Question 1

Your esteemed boss Hugo has just leased a new nut-grinding machine that has the capacity to produce 100 units of hugonuts per day. The daily lease cost is $405. The market price for a unit of hugonuts is $120, and no one expects this to change in the foreseeable future. When running the machine, you observe that daily total costs follow the pattern: \( C(Q)=405+20Q+5Q^2 \), where \( Q \) is the number of hugonuts produced and \( C(Q) \) is in $.

a) Hugo has decided to minimize his total costs. How much should he produce? What are his total profits?

C increases with \( Q \) and reaches a minimum when \( Q \) is just above 0, say at 0.01, at which point profits = 1.2-(405+.2+.005) \approx -404.

b) Hugo is back in the shop. He understands the importance of amortizing the daily lease cost of the machine over a large production run. His brother-in-law the VP of marketing has convinced Hugo of the importance of dominating the market and getting as much market share as he can with his existing machine. Hugo decides to follow this friendly advice, believing that not only will he make great heaps of money but he will also achieve much more pleasant conditions at the next gathering with his in-laws. How much does he instruct you to produce in order to maximize sales? What are his total profits (i.e. his total revenues minus total costs)?

Running at capacity of 100, \( TC=52,405 \). \( TR=100*120=12,000 \). 
Profits=TR-TC= -$40,405

c) You notice Hugo has sprouted a few more gray hairs. Coincidentally, the marketing VP was called away to investigate a potential new client in Tierra del Fuego. Hugo has given the matter more thought, and instructs you to
minimize the average cost of production. How much do you produce? What are your profits?

You have a choice of two methods to answer this question. At its minimum, you know AC is equal to MC. If the marginal cost of producing the next unit were below average cost, then AC would still be falling. If the marginal cost of producing the next unit were above the average cost, then AC would be increasing. So AC must reach a minimum where AC=MC. AC is found by dividing TC by Q, yielding \( AC = \frac{405}{Q} + 20 + 5Q \). MC is found by taking the derivative of TC with respect to Q, yielding \( MC = 20 + 10Q \). Setting \( MC = AC \) and solving for Q you find that \( Q = 9 \). At this point profits = 90 (\( =1080-990 \)).

Alternatively, AC reaches a minimum where its derivative with respect to Q (\( AC'(Q) = -\frac{405}{Q^2} +5 \)) is zero. This occurs at \( Q = 9 \).

d) Hugo is looking a bit better. He almost smiles now, especially when he shows everyone those spectacular post-cards of the fog and ice his brother-in-law keeps sending from Tierra del Fuego. You spot your opportunity and recommend that the time has come to maximize profits. Hugo is feeling so good, that he forgives you your MBA and follows your advice. How much do you produce? What are your profits?

As long as an additional unit of production adds more to revenues than to costs, profits (the difference between revenues and costs) are increasing. For a price-taking firm, maximum profits are achieved when price equals marginal cost. In this case, Hugo can sell each additional unit at a price of $120. \( MC = 20 + 10Q \). Setting \( 20 + 10Q = 120 \), you find that profits are maximized by producing 10 units. At this point your profits are $95 (\( =10*(120-405-200-500) \)). Hugo is impressed that your strategy has delivered greater profits than the alternatives of minimizing costs, maximizing market share, or minimizing average costs.

**Question 2**

You own the only carrot juice bar in Berkeley, which appears to be a valuable franchise. The daily demand you face for carrot juice is \( Q = 100 - P \). The total daily cost of operating the juice bar is \( TC = 100+10Q \). (The daily fixed cost of operating is 100. That is an avoidable fixed cost if the juice bar is not open).

a) What is your firm’s marginal revenue schedule. (Remember that marginal revenue (MR) is the change in revenue associated with a one unit change in quantity, not a one unit change in price)? How much should you produce in order to maximize profits? How much will your daily profit be?
Q = 100-P \rightarrow P = 100 - Q \rightarrow Total Revenue = P(Q)xQ = 100Q - Q^2 \rightarrow Marginal Revenue = 100 - 2Q. (Recall that you can get this either by taking the derivative of revenue with respect to quantity, or by remembering the rule that MR has the same intercept as the demand curve and twice its slope).

\[ MC = 10 \] (take the derivative of the TC function with respect to Q). Setting MR = MC implies 100 – 2Q = 10 \rightarrow Q = 45 \rightarrow P = 55.

Profits are Total Revenue – Total Costs = 55x45 – (100+10x45) = 1925.

b) You have brought in a hot-shot production consultant with a Stanford MBA who explains to you that your cost function is not what you thought. She says that your cost function is actually a bit more complicated. Your daily fixed cost is indeed 100, but your marginal cost depends on the quantity you produce:

\[ MC=10 \text{ for } Q \leq 20 \]
\[ MC=8 \text{ for } 20<Q \leq 50 \]
\[ MC=6 \text{ Q>50} \]

If this is so, how much should you produce in order to maximize profits? How much will your daily profit be?

Note that for Q\leq20, MR>MC (MC=10 and MR\geq100-2x20 = 60). At Q=20, MR=60 and at Q=50, MR=0, so at some 20<Q<50, there is a Q for which MR=MC. We know MC=8 in this whole range. Setting 100-2Q=8 \rightarrow Q=46 \rightarrow P=54 \rightarrow Profits = 54x46 -(100+10x20+8x26)=1976.

Note that in the last step, we are calculating variable costs by summing up the marginal costs for all of the units we have produced. This is akin to calculating variable costs by summing the area under the MC curve, as we have done in class.

c) Finally, you bring in a Berkeley-Haas MBA, who is confident without an attitude! She explains that your cost function is actually \( TC = 100 + Q^2 \), so your marginal cost rises as you produce more, \( MC = 2Q \). She also says that the demand function you face is actually \( Q = 20 - P \). If this is so, how much should you produce in order to maximize profits? How much will your daily profit be?

\[ TC=100+Q^2 \rightarrow MC=2Q \text{ and} \]
\[ Q=20-P \rightarrow MR=20-2Q. \]
Setting MR=MC gives Q=5 and P=15.
At Q=5, Profits =5x15-(100+5^2)=-50.
Even when the company maximizes profits, it still loses money. It should shut down.
d) Finally, you recognize three things. First, your initial estimate of costs were correct so that when you produce a total of \(Q\) units then our costs are \(TC = 100 + 10Q\). Second, your initial estimate of Haas demand was true, and is equal to \(Q_H = 100 - P\) (where \(Q_H\) represents “Haas generated demand”). Third, the recent student and faculty walkout has generated a huge amount of tourism right outside the Haas school, and these tourists form another source of demand given by \(Q_T = 75 - P\). Since these tourists do not have Berkeley IDs, you realize that you can set a price \(P\), and offer a “Cal Discount” to your original customers. Would you like to do this? If so, what price would you charge, what would the discount be, and what would your profits be? Compared to part (a), has the discount made your Haas clientele better off? Why?

Yes. Your objective is to set \(MR_H = MR_T = MC\); which implies that each market price could be solved independently. In other words, since either market’s demand does not affect marginal cost, we can treat it as two separate optimization problems. \(MR_H = MC\) is one equation with one unknown, \(Q_H\), which is again solved as in part (a) so that \(P_H = 55\), \(Q_H = 45\). Next, from \(MR_T = MC\) we solve (remember to invert demand to find MR) \(150 - 2Q_T = 10\), which gives \(Q_T = 35\) and then from the demand we have \(P_T = 80\). In one sense we could say the Haas discount is $25 (i.e., \(P_T > P_H\)). However, in reality we are simply able to charge the tourist a surcharge (and it is more profitable to do so) because their demand does not affect marginal cost and thus does not changing the optimal price to charge Haas folks.

Total profits are then \(80 \times 35 + 55 \times 45 - (100 + 10 \times (35 + 45)) = $4,375\).

Question 3

Toscanini’s, a Boston firm, has the patent on gingersnap molasses ice cream. It decides to introduce it to the Bay Area, where it proves to be an enormous hit in both San Francisco County and Marin County. Ice cream does not travel well, and the congestion on the Golden Gate Bridge is so bad that it is impossible to transport the ice cream across the bridge. Consequently, Toscanini’s builds two identical plants, one in San Francisco and the other in Sausalito. For either plant, the daily cost of \(Q\) gallons of ice cream is \(2Q\). The daily demand for gingersnap molasses ice cream in San Francisco County is \(Q_{SF} = 35,000 - 5000p\). The daily demand for gingersnap molasses ice cream in Marin County is \(Q_M = 25,000 - 2500p\).

a) A California state law prohibits charging different prices in adjacent counties. What single price \(p\) should Toscanini’s charge to maximize its profit from gingersnap molasses ice cream? What is its profit?
For \( p \geq 10 \), no ice cream is sold in either county. For \( 10 > p \geq 7 \), ice cream is sold in Marin county only. Finally, for \( p < 7 \), ice cream is sold in both counties. We need, consequently, to consider two cases: \( p < 7 \) and \( 10 > p \geq 7 \).

For \( 10 > p \geq 7 \), ice cream is only sold in Marin county, so total demand is \( Q = 25,000 - 2500p \Rightarrow p = 10 - Q/2500 \Rightarrow \text{Revenue} = 10Q - Q^2/2500 \Rightarrow \text{MR} = 10 - Q/1250 \).

To figure out whether MR crosses MC when only Marin county buys, note that at \( p = 7 \), \( Q = 25,000 - 2500(7) = 7500 \) and at \( Q = 7500 \), \( \text{MR} = 4 \), which is still higher than \( \text{MC} = 2 \), so the monopolist does not want to stop producing at \( Q = 7500 \).

For \( p < 7 \), total Bay Area demand is the sum of the two demand curves, or \( Q = 60000 - 7500p \Rightarrow p = 8 - Q/7500 \Rightarrow \text{Revenue} = 8Q - Q^2/7500 \Rightarrow \text{MR} = 8 - Q/3750 \). Equating marginal revenue to marginal cost (i.e., 2), we have \( 8 - Q/3750 = 2 \Rightarrow Q^* = 22500 \Rightarrow P^* = 5 \).

Toscanini’s profits are \( P^* \times Q^* - \text{Costs}(Q^*) = 5 \times 22500 - 2 \times 22500 = 67,500 \).

In case you’re curious, here is a graph of what demand and MR looks like in this example. Note that marginal revenue jumps up at \( Q = 7500 \). This is because when the firm lowers its price just below 7, it not only sells to several more Marin county residents, but also taps into the San Francisco demand.

b) Responding to a suit brought by Toscanini’s, a federal judge rules that San Francisco and Marin counties are not adjacent because they are separated by the Golden Gate. Toscanini’s is now free to charge different prices in the two counties, \( p_{\text{SF}} \) and \( p_{\text{M}} \). What prices \( p_{\text{SF}} \) and \( p_{\text{M}} \) maximize Toscanini’s profits? What are the resulting profits?
We can treat these as two separate uniform-pricing problems.

We saw that for Marin, \( MR_M = 10 - \frac{Q_M}{1250} \). Equating marginal revenue to marginal cost yields \( Q_M = 10,000 \Rightarrow p_m = 6 \). Profit in Marin is \( 6 \times 10,000 - 2 \times 10,000 = 40,000 \).

The demand in San Francisco is \( Q_{SF} = 35,000 - 5000p_{SF} \Rightarrow p_{SF} = 7 - \frac{Q_{SF}}{5000} \Rightarrow MR_{SF} = 7 - \frac{Q_{SF}}{2500} \). Setting MR=MC yields \( Q_{sf}^* = 12,500, \Rightarrow p_{sf} = 4.5 \). Profit in San Francisco is \( 4.5 \times 12,500 - 2 \times 12,500 = 31,250 \).

Total profit is 71,250, higher than in part a).

c) If the suit had not been brought by Toscanini’s, which county government would have had an incentive to bring the suit?

If Toscanini’s had not brought the suit, then the people of San Francisco would have had an incentive to bring it since they get a lower price with separate pricing.

Question 4

Your software company has just completed the first version of SpokenWord, a voice-activated word processor. As marketing manager, you have to decide on the pricing of the new software. You commissioned a study to determine the potential demand for SpokenWord. From this study, you know that there are essentially two market segments of equal size, professionals and students (one million each). Professionals would be willing to pay up to $400 and students up to $100 for the full version of the software. A substantially scaled-down version of the software would be worth $50 to students and worthless to professionals. It is equally costly to sell any version. In fact, other than the initial development costs, production costs are zero. Assume that you cannot use third-degree price discrimination. In other words, you cannot identify students separately from professionals other than by the prices they are willing to pay.

a) What are the optimal prices for each version of the software?

It is optimal to price the full version at $400 and the scaled-down version at $50. Total profits are $450 million since you sell 1 million of each. Selling the full version to everyone at a price of $100 would yield $200 million in profits. Selling the full version to professionals only would yield $400 million in profits.

Suppose that instead of the scaled-down version, the firm develops an intermediate version that is valued at $200 by professionals and $75 by students.

b) What are the optimal prices for each version of the software? Is the firm better off by selling the intermediate version instead of the scaled-down version?
One possibility would be to price the intermediate version at $75 and the full version at $400. However, this would lead professionals to choose the intermediate version since the difference between willingness to pay and price is greater for the intermediate version. In order to induce professionals to buy the full version, the full version’s price would need to be $75 + (400 - 200) = $275, where the value in parentheses is the difference in professionals’ willingness to pay for the full version and their willingness to pay for the intermediate version. This would lead to a total profit of $275 + $75 = $350 million, which is lower than initially. Still another possibility would be to price the full version at $400 and the intermediate version at $400 - (400 - 200) = $200. In this case, professionals would buy the full version but students would not buy the intermediate version. Profits would then be $400 million: better than $350 million but still less than the $450 million the firm would get with the truly scaled-down version.