1. Rose, who is currently earning an income of $47,000, is considering quitting her job and opening her own restaurant. To do so, she will have to cash in her life savings of $200,000, which have been in a certificate of deposit paying 6% per year. She will need this $200,000 to purchase equipment for her restaurant operations. She estimates that she will have to spend $4000 during the year to maintain the equipment so as to preserve its market value at $200,000. Fortunately, she owns a building suitable for the restaurant. She currently rents out this building on a month-by-month basis for $2500 per month. She anticipates that she will spend $50,000 for food, $40,000 for extra help, and $14,000 for utilities and supplies during the first year of operations. There are no other costs involved in this business. To keep things simple, suppose that Rose lives in Bushtopia, a country without any taxation whatsoever. What are the economic costs of operating the restaurant during the first year? In other words, what level of revenues will Rose need to achieve in the first year to make the first year profitable in an economic sense?

**Answer:** Rose has opportunity costs equal to the forgone interest on her $200,000, which is $12,000, plus her forgone income, $47,000, and plus her forgone rent, $30,000. Add to these numbers her expenses, $4000 for maintenance, $50,000 for food, $40,000 for help, and $14,000 for utilities. The total is $197,000. She needs to earn a revenue of at least $197,000 to be profitable.

2. Jack and Jill own a large confection plant that specializes in the production of 2 oz. vintage style pails that are made out of 72% cocoa dark chocolate and are filled with a soft center of mocha mousse (they produce no other products). The shop’s weekly overhead cost, that includes rent, local taxes and preventive maintenance, is $36,000. The marginal cost of producing a pail is constant at $1.20, up to the maximum capacity of 100,000 chocolate pails a week. The demand for pails comes from about 50,000 potential Bay area customers, each of whom has the following demand curve (per week):

\[ q^d(P) = 6 - 2P \quad (1) \]

where \( p \) is the per-unit price. Assume that Jack and Jill can only use simple pricing (aka uniform per-unit pricing).

1
(a) What is the profit-maximizing price for the bakery to charge? How many chocolate pails does it sell at that price (abstract from the fact that people buy an integer amount of chocolate pails)? What are its maximized profits?

(b) Following the weakness in the housing market, Jack and Jill negotiated a lower rent with their landlord which reduced overhead by $3,000 per week. How are the optimal price and profits affected by this change (compared to part (a) of this question)?

(c) Due to an accidental fire, 25% of the plants production capacity has been destroyed, and city regulations prevent Jack and Jill from reconstructing their plant. This has not affected the marginal cost of production or the plant’s overhead, but reduced capacity to 75,000 chocolate pails a week. How are the optimal price and profits affected by this change (compared to part (a) of this question)?

(d) An increase in the price of cocoa increases the marginal cost by $0.30. How are the optimal price and profits affected by this change (compared to part (a) of this question)?

**Answer:** a) First we need to determine marginal revenue and marginal cost. Notice that since there are 50,000 customers, and the individual customer demands \( q^d(P) = 6 - 2P \) chocolate pails at price \( P \), the total demand at price \( P \) must be 50,000\( q^d(P) \) = 50,000 \( 6 - 2P \) = 300,000 − 100,000\( P \). We need to invert the demand curve before we can calculate marginal revenue:

\[
300,000 - 100,000P = Q \iff P = \frac{300,000 - Q}{100,000} = 3 - \frac{Q}{100,000}.
\]

So the inverse demand curve for the whole market is \( P(Q) = 3 - \frac{Q}{100,000} \). Marginal revenue is then

\[
\frac{\partial}{\partial Q} [P(Q) \cdot Q] = \frac{\partial}{\partial Q} \left[ 3Q - \frac{Q^2}{100,000} \right] = 3 - \frac{Q}{50,000}.
\]

Marginal cost is constant at $1.20 (directly from the text).

The optimal quantity to produce is found by setting \( MC = MR(Q) \):

\[
1.20 = 3 - \frac{Q}{50,000} \iff Q^* = (3 - 1.2) \cdot 50,000 = 90,000.
\]

So the capacity constraint of 100,000 is not an issue here. Finally, the optimal simple price is

\[
P^* = P(Q^*) = 3 - \frac{1}{100,000} \cdot 90,000 = 2.1.
\]
Total costs, as a function of output, are $TC(Q) = 36,000 + 1.2 * Q$. The maximized weekly profits are

$$P^* * Q^* - (36,000 + 1.2 * Q^*) = 2.1 * 90,000 - (36,000 + 1.2 * 90,000) = 189,600 - 144,000 = 45,000.$$ 

b) Overhead has no implications on the optimal price and quantity produced, but weekly profits are increased by the amount of the decrease in overhead (to 48,000).

c) With a change in capacity, Jack and Jill are no longer able to produce the profit maximizing quantity of 90,000 pails. Because the marginal revenue function is always above the constant marginal cost up until 90,000 they need to produce as close to the optimal amount as possible. The closest they can get is 75,000. Since the optimal quantity is now 75,000, we need to return to the price function and solve for what price would generate a demand for 75,000 pails.

$$P^* = P(Q^*) = 3 - \frac{1}{100,000} * 75,000 = 2.25.$$ 

Since we know that producing 90,000 pails will maximize profit, we know that producing any other amount of pails, should result in less profit. This is the case even though the price went up, because the drop in demand overcompensates for the increase in price. We can verify this by solving for profit again with the new price.

$$P^* * Q^* - (36,000 + 1.2 * Q^*) = 2.25 * 75,000 - (36,000 + 1.2 * 75,000) = 168,750 - 126,000 = 42,750.$$ 

d) With a change in marginal cost, we have to re-optimize the price. As there is no change in demand, marginal revenue is unaffected. The optimal quantity is again solved from $MC = MR(Q)$:

$$1.5 = 3 - \frac{1}{50,000} Q \iff \frac{Q^*}{50,000} = 3 - 1.5 \iff Q^* = 75,000.$$

The new optimal price is

$$P(Q^*) = 3 - \frac{1}{100,000} * 75,000 = 2.25.$$
This weekly profits are reduced to

\[ P^* \cdot Q^* - (36,000 + 1.5 \cdot Q^*) = \\
2.25 \cdot 75,000 - (36,000 + 1.5 \cdot 75,000) = \\
168,750 - 148,500 = \\
20,250. \]

3. A bakery in a remote town faces essentially no competition. The weekly demand for its famous morning buns is given by \(800 - 200p\), where \(p\) is the price per bun. Currently, the bakery is charging $2.55 per bun. Assume that is the profit-maximizing price. Half of the bakery’s marginal cost comes from is the wholesale price it pays for flour. Due to recent events in the wheat market, the wholesale price of flour rises by 11%.

(a) What is the new profit-maximizing price for morning buns?
(b) By how much does the bakery’s profit per week rise or fall?
(c) What does your analysis suggest about the validity of supposition that retailers get rich when they raise their prices? (For example, bakeries when the price of flour rises)?

**Answer:** a) We start by uncovering the MC from the data in two steps. First, we will find the quantity produced, which we are told is the optimal (profit maximizing) quantity. Since the optimal price is $2.55 it must follow from the demand that

\[ Q = 800 - 200P \]
\[ = 800 - 200 \times 2.55 = 290 \]

Second, we will look for the MC that makes this choice of \(Q = 290\) optimal, that is, what is the marginal costs for which \(MR = MC\) at \(Q = 290\). We know that demand is given by

\[ Q = 800 - 200P \]
\[ \iff P = \frac{800 - Q}{200} = 4 - \frac{Q}{200}. \]

Since the demand is linear, we know the MR has the same intercept as demand, with twice the value of the slope, so:

\[ MR = 4 - \frac{Q}{100}. \]
Thus, at \( Q = 290 \) we have \( MR(290) = 4 - \frac{290}{100} = 1.10 \). Given that the firm is maximizing profit, it must be that \( MC = 1.10 \). Since it says that half the bakery’s marginal cost of selling a morning bun is the wholesale price it pays for flour, it must be that the marginal cost of the flower for a morning bun is $0.55 to make the price of $2.55 optimal.

Now that we have the old \( MC \) we know that the new \( MC = .55 + .55 \times 1.11 = .55 + .61 = 1.16 \), and the new optimal quantity equates \( MR = MC \):

\[
4 - \frac{Q}{100} = 1.16,
\]
yielding \( Q = 284 \), and the optimal price is

\[
P = \frac{800 - 284}{200} = 2.58
\]

b) The old profit (before any fixed costs) was:

\[
\pi = (P - MC) \times q = (2.55 - 1.1) \times 290 = $420.5
\]

and the new profit is

\[
\pi = (P - MC) \times q = (2.58 - 1.16) \times 284 = $403.28
\]

c) Clearly, profits dropped. Flour is just an input for bakeries, and if they are profit maximizers then a raise in the price of their inputs would cause a raise in their price, but the drop in quantity would more than overcome the price hike, implying lower profits.¹

4. Ann & Bob’s bakery produces very special cookies (and no other products). The bakery’s weekly overhead cost is $24,000. The marginal cost of producing one more cookie is constant at $0.80, up to the maximum capacity of 45,000 cookies a week. The demand for cookies comes from 10,000 potential customers, each of whom has the following demand curve (per week):

\[
q_d(p) = 4 - p/4
\]

where \( p \) is the per-unit price. Assume that Ann & Bob’s can only do simple pricing (aka a single per-unit price for all units).

(a) What is the profit-maximizing price the bakery should charge? How many cookies are sold at that price? What are its profits?

¹Another interesting argument is a “revealed preference argument” as follows: When \( MC = 1.10 \), the bakery could have charged \( P = $2.58 \), but it was more profitable to charge \( P = $2.55 \). Now, not only are they choosing \( P = $2.58 \), but their costs are higher too. Hence, it must be that profits are lower!
(b) An increase in rent increases Ann & Bob’s overhead by $2,000 per week. How are the optimal price and profits affected by this change (compared to part a.)?

(c) An increase in the price of cardamon increases the marginal cost by $0.10. How are the optimal price and profits affected by this change (compared to part a.)?

**Answer: a)** First we need to determine marginal revenue and marginal cost. Notice that since there are 10,000 customers, and the individual customer demands $q^d(P) = 4 - P/4$ cookies at price $P$, the total demand at price $P$ must be $10,000q^d(P) = 10,000 (4 - P/4) = 40,000 - 2,500P$. We need to invert the demand curve before we can calculate marginal revenue:

$$40,000 - 2,500P = Q \iff P = \frac{40,000 - Q}{2,500} = 16 - 0.0004Q.$$  
So the inverse demand curve for the whole market is $P(Q) = 16 - 0.0004Q$. Marginal revenue is then

$$\frac{\partial}{\partial Q} [P(Q) * Q] = \frac{\partial}{\partial Q} [16Q - 0.0004Q^2] = 16 - 0.0008Q.$$  
Marginal cost is constant at 0.8 (directly from the text).

The optimal quantity to produce is found by setting $MC = MR(Q)$:

$$0.8 = 16 - 0.0008Q \iff Q^* = (16 - 0.8)/0.0008 = 19,000.$$  
So the capacity constraint of 45,000 is not an issue here. Finally, the optimal simple price is

$$P^* = P(Q^*) = 16 - 0.0004 * 19,000 = 8.4.$$  
Total costs, as a function of output, are $TC(Q) = 24,000 + 0.8 * Q$. The maximized weekly profits are

$$P^* * Q^* - (24,000 + 0.8 * Q^*) =$$

$$8.4 * 19,000 - (24,000 + 0.8 * 19,000) =$$

$$159,600 - 39,200 =$$

$$120,400.$$  

**b)** Overhead has no implications on the optimal price and quantity produced, but weekly profits are reduced by the amount of the increase in overhead (to 118,400).
c) With a change in marginal cost, we have to re-optimize the price. As there is no change in demand, marginal revenue is unaffected. The optimal quantity is again solved from $MC = MR(Q)$:

$$0.9 = 16 - 0.0008Q \iff Q^* = (16 - 0.9)/0.0008 = 18,875.$$ 

The new optimal price is

$$P(Q^*) = 16 - 0.0004 \times 18,875 = 8.45.$$ 

This weekly profits are reduced to

$$P^* \times Q^* - (24,000 + 0.9 \times Q^*) =$$

$$8.45 \times 18,875 - (24,000 + 0.9 \times 18,875) =$$

$$159,494 - 40,987.5 = 118,506.$$

5. A gas station in a more rural part of the state faces essentially no competition. Its weekly demand is given by $82,500 - 30,000p$, where $p$ is the price per gallon that it charges. Currently, the gas station is charging $2.63 per gallon. Assume that is the profit-maximizing price. For all extents and purposes, the gas station’s marginal cost of selling a gallon of gasoline is the wholesale price it pays per gallon. Suppose, due to recent events, the wholesale price of gasoline rises by 5%.

(a) What is the new profit-maximizing price for the gas station to charge its customers?

(b) By how much does the gas station’s profit per week rise or fall?

(c) What does your analysis suggest about the validity of allegations that gas stations get rich when the price of oil rises?

**Answer:** a) We start by uncovering the MC from the data in two steps. First, we will find the quantity produced, which we are told is the optimal (profit maximizing) quantity. Since the optimal price is $2.63 it must follow from the demand that

$$Q = 82,500 - 30,000p = 82,500 - 30,000 \times 2.63 = 3,600$$

Second, we will look for the MC that makes this choice of $Q = 3,600$ optimal, that is, what is the marginal costs for which $MR = MC$ at $Q = 3,600$. We know that demand is given by

$$Q = 82,500 - 30,000p \iff P = \frac{82,500 - Q}{30,000} = 2.75 - \frac{Q}{30,000}. $$
Since the demand is linear, we know the MR has the same intercept as demand, with twice the value of the slope, so:

\[ MR = 2.75 - \frac{Q}{15,000}. \]

Thus, at \( Q = 3,600 \) we have \( MR(3,600) = 2.75 - \frac{3600}{15000} = 2.51 \). Since it says that “the gas station’s marginal cost of selling a gallon of gasoline is the wholesale price it pays per gallon,” it must be that \( MC = 2.51 \) to make the price of $2.63 optimal.

Now that we have the old \( MC \) we know that the new \( MC = 2.51 \times 1.05 = 2.6355 \), and the new optimal quantity equates \( MR = MC \):

\[ 2.75 - \frac{Q}{15,000} = 2.6355, \]

yielding \( Q = 1,717.5 \), and the optimal price is

\[ P = \frac{82,500 - 1,717.5}{30,000} = 2.6928 \]

**b)** The old profit (before any fixed costs) was:

\[ \pi = (P - MC) \times q = (2.63 - 2.51) \times 3,600 = $432 \]

and the new profit is

\[ \pi = (P - MC) \times q = (2.6928 - 2.6355) \times 1717.5 = $98.41 \]

**c)** Clearly, profits dropped. Oil is just an input for gas stations, and if they are profit maximizers then a raise in the price of their inputs would cause a raise in their price, but the drop in quantity would more than overcome the price hike, implying lower profits.\(^2\)

**6.** You are asked to help with the pricing at a golf course. The course has 10,000 customers, each with an individual monthly demand curve for games played \( q(P) = 10 - P \). The marginal cost for a game played is $2, and monthly overhead costs are $150,000. The course has enough capacity to serve all potential customers without overcrowding the course. Assume that the fixed cost for serving each customer (such as the cost of printing a membership card) is negligible, so that all variable costs are due to games played.

\(^2\)Another interesting argument is a “revealed preference argument” as follows: When \( MC = 2.51 \), the gas station could have charged \( P = $2.69 \), but it was more profitable to charge \( P = $2.63 \). Now, not only are they choosing \( P = $2.69 \), but their costs are higher too. Hence, it must be that profits are lower!
1. (a) The owner of the course is an old-fashioned businessperson and insists on a simple pricing strategy, i.e., a fixed price per game played. Your job is just to figure out the optimal price. What price would you set, and what would be the monthly profits?

(b) Emboldened by your success in part a), you decide to show the owner that the course would be better off by implementing a membership fee and a price per game. What would be the optimal prices, and what would be the monthly profits? Also, try to briefly explain to the owner the intuition of why this works better than simple pricing.

**Answer: a)** There are 10,000 customers so the aggregate demand curve is \( Q(P) = 10,000 \times (10 - P) \).

Solving for inverse demand (or the price as a function of demand) gives \( P(Q) = 10 - 0.0001Q \). Multiplying price times quantity gives you revenue of \( R(Q) = 10Q - 0.0001Q^2 \). Take the derivative of revenue, we get \( MR(Q) = 10 - 0.0002Q \). \( MC = 2 \), so

\[
\begin{align*}
MR(Q) & = MC \\
10 - 0.0002Q & = 2 \\
8 & = 0.0002Q \\
Q^* & = 40,000 \\
P^* & = P(Q^*) = 6
\end{align*}
\]

The optimal simple price is $6 per game, it induces the customers to play 40,000 games per month (that’s 4 per customer). Profits are \( P \times Q^* - C(Q^*) = 240,000 - (150,000 + 80,000) = 10,000 \).

**b)** Two-part tariff is perfect here, since we have just one customer type. Set price of games equal to \( P^* = MC = 2 \). Set golf club monthly membership fee equal to what would be a consumer’s surplus at a simple price of 2. At this price, individuals would play \( q^* = q(2) = 8 \) games per month. (The fee is paid by individuals, so we need the individual inverse demand curve here, \( p(Q) = 10 - Q \). CS would be \( [p(0) - P^*]q^* = (10 - 2) \frac{8}{2} = 32 \).

Optimal two-part pricing is a monthly membership fee $32, which gives the right to play as many games as one likes at $2/game. With unit price equal to constant MC, the revenue from games played just exactly cancels out the variable cost of producing them, so profit = revenue from membership fees – overhead costs. Since there are 10,000 members, profits are \( 10,000 \times 32 - 150,000 = 170,000 \). This is a big improvement over the simple pricing strategy!