\), the difference between Rex’s expected value of making his choice with the information and his expected value without the information. The information provided by the decision in the lawsuit will help Rex if he learns that the takeover defense is legal, because in this case, he decides not to try the takeover.

Question 3

AK Steel Holding Corporation is a producer of flat-rolled carbon, stainless and electrical steels and tubular products through its wholly owned subsidiary, AK Steel Corporation. The 2008 surge in the demand for steel significantly increased AK’s profits, so it engaged in a research project to improve its production of rolled steel. The research involves three distinct steps, each of which must be successfully completed before the firm can implement the cost-saving new production process. If the research is completed successfully, it will save the firm \(\$4\) million. Unfortunately, there is a chance that one or more of the research steps might fail, in which case the project is worthless. The three steps are done sequentially so that the firm knows whether one step was successful before it has to invest in the next step. Each step has a 0.8 probability of success and each step costs \(\$500,000\). The risks of failure in the three steps are uncorrelated with one another. Assume that AK Steel is a risk neutral company (i.e., that it maximizes expected values). In case you are worried about such things, the interest rate is zero.

(a) Draw the decision tree for the firm.

See figure 2.

(b) If the firm proceeds with this project, what is the probability that it will succeed in implementing the new production process?

For the project to be successful, each of the three independent steps must be completed. Since the probability of success in each stage is 0.8 and the probabilities are independent, the probability of three successes is

\[ Pr = 0.8 \times 0.8 \times 0.8 = 0.8^3 = 0.512, \text{ just over one-half.} \]

(c) If the research were costless, what would be the firm’s expected gain from it before the project began?
\[ E[gain] = 0.512 \cdot 4,000,000 + 0.488 \cdot 0 = 2,048,000 \]

(d) Should the firm begin the research, given that each step costs $500,000?

The expected cost of the project is

\[ 0.2 \cdot 500,000 + 0.8 \cdot 0.2 \cdot 1,000,000 + 0.8 \cdot 0.8 \cdot 1,500,000 = 1,220,000. \]

The first term is the probability times cost of a failure in the first step. The second term is the probability times cost of success in the first step and failure in the second step. The third term is the probability times cost of success in the first step and success in the second step (success or failure in the third step does not affect the cost of the project, just the gain from it). The expected cost is less than the expected gain (by $828,000). Since the company is not risk averse, it should begin the project.

Note that this is not the only way to do the calculation. An alternate approach would be to aggregate the costs and benefits of each possible outcome:

\[
0.8 \cdot 0.8 \cdot 0.8 \cdot (4,000,000 - 500,000 - 500,000 - 500,000) + 0.8 \cdot 0.8 \cdot 0.2 \cdot (-500,000 - 500,000 - 500,000) + 0.8 \cdot 0.2 \cdot (-500,000) + 0.2 \cdot (-500,000) = 828,000.
\]

Either way, the expected net gain is $828,000.

(e) Once the research has begun, should the firm quit at any point even if it has had no failures? Should it ever continue the research even if it has had a failure?

\[
\text{NO to both. Obviously, if one stage fails, then the project cannot be completed successfully, so any more expenditures on it are a waste. If no stage has failed and at least one has succeeded, then the benefit/cost comparison of going forward with the project is even more favorable than when the project began.}
\]

After the firm has successfully completed steps one and two, it discovers an alternate production process that would cost $150,000 and would lower production costs by $1,000,000 with certainty. This process, however, is a substitute for the three-step cost-saving process; they cannot be used simultaneously. Furthermore, to have this process available, the firm must spend the $150,000 before it knows if it will successfully complete step three of the three-step research project.

(f) Draw the augmented decision tree that includes the possibility of pursuing this alternate production process.
See figure 3.

(g) If the firm continues the three-step project, what is the chance it would get any value from also developing the alternate production process?

The alternate process would be used only if step three of the current project failed, which has a 0.2 probability.

(h) If developing the alternate production process were costless and if the firm continues the three-step project, what is the expected value that it would get from having the alternate production process available (at the beginning of research step 3)? (This is known as the option value of having this process available.)

There is a 0.2 probability that the alternate process would be used and a $1,000,000 value if it is used, so the option value of having the alternate process available is $200,000.

(i) Should the firm:
   i. Pursue only the third step of the three-step project
   ii. Pursue only alternate production process
   iii. Pursue both the third step of the three-step project and the alternate process

Since the option value of the alternate process is greater than the cost of having this option, the alternate process should be developed if one continues with the three-step project. The net value of developing this option is $200,000 − $150,000 = $50,000. Of course, the alternate process would also be developed if the three-step project were unavailable, since it will be used with certainty and the net value of the alternate process would then be $850,000. The remaining question is whether AK should drop the three-step project rather than attempting the third step. Given that the alternate process will be developed, the extra (or marginal) value of successfully completing the three-step project would be $3,000,000, because it would save $3,000,000 more than the alternate process. The expected value of attempting the third step is then 0.8 · $3,000,000 = $2,400,000. This is greater than the $500,000 cost of the third step, so AK should proceed with the three-step project as well as the alternate process, i.e., take strategy III.

(j) If the firm had known of the alternate production process before it began the three-step research project, what should it have done?
We know that AK should pursue the alternate process: It was worth doing after successful completion of steps one and two (see (i)) and would have greater expected value if the probability of the three-step project failing were higher. In fact, the option value of the alternative process declines with each step of success in the three-step project. At the beginning of step three AK would pay up to $200,000 for the alternate process. Convince yourself that it would be willing to pay up to $360,000 for the alternate process at the beginning of step two and up to $488,000 for the alternate process at the beginning of step one, assuming in each case that it couldn’t wait to develop the alternate later. In fact, the option to wait until the beginning of the third period to develop the alternate process could itself be valuable, but it isn’t in this case, when the process costs $150,000. The other question is whether AK should pursue the three-step project given that it will have the alternate process available with certainty. As in (i), the marginal value of successfully completing the three-step project would be $3,000,000, because it would save $3,000,000 more than the alternate process. The expected value of attempting the three-step project is then 0.512 · $3,000,000 = $1,536,000. This is greater than the expected cost of pursuing the three-step project, which is 0.2 · 500,000 + 0.8 · 0.2 · 1,000,000 + 0.8 · 0.8 · 1,500,000 = 1,220,000, so AK should proceed with the three-step project as well as the alternate process. This is the same calculation as in 2(c) and (d) except the benefit of success is now $3,000,000 instead of $4,000,000.
Figure 1

Takeover Succeeds [p=0.1]

New Defense is Legal [p=0.5]

EV=80,000 1,000,000

Takeover Fails [p=0.9]

-200,000

Try Takeover Now

EV=160,000

Takeover Succeeds [p=0.5]

New Defense is Illegal [p=0.5]

EV=400,000 1,000,000

Takeover Fails [p=0.5]

-200,000

Wait Until Lawsuit Decided

EV=175,000

Takeover Succeeds [p=0.1]

Try Takeover

EV=90,000 900,000

Takeover Fails [p=0.9]

-200,000

Do Not Try Takeover

0

Takeover Succeeds [p=0.5]

Try Takeover

EV=350,000 900,000

Takeover Fails [p=0.5]

-200,000

Do Not Try Takeover

0

0