Final Exam - Draft

Question 1: Warm Ups [20 points]

a. [10 points] Two firms have to set their output capacity in a market that they are about to enter. The resulting payoffs are described by the following matrix:

<table>
<thead>
<tr>
<th></th>
<th>Column Ltd.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row Inc.</td>
<td></td>
</tr>
<tr>
<td>Small</td>
<td>(6, 5)</td>
</tr>
<tr>
<td>Mid</td>
<td>(7, 3)</td>
</tr>
<tr>
<td>Large</td>
<td>(10, 2)</td>
</tr>
<tr>
<td>Small</td>
<td>(4, 6)</td>
</tr>
<tr>
<td>Mid</td>
<td>(5, 5)</td>
</tr>
<tr>
<td>Large</td>
<td>(4, 3)</td>
</tr>
<tr>
<td>Small</td>
<td>(3, 9)</td>
</tr>
<tr>
<td>Mid</td>
<td>(4, 4)</td>
</tr>
<tr>
<td>Large</td>
<td>(2, 1)</td>
</tr>
</tbody>
</table>

What is (or are) the Nash equilibrium of this game?

b. [10 points] Alice is a bidder in a sealed-bid auction for a classic baseball card. The value of the card to Ann is $5,000. She doesn’t care about how much others value this card because she intends to keep it for herself. The auction rules state that the highest bidder gets the card and pays the amount of the second highest bid. Explain to Ann, who does not know economics, why she should bid exactly $5,000.

(a) Choosing “Small” is a dominated strategy for both players, i.e., not the best response to anything the other player could do, so it can be removed from the game. In the remaining $2 \times 2$-game both players have a dominant strategy in “Mid,” i.e., it is the best response to all strategies by the other player. Therefore (Mid, Mid) is the unique Nash equilibrium.

(b) Call the highest bid by any other bidder besides Ann $X$. If Ann overbids to, say, $5100$ then compared to truthful bidding that i) makes no difference if $X < 5000$, Ann still gets the card and pays $X$, ii) makes no difference if $X > 5100$, Ann still doesn’t get the card, iii) hurts Ann if $5000 < X < 5100$, because she gets the card but has to pay more than she values it. If Ann underbids to, say, $4900$, then compared to truthful bidding it i) makes no difference if $X > 5000$, because Ann still doesn’t get the card, ii) if $X < 4900$ makes no difference, Ann still gets the card and pays $X$, iii) hurts Ann $4900 < X < 5000$ because now she doesn’t get the card but with a truthful bid she would have won and paid less than her value.

Question 2: Bundling Cable Services [20 points]
The sole cable service provider in a remote town can provide cable TV (service \( x \)) and internet access (service \( y \)). The company realizes that there are four types of consumers, each with an equal number of customers. The valuations of each type for the two services is given in the following table.

<table>
<thead>
<tr>
<th>Type ( t )</th>
<th>WTP for Cable-TV</th>
<th>WTP for Internet</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_1 )</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>( t_2 )</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>( t_3 )</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>( t_4 )</td>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

The fixed costs of installing the network have been made, and the marginal costs of providing the service are zero for every consumer.

a. [5 points] If the company is constrained by regulation to offer a price per service and cannot sell both services together as a bundle, what would be the profit-maximizing prices? What is the profit?

b. [5 points] If the company is constrained by regulation to offer a price for both services combined (a bundle price) and cannot sell each service independently, what would be the profit-maximizing bundle price? What is the profit?

c. [10 points] If the company is not constrained by regulation so that it can offer both price per service and bundle prices, what would be the profit-maximizing prices? What is the profit?

(a) \( p_x = p_y = 7 \), profits = \( 28t_1 \)
(b) \( p_B = 9 \), profits = \( 36t_1 \)
(c) \( p_x = p_y = 8 \), \( p_B = 11 \), profits = \( 38t_1 \)

Question 3: The New Finance Textbook [25 points]

A textbook publisher is considering signing up Hank Paulson and Ben Bernanke to write a new finance textbook titled “Just the Two of Us: Bailouts, Rescues, and Your Asset Portfolio.” The authors are asking to be paid 4 million dollars together, the promotion campaign would cost another million dollars and the marginal costs of production and shipping is $20 per book. There are two different markets: 100,000 undergraduates and 20,000 MBA students. Market research suggests that the willingness to pay for this book is $80 for the undergraduates and $100 for the MBAs.

a. [5 points] If the contract is signed, at what price would you recommend that the publisher sell the book? Is it worth signing the contract?
You can enhance the book by creating a CD version of the text. Both populations would benefit from having the CD, but the MBA students who all own cars would get more out of it as they drive on their weekend getaways. The willingness to pay for the book+CD package is $85 for the undergraduates and $130 for the MBAs. The costs of recording the CD are $100,000, and the marginal costs are $0.5 per CD.

b. [10 points] If the publisher would offer two options for purchase, a book-only and a book+CD version, what prices would you recommend that it charge for each option? Would you suggest that they do this compared to the book-only case in part (a) above?

If the publisher uses thicker and heavier paper then the book will be less desirable. The undergraduates will not suffer as much since they carry huge backpacks, but the MBAs would just leave the book at home, and their willingness to pay for the book alone drops a lot, while less so for the book+CD option. In particular, the new willingness to pay data for each group is:

<table>
<thead>
<tr>
<th></th>
<th>WTP UG</th>
<th>WTP MBA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Book only</td>
<td>79</td>
<td>80</td>
</tr>
<tr>
<td>Book+CD</td>
<td>84</td>
<td>125</td>
</tr>
</tbody>
</table>

Using the heavier paper will increase the marginal costs to $21 per book.

c. [10 points] If the publisher would offer two options for purchase as in part (b) but with the heavier paper, what prices would you recommend that it charge for each option? Would you suggest that they switch to the heavier paper?

Answer:

(a) Charge $80 and sell to all. Selling only to MBAs at $100 adds $20 on 20K people but loses $60 on 100K people. Profits are \((80 - 20) \times 120,000 - 4,000,000 - 1,000,000 = 2,200,000\).

(b) If you charge $80 for book only then you can’t charge more than $110 for book+CD. Total profits are then \((80 - 20) \times 100,000 + (110 - 20.5) \times 20,000 - 5,000,000 - 100,000 = 2,690,000\).

Now if you sell the book+CD version to all students your profit is: \((90 - .5 - 20) \times 120000 - 500000 - 100000 = 3.24 \times 10^6\), which is better than selling two different packages. So I would recommend only selling the CD version of the book.

Note if UG TWP is $81 for book+CD, then we get selling this to everyone is: \((85 - .5 - 20) \times 120000 - 500000 - 100000 = 2.64 \times 10^6 = 2.16 \times 10^6\), which is worse than any of the above options.
(c) Now you charge $79 per book only and $124 for book+CD and total profits are $(79 - 21) \times 100,000 + (124 - 21.5) \times 20,000 - 5,100,000 = 2,750,000$.

So this is better. However, again selling the book with the CD to all with regular paper would be better than heavier paper under any scenario $$(84 - .5 - 20) \times 120000 - 5000000 - 100000 = 2.4 \times 10^6.$$ 

**Question 4: R&D Investing in Competitive Markets [35 points]**

The market for a certain mineral has a yearly demand given by $Q = 2250 - 200p$, where $Q$ is the total quantity demanded in tons, and $p$ is the price per-ton. You are one of 100 identical producers of this mineral, where all act as price takers and all use the standard technology represented by the cost function $TC(q) = 8 + 2q^2$ where $q$ is the quantity your firm produces. The current market price is 10 and demand equals supply.

a. [5 points] Show that at this price of 10, the optimal choice of each firm results in total supply that equals demand.

b. [5 points] Explain why this is no the long-run equilibrium of the market, and proceed to compute the long run equilibrium in terms of price, quantity per firm, number of firms and total quantity demanded.

You are now in the long run equilibrium of this market. Your R&D department recommends a project that, if successful, will result in a technology with total costs given by $TC(q) = 4 + \frac{1}{2}q^2$. The results of the project will have intellectual property protection (IP) for 10 years, and it is safe to assume that other R&D projects are not on the horizon for you or your competitors. After 10 years any firm can use the technology. It takes a year for firms to enter or exit with the old (standard) technology. As a result, if you change your technology then for a full year the number of your competitors will be the same as calculated in part b.

c. [10 points] What will your profits be in the first year after the realization of the R&D project if the project succeeds? If it fails?

d. [10 points] What will be the new long run equilibrium a year after the project succeeds (price, your quantity and total quantity? What are your profits in each of the following 9 years (after the first year) before the IP expires? What are you profits after it expires?

e. [5 points] If the R&D project will cost $140, how likely will it have to be for it to succeed for you to make that investment? (Assume risk neutrality and a discount rate of 0 for simplicity)
Answer:

(a) \( MC = 4q \), at \( p = 10 \) each firm produces 2.5 units totalling 250, which is total demand.

(b) \( AC = \frac{8}{q} + 2q \) and min-\( AC \) is at \( q = 2 \), with \( MC = 8 \). At the LR equilibrium price of 8 the total demand is 650 and the LR number of firms is 325.

(c) If succeeds your new \( MC = q \) and the other 324 firms have supply \( q = \frac{p}{4} \), so total supply is \( Q_S = \frac{324p}{4} + p = 82p \), and equilibrium is \( p = 7.98 \). Your profits are \( 7.98^2 - 4 - \frac{1}{2}7.98^2 = 27.83 \). If it fails you have the old technology so your profits are zero.

(d) New LR price has to be 8 (subject to integers working) because of the fringe firms who each produce 2 at their min-\( AC \). The demand from part (b) at this price is 650. You supply 8 units and 642 units are produced by \( n = 642 \div 2 = 321 \) firms. Your profits are \( 8^2 - 4 - \frac{1}{2}8^2 = 28 \).

Why not price at 7.99 and take the whole market? The market would not clear because demand would be roughly 650 but you would only want to produce roughly 8, and thus this is not an equilibrium.

(e) The NPV with discount rate is just $280, so the probability needed is \( \frac{140}{280} = 0.5 \).

Question 5: The Truffle Market [30 points]

There are two Italian firms that are the sole producers of truffles worldwide: Alicia LTD (A) and Boberto Inc. (B). The annual world demand for truffles is \( Q = 1000 - p \) and not expected to change. Both pigs and dogs can be used to sniff out truffles. Pigs are less costly to maintain, but less efficient at finding truffles, while dogs are more costly to maintain and more efficient at finding truffles. Firm A uses pigs and its annual costs are given as \( TC(q_A) = 10,000 + 4q_A^2 \), while firm B uses dogs and its annual costs are given as \( TC(q_B) = 50,000 + q_B^2 \).

a. [5 points] If firm A were the only firm in the market, how much would it produce and what price would it charge? What are its profits?

b. [5 points] If firm B were the only firm in the market, how much would it produce and what price would it charge? What are its profits?

c. [10 points] If both firms were in the market, but they were owned by the same company and acted to maximize joint profits, how much would the company produce and what price would it charge? How does its profits compare to the profits in (a) and (b) above? Briefly explain your finding.

d. [10 points] Both firms are in the market, but they are separately owned and acting non-cooperatively. Each chooses its quantity to produce and the total quantity in the market determines the price along the market demand curve. That is, they are Cournot competitors. What is the one-shot Cournot equilibrium in this market? (Quantities, price and profits.)
Answer:

(a) \( MC = 8q, 8q = 1000 - 2q \Rightarrow q = 100, p = 900, \pi_A = 100 \times 900 - 10000 - 4 \times 100^2 = 40,000. \)

(b) \( MC = 2q, 2q = 1000 - 2q \Rightarrow q = 250, p = 750, \pi_B = 250 \times 750 - 50000 - 250^2 = 75,000. \)

(c) The key is to observe that if you are producing optimally with two firms then \( MC \) must be the same in both firms and it is equal to \( MR \) (the mirror image of 3rd degree discrimination). The two equations are \( 8q_A = 2q_B \) and \( 8q_A = 1000 - 2(q_A + q_B) \Rightarrow q_A = 55\frac{5}{2}, q_B = 222 \frac{1}{2}, p = 722 \frac{1}{2} \) so total profit is \( \pi = 277 \frac{1}{2} \times 722 \frac{1}{2} - 60000 - 4 \times (55\frac{5}{2})^3 - (222 \frac{1}{2})^2 = 78,889. \) This is more than either firm alone despite paying twice the fixed costs. The reason is that by splitting the production you use the more efficient part of each firm’s variable cost, which more than makes up for the added fixed cost.

(d) The best response of firm A is \( q = \frac{1000 - q_B}{4} \) and the best response of firm B is \( q_B = \frac{1000 - q_A}{4} \) and the solution yields \( q_A = 76.92, q_B = 230.77, p = 692.31. \) Profits are \( \pi_A = 76.92 \times 692.31 - 10000 - 4 \times 76.92^2 = 19,586, \) and \( \pi_B = 230.77 \times 692.31 - 50000 - 230.77^2 = 56,510. \)

Question 6: Local Restaurants [20 points]

Berkeley and Oakland are filled with interesting, and not-so-interesting, restaurants. They are all different, but they clearly compete with one another, so it would be reasonable to view them as monopolistically competitive. Assume that the industry was in long-run equilibrium before the changes discussed below. Berkeley is considering increasing its restaurant tax from 5\% of the price of the meal to 10\%. (The tax is paid by the restaurant, but it knows the tax when it sets its price.) Oakland would make no change to its tax rate.

a. [5 points] In the short-run, before any restaurants can enter or exit the market, what effect would this tax change have on the price of restaurant meals in Berkeley? What effect would this have on the price of restaurant meals in Oakland?

b. [5 points] In the long run, what effect would this have on the number of restaurants in Berkeley? What effect would this have on the number of restaurants in Oakland?

Berkeley decides not to increase the restaurant tax, but instead increases the restaurant license fee from $10,000 per year to $20,000 per year. This fee has to be paid regardless of how many meals the restaurant sells or how much revenue it receives.

c. [5 points] In the short-run, before any restaurants can enter or exit the market, what effect would this fee change have on the price of restaurant meals in Berkeley? What effect would this have on the price of restaurant meals in Oakland?
d. [5 points] In the long run, what effect would this have on the number of restaurants in Berkeley? What effect would this have on the number of restaurants in Oakland?