Notes on Cournot Competition

MBA 299 — Spring 2003

1 Introduction

These notes explore the Cournot model in greater depth than we covered it in lecture.

2 The Model

Although we can analyze the Cournot model under fairly arbitrary assumptions about demand conditions, cost structures, and possibly heterogeneity across firms, we will here limit ourselves to the following model:

- There are $N \geq 2$ identical firms who produce a homogeneous product.
- Each firm has a constant marginal cost of c.
- The *market* demand for the good in question is linear; specifically, assume $D(p) = A - Bp$, where p is price and A and B are fixed positive constants.

In what follows it is important to consider the *inverse* demand function; that is, price as a function of quantity. If $D(p) = A - Bp$ is the demand function, then the inverse demand function is

$$P(Q) = \frac{A}{B} - \frac{1}{B}Q = a - bQ\,,$$

where $a = A/B$ and $b = 1/B$ are constants. So that some market exists, assume $a > c$ (if not, then even at the maximum price, $a - b \times 0$, price would be below cost and it could not possibly be profitable to be in business).

Let $n = 1, \dots, N$ index the firms. The firms simultaneously decide how much output to produce (loosely, this can be thought of as decisions about capacity). Let q_n be the output of firm n. Total output, Q, is, thus,

$$Q = q_1 + \cdots + q_N = \sum_{n=1}^{N} q_n\,.$$

The market mechanism is somewhat artificial in the Cournot model. All the output is brought to market and sold at a price of $P(Q)$ per unit. Although artificial, this aspect of the model can be reconciled with a market in which Bertrand competition follows the firms simultaneously setting their capacities.[1]

[1] How it is reconciled is, however, beyond the scope of this course.

3 A Firm's Problem

In considering how much to produce, each firm n must form expectations about how much it believes its rivals will produce. Let Q_{-n} denote the *total* produced by firm n's rivals (*e.g.,* if $N = 3$ and $n = 2$, then $Q_{-2} = q_1 + q_3$).

Each firm wants to choose its production as a *best response* to what it projects its rivals will produce. A best response is the level of production that maximizes a firm's profit given its projection of its rivals' output. Profits, recall, are revenue minus cost. Given the assumption of constant marginal cost, c, cost is cq_n. Revenue is the price received, $P(Q)$, times quantity sold, q_n; hence, revenue is $P(Q)q_n$. The firm's profit given its projection of its rivals' output is, thus,

$$\begin{aligned}\pi_n &= P(Q)q_n - cq_n \\ &= (a - bQ)q_n - cq_n \\ &= (a - bQ_{-n} - bq_n)q_n - cq_n \,, \end{aligned} \tag{1}$$

where the third line follows because $Q = Q_{-n} + q_n$. A best response is the q_n that maximizes expression (1).

To maximize expression (1), we equate marginal revenue to marginal cost. Marginal cost we know, of course, is c. How to determine marginal revenue?

To begin, note, from the perspective of firm n, that $a - bQ_{-n}$ is a constant (*i.e.,* not something *its* action affects). So expression (1) has the familiar form of a linear inverse demand. That is, revenue is of the form

$$\big(\tilde{a} - bq\big)q \,.$$

Recall that marginal revenue consists of two components, the revenue generated by the last unit sold (*i.e.,* the price) and the driving-down-the-price effect. This second component reflects that to increase sales, a firm must lower price. How much must it lower price? Well the price it can charge if it wishes to sell $q - 1$ units is

$$\tilde{a} - b(q - 1) \,.$$

The price it can charge if it wishes to sell q units is

$$\tilde{a} - bq \,.$$

The difference is b; that is, it must lower price by b to sell an additional unit. Of course, the firm doesn't just lower the price on the last unit — it lowers them on all units — hence, the driving-down-the-price effect is $b \times q$; that is, a price reduction of b per unit on all q units. Hence, if inverse demand is $\tilde{a} - bq$, then the marginal revenue of the qth unit is

$$\underbrace{\tilde{a} - bq}_{\text{price of add'l unit}} \quad - \quad \underbrace{bq}_{\text{driving down price effect}} \quad = \tilde{a} - 2bq \,. \tag{2}$$

Using expression (2), we see that marginal revenue in expression (1) is

$$\underbrace{a - bQ_{-n}}_{\tilde{a}} - 2bq_n\,.$$

Equating marginal revenue to marginal cost:

$$a - bQ_{-n} - 2bq_n = c\,.$$

Solving for q_n:

$$q_n = \frac{a - bQ_{-n} - c}{2b}\,. \tag{3}$$

Note that as we vary firm n's projection of its rivals' output, Q_{-n}, we can use expression (3) to see how its best response varies. In particular, observe that if firm n expects its rivals to produce more (*i.e.,* Q_{-n} is larger), then firm n's best response is to produce less itself. Conversely, if it expects less output from its rivals, then its best response is to produce more itself.

4 Equilibrium

In equilibrium, firms must be playing best responses to their rivals' best responses. That is, an equilibrium is a situation of mutual best responses. To express this mathematically, let $\sum_{j\neq n}$ denote the sum of all N terms *except* the nth. Hence, we can write $Q_{-n} = \sum_{j\neq n} q_j$. The situation of mutual best responses can thus be expressed as the following set of N simultaneous equations in the N unknowns $q_1, \dots, q_N$:

$$\begin{aligned} q_1 &= \frac{a - b\sum_{j\neq 1} q_j - c}{2b} \\ q_2 &= \frac{a - b\sum_{j\neq 2} q_j - c}{2b} \\ &\vdots \\ q_N &= \frac{a - b\sum_{j\neq N} q_j - c}{2b} \end{aligned} \tag{4}$$

In general, solving a system of N equations like (4) is rather difficult. Fortunately, here we can exploit the fact that all N firms are the same. This symmetry suggests that, in equilibrium, $q_1 = q_2 = \cdots = q_N$. If that's the solution, then we can write

$$q_n = \frac{a - b(N-1)q_n - c}{2b}\,.$$

A little algebra reveals:

$$q_n = \frac{a - c}{(N+1)b}\,. \tag{5}$$

¿From expression (5), it is clear that each firm produces less in equilibrium the more firms there are in the industry.

5 Prices and Profits

The equilibrium price is given by $P(Q)$ where, given symmetry, Q equals N times the q_n found in expression (5). Hence equilibrium price, p^e, equals:

$$\begin{aligned} p^e &= a - bQ \\ &= a - bNq_n \\ &= a - bN\frac{a-c}{(N+1)b} \\ &= \frac{a+Nc}{N+1}\,. \end{aligned} \tag{6}$$

¿From (6), it is readily shown that p^e is decreasing in N;[2] that is, equilibrium price is falling in the number of firms in the industry. Indeed, as N gets arbitrarily large, the equilibrium price is trending toward marginal cost.[3]

A firm's profit is revenue minus cost. Revenue is equilibrium price times quantity and cost is c times quantity:

$$\begin{aligned} \pi_n &= p^e q_n - cq_n \\ &= \frac{a+Nc}{N+1} \times \frac{a-c}{(N+1)b} - c\frac{a-c}{(N+1)b} \\ &= \left(\frac{a+Nc}{N+1} - c\right)\frac{a-c}{(N+1)b} \\ &= \frac{(a-c)^2}{(N+1)^2 b}\,. \end{aligned} \tag{7}$$

¿From expression (7), it is clear that each firm's profit is decreasing in the number of firms in the industry. In the limit, as N gets arbitrarily large, profits are driven to zero.

6 Comparison with Monopoly

If this industry had just one firm, a monopoly, then Q_{-n} would be zero for it (there are no other firms). The same logic as before gets us to expression (3), which is here

$$q^M = \frac{a-c}{2b}\,,$$

where the superscript M denotes monopoly (recall $Q_{-n} = 0$ for monopoly). Note this matches expression (5) for $N = 1$ (*i.e.*, monopoly).

[2]This readily shown by calculus: Differentiating (6) with respect to N yields

$$\frac{dp^e}{dN} = \frac{c-a}{(N+1)^2} < 0\,,$$

where the inequality follows because $a > c$ (otherwise there's no market).

[3]Mathematically, we have $\lim_{N\to\infty} p^e = c$.

Since the firm is a monopoly, $Q = q^M$. Hence, equilibrium price is

$$p^M = a - bq^M = \frac{a+c}{2}\,.$$

Note this matches expression (6) for $N = 1$. Consequently, we can conclude that the monopoly price is greater than the price that would prevail given Cournot competition (*i.e.,* if $N \geq 2$).

The monopolist's profit is $p^M q^M - cq^M$ or

$$\pi^M = \frac{(a-c)^2}{4b}\,.$$

Again, this is consistent with (7) for $N = 1$. Hence, we see that a monopolist earns larger profits than does a Cournot competitor.

7 Conclusions

Cournot competition yields positive profits for each firm. These profits shrink, however, as the number of competitors increases. The equilibrium price in Cournot competition is likewise shrinking with the number of firms. Finally, compared to monopoly, the equilibrium price is lower and individual firm profits smaller.