1. Assume the White House needs to be repainted and Acme is the only contractor capable of doing the job. Its costs would be $C = \beta - e$, where $\beta$ is the firm’s efficiency and $e$ is its effort. The Department of the Interior (DOI), which is overseeing the project, can only observe $C$. If it signs a contract with Acme, it will pay Acme’s cost and, in addition, give it a net payment $t$, which may depend on $C$. Acme will only accept $(t, C)$ contracts for which $U = t - \psi(e) \geq 0$, where $\psi$ is convex and increasing and $\psi(0) = 0$, and $e$ is cost-reducing effort equal to $(\beta - C)$. The value to society of finishing the project would be $S$, and the shadow cost of public funds is $\lambda > 0$. The efficiency parameter $\beta$ equals $-\beta$ with probability $\upsilon$ and $\_\beta$ with probability $(1 - \upsilon)$ where $\beta < \_\beta$. The DOI is interested in maximizing (expected) total surplus.

   a. If spies tell the DOI Acme’s value of $\beta$, what contract should it offer Acme conditional on that value?

   b. If the DOI does not know $\beta$ but does know that the project is not worth completing unless $\beta = \_\beta$, what contract should the DOI offer Acme?

   c. For the case in which the DOI does not know $\beta$ and $S$ is large so that it is worth completing the project even if $\beta = \_\beta$, discuss the problems that would arise if (i) Acme’s cost could not be observed, or (ii) the DOI and Acme expect that there would be future tunnel projects for which Acme would be the only qualified contractor and for which Acme’s costs would again be $(\beta - e)$, where $e$ is project-specific effort and $\beta$ is project-independent efficiency.

2. Consider two products that can be produced by a firm according to the following cost function:

   \[ C(q_1, q_2) = 89 + q_1 + q_2 \]

   \[ C(0,0) = 0 \]

   where $q_1$ is the quantity of product 1 supplied and $q_2$ is the quantity of product 2 supplied. The inverse market demand function for these products are given by:

   \[ p_1 = 19 - q_1 \]
\[ p_2 = 9 - 2q_2 \]

a) Is the production of this product a “natural monopoly”? Why or why not?

b) What are the Ramsey prices for these two products? (Hint: find the profit maximizing linear prices first.)

c) Do the Ramsey prices involve cross-subsidization according to the definitions we discussed in class? Will that generally be true?

3. Prove that declining ray average cost (DRAC) at any output vector \( Q \) implies that pricing that output vector at marginal cost would involve losses. What does this imply about natural monopoly pricing?

4. Suppose a regulated industry (such as pre-divestiture long-distance telephone service in the US) is opened to new entrants. Does the fact that some firms find it profitable to enter show that the regulated firm is inefficient?