1. Assume the White House needs to be repainted and Acme is the only contractor capable of doing the job. Its costs would be \( C = \beta - e \), where \( \beta \) is the firm’s efficiency and \( e \) is its effort. The Department of the Interior (DOI), which is overseeing the project, can only observe \( C \). If it signs a contract with Acme, it will pay Acme’s cost and, in addition, give it a net payment \( t \), which may depend on \( C \). Acme will only accept \((t,C)\) contracts for which \( U = t - \psi(e) \geq 0 \), where \( \psi \) is convex and increasing and \( \psi(0) = 0 \), and \( e \) is cost-reducing effort equal to \((\beta - C)\). The value to society of finishing the project would be \( S \), and the shadow cost of public funds is \( \lambda > 0 \). The efficiency parameter \( \beta \) equals \( \tilde{\beta} \) with probability \( \upsilon \) and \( _{\beta} \) with probability \((1-\upsilon)\) where \( \beta < _{\beta} \). The DOI is interested in maximizing (expected) total surplus.

a. If spies tell the DOI Acme’s value of \( \beta \), what contract should it offer Acme conditional on that value?

We know rent is costly, so \( t = \psi(\beta - c) \). Therefore, DOI solves:

\[
\text{Max}_{t,e} \ S - (1+\lambda)(\psi(e) + \beta - e)
\]

\[
\text{FOCs: } (1+\lambda)(\psi'(e') - 1) = 0 \Rightarrow \psi'(e') = 1
\]

So, DOI offers to pay \( t^* = \psi(\beta - C^*) \) for observed cost \( C^* \)

b. If the DOI does not know \( \beta \) but does know that the project is not worth completing unless \( \beta = \tilde{\beta} \), what contract should the DOI offer Acme?

The first-best contract will work: \( t^*(\beta) = \psi(\beta - C^*) \) for observed cost \( C^*(\beta) = \beta - e^* \)

Note how the shutdown option eliminates the rents to the low type.

c. For the case in which the DOI does not know \( \beta \) and \( S \) is large so that it is worth completing the project even if \( \beta = _{\beta} \), discuss the problems that would arise if (i) Acme’s cost could not be observed, or (ii) the DOI and Acme expect that there would be future tunnel projects for which Acme would be the only qualified contractor and for which Acme’s costs would again be \((\beta - e)\), where \( e \) is project-specific effort and \( \beta \) is project-independent efficiency.

(i) If Acme’s costs \( (C) \) are not observable, then the DOI cannot offer \((t,C)\) contracts as we have specified above. For these schemes to work, there needs to be some signal of a firm’s efficiency. In the real world, for instance, accounting costs may be imperfect indicators of true costs, but as long as they are correlated with a firm’s true costs, we can separate inefficient from efficient firms.

(ii) If Acme could be involved in future projects, then it may want to pose as a high cost firm in period (1) in order to influence the DOI’s perception of the probability that it is a low cost firm. This is known as the “Ratchet Effect.”
2. Consider two products that can be produced by a firm according to the following cost function:

\[ C(q_1, q_2) = 89 + q_1 + q_2 \]

\[ C(0,0) = 0 \]

where \( q_1 \) is the quantity of product 1 supplied and \( q_2 \) is the quantity of product 2 supplied. The inverse market demand function for these products is given by:

\[ p_1 = 19 - q_1 \]

\[ p_2 = 9 - 2q_2 \]

a) Is the production of this product a "natural monopoly"? Why or why not?

Yes, the cost function is subadditive \( C(\sum q_i) = 89 + q_1 + q_2 < \sum C(q_i) = 178 + q_1 + q_2 \).

b) What are the Ramsey prices for these two products? (Hint: find the profit maximizing linear prices first.)

Following the hint, first find the profit-maximizing linear prices:

\[ \Pi = p_1(19-p_1) + p_2(4.5-.5p_2) - 89 - (19-p_1) - (4.5-.5p_2) \]

FOCs

\[ 19 - 2p_1 +1 = 0 \Rightarrow p_1 = 10 \]

\[ 4.5 - p_2 +.5 = 0 \Rightarrow p_2 = 5 \]

So, \( \Pi = 10*9 + 5*2 - 89 - 9 - 2 = 0 \).

Therefore, the only prices that allow the firm to break even are the monopoly prices, so these are also the Ramsey prices.

c) Do the Ramsey prices involve cross-subsidization according to the definitions we discussed in class? Will that generally be true?

The Ramsey Prices do not involve cross-subsidization. Recall that the definition of subsidy-free pricing discussed in class had two parts: (i) a breakeven constraint, and (ii) the revenue from any subset or products had to be less than or equal to the standalone costs of producing this subset \( S: P'Q^* \leq C(Q^*) \). We know the breakeven constraint is satisfied by the definition of Ramsey Prices. Check the second part for product 1: \( 10*9 \leq 89 + 9 \)

Since Ramsey Prices will always allocate part of the fixed costs to each product, this will generally be the case (see Spulber, Chapter 6).

3. Prove that declining ray average cost (DRAC) at any output vector \( Q \) implies that pricing that output vector at marginal cost would involve losses. What does this imply about natural monopoly pricing?

A multiproduct cost function \( C(Q), Q=\{q_1,\ldots,q_n\} \), has declining ray average cost (DRAC) at output vector \( Q \) if:
\[
\frac{d}{d\lambda} \left( \frac{C(\lambda Q)}{\lambda \sum q_i} \right) \bigg|_{\lambda=1} < 0 \iff \left[ \sum_{i=1}^{N} \frac{\partial C(\lambda q_i)}{\partial q_i} q_i - C(\lambda q) \sum q_i \right] \left( \lambda \sum q_i \right)^2 < 0 \bigg|_{\lambda=1} 
\]

\[
\iff \sum_{i=1}^{N} \frac{\partial C(\lambda q_i)}{\partial q_i} q_i - C(q) < 0
\]

So, if price is set equal to marginal cost

\[
\Pi = \sum_{i=1}^{N} \frac{\partial C(q_i)}{\partial q_i} q_i - C(q)
\]

which we know is less than zero with DRAC.

Note that DRAC is neither necessary nor sufficient for subadditivity. Some notion of economies of scope must also hold. If economies of scope are present, (linear) pricing at marginal cost will lead to losses.

4. Suppose a regulated industry (such as pre-divestiture long-distance telephone service in the US) is opened to new entrants. Does the fact that some firms find it profitable to enter show that the regulated firm is inefficient?

Not necessarily. The regulated firm’s pricing structure may involve some form of cross-subsidization, for instance, between products or between consumers. Therefore, even entrants with inefficient cost structures may find it profitable to enter one or more segments of the market.