Persuasion Contest: Disclosing Own and Rival Information*

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Abstract

This paper investigates a contest in information revelation between firms that seek to persuade consumers by revealing positive own information and negative information about the rival. In the face of limited bandwidth, firms are forced to make a trade-off between disclosing their own positive information and their rival's negative information. A negative-communication equilibrium, in which firms disclose rival's negative information whenever possible, exists when consumers have poor outside options or when firms are better informed. Bandwidth limitations make competitive firms more likely to disclose information compared to when they have no limitations. When firms strictly prefer consumers to choose the outside option over the rival (as in political contests), there is a greater prevalence of the negative communication equilibrium while the incidence of positive communication is lowered. Competition in information disclosure leads to greater consumer surplus compared to the unilateral disclosure case, whereas bandwidth limitations reduce consumer surplus. Finally, when firms are asymmetric in their ex-ante quality valuations, the higher quality firm is less likely to engage in negative communication.

1 Introduction

In a broad range of contexts from product markets, political competition, and social interactions, participants choose how to communicate information about themselves and about their rivals. In product markets, firms may have credible information about the quality of their product as well as that of competitors which they may release in order to persuade consumers. For example, a pharmaceutical firm may have information that the rival's drug has a particular side effect that its own drug does not suffer from. Disclosure of this information to the market is a strategic choice and has the potential to change consumers' relative valuations of the drugs. Politicians may try to persuade voters by using positive information about their own background or capabilities or by digging up adverse information about their rivals. Or in a hiring-committee meeting, a faculty member may have some positive signals about his preferred candidate and adverse signals about a non-preferred candidate. What strategic considerations determine the choice of disclosure of positive versus negative information by players? Would providing negative information about one's rival provoke skepticism among consumers (voters/receivers) and lead to adverse inferences that in fact hurt the sender? Consider the following examples:

- In 2018, Google disclosed a security flaw in Microsoft Edge before Microsoft could take actions to fix the bug.[1] On the contrary, Microsoft claimed it acted responsibly when it discovered security vulnerabilities on Chrome, by disclosing it to Google on September 14, 2017. However, earlier in 2013, Microsoft also ran the "Scroogled" campaign to evoke privacy concerns and to advertise Google's practice of scanning all the emails in Gmail accounts.[2] The advertising messages included a statement that Outlook does not scan customer emails.

- In the 2018 midterm elections, Democratic congressional candidates in swing districts successfully emphasized their policies to tackle economic disparities and the healthcare overhaul rather than focus on the special counsel investigation of the 2016 election. More recently, on the same day President Trump faced potentially damaging revelations from the House impeachment inquiry, the top Democratic presidential candidates focused on their policy proposals on a range of issues covering healthcare, housing, education costs, and child care. Focusing on the benefits of their policy proposals rather than on attack ads was seen as a more effective strategy. What motivates candidates in this case to not focus on highlighting

[1] www.theverge.com/2018/2/19/17027138/google-microsoft-edge-security-flaw-disclosure; accessed 2/6/2020.
[2] www.pcmag.com/article2/0,2817,2427459,00.asp; accessed 2/6/2020.

the negative information about the rival (see Stein, 2019)?

- In the mattress industry, Casper Sleep launched the direct-to-consumer (DTC) mattress business and positioned itself as the "One Perfect Mattress for Everyone," delivered in a box. Launching in April 2014, Casper hit a $100 million run rate by June 2015. Competitors responded by highlighting potentially adverse aspects of Casper's product offering. For example, advertising tag lines from Saatva, a leading competitor, included "Why buy a mattress in a box?" and "Luxury can't be stuffed in a box." Some of the "luxury" features claimed in the ad are technological attributes such as quality of springs, individually wrapped pocket coils, memory foam for support, and airflow. What aspects of the DTC mattress market can lead to such negative advertising strategies?

- In non-market settings, individuals may vie for influence within a group. In hiring-committee meetings, individual faculty may want to convince the hiring committee to hire their preferred candidate; and lawyers may try to convince the jury of whether the defendant is guilty. In these settings, individuals often try to make their positions stronger in comparison to their rival by disclosing some weakness of the rival's position and/or by disclosing some positive information that supports their position.

The above examples highlight the following general features: Competitive players choose the type of information they must reveal in order to persuade the market. In each case, firms/senders decide whether to send positive information about itself and/or negative information about the rival in order to increase the likelihood of being chosen. Consumers/receivers use the revealed information, but they may also be skeptical of information that remains undisclosed and make rational inferences. For instance, when a politician goes negative on a rival, voters may wonder whether the politician has anything positive to offer. This competitive revelation of information to influence the beliefs of skeptical and rational consumers can be interpreted as a contest in persuasion between rival firms.

What determines the incentives of competing firms to reveal positive information about itself or negative information about rivals? This choice is particularly germane when there are potential bandwidth constraints on communication either due to limited budgets or due to consumer processing and attention constraints. Both favorable information about a firm and adverse information about its rival can persuade them to choose the firm over its rival. So does it matter which type of information is chosen by the firm? Are these two types of information substitutes in changing consumer beliefs or do they have distinct effects?

Negative communication about one's rival may potentially lead to adverse consumer inferences about the firm's own unobserved quality resulting in lowered consumer valuations. A body of research in political advertising documents the reduction of voter valuations when campaigns use negative communications, and may also suppress voter turnout. This is labeled as the demobilization effect of negative advertising (Ansolobehere et al., 1994). We investigate the equilibrium basis for this phenomenon? In other words, how does the equilibrium balance of negative and positive communication in a product market affect consumer valuations and the total demand (or in political markets, the extent of voter turnout)?

Some specific aspects of the above examples are captured in the model. First, a firm may (but not always) have credible information about how its product compares with a competing product. Moreover, a firm having some information does not necessarily mean that the other firm also has the same information. Second, firms may have bandwidth constraints (or communication costs), due to which they may not be able to communicate all the available information that they have. It might be that the available information is unfavorable for the firm leading to non disclosure. And even if the information is favorable, the bandwidth constraints may bite and prevent full disclosure. Lastly, consumers are rational in the sense that their purchase decisions depend not only on the information that the firms disclose, but also on the information they believe firms cannot or will not disclose. Consumers understand firms may pretend to be uninformed when they actually have some private information. We characterize the important role that this rational consumer inference plays in determining firms' disclosure strategies.

The model consists of two competing firms that may have favorable or unfavorable valuation shocks. Nature independently reveals the valuation shocks to firms with some probability. Firms simultaneously decide the content of information they reveal to consumers or whether to stay silent. Such information revelation has the potential to convince consumers to choose one firm over its rival. We study firms' incentives to reveal information to consumers, who update their product valuations using the disclosed information and the inferences about the undisclosed information.

We examine the effect of two important types of restrictions on information transmission. The first is the case of unilateral information disclosure, in which one firm may be restricted from disclosing any information. This case may, for example, represent scenarios in which one firm (e.g., a foreign firm operating in a controlled economy) has no voice. Any information that is revealed is due to the other firm that faces no restriction on disclosure. Yet, as we show, even the presence of a passive rival has consequential effects on a firm's information-disclosure strategies. The second type of restriction is when firms face limited bandwidth for disclosure. Firms may face limited advertising budgets, limited consumer attention, or increasing disclosure costs that force them to limit the amount of information they disclose. In the model, the limited disclosure bandwidth is represented as allowing firms to disclose the valuation-shock information for at most one firm.

Comparing the incentives for information disclosure under unilateral information disclosure with incentives when both firms are strategic leads to an interesting result: An individual firm is more likely to disclose information (rather than stay silent) under unilateral disclosure than when it faces a strategic rival. The stronger incentive to disclose information when facing a passive rival results from the fact that the silence of a firm that could potentially disclose information is punished relatively more by the consumers through their equilibrium belief updating. Staying silent leads to a stronger inference that the firm is hiding unfavorable information about itself when the rival is unable to disclose information.

The main results pertain to situations in which firms face bandwidth constraints as this case highlights the trade-off between revealing own versus rival information. In the presence of limited bandwidth, firms are at least as or even more likely to disclose some information than in the case in which firms face no such limitation. Paradoxically, a limitation on the amount of information disclosure actually increases the individual firm's equilibrium likelihood of disclosure. The intuition runs as follows: Suppose no bandwidth constraints exist, and firms could disclose all the information that they have. Now any partial disclosure will induce the belief that the undisclosed information must be unfavorable. Firms therefore have the incentive to disclose only when all the information they have is favorable, and would pretend to be ignorant otherwise. However, with bandwidth limitations, partial information disclosure is possible and a firm may disclose information not only in the case in which all the information is favorable, but also when only partial information is favorable. Bandwidth constraints lead to less adverse consumer beliefs about a firm when it reports unfavorable news about its rival. In other words, bandwidth constraints make negative communication about the rival relatively more persuasive. As a result, firms become more likely to disclose information when the amount of information they can disclose is restricted.

Two distinct types of communication strategies can emerge in markets with limited bandwidth: Firms can engage in "positive communication" in which they disclose their own positive valuation shock whenever possible. Alternatively, they can engage in "negative communication" in which they disclose their rival's negative valuation shock whenever possible. Our analysis connects the equilibrium existence of positive versus negative communication to two general market features. Whether positive or negative communication emerges in the equilibrium turns upon the attractiveness of the consumers' outside option and the extent to which firms are informed about the valuation shocks.

Firms use negative communication in equilibrium only if the consumers' outside option is sufficiently small and when the firms are better informed. When a firm engages in negative communication, consumers not only become informed about the rival's negative information, but also form a belief that information about its own valuation is not necessarily positive. If consumers have a strong outside option, they switch to buying from their outside option instead of buying from firms that engage in negative communication. Therefore, in the presence of a significant outside option for the consumers, firms become less likely to engage in negative communication. Firms are more likely to use negative communication if consumers expect firms to be more likely informed about the valuation shocks. The consumers' belief about the valuation (if it remains undisclosed) of the firm that uses negative communication is less extreme if the likelihood that firms are informed is high. The reason is that if the firm actually has a negative valuation shock, the rival firm will likely disclose it, and if the rival is silent, chances are that the firm's valuation shock is positive.

Considering the amount of information reaching consumers and consumer welfare, we find that competition leads to more total equilibrium information transmission to consumers and therefore to higher consumer surplus. Bandwidth limitations, however, hurt consumer welfare irrespective of whether firms engage in negative or positive communication. Finally, a comparison of consumer surplus in the positive and negative communication equilibria suggests that consumers prefer negative communication when firms have sufficiently low levels of knowledge, while the converse is true for positive communication.

Next we consider the variation in how the outside options affect firm payoffs across different markets. In product markets, while a firm gets a positive payoff from a consumer who buys from it, the firm gets zero payoff otherwise; i.e., it is indifferent between whether a consumer chooses the outside option or the rival firm. However, in political contests or non-market contexts, this is no longer the case. For example, a politician would strictly prefer that a voter who does not vote for him exercise the outside option of not turning out to vote rather than voting for the rival. In section 4, we analyze a generalized model to show that this feature of political markets imply a greater prevalence of the negative communication equilibrium in political markets. In fact, when the firms' relative payoff from choosing the outside option over the rival becomes sufficiently large the positive communication equilibrium no longer exists. This is in contrast to the case when the firms' are indifferent between the outside option and the rival (or relative payoff is small), in which case the positive communication equilibrium always exists.

We analyze an extension with firms that have asymmetric quality/valuations to show that the firm with the higher quality has lower incentives to deploy negative communication. We also investigate firms' disclosure incentives if their information shocks are correlated. If firms are allowed to disclose all the information, they can possibly hide some negative information from consumers by staying silent. However, if firms face bandwidth limitation and can disclose only one of the two valuation shocks they are informed about, they are unable to hide any information from consumers. Information reaches consumers either as a result of firms' direct disclosures or due to consumers' inference. Similar to the case of independent information shocks, bandwidth restriction results in more communication. In addition, correlation in information shocks helps consumer inference, resulting in greater information transmission from firms to consumers. Finally, we also investigate the implications of better information about own valuations as well as firms' information acquisition incentives.

1.1 Related Literature

The disclosure literature originates in the seminal papers of Grossman (1981) and Milgrom (1981), who establish the classic unraveling result that all quality levels are separated and revealed in equilibrium because the highest types in any potential pooling set will have the incentive to reveal their type through disclosure. The subsequent literature focused on identifying mechanisms that can mitigate information unraveling, and identified the reasons for why firms are able to prevent full revelation. One strand in the literature (Dye, 1985; Jung and Kwon, 1988) has looked at the role of information endowment to show a firm may be able to credibly suppress bad information if the market is uncertain about whether the firm is endowed with information.[3] Lauga (2010) characterizes persuasive advertising as affecting consumers' recollections of product quality and shows that such advertising can mitigate full unraveling. A second strand beginning with Jovanovic (1982) and Verrecchia (1983) introduces the role of communication costs as a rationale for the non-prevalence of the unraveling result. This paper brings together the effects of both the uncertainty about infor-

[3] In media markets Anderson and McLaren (2012) examines the incentives of a news provider to disclose information that is pejorative to the owner's political preferences. Because the news provider may not have the information for sure, it is still possible for the provider to affect the public opinion. Consumers do not know whether silence is due to a lack of information or because it was strategically withheld.

mation endowment as well as communication costs to show that this is crucial for understanding the trade-offs involved in competitive information revelation. Specifically, uncertainty about information endowment prevents the full communication of the firms' private information to the market, whereas bandwidth constraints, which are analogous to communication costs, create a meaningful trade-off between revealing own versus rival's information.

A stream of research investigates information-disclosure incentives in oligopoly. The early work in this area (e.g., Vives, 1984; Gal-Or, 1985) focused on information sharing of private demand shocks between competing firms. The subsequent papers have investigated competing firms' incentives to disclose private quality information to consumers.[4] Anderson and Renault (2009) relax the assumption that firms can disclose only their own product information and allow firms to disclose comparative information. They show the use of comparative disclosure of horizontal match information by an inferior-quality firm helps consumers by transmitting more information and by reducing the price set by the better-quality firm. In this entire stream of research, consumers are not strategic in the sense that they do not make rational inferences. In our analysis of competitive information disclosure, consumers are strategic and make rational inferences about the competing firms based not only on the firms' equilibrium disclosure, but also on what is not disclosed. In this way, it brings together competitive information disclosure and strategic consumer inference. Doing so helps us examine competitive persuasion through strategic information disclosure.

Starting with Butters (1977) and Grossman and Shapiro (1984), a large literature investigates the role of advertising as information: Advertising informs consumers about the existence of the product characteristics and makes them consider the product. The higher the levels of advertising, the greater the probability that consumers consider the product. Our model of positive and negative disclosure can be seen as one of persuasive competitive communication, and it is distinct from the informative-advertising models in two important ways. First, in our setup, competing firms choose advertising content: whether to reveal their own (positive) and/or their rival's (negative) information.[5] Informative-advertising models are not about the content of advertising. Second, unlike in

[4] For example, Guo and Zhao (2009) show that privately informed firms in a competitive market reveal less information than a monopolist, while Gu and Xie (2013) show competition increases fit-revelation incentives for the high-quality firm but lowers it for the low-quality firm. In Board (2009), firms that are informed about the quality of both their own and their rival's products decide whether to disclose only their own quality information. Competition undermines full-disclosure (which would otherwise obtain under monopoly) leading to disclosure by only the higher-quality firm. Also related is Ofek and Turut (2013) who examine an incumbent firm's decision to preannounce its new product development when facing a competitive entry and show that an interplay between competition-related and demand-related considerations incentivize truthful communication. There is also a related stream of research on information disclosure and sharing incentives in distribution channels (e.g., Gal-Or et al., 2008; Guo, 2009; Guo and Iyer, 2010; and Sun and Tyagi, 2020).

[5] Anderson and Renault (2006) study a monopoly firm's choice of advertising content of whether to disclose

informative-advertising models where consumers are passive recipients of information, consumers in our analysis are strategic and make rational inferences about undisclosed information. These features distinguish our analysis from informative advertising and specifies one type of characterization of persuasive communication by competing firms. Our paper is therefore also related to the strand of research on advertising competition in which persuasive advertising directly enters the utility function and changes product preferences (i.e., the complementary view of Becker and Murphy, 1993). For instance in Baye and Morgan (2009) advertising creates loyal consumers for a firm, whereas in Chen et al. (2009) advertising shifts consumer preferences and product demand. In a recent paper, Bostanci et al. (2020) analyze the role of positive and negative (comparative) advertising as affecting consumer utilities for own and rival products and the resulting effects on competitive positioning.

Sun (2011) investigates disclosure incentives of a multi-attribute monopolist whose quality type is common knowledge, and finds that the high-quality firm may be less likely to disclose horizontal product attribute. Zhang (2014) shows that a mandate on the disclosure of product content can be harmful to consumers because it can make consumers excessively concerned about the product and limit consumption. Our analysis can also be seen as one in which firms disclose multiple dimensions (own and rival information), but in contrast to the above papers, we study disclosure under competition.

This paper also contributes to the literature on disclosure incentives when disclosure is costly. A common finding in this literature is that less information is transmitted to consumers if information disclosure is costly. As Grossman and Hart (1980) show, if disclosure is costly, only quality levels above a threshold are disclosed. Cheong and Kim (2004) show even a small disclosure cost can result in no information being transmitted to consumers if a large number of firms are competing in the market. In a team context, Dewatripont and Tirole (2005) show that cost of communication lowers transmission of information. In the context of product safety, Iyer and Singh (2018) show that a firm may seek costly certification as information disclosure to influence consumers' beliefs about its product-safety type. Dai and Singh (2020) show that an expert incurs a cost by not performing diagnostic tests to reveal own diagnostic ability. Our contribution to this literature arises from analyzing the effect of competition on information disclosure when communication is costly.

product and/or price information. However, because of the monopoly analysis, that paper contains no own versus rival information trade-off.

The implications of limited bandwidth have been analyzed in several marketing contexts. For example, Bhardwaj et al. (2008) assume a limited bandwidth between firms and consumers and show that competing firms' profits are higher in the case of buyer-initiated information revelation than seller-initiated information revelation. Mayzlin and Shin (2011) show that uninformative advertising and an invitation to search can act as a signal of high product quality when advertising bandwidth is limited. Our focus is on the trade-offs between disclosing own positive and rival's negative information in the face of limited bandwidth.

The rest of the paper is organized as follows. Section 2 introduces the model and compares the case in which one of two firms is restricted from communicating information with the case in which there is no such restriction. Section 3 presents disclosure under limited bandwidth and its implications on firms' incentives to disclose information. Section 4 examines an alternative generalized firm payoff. Section 5 presents extensions, and section 6 concludes.

2 The Model

Consider two ex-ante identical firms $i = 1, 2$ competing in a market consisting of a unit mass of homogeneous consumers. We refer to firm i's rival as firm j. The consumers' valuation of firm i consists of an observable (quality) component v that is common across the firms and a random shock δ_i. The expected valuation $E(v_i)$ is therefore $v + E(\delta_i)$. Nature draws δ_i, which can be either δ or $-\delta$, independently for both firms from the same distribution, and assigns it to firms at the start of the game. The probability that $\delta_i = \delta$ (or $\delta_i = -\delta$) is $1/2$. The possible outcomes for draws $\{\delta_1, \delta_2\}$ represent four possible states of the world: $\{\delta, \delta\}$, $\{\delta, -\delta\}$, $\{-\delta, \delta\}$, and $\{-\delta, -\delta\}$. The probability that firm i is independently informed about the state of the world is α, and it is ignorant about the state of the world and does not have any information about the valuations with the complementary probability.[6] Information, when available to firms, is hard and may be verifiably disclosed. We use $s_i = 1$ ($s_i = 0$) to denote that firm i is (is not) informed about the state of the world. Consumers are uninformed about the state of the world and whether or not a particular firm is informed, and firms are uninformed regarding whether or not the rival is informed about the state of the world.

Note we have assumed a firm, if informed, will have information about the valuations of both firms. This assumption is deliberate because it generates the essential trade-off for a firm between

[6] When a firm is informed about its rival's draw, the rival may also be informed about its own draw. The case of this type of correlation in information availability across the firms is examined in section 4.1.

revealing its own or its rival's information. In other words, under bandwidth constraints, it is when firms have information on both the valuation shocks, that they have to make the choice between either revealing their own information or that of the rival. Nevertheless, in section 4.2, we allow for the possibility that firms may have only private information about themselves and no information about their competitor, and show all the main results continue to hold.

Both firms simultaneously decide the information m_i they want to disclose to consumers. In this basic analysis, we assume firms have no bandwidth constraints and can choose to disclose all the available information. Because any disclosure is hard information, if a firm is uninformed about the state, it stays silent, that is, $m_i \mid_{s_i=0} = \phi$. If a firm is informed about the state, it can still choose to stay silent, or it can disclose the valuation-shock information of one or both firms, that is, $m_i \mid_{s_i=1} \subset \{\delta_i, \delta_j\}$. Consumers observe the valuation-shock information that is disclosed by the two firms and rationally update their beliefs about the information that remains undisclosed, if any, using Bayes' rule.

A consumer's utility from choosing firm i is $E(v_i) + \varepsilon_i$, where ε_i is the random part of consumer utility that is drawn from an independent and identically distributed (i.i.d.) extreme value distribution of zero mean and unit variance. Consumers choose between either of the two firms (either i or j) or else choose an outside option which gives them a fixed utility of v_0. In political markets, this could mean voters choosing not to turn out, and in product markets, it could mean consumers choosing an outside alternative or not buying at all. In non-market examples such as in the faculty hiring scenario, it could mean the expected utility from postponing hiring to the next cycle.

Consider the firm's expected payoff to be:

$$\pi_i = \frac{e^{E(v_i)}}{e^{E(v_i)} + e^{E(v_j)} + e^{v_0}}. \tag{1}$$

Thus firm i's payoff is linear in the share of consumers who choose the firm. Specifically, the π_i can be thought of as the expected proportion of consumers choosing i times a unit margin per consumer. Note equation (1) implies that the firm receives a positive payoff if a consumer chooses it, while zero payoffs if the consumer chooses either the rival or the outside market. The firm is indifferent between whether the consumer chooses the rival or the outside option. This best represents the type of product market interactions between firms that motivate this paper.[7] However, in other

[7]The focus of the paper is on the competitive communication strategies of the firms and so we assume that prices are fixed in the payoffs. This model can be seen as representing markets in which prices are non-strategic while firms make communication decisions (for example, pharmaceutical and health-care markets), or markets where prices are not relevant (for example, political markets). Allowing prices to be non-strategic helps us to focus on the incentives

contexts such as in political contests this assumption may not hold. For example, in an election, a politician may strictly prefer that voters (if they were not to choose him) choose the outside option and do not vote rather than vote for the rival. We examine this alternative firm payoff case in section 4.

We look for symmetric perfect Bayesian equilibrium (PBE) of the game in which both firms make the same disclosure decisions if they face the same information environment. The equilibrium consists of a specification of firms' disclosure decisions in every possible information state and the consumers' beliefs about any valuation shocks that remain undisclosed after firms have made their disclosures. If consumers were to observe an off-equilibrium message from any firm that cannot occur in the equilibrium, they would ascribe the worst possible belief for any undisclosed information about the firm.

Figure 1 summarizes the timing of the actions. First, nature draws the valuation shocks for both firms and independently reveals them to firms with probability α. Then, both firms, uninformed about the rival's information state, simultaneously make their disclosure decisions m_i. Next, consumers observe the disclosed information and update their beliefs $E(\delta_i|m_i, m_j)$ about the information that remains undisclosed. In the next stage, consumers choose one of the two firms or their outside option. Payoffs are realized in the last stage.

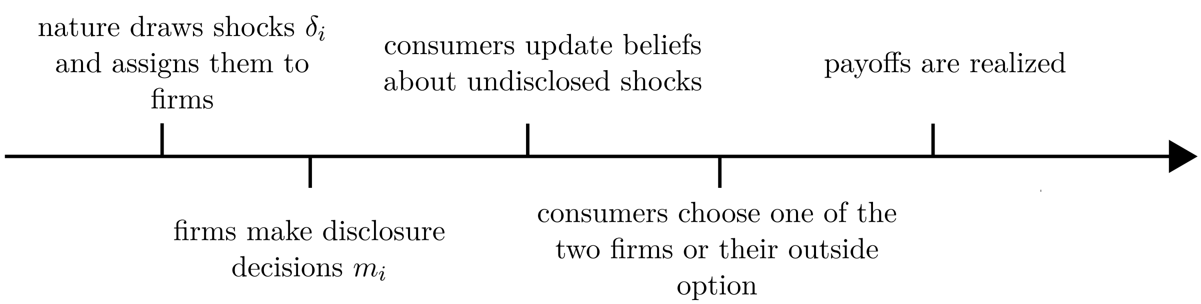


Figure 1: Timing

2.1 Unilateral Information Disclosure

We start the analysis with the case in which one of the firms (say, firm 2) is passive or non-strategic and cannot reveal any information. This can be interpreted as firm 2 being restricted from disclosing any information or that the cost of disclosing information is prohibitively high for the firm. For example, in the more controlled economies, foreign firms may face restrictions on communication

related to competitive information disclosure. In the Conclusion and Discussion section, we discuss how endogenous pricing choices and their timing may affect the firm's disclosure decisions.

that domestic firms are not subject to. Similarly, domestic firms may try to create entry barriers for overseas firms by having an advantage in disclosing information to domestic policymakers. Thus, $m_2 = \phi$, and only firm 1 can strategically disclose information if it is informed. Firm 1 may choose to disclose all or part of its information $m_1 \subset \{\delta_1, \delta_2\}$. Because firm 1 can disclose information only when it is informed and consumers do not observe the firm's information type, firm 1 can pretend to be uninformed and stay silent when its information is unfavorable. If firm 1 stays silent, consumers are unable to figure out if it is silent because it is uninformed or because its information is unfavorable. Therefore, silence can be used to hide unfavorable information from consumers.[8] Of course, consumers understand firm 1's incentives and take it into account when updating their beliefs about undisclosed information when firm 1 stays silent.

In this setting, firm 1 either stays silent or all the information is transmitted to consumers in the equilibrium. If firm 1 discloses only part of the information, consumers will rationally infer that the undisclosed information must be unfavorable (otherwise, firm 1 would have disclosed it). Therefore, in this setting without bandwidth constraints, partial disclosure and full disclosure are essentially equivalent. We assume the firm directly discloses all the information instead of using partial disclosure.

The equilibrium specifies firm 1's strategies in all the possible information states and consumers' beliefs about the realized valuation shocks for both firms if firm 1 stays silent. We provide the full statement of the equilibrium as well as a formal proof in the Appendix. Lemma 1 below describes conditions under which, in the equilibrium, firm 1 discloses both firms' valuation draws to the consumers.

Lemma 1 *In the case of unilateral information disclosure, $m_1 = \{\delta_1, \delta_2\}$ is an equilibrium disclosure strategy if firm* 1 *is informed and (1) $\delta_1 = \delta$, or (2) $\delta_1 = -\delta$, $\delta_2 = -\delta$, and $v_0 < v + ln\left(\frac{e^{\frac{2\alpha\delta}{4-3\alpha}}-1}{e^{\delta}-e^{\frac{\alpha\delta}{4-3\alpha}}}\right)$. The existence condition implies that firm* 1 *discloses both negative draws only if v_0 is sufficiently small and is more likely to disclose them if α is higher.*

As expected, if firm 1 is informed and $\delta_1 = \delta$, it discloses both δ_1 and δ_2. And firm 1 stays silent if $\delta_1 = -\delta$ and $\delta_2 = \delta$. Of greater interest, however, is that if δ_1 and δ_2 are both $-\delta$, firm 1 may still disclose both δ_1 and δ_2. The intuition is the following. If firm 1 stays silent, consumers'

[8]This is analogous to Dye (1985) and Jung and Kwon (1988) where a manager can hide information from the market if the market is not sure whether the manager is endowed with information.

inferences of the expected valuation shocks are $E(\delta_1|m_1 = \phi) < 0$ but $E(\delta_2|m_1 = \phi) > 0$. This inference is driven by the facts that i.) firm 2 cannot disclose any information and ii.) firm 1 stays silent if $\delta_1 = -\delta$ and $\delta_2 = \delta$. Surprisingly, firm 1's silence puts it at a disadvantage and can help firm 2 precisely when the latter cannot disclose information. If firm 1 discloses both $\delta_1 = -\delta$ and $\delta_2 = -\delta$, consumers would know both firms are equivalent but would find both of them relatively less attractive than the outside option. Therefore, if the outside option is not too attractive, firm 1 finds it optimal to disclose the negative valuation information of both firms. But if consumers' outside option is sufficiently attractive, the disclosure of negative draws of both firms makes consumers likely to switch to their outside option. The resulting reduction in firm 1's payoff on the extensive margin outweighs the adverse inference effect from staying silent. In this case, firm 1 stays silent. Firm 1 is more likely to disclose both negative draws if α is higher because in this case firm 1's silence induces the belief that firm 1's own information is more negative whereas rival firm's information is more positive.

2.2 Competitive Information Disclosure

In this section, we consider the competitive information disclosure case in which both firms are strategic. Firms therefore play in a persuasion contest. Since there is no bandwidth restriction on the disclosure of any of the two firms, firms can withhold information only by pretending to be ignorant and not through partial disclosure. Therefore, in any of the possible information states, the equilibrium firm strategy involves either disclosing all the information or staying silent. The full characterization of equilibrium is presented in the Appendix. Lemma 2 below describes conditions under which disclosing information is an equilibrium strategy for firms.

Lemma 2 *In the case of competitive information disclosure, $m_i = \{\delta_i, \delta_j\}$ is an equilibrium disclosure strategy if firm i is informed and $\delta_i = \delta$.*

In equilibrium, firms disclose both their own and their rival's valuation draws if their own draw is positive. If the rival's draw is negative, disclosing both own positive and the rival's negative draws results in the highest possible payoff for firm i. If the rival's draw is also positive, firm i's disclosing only its own positive draw and disclosing both draws are equivalent in the sense that in both cases all the information is transmitted to the consumers. Similar to section 2.1, we assume the firms disclose both valuation draws in such situations. Firms choose to stay silent and withhold information if their own draw is negative. Now suppose consumers observe that both firms are silent. Because consumers know firm i (and j) is silent either because its own draw is negative or because it is ignorant, consumers' inferred expected valuations would be equal, negative, and larger than $-\delta$ for both firms.

A comparison of firm i's disclosure strategies in the cases in which only firm i can unilaterally disclose information (presented in section 2.1) and in which both firms can disclose information leads us to the following proposition (all proofs are in the Appendix.)

Proposition 1 *Firm i is (weakly) less likely to disclose information in a competitive information-disclosure setting than in a unilateral information disclosure setting.*

This proposition identifies an interesting result from the analysis: Counter to what one might expect, competition reduces a firm's incentive to disclose information. If the rival firm is non-strategic and cannot disclose any information, firm i's silence induces a favorable belief about the non-strategic firm j's valuation $E\left(\delta_j|m_i=\phi\right) > 0$ but an unfavorable belief about firm i's valuation $E\left(\delta_i|m_i=\phi\right) < 0$. This punishment for silence in the form of a more negative consumer inference for the firm i (that has the ability to communicate) induces more communication from this firm in the unilateral disclosure case. As a result, in the unilateral information-disclosure setting, firm i may disclose information even in the case in which the valuation draws are negative for both firms.[9] However, in the competitive information-disclosure setting in which both firms are strategic, the beliefs induced by the silence of both firms are symmetric $E\left(\delta_i|m_i=\phi, m_j=\phi\right) = E\left(\delta_j|m_i=\phi, m_j=\phi\right)$ and larger than $-\delta$. Firms choose to stay silent when they are informed that both firms have negative valuation draws, because of the threat that the consumer might switch to the outside option if both negative draws are disclosed.[10]

3 Competitive Disclosure with Limited Bandwidth

We now proceed to analyze the effect of limited bandwidth. As already discussed, the inability to credibly disclose all their information may arise due to convex disclosure costs, limited airtime or the inability of firms to communicate multiple messages contemporaneously, or consumers' limited

[9]In this case, firm i stays silent if the outside option v_0 is sufficiently large. Therefore, in a region of parameter space, firm i may also be equally likely to disclose information in the two settings.

[10]In a related vein, Zhu and Dukes (2015) show that competition among media firms may result in consumers receiving fewer facts.

attention. In the current setup, the firms' limited disclosure bandwidth can be captured as the firms being constrained to disclose only one of the two available pieces of information. Although an informed firm learns both its own and its rival's valuation draws, it can disclose at most one of those draws, $|m_i| \leq 1$. This competitive interaction in which each firm (in its attempt to persuade the market) can disclose at most one valuation draw is a persuasion contest with limited bandwidth. Similar to the previous section, an informed firm can always choose to stay silent and pretend to be uninformed. In addition to making a decision about whether to disclose information, firms also decide whether to disclose information about their own valuation or that of their rival. Because only hard information can be disclosed, an uninformed firm stays silent.

A firm discloses its own valuation draw only if it is positive, and it discloses its rival's draw only if it is negative. Firms prefer to stay silent (and pretend to be uninformed) instead of disclosing their own information if it is negative or disclosing their rival's information if it is positive.[11] If a firm's own valuation information is positive but that of its rival is negative, the firm makes a trade-off between disclosing its own and its rival's information. Similarly, when the information about both its own and its rival's valuation draw is negative, the firm makes a trade-off between disclosing the rival's negative information and staying silent. However, if both draws are positive, the decision is straightforward: The firm discloses its own positive valuation draw. Given firm strategies, consumers observe firm i making one of the three disclosure decisions: i.) disclose its own positive valuation draw ($m_i = \delta_i = \delta$), ii.) disclose its rival's negative valuation draw ($m_i = \delta_j = -\delta$), or iii.) stay silent ($m_i = \phi$).

Once both firms have made their disclosure decisions, consumers observe the disclosed information and infer the information that remains undisclosed. For example, if both firms disclose their own valuation draws ($m_i = \delta_i$ and $m_j = \delta_j$) or if both disclose their rival's valuation draws ($m_i = \delta_j$ and $m_j = \delta_i$), consumers would be fully informed ex post. However, if one or both firms stayed silent ($m_i = \phi$ and/or $m_j = \phi$), either because they were uninformed or made a strategic decision to stay silent, consumers would make inferences about the undisclosed information. All the possible situations in which consumers need to make inferences about some undisclosed valuation draw can be captured in three cases: i.) Both firms stay silent ($m_i = m_j = \phi$); ii.) one firm discloses its own positive valuation draw but the other stays silent (e.g., $m_i = \delta_i = \delta$ and

[11] Suppose firm i is informed and the rival's draw δ_j is δ. The realized draws must be either $\{\delta, \delta\}$ or $\{-\delta, \delta\}$. Regardless of whether $m_j = \delta_i$, δ_j or ϕ, firm i is better off disclosing own draw or staying silent than disclosing the rival's positive draw. Therefore, firm i never discloses $m_i = \delta_j = \delta$ along the equilibrium path and if it does then consumers infer that firm has deviated from the equilibrium. Similarly, one can argue that firm i never discloses $m_i = \delta_i = -\delta$ along the equilibrium path.

$m_j = \phi$); and iii.) one firm discloses its rival's negative valuation draw, but the other stays silent (e.g., $m_i = \delta_j = -\delta$ and $m_j = \phi$). Consumers make rational inferences about the undisclosed information given firms' equilibrium disclosure strategies. We discuss the consumers' inference of undisclosed valuation information for specified equilibrium disclosure strategies in sections 3.1 and 3.2.

We refer to a firm's disclosure strategy in which the firm discloses its own positive valuation draw whenever possible as *positive communication*. Similarly, a *negative communication* is a disclosure strategy in which the firm discloses its rival's negative valuation draw whenever possible. Similar to section 2 we look for a symmetric equilibrium in which both firms use the same disclosure strategies if they are in the identical information state. For example, in the equilibrium, if firm i when informed stays silent in the state $\{\delta_i = -\delta, \delta_j = \delta\}$, firm j when informed would also stay silent in the corresponding $\{\delta_i = \delta, \delta_j = -\delta\}$ state. In the next section, we examine firms' incentives to engage in negative communication.

3.1 Negative Communication

Limited bandwidth raises the possibility that firms may go negative and disclose negative information about their rivals whenever possible and at the expense of disclosing positive information about themselves. In this section, we examine the interesting features of the consumers' belief updating of the undisclosed valuation draws and how it affects the existence of such an equilibrium. Because consumer inferences of undisclosed information depend on firms' equilibrium disclosure strategies, we start by specifying the firm's disclosure strategies in a candidate equilibrium as the following. Suppose firm i is informed ($s_i = 1$). The realized state may be such that the information about both firm i and the rival are positive, $\{\delta_i, \delta_j\} = \{\delta, \delta\}$. In this case, firm i discloses its own valuation information ($m_i = \delta_i$). If the realized state is such that firm i's own information is positive but that of the rival is negative, that is, the state is $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$, the firm may be indifferent between $m_i = \delta_i$ and $m_i = \delta_j$. Suppose in this case firm i discloses its own positive information, $m_i = \delta_i$, with probability $\rho \in [0, 1]$.[12] If the realized state is $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$, both own and rival's valuation information is unfavorable to firm i. In this case, firm i stays silent ($m_i = \phi$) and pretends to be uninformed. However, if the realized state is $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, firm i discloses its rival's

[12] We consider this more general specification and show that a pure negative-communication equilibrium (i.e., $\rho = 0$) and a mixed negative-communication equilibrium (i.e., $\rho \in (0, \frac{1}{2})$) exist. In addition, we show $\rho = 1$ does not constitute an equilibrium, because if $\rho = 1$ and the realized state is $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, firm i would be better off deviating (and staying silent) than engaging in negative communication (and disclosing $m_i = \delta_j$). For details, see the proof of proposition 2 in the Appendix.

negative information ($m_i = \delta_j$). Firm j's disclosure strategies are symmetric. An uninformed firm obviously does not disclose any information.

Having specified the firms' disclosure strategies in a candidate equilibrium, we now describe consumers' ex-post beliefs (about any undisclosed draws) induced by the specified strategies. At the time of belief updating, along the equilibrium path, consumers will correctly infer both firms' strategies in all information states. If some valuation draw remains undisclosed, consumers will be uninformed about the realized information state. Consumers will update their beliefs about the realized information state given firms' disclosure strategies and take them into account (along with firms' equilibrium strategies in all states regardless of whether they are realized) when updating their beliefs about any undisclosed valuation draws. The following lemma presents consumers' ex-post beliefs about the undisclosed draws in all possible contingencies.

Lemma 3 *Given the firms' candidate strategies in the negative-communication equilibrium specified above, consumers' ex-post beliefs about the undisclosed valuation draws are the following:*

(a) if $m_i = \delta_j$ *and* $m_j = \phi$, $E(\delta_i|m_i, m_j) = \frac{\alpha-\rho}{2-\alpha-\rho}\delta$,

(b) if $m_i = \delta_i$ *and* $m_j = \phi$, $E(\delta_j|m_i, m_j) = \frac{1-\alpha-\rho}{1-\alpha+\rho}\delta$, *and*

(c) if $m_i = \phi$ *and* $m_j = \phi$, $E(\delta_i|m_i, m_j) = E(\delta_j|m_i, m_j) = 0$.

Negative communication has a positive effect on the firm's own valuation. Precisely because consumers know firms in equilibrium are focusing on negative communication, they also understand that positive information about firms may remain undisclosed. Therefore, their belief about the undisclosed draws becomes more favorable with an increase in negative communication. It is noteworthy that in a negative communication equilibrium (when $\rho = 0$), if $m_i = \delta_j = -\delta$, and the other firm is silent, consumers' belief about firm i's information $E(\delta_i) < \delta$. In this case, consumers are not fully convinced that firm i's own information is positive. They account for the possibility that firm i's information may be negative and firm j may be uninformed. However, if $m_i = \delta_i = \delta$, consumers' belief about rival's undisclosed information $E(\delta_j) = \delta$. Consumers understand that if firm i's information about firm j's valuation was negative, firm i would have disclosed it. We will see that this rational inference plays an important role in how the existance of negative communication equilibrium depends of consumers' outside option v_0.

The effectiveness of negative communication depends on the relative extent of damage (or, $E(\delta_i) - \delta_j$) that the negative information can cause to the rival. As expected, the effectiveness of negative communication is increasing in the magnitude of the negative draw. Another aspect of consumers' inferences relates to their expectation of a firm's disclosure. Consumers understand a firm's silence does not necessarily mean the firm's private information is unfavorable. The firm may not have any private information. However, if consumers expect firms are very likely to be informed (i.e., α is high), they impose a smaller penalty on the silent firm (i.e., they lower their beliefs about the undisclosed draw by a smaller amount). The reason is that if α is high, negative information about a firm will most likely be disclosed by the rival. And, if the information remains undisclosed, it is more likely that the information is actually positive.

The above observations about the consumers' beliefs about undisclosed valuation help in our understanding of the negative-communication equilibrium. A property of the equilibrium characterization is that (at the time of updating beliefs about undisclosed valuation draws) consumers take firms' disclosure strategies in all information states into account regardless of which information state is actually realized. Therefore, consumers' beliefs about firms' equilibrium strategies must be consistent in all information states. In other words, firms must have no profitable deviations from their proposed strategy in any information state regardless of which state is realized. Suppose firm i is informed ($s_i = 1$) and the realized state is $\{\delta_i, \delta_j\} = \{\delta, \delta\}$. In this case, firm i's equilibrium payoff from disclosing own positive valuation draw ($m_i = \delta_i$) is given by

$$\pi_i = \alpha \frac{e^{v+\delta}}{e^{v+\delta} + e^{v+\delta} + e^{v_0}} + (1-\alpha) \frac{e^{v+\delta}}{e^{v+\delta} + e^{v+\frac{1-\alpha-\rho}{1-\alpha+\rho}\delta} + e^{v_0}}, \tag{2}$$

where the first term represents the possibility that firm j may be informed with probability α and would disclose its own positive valuation draw, and the second term represents the possibility that firm j may be uninformed with probability $1-\alpha$ and stay silent. In this information state, firm i must not deviate from disclosing its own positive valuation draw. Similarly, firm i's payoff can be written in all the possible information states and the no-deviation conditions can be written for all possible deviations in all information states. The following proposition specifies firms' symmetric equilibrium strategies and presents the existence condition for the negative-communication equilibrium. Consumers' equilibrium beliefs about any undisclosed valuation draws can be obtained by substituting $\rho = 0$ in Lemma 3.

Proposition 2 *If $v - v_0 > ln\left(\frac{e^{\frac{-\alpha\delta}{2-\alpha}} - e^{-\delta}}{1-e^{\frac{-2\delta}{2-\alpha}}}\right)$, a negative-communication equilibrium exists in which, if firm i is informed and*

(1) $\delta_i = \delta$ and $\delta_j = \delta$, it discloses its own positive valuation draw,

(2) $\delta_i = \delta$ and $\delta_j = -\delta$, it discloses its rival's negative valuation draw information,[13]

(3) $\delta_i = -\delta$ and $\delta_j = \delta$, it remains silent, and

(4) $\delta_i = -\delta$ and $\delta_j = -\delta$, it discloses its rival's negative valuation draw.

The existence condition implies that firms (i) engage in negative communication only if the consumers' outside option is sufficiently small, and (ii) are more likely to engage in negative communication if α is higher.

In the negative-communication equilibrium described in the proposition, firms disclose their rival's negative information whenever possible. In other words, firm i discloses $m_i = \delta_j$, if $s_i = 1$ and $\delta_j = -\delta$. The first important insight from the proposition is that the negative-communication equilibrium exists when the consumers' outside option v_0 is sufficiently small. Figure 2 represents the existence conditions. The negative-communication equilibrium generates an endogenous form of negative externality at the extensive margin between the two competing firms and the outside option. Firms engage in negative communication in order to reduce the valuation of the rival. If both firms disclose negative information, consumers become less willing to buy from either of the two competing firms. This favors the outside option and if it is attractive enough, consumers switch to the outside option, abandoning the two competing firms altogether. Even when only one of the two firms is informed and reveals its rival's negative information, consumers cast doubt on whether the valuation information of the informed firm is definitely positive.

Specifically, as discussed after Lemma 3, in a negative communication equilibrium when firm i sends a negative message and firm j is silent we have that $E[\delta_i|m_i, m_j] < \delta$. This along with the disclosed negative information that consumers have along the equilibrium path increases the likelihood that consumers will choose the outside option. This is consistent with the characterization of the demobilization effect on voter turnout that is identified in the empirical literature in political science such as Ansolabehere et al. (1994) and Finkel and Geer (1998). While the empirical papers identify the demobilization effect, this paper can be seen as a providing a competitive communication based rationale for the effect.

The above result supports the often invoked idea that when two rivals go negative on each other, it may benefit a third party who is not part of contest. Anecdotal evidence in product markets sug-

[13] In this case, a mixed-strategy negative-communication equilibrium may also exist in which the firm discloses its own positive valuation draw with probability $\rho \in \left(0, \frac{1}{2}\right)$. Thus, in any mixed-strategy equilibrium, the probability of negative communication is higher. The formal analysis is provided in the Appendix.

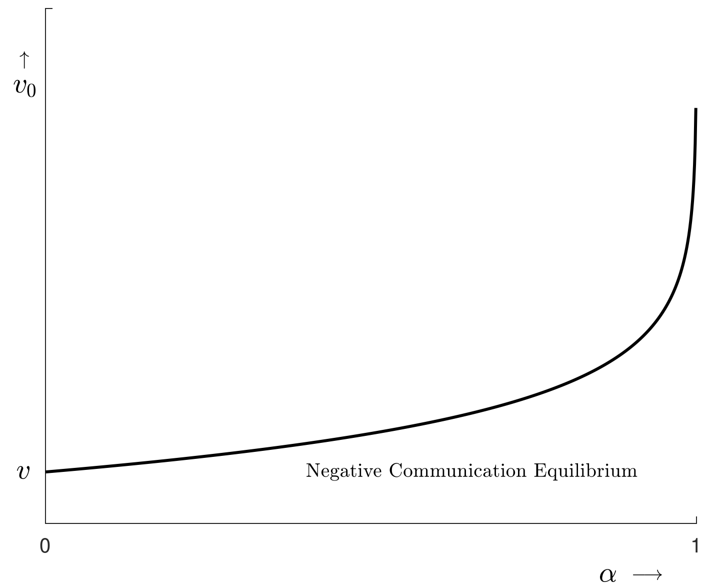


Figure 2: Existence of the negative-communication equilibrium

gests that firms in markets with fewer dominant players engage in negative advertising. Examples of competing firms that regularly engage in negative advertising are often markets with two dominant players. It seems harder to find instances of negative advertising in more fragmented markets with a larger number of competing firms. Competing firms may avoid negative communication when they expect consumers' switching to the outside option may result in an unfavorable outcome. For example, the realization that negative communication can reduce swing-voter turnout and affect the election outcome may have been the driving force behind Democratic congressional candidates' strategy of emphasizing their economic and healthcare-related policies instead of criticizing Trump in the 2018 midterm elections.[14]

The other implication of the proposition is that the negative-communication equilibrium is more likely if firms are better informed about the valuation shocks (i.e., α is high). Suppose α is high. In this case, if firms use negative communication, the consumers' belief about any undisclosed valuation draw becomes less extreme. This is because if a firm actually has a negative valuation shock, the rival firm will likely disclose it, and if the rival is silent, chances are that the firm's valuation shock is positive. Therefore, a higher α helps sustain negative communication over a larger parameter space. The example of the DTC mattress market described in the introduction might suggest this result. The mattress product is complex because it has numerous technological

[14] See Stein (2019) and www.wbur.org/hereandnow/2020/01/14/swing-voters-us-politics; accessed 2/6/2020.

or "luxury" features (material type, support, air flow, comfort) and consumers expect firms to have good information about the existence of these features in rival products. This might support the deployment of negative communication.

3.2 Positive Communication

In the equilibrium described in the previous section, a firm disclosing its own positive valuation draw ($m_i = \delta_i$) in the $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$ state cannot be part of the equilibrium. If a firm were to indeed disclose its own positive valuation when it had the option to disclose its rival's negative valuation draw, it would be optimal for the firm to stay silent ($m_i = \phi$) instead of disclosing its rival's negative draw in the $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$ information state. Consider, therefore, a candidate equilibrium in which firms stay silent instead of disclosing their rival's negative valuation draw (as was the case in the negative-communication equilibrium) in the $\{-\delta, -\delta\}$ information state. Firms disclose own positive valuation draw in the $\{\delta_i, \delta_j\} = \{\delta, \delta\}$ state. But if the realized information state is $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$, firm i discloses its own positive valuation draw with probability $\rho' \in [0, 1]$.[15] If the realized state is $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$, firm i obviously pretends to be uninformed and stays silent. Firm j's strategies are symmetric.

Having specified the firms' strategies in a candidate positive-communication equilibrium, we can derive consumers' ex-post beliefs about undisclosed draws and then examine the existence of the candidate equilibrium. Similar to Lemma 3 presented above for the negative communication equilibrium, the following lemma presents consumers' ex-post beliefs in the candidate positive-communication equilibrium.

Lemma 4 *Given the firms' candidate positive-communication equilibrium strategies specified above, consumers' ex-post beliefs about the undisclosed valuation draws are the following:*

(a) if $m_i = \delta_j$ *and* $m_j = \phi$, $E(\delta_i | m_i, m_j) = \begin{cases} \delta & if\ \rho' \in [0, 1) \\ -\delta & if\ \rho' = 1 \end{cases}$,

(b) if $m_i = \delta_i$ *and* $m_j = \phi$, $E(\delta_j | m_i, m_j) = \frac{1-\alpha-\rho'}{1-\alpha+\rho'}\delta$ *, and*

(c) if $m_i = \phi$ *and* $m_j = \phi$, $E(\delta_i | m_i, m_j) = E(\delta_j | m_i, m_j) = -\frac{\alpha}{2-\alpha}\delta$.

This lemma provides an important insight on the equilibrium consumer-inference mechanism that supports positive communication. Central to the positive-communication equilibrium (de-

[15] We start with this more general specification and show only $\rho' = 1$ can be sustained in the equilibrium.

scribed below in Proposition 3) is the consumers' inference $E(\delta_i|m_i, m_j)$ of the firm i's valuation draw if it discloses its rival's negative information and firm j is silent. Because firm i may potentially disclose $m_i = \delta_j$ only in the $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$ information state, the message pair $m_i = \delta_j$ and $m_j = \phi$ will induce the belief $E(\delta_i|m_i, m_j) = \delta$ for all $\rho' \in [0, 1)$. However, if $\rho' = 1$, consumers will be able to perfectly identify a deviation from the positive-communication equilibrium upon observing $m_i = \delta_j$ and will therefore assign an unfavorable out-of-equilibrium belief $E(\delta_i|m_i, m_j) = -\delta$. Thus, along the equilibrium path, the firm does not have the incentive to disclose $m_i = \delta_j$. It is this ability of consumers to perfectly detect a deviation in the $\rho' = 1$ case and punish it by assigning an unfavorable belief that results in the existence of the positive-communication equilibrium.

Positive communication leads to an interesting form of negative externality across the firms. Consumers understand that negative information about valuations remain undisclosed when firms are engaged in positive communication. Therefore, consumers' inference about undisclosed information is negative. However, because consumers also recognize the possibility that a firm whose information remains undisclosed may be uninformed about its own positive information, their equilibrium inference of a rival firm's undisclosed draw $E(\delta_j|m_i = \delta_i, m_j = \phi)$ is always higher than $-\delta$. The implication is that firms' position in relation to consumers' outside option becomes stronger as a result of their focus on disclosing their own positive information. In addition, firms find engaging in positive communication is more effective when they are able to disclose strong positive information about themselves. The following proposition presents informed firms' equilibrium disclosure strategies. Consumers' equilibrium beliefs about any undisclosed valuation draws can be obtained by substituting $\rho' = 1$ in Lemma 4.

Proposition 3 *A positive-communication equilibrium exists in which, if firm i is informed and*

(i) $\delta_i = \delta$ and $\delta_j = \delta$, or $\delta_i = \delta$ and $\delta_j = -\delta$, it discloses own positive valuation draw,

(ii) $\delta_i = -\delta$ and $\delta_j = \delta$, or $\delta_i = -\delta$ and $\delta_j = -\delta$, it remains silent.

In this positive-communication equilibrium, firms disclose their own positive valuation whenever possible (i.e., if firms are informed and their own valuation draw is δ). In addition, if they cannot disclose their own positive information (either because they are uninformed or their own draw is $-\delta$), they stay silent. Because this equilibrium strengthens firms' position relative to the outside option, it exists regardless of the attractiveness of consumers' outside option. This may imply a more widespread prevalence of positive communication compared to negative communication across real-world settings.

A comparison of firms' disclosure strategies in the negative- and positive-communication equilibria shows firms are more vocal in the negative-communication equilibrium. The increased vocality under negative communication comes from the difference in behavior in the $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$ state. In the positive-communication equilibrium, if their own valuation draw is negative, firms always stay silent. By doing so, they not only avoid being perceived for sure as a firm with a $-\delta$ draw, but also help the rival (which consumers believe has an expected valuation larger than $-\delta$). The hand-holding between firms, by not disclosing their rival's negative valuation draw, results in reduced communication in the positive-communication equilibrium relative to the negative-communication equilibrium.

3.3 Effect of Limited Bandwidth

A limitation on the firms' communication bandwidth restricts the amount of information firms can directly transmit to consumers, and one might expect this direct effect to dampen information transmission. What is the effect of such limitations on the firms' likelihood of disclosing information? This question is relevant for policy seeking to facilitate greater flow of information to consumers. A comparison of the firms' likelihood of disclosing information in the absence (section 2.2) and in the presence (section 3) of a limited disclosure bandwidth leads us to the following proposition.

Proposition 4 *Firms are as likely (in the case of positive communication) or more likely (in the case of negative communication) to disclose information in the presence of bandwidth limitation as in the absence of bandwidth limitation.*

Paradoxically, a limitation on communication bandwidth may actually result in more information disclosure. In the absence of bandwidth limitation, firms can disclose both valuation draws. In this case, any partial disclosure will induce the belief that the undisclosed information must be negative. This has perverse consequences for the extent of information disclosure. Firms disclose information only when disclosing both draws is favorable, and restrict information transmission otherwise. As shown in Lemma 2, firms disclose both draws only if their own valuation draw is positive; otherwise, they prefer to not transmit any information. Limited bandwidth can make firms relatively more likely to disclose information. The intuition is that if firms encounter bandwidth limitations, they have the incentive to disclose information not only when disclosing both draws is favorable, but also when disclosing one of the two draws is favorable. Bandwidth limitation shields firms from the consequences of negative consumer inference in the event of partial disclosure.

3.4 Consumer Information and Welfare

In this section, we comment on the implications of competitive disclosure and limited bandwidth for the information reaching consumers and consumers welfare. At the time of consumer's decision making, her anticipated utility W_i from choosing firm i will be $v + E(\delta_i) + \varepsilon_i$. However, the actual ex-post utility will depend upon the realized state. For example, if the realized state is $\{\delta, \delta\}$, the actual utility U_i is $v + \delta + \varepsilon_i$. Denote the difference between the actual and the anticipated utility by $d_i = U_i - W_i$. The difference d_i can be calculated for any realized state $\{\delta_i, \delta_j\}$ and the realized information (s_i and s_j). Let i^* denote the firm chosen by the consumer to maximize anticipated utility W_i. The actual ex-post consumer surplus for a given realized state $\{\delta_i, \delta_j\}$ and realized information (s_i and s_j) of firms will be $E(U_{i^*})$. Assume that the marginal utility of income is constant and is equal to unity and following Train (2015) we have that,

$$\begin{aligned} E(U_{i^*}) &= E\left[W_{i^*} + d_{i^*}\right] \\ &= E\left[\max_i \left(v + E(\delta_i) + \varepsilon_i\right) + d_{i^*}\right] \\ &= \ln \sum_i e^{v+E(\delta_i)} + \sum_i P_i d_i. \end{aligned}$$

Note, $E(W_{i^*})$ is the expectation of the maximum value of the anticipated utility W_i over all possible values of ε_i. From Small and Rosen (1981), if each ε_i is i.i.d. and extreme value distributed and if the marginal utility of income is constant this expectation becomes the log-sum term as shown above. In addition, $E(d_{i^*})$ is the average difference between actual and anticipated consumer utility.

The expected consumer surplus CS_m is the expectation of $E(U_{i^*})$ over the information realizations $\{s_i, s_j\}$ and over the state realizations $\{\delta_i, \delta_j\}$. (The subscript m identifies the specific equilibrium: $m = u$ for unilateral disclosure, $m = c$ for competitive disclosure, $m = n$ for negative communication, and $m = p$ for positive communication equilibria.) The expected consumer surplus CS_m can be expressed as

$$CS_m = E_{\{\delta_i, \delta_j\}}\left[E_{\{s_i, s_j\}}\left(E(U_{i^*})\right)\right].$$

We present the detailed calculations for the expected consumer surplus in the supplementary

Online Appendix. Consumer surplus comparisons are graphically presented in Figure 3 in the Appendix.

Consider first the effect of competition: From Proposition 1, we can note that although the (individual) firm i is weakly less likely to disclose information under competitive information disclosure when compared to the unilateral case, consumers do not necessarily receive more information and may not be better off in the unilateral disclosure case. Specifically, if the consumers' outside option v_0 is large enough, then they will receive less information in the unilateral information disclosure case than under competition. An individual firm i is equally likely to disclose information in both cases. However, because in the unilateral disclosure case, firm j is always silent, less information reaches consumers and so consumers are unambiguously better off under competitive disclosure. Now consider the alternative case when the outside option v_0 is sufficiently small ($v_0 < v + ln\left(\frac{e^{\frac{2\alpha\delta}{4-3\alpha}}-1}{e^{\delta}-e^{\frac{\alpha\delta}{4-3\alpha}}}\right)$). If outside option is small, firm i is more likely to disclose information in the unilateral disclosure case. However, because firm j is always silent, the likelihood that consumers are fully informed about realized draws is higher in the competitive disclosure case compared to unilateral disclosure case. As expected and confirmed by numerical simulations, shown graphically in Figures 3(a) and 3(b) in the Appendix, equilibrium consumer surplus is always higher under competitive disclosure.

Next consider the effects of bandwidth limitations under competition: In the positive communication equilibrium, an individual firm is as likely to disclose information as it does in the absence of bandwidth limitation (i.e., in the competitive disclosure setting). However, the constraint on bandwidth limits the amount of information that the individual firm discloses. Therefore, the amount of information reaching consumers is less in the positive communication equilibrium (with limited bandwidth) than in the competitive disclosure case and as a result consumers are better off when there are no bandwidth limitations (See Table 1 for a comparison of information reaching consumers with and without bandwidth limitations).

In the negative communication equilibrium, an individual firm is more likely to disclose information than it does in the absence of bandwidth limitations. Across the different states an individual firm is more likely to disclose information in the presence of bandwidth limitations such that the expected number of valuation draws fully disclosed to (or perfectly inferred by) consumers is exactly the same as in the competitive disclosure case without bandwidth limitations. Although the expected number of draws that are disclosed to consumers are the same, the actual draws and the states in which they are disclosed are different in the two cases (i.e., the information is not ordered).[16] Once again we need to numerically compare the consumers' expected utility with and without bandwidth constraints. As shown in Figures 3(e) and 3(f), the consumers are again better off under competition without any bandwidth limitations than under negative communication. Taken together we get the overall result that bandwidth limitations reduce consumer welfare irrespective of whether there is a positive or negative communication equilibrium.

	Valuation draws received by consumers					
	Competitive disclosure		Negative Communication		Positive Communication	
Realized state	$s_i = 1, s_j = 1$	$s_i = 1, s_j = 0$	$s_i = 1, s_j = 1$	$s_i = 1, s_j = 0$	$s_i = 1, s_j = 1$	$s_i = 1, s_j = 0$
$\delta_i = \delta,\ \delta_j = \delta$	$\delta_i,\ \delta_j$	$\delta_i,\ \delta_j$	$\delta_i,\ \delta_j$	$\delta_i,\ \delta_j$	$\delta_i,\ \delta_j$	δ_i
$\delta_i = \delta,\ \delta_j = -\delta$	$\delta_i,\ \delta_j$	$\delta_i,\ \delta_j$	δ_j	δ_j	δ_i	δ_i
$\delta_i = -\delta,\ \delta_j = \delta$	$\delta_i,\ \delta_j$	none	δ_i	none	δ_j	none
$\delta_i = -\delta,\ \delta_j = -\delta$	none	none	$\delta_i,\ \delta_j$	δ_j	none	none

Table 1: Valuation draws received by consumers in competitive disclosure, negative communication, and positive communication equilibria

Finally, in Figures 3(g) and 3(h), we provide the comparison of consumer surplus in the negative and positive communication equilibria. In high information environments where firms are known to have sufficiently high knowledge levels (α), consumers prefer positive communication equilibrium. In contrast, in poor information environments when firms do not have sufficiently high levels of knowledge, consumers prefer negative communication. The intuition is the following. In a high-information environment, firms find hiding unfavorable information difficult and consumers can more effectively infer undisclosed information. Now notice that under negative communication the likelihood of firms' directly disclosing information across the states is higher than under positive communication because of the additional disclosure of the rival's negative information in the $(-\delta, -\delta)$ state. This creates an asymmetry in consumer inference when consumers do not receive any directly disclosed information: i.e., when consumers do not receive any information from firms in the negative communication equilibrium their inference is $E(\delta_i) = E(\delta_j) = 0$ or no different from their prior. However, their inference is $\frac{-\alpha\delta}{2-\alpha}$ in the positive communication equilibrium. In other words, consumers are able to infer more information and their surplus is higher in the positive communication than in the negative communication equilibrium. In contrast, a poor-information environment allows firms to more effectively hide their unfavorable information by pretending ignorance. In this case, consumers find it difficult to make inferences about the valuation draws that are not directly revealed by firms. Therefore, consumers prefer negative communication, which

[16] See Jerath and Ren (2021) for the use of an alternative entropy-based metric of information content reaching consumers.

directly discloses more information compared to positive communication. In summary, the overall information reaching consumers (including both direct disclosure and from consumer inference) may become higher in the positive communication equilibrium when α is sufficiently high resulting in higher consumer surplus, while the converse is true when α is sufficiently low. Finally, the numerical analysis also indicates that consumer surplus is higher under negative communication over a larger range for larger outside options and for smaller δ's.

4 Generalized (Alternative) Firm Payoffs

The model in the previous section has the implicit assumption that while a firm gets a positive payoff from a consumer choosing it, it gets zero payoff otherwise, i.e., it is indifferent between whether the consumer chooses the rival or the outside option. This characterization is more relevant for interactions between firms in product markets. However, in some of the other important contexts that motivate this paper such as political elections or non-market situations such as influencing a jury or faculty hiring this characterization may not be fully representative.

Consider for example an electoral contest between rival political candidates i and j: In this case a voter (consumer) can choose to vote for either of the two politicians or else exercise her outside option choosing to abstain from voting. In political contests, when a voter does not choose a political candidate i, the candidate is no longer indifferent between choosing the rival j and the outside option of abstention. The more natural assumption is that a candidate i would strictly prefer a voter to abstain and not vote rather than voting for the rival.[17] To capture this idea note that in the basic model (sections 2 and 3), the probability (or the expected proportion) of a consumer choosing firm i can also be denoted by $P_i = \frac{e^{E(v_i)}}{e^{E(v_i)}+e^{E(v_j)}+e^{v_0}}$. Then the alternative generalized firm payoff function can be represented as:

$$\Pi_{ia} = P_i \times 1 + (1 - P_i)\frac{e^{v_0}}{e^{v_0} + e^{E(v_j)}}k = \frac{e^{E(v_i)} + ke^{v_0}}{e^{E(v_i)} + e^{E(v_j)} + e^{v_0}}. \tag{3}$$

The first term is the probability with which firm (candidate) i is chosen by the consumer (voter), in which case the firm gets a unit payoff. The second term denotes the event in which the consumer does not choose i in which case the firm prefers the consumer to choose the outside option. And

[17]This assumption is also consistent with other non-market contexts, for example jury trials: A jury trial may result in a defendant being found guilty, not guilty, or the trial may result in a hung jury (the outside option for the defendant). The defendant may find the outside option of a hung jury to be strictly preferred to the prosecution winning a guilty verdict.

the outside option gives the firm a lower payoff of $k < 1$. Finally, when the consumer chooses the rival the firm gets zero payoff. Notice that when $k = 0$, the firm is indifferent between choosing j and the outside option and we will recover the basic case discussed in sections 2 and 3, whereas at the other extreme with $k = 1$ the choice of the outside option over the rival gives a firm the same payoff as the choice of the firm.[18] We start with the case of competition with no bandwidth limitations and derive the symmetric pure strategy PBE of the game.

Proposition 5 *The equilibrium for the competitive information disclosure with no bandwidth constraints is as follows:*

[a] When $k \leq \frac{1}{2}$, then $m_i = \{\delta_i, \delta_j\}$ is an equilibrium strategy if firm i is informed and $\delta_i = \delta$.

[b] When $k \geq \frac{1}{2}$, and if firm i is informed, then its equilibrium strategy is:

(i) $\delta_i = \delta$ and $\delta_j = \delta$, it remains silent,

(ii) $\delta_i = \delta$ and $\delta_j = -\delta$, it discloses $m_i = \{\delta_i, \delta_j\}$,

(iii) $\delta_i = -\delta$ and $\delta_j = \delta$, it remains silent, and

(iv) $\delta_i = -\delta$ and $\delta_j = -\delta$, it discloses $m_i = \{\delta_i, \delta_j\}$.

When the relative preference for the outside option is sufficiently small ($k \leq \frac{1}{2}$), we recover an equilibrium that is fully consistent with the result in the basic model (with $k = 0$) described in Lemma 2. Disclosing information in the state where both firms have positive valuation draws or keeping silent when both firms have negative valuation draws suppresses the possibility of the consumer choosing the outside option. This becomes optimal in equilibrium when the relative payoff for the firm of the consumer choosing the outside option is sufficiently low.

When $k \geq \frac{1}{2}$, the value of the consumer choosing the outside option over the rival is sufficiently high. We see a qualitative reversal in the nature of the firms' equilibrium behavior. In contrast to the previous case, each firm chooses to stay silent when both have positive valuation draws and indeed to disclose the information when both valuation draws are negative. There is an equilibrium incentive to cloak (symmetric) positive information while revealing negative information. Intuitively, this strategy encourages the consumers' choice of the outside option over the rival.

Next, to complete the analysis, we investigate the role of bandwidth limitations in the following

[18] An alternative formulation that is relevant in some political contexts is that politician i's payoff may be based only on the share of the voters who choose i over j and not depend on the outside option. Specifically, suppose the payoff function is: $\pi_i = \frac{e^{E(v_i)}}{e^{E(v_i)}+e^{E(v_j)}}$. The results for this formulation can be obtained as a special case of the generalized firm payoff function in (3) when $v_0 = -\infty$. The main insights of this section continue to hold for this alternative formulation.

Proposition:

Proposition 6 *The equilibrium for the competitive information disclosure with limited bandwidth is as follows:*

[a] When $k \leq \frac{1}{2}$, a positive communication equilibrium always exists in which if firm i is informed it has the following disclosure strategies: i) if $\delta_i = \delta$ and $\delta_j = \delta$ or $\delta_i = \delta$ and $\delta_j = -\delta$, it discloses own positive information, and ii) if $\delta_i = -\delta$ and $\delta_j = \delta$ or $\delta_i = -\delta$ and $\delta_j = -\delta$ it remains silent.

[b] When $k \leq \frac{1}{2}$ and $v_0 < v + \hat{v}_k$,[19] a negative communication equilibrium exists in which if firm i is informed it has the following disclosure strategies: i) if $\delta_i = \delta$ and $\delta_j = \delta$, it discloses its own positive valuation draw, ii) if $\delta_i = \delta$ and $\delta_j = -\delta$, it discloses its rival's negative valuation draw, iii) if $\delta_i = -\delta$ and $\delta_j = \delta$, it remains silent, and iv) if $\delta_i = -\delta$ and $\delta_j = -\delta$, it discloses its rival's negative valuation draw.

[c] When $k \geq \frac{1}{2}$, there is an "extreme" negative communication equilibrium exists in which if firm i is informed it has the following disclosure strategies: i) if $\delta_i = \delta$ and $\delta_j = \delta$, it stays silent, ii) if $\delta_i = \delta$ and $\delta_j = -\delta$, it discloses the rival's negative valuation draw, iii) if $\delta_i = -\delta$ and $\delta_j = \delta$, it stays silent, and iv) if $\delta_i = -\delta$ and $\delta_j = -\delta$, it discloses the rival's negative valuation draw.

With bandwidth limitations, we get interesting insights into the role of the firms' relative importance k of the consumer's choice of the outside option. When k is small enough ($k \leq \frac{1}{2}$) and the firm's relative preference for the outside option over the rival not too high, the nature of the equilibrium is qualitatively similar to the basic model (in section 3 with $k = 0$). In this range, positive communication equilibrium, similar to that in Proposition 3, always exists. We also have a negative communication equilibrium exactly along the same lines as in Proposition 2: i.e., when the outside option is not too large.

But when $k \geq \frac{1}{2}$, i.e., the firms' relative payoff from the consumer choosing the outside option over the rival becomes sufficiently large, we see a distinct shift in the nature of the equilibrium behavior. The positive communication equilibrium no longer exists. The equilibrium that does exist is an extreme form of negative communication: Not only do the firms choose to disclose negative information about their rivals whenever possible, but what is interesting is that they choose to remain silent (rather than disclosing their own positive information) when both firms

[19] As shown in the Appendix, $\hat{v}_k = ln\left(\frac{1-e^{\frac{-2\delta}{2-\alpha}}}{e^{\frac{-\alpha\delta}{2-\alpha}}-e^{\delta}+k\left(-2e^{\frac{-\alpha\delta}{2-\alpha}}+e^{-\delta}+e^{-2\delta-\frac{-\alpha\delta}{2-\alpha}}\right)}\right)$

have positive valuation draws. As the payoff from the consumers choosing the outside option over the rival increases, each firm has the incentive to choose negative communication as this promotes the choice of the outside option. Similarly, choosing to suppress own positive information when both firms have positive draws is also a strategy which induces the choice of the outside option over the rival.

Our analysis of the generalized alternative firm payoff functions provides insights into the motivations of firms (or more generally senders of information) to deploy negative or positive communication strategies in a range of markets. Many observers might agree that there is a greater prevalence of negative communication in politics and political markets as compared to product markets. Our analysis provides one rationale for this pattern and traces it to the nature of the outside option and its effect on the sender's payoffs. The payoff function of a political candidate is the relative vote share advantage over the rival, and so in the event a voter does not choose the candidate, he would strictly and strongly prefer that the voter abstain and not turn out to vote, rather than voting for the opponent. As we have shown in Propositions 5 and 6, this leads to a greater prevalence of negative communication. Indeed when the politicians' incentive to reduce voter turnout (i.e., the relative preference for the outside option over the rival) becomes sufficiently high, there is no longer an equilibrium in positive communication, and an extreme form of negative communication exists in which the candidate suppresses own positive information.

5 Extensions

5.1 Correlated Information Availability

In the models presented above, we assume the firms are independently informed about the valuation draws. In this section, we assume the information shocks are correlated and investigate the effect of this correlation on the firms' disclosure incentives. Suppose firms' information availability is perfectly correlated: When a firm is informed about the rival's draw, the rival is also informed about its own valuation draw. Equivalently, either both firms are informed or neither one is informed. This represents the plausible scenario in which a rival cannot know more about a firm than the firm itself.

Specifically, both firms are informed about both valuation draws with probability β and both are uninformed with probability $1-\beta$. Other assumptions are the same as in the basic model (sections 2 and 3). A comparison of firms' equilibrium disclosure strategies in the absence and in the presence of a limitation on the disclosure bandwidth leads us to the following proposition.

Proposition 7 *In the absence of any bandwidth limitations, firms can possibly hide some unfavorable information from consumers by staying silent. However, if the disclosure bandwidth is limited, all the valuation information is transmitted to consumers (either directly through firms' disclosures or indirectly through consumer inferences.)*

Even with the correlation of information across the firms, more information can be transmitted from firms to consumers under limited bandwidth. In the absence of any bandwidth restriction, two equilibria exist. All the information is transmitted from firms to consumers in one of them but not the other. In the presence of bandwidth restriction, all the information is always transmitted to consumers. Similar to the case of independent information shocks, bandwidth restriction makes firms more likely to disclose information. Correlation in information shocks helps consumer inference, resulting in full information transmission from firms to consumers.

5.2 Private Information about Own Valuation

In the analysis presented above, we assumed an informed firm knows about both its own and its rival's valuation shocks. This assumption helps us clearly understand a competing firm's trade-offs between disclosing own and rival information. The decision of whether to disclose its own or its rival's information becomes relevant only when the firm is informed about both own and rival's valuation draws. In this extension of the basic model (of sections 2 and 3), we relax the above assumption and allow for the possibility that a firm may sometimes be informed only about its own valuation draw. Specifically, we assume a firm may be i) informed about both its own and its rival's valuation draws with probability α, ii) informed only about its own valuation draw with probability γ, and iii) uninformed with probability $1 - \alpha - \gamma$. Other assumptions are the same as in the basic model. The analysis of this case and a sketch of the proof is provided in the Appendix.

If a firm is privately informed only about its own valuation shock, the firm's decision is straightforward. If the private information is favorable ($\delta_i = \delta$), the firm discloses it ($m_i = \delta_i$). However, if the private information is unfavorable ($\delta_i = -\delta$), the firm pretends to be ignorant and stays silent ($m_i = \phi$). The possibility that a firm may be informed only about its own information shock also changes the firm's equilibrium strategy in states in which it is informed about both information shocks. In particular, when firm i is informed that the realized state is $\{\delta_i, \delta_j\} = \{\delta, \delta\}$, it discloses only its own valuation draw ($m_i = \delta_i$), regardless of whether bandwidth is limited, and pretends to be ignorant about its rival's information. Hiding information in the $\{\delta, \delta\}$ state becomes possible for the firm because consumers cannot figure out if the firm is hiding information or it actually knows only about its own valuation shock. Equilibrium strategies in all the other information states remain the same as those described in sections 2 and 3.

An examination of equilibrium strategies and existence conditions for unilateral information disclosure and competitive information disclosure (both with and without bandwidth limitations) cases reveals that all the main results (presented above in propositions 1-4) continue to qualitatively hold in the more general model setup of this section. The equilibrium strategies in the new information states of this section (i.e., if firm i knows only δ_i) are exactly the same in both unilateral and competitive disclosure cases. In the basic model, if the realized state was $\{\delta, \delta\}$, firms disclosed as much information as possible. In the setup of this section, if $|m_i| \leq 2$, firms definitely disclose their own but not their rival's information in the $\{\delta, \delta\}$ state. However, if $|m_i| \leq 1$ and the realized state is $\{\delta, \delta\}$, equilibrium strategies are exactly the same as in the basic model. Therefore, the result that a limitation on bandwidth makes firms more vocal actually becomes stronger.

5.3 Asymmetric Valuations

In the models and extensions above, we assume the two competing firms are ex-ante symmetric. In this section, we present a model extension which studies the effect of asymmetric valuations on the firms' information disclosure incentives. We assume the observable component of consumers' valuation for firm i is v but for firm j it is $v - \triangle_v$. Firm j is essentially the weaker of the two firms and has a valuation disadvantage of $\triangle_v$. Other assumptions are the same as in sections 2 and 3. This model extension helps us to analyze whether it is the higher-valuation (stronger) or the lower-valuation (weaker) firm that has a higher incentive to engage in negative/positive communication.

We describe how the existence of various equilibria (described in sections 2 and 3) respond to the extent of the valuation disadvantage $\triangle_v$ of firm j. We find that in the unilateral information disclosure setting, when facing a weaker non-strategic rival firm j, firm i discloses draws in the $\{-\delta, -\delta\}$ state over a smaller parameter space. The valuation asymmetry does not affect disclosure incentives in other states. The disclosure by firm i in the $\{-\delta, -\delta\}$ state helps the outside option more when firm j is weaker. Therefore, firm i becomes more likely to stay silent instead of disclosing both firms' negative draws. Recall, in the competitive information setting in which both firms are strategic, a firm discloses information only when its own draw is positive. They stay silent in the $\{-\delta, -\delta\}$ state. Therefore, there is no effect of valuation asymmetry on the existence of equilibrium in the competitive disclosure setting.

In the presence of limited disclosure bandwidth, the positive communication equilibrium always exists regardless of the level of asymmetry in valuations. However, the negative communication equilibrium exists over a smaller region as $\triangle_v$ increases. As the rival firm becomes weaker (i.e., $\triangle_v$ increases) the disclosure of its negative information ($m_i = \delta_j = -\delta$) becomes less desirable than disclosing own positive information ($m_i = \delta_i = \delta$) for firm i in the $\{\delta, -\delta\}$ state. This is because the disclosure of rival's negative information helps the outside option even more when the rival has lower valuation. Conversely, it becomes more attractive to reveal own positive information when there is significant asymmetry. We conclude that when valuations are asymmetric the stronger (higher-valuation) firm is less likely to engage in negative communication than the weaker (lower-valuation) firm. There is no effect of asymmetry in valuations on their incentives to engage in positive communication.

5.4 Information Acquisition Incentives

In this section, we seek to understand if better information (i.e., a higher α) is desirable for firms and if firms have more incentives to acquire more information when they engage in positive or negative communication. Consider an initial stage of the game in which firms simultaneously decide whether or not to acquire information before they choose their disclosure strategies. If firms acquire information then they learn about valuation draws with probability α, otherwise they remain uninformed. To highlight the strategic incentives that are not cost driven, assume for now that acquiring information is costless for firms. Three types of information acquisition pure-strategy equilibria are possible: i) both firms choose to acquire information, denoted by $\{\alpha, \alpha\}$, ii) only one firm acquires information, denoted by $\{\alpha, 0\}$, or $\{0, \alpha\}$, (i.e., equivalent to the unilateral disclosure case), and iii) no firm acquires information, denoted by $\{0, 0\}$.

Given the nature of the payoff functions comparing the expected profits at the information acquisition stage is intractable. However, we can identify and solve for the information acquisition equilibria through numerical analysis (See Appendix for the details). Figure 4 in the Appendix represents the three possible types of information acquisition equilibria under positive and negative communication. Overall, firms are more likely to acquire information in the positive communication equilibrium than in the negative communication equilibrium. Informed firms more effectively exploit their information advantage when they engage in positive communication (which not only helps them, but also suppresses the choice of the outside option) than negative communication (which not only hurts the rival, but also helps the outside option). An additional mechanism for the firms' higher information acquisition incentives in the positive communication equilibrium is the consumers' inference of undisclosed information. Because firms are more vocal when they engage in negative communication, their silence is punished more (when they are better informed) by consumers' belief updating compared to when firms engage in positive communication. Therefore, firms are more likely to acquire information in the positive communication equilibrium.

Figure 4 also indicates that firms have higher incentives to acquire information (for both positive- and negative-communication equilibria), i) if δ is smaller, and ii) if v_0 is larger. Recall, in the unilateral information disclosure case, $E\left(\delta_1|m_1=\phi\right)=-E\left(\delta_2|m_1=\phi\right)=-\frac{\alpha}{4-3\alpha}\delta$ (see proof of Lemma 1). If δ is smaller, consumers impose a smaller penalty on the firm that acquires information but stays silent. At the same time, the belief about the valuation of the firm that does not acquire information becomes less favorable. A deviation from the $\{0,0\}$ equilibrium becomes more attractive while one from the $\{\alpha,\alpha\}$ equilibrium less attractive. As a result, firms become more likely to acquire information when δ is smaller. The effect of v_0 is the opposite: a larger v_0 makes information acquisition more desirable. The reason is that, in the unilateral information disclosure case, a larger v_0 dampens the effect of both unfavorable beliefs about the firm that acquires information and favorable beliefs about the firm that stays uninformed. Therefore, as v_0 increases, a firm becomes more likely to acquire information regardless of its belief about the rival's information acquisition.

We note that even if the information acquisition decision is costly and involved a positive investment cost the insights of this section would continue to hold. In general, positive acquisition costs would shrink the parameter space where the investment equilibria (involving both or one of the firms acquiring information) exist. Nevertheless, the firms will continue to be relatively more likely to acquire information in the positive communication equilibrium. Similarly the effects pertaining to δ and v_0 will be qualitatively be the same.

6 Conclusion and Discussion

In a range of situations spanning product markets, political contests, and social interactions, players compete by choosing whether to reveal own positive information and/or negative information about rivals. In designing their communication strategies to persuade consumers, firms may choose between focusing on valuable characteristics of their products or adverse characteristics about their competitors' products. Politicians have the choice of whether to focus on the positive aspects of their candidacy or focus on negative communication about their rivals. Similar considerations are also relevant in a number of organizational and social interactions. These choices and the trade-offs involved in revealing negative versus positive information are particularly sharp when firms have limited bandwidth constraints.

We characterize these situations as a persuasion contest in which firms reveal positive or negative information in order to influence the beliefs of skeptical and rational consumers. The analysis links the choice of negative- and positive-information revelation to the extent of information endowment of the firms and the attractiveness of the outside options. Bandwidth limitation leads to two distinct types of equilibria: a negative-communication equilibrium in which firms disclose their rival's negative information whenever possible, or alternatively a positive-communication equilibrium in which they disclose their own positive information whenever possible. Although the positive communication always exists, negative communication does not always exist and it has strategic costs. Consistent with the empirical findings from the political advertising literature, we show that in the equilibrium, firms use negative communication when consumers' outside options are not attractive enough. In the presence of an attractive outside option for consumers, firms worry that consumers may switch to their outside option instead of choosing one of the two competing firms. An implication is that negative communication is more likely in industries that are more concentrated with, for example, two major competitors. This seems to bear out, for example, in markets such as telecommunications (AT&T vs. Verizon) and email (Google vs. Microsoft). Firms are more likely to engage in negative communication when they are better informed and are more likely to have valuation information. Another important insight of the analysis is that limitations on the amount of information firms can disclose can make them more likely to disclose information.

We also compare a firm's disclosure strategies when facing a non-strategic and when facing a strategic rival. The analysis reveals a firm is more likely to disclose information when facing a non-strategic rival or in a unilateral disclosure setting. Firms become more vocal when facing a silent rival, because the silence of a strategic firm is punished more by consumers in their belief updating when facing a non-strategic rival than when facing a strategic rival.

In this paper, prices were assumed to be fixed and this allowed us to focus on the choice of competitive information disclosure strategies by the firms. A natural extension would be to consider the strategic choice of pricing by the firms and see how this would affect the nature of information disclosure. Within the logit like consumer choice formulation of this paper a full analytical treatment of this problem with strategic prices is intractable. Regardless we can comment on some of the potential implications: If prices were endogenously chosen prior to the revelation of information to the firm, then there may exist a potential surplus extraction and competition trade-off. If firms try to increase prices to extract surplus, then this might also increase the competition with the outside option as more consumers choose not to buy. This can then affect the prevalence of the negative communication equilibrium. If prices are chosen after the information revelation, then they may have a role in communicating firms' information in addition to the disclosure strategies. This is an interesting possibility for future investigation though a model of product differentiation which is more natural for incorporating prices.

To study competing firms' incentives to disclose own and rival's information, this paper assume firms may be privately informed about positive or negative valuation draws, which they decide whether they want to disclose to consumers. Another way to formulate the model could be to consider multi-attribute firms similar to Anderson and Renault (2006) and Sun (2011). Each firm's product may have positively and negatively valued attributes and firms may be independently informed about a subset of attributes for its own and rival's product. After that firms simultaneously make disclosure decisions. Bostanci et al. (2020) exploit a similar micro-foundation to capture the effect of positive and negative advertising in a comparative advertising setting. In the context of information for product design modifications, Iyer and Soberman (2000) consider innovations of firms that add value to own customers than adding value to rival's customers which can be seen as highlighting own positive or rival's negative attributes.

It might also be useful to consider the possibility that firms when informed about the state are not be perfectly informed. Rather they have noisy but informative signals of the true state. Suppose firms play a disclosure game in deciding whether and what signals to reveal. At the one extreme if the precision of the signals were to be zero then the consumers would ignore any disclosed information. At the other extreme when the precision of the signal is perfect, we will get the model that is analyzed in the paper where consumer inferences are a response to the equilibrium communication strategies given the knowledge level α. In between the extremes, the firms will have imperfect signals about the valuation shocks and when consumers make their inferences they will not only consider the strategic disclosure motives of the firms but also the fact that the firms' information is noisy. All else being equal this might lead to outcomes which are equivalent to when the firms had a lower level of knowledge.

Appendix

Proof of Lemma 1 (Unilateral Information Disclosure)

Firm $i = 1$ is strategic, whereas firm $j = 2$ is non-strategic. Consider a candidate unilateral information-revelation equilibrium in which the informed firm 1 discloses both valuation draws $m_1 = \{\delta_1, \delta_2\}$ with (1) probability λ, if $\delta_1 = \delta$ and $\delta_2 = \delta$, (2) probability 1, if $\delta_1 = \delta$ and $\delta_2 = -\delta$, (3) probability zero, if $\delta_1 = -\delta$ and $\delta_2 = \delta$, and (4) probability μ, if $\delta_1 = -\delta$ and $\delta_2 = -\delta$. An uninformed firm 1 will stay silent. Firm $j = 2$ always stays silent in this setup.

First, we derive consumers' beliefs about the undisclosed valuation draws. In the equilibrium, if consumers observe a silent firm 1, they update their beliefs about the draws using Bayes' rule. It is straightforward to show that if firm 1 is silent,

$$E(\delta_1|m_1 = \phi) = -E(\delta_2|m_1 = \phi) = \frac{-\alpha(1+\lambda-\mu)\delta}{4(1-\alpha)+\alpha(3-\lambda-\mu)}.$$

Therefore, $E(\delta_1|m_1 = \phi) < 0$ and $E(\delta_2|m_1 = \phi) > 0$. Next, we examine the four possible information states of an informed firm 1.

Case 1. Suppose firm 1 is informed and $\{\delta_1, \delta_2\} = \{\delta, \delta\}$.

If $\lambda = 1$, firm 1's profit

$$\pi_1 = \frac{e^{v+\delta}}{e^{v+\delta}+e^{v+\delta}+e^{v_0}}.$$

If the firm deviates and stays silent, it makes the profit of

$$\pi_1^D = \frac{e^{v+E(\delta_1|m_1=\phi)}}{e^{v+E(\delta_1|m_1=\phi)}+e^{v+E(\delta_2|m_1=\phi)}+e^{v_0}}.$$

Because $\pi_1 > \pi_1^D$, $\lambda = 1$ can be part of the equilibrium. Similarly, it can be shown that $\lambda = 0$ cannot be part of the equilibrium, because the deviation to disclosing both draws is profitable. Also, any $\lambda \in (0,1)$ is not part of the equilibrium, because firm 1 is not indifferent between disclosure and silence. As a result, $\lambda = 1$ is the unique equilibrium strategy for firms in the $\{\delta, \delta\}$ information case.

Case 2. Suppose firm 1 is informed and $\{\delta_1, \delta_2\} = \{\delta, -\delta\}$.

In this case, firm 1's profit from disclosing both draws is $\frac{e^{v+\delta}}{e^{v+\delta}+e^{v-\delta}+e^{v_0}}$, which is the highest possible profit for a firm in this model setup. No deviation can be more profitable.

Case 3. Suppose firm 1 is informed and $\{\delta_1, \delta_2\} = \{-\delta, \delta\}$.

This state is the most unfavorable one for firm 1. Firm 1 stays silent and makes a profit of $\frac{e^{v+E(\delta_1|m_1=\phi)}}{e^{v+E(\delta_1|m_1=\phi)}+e^{v+E(\delta_2|m_1=\phi)}+e^{v_0}}$. A deviation to disclosing draws results in the lowest profit possible.

Case 4. Suppose firm 1 is informed and $\{\delta_1, \delta_2\} = \{-\delta, -\delta\}$.

From Case 1, we know only $\lambda = 1$ is part of the equilibrium. If firm 1 discloses both valuation draws,

$$\pi_1 \left(m_1 = \{\delta_1, \delta_2\} \right) = \frac{e^{v-\delta}}{e^{v-\delta} + e^{v-\delta} + e^{v_0}},$$

and if it stays silent,

$$\pi_1 \left(m_1 = \phi \right) = \frac{e^{v+E(\delta_1|m_1=\phi)}}{e^{v+E(\delta_1|m_1=\phi)} + e^{v+E(\delta_2|m_1=\phi)} + e^{v_0}}.$$

A straightforward comparison reveals that if the realized information state is $\{-\delta, -\delta\}$, in the equilibrium, firm 1

(1) discloses both draws (i.e., $\mu = 1$), if $v_0 < \hat{v}_0 \equiv v + ln \left(\frac{e^{\frac{2\alpha\delta}{4-3\alpha}} - 1}{e^{\delta} - e^{\frac{\alpha\delta}{4-3\alpha}}} \right)$,

(2) stays silent (i.e., $\mu = 0$), if $v_0 > \tilde{v}_0 \equiv v + ln \left(\frac{e^{\frac{\alpha\delta}{2-\alpha}} - 1}{e^{\delta} - e^{\frac{\alpha\delta}{2-\alpha}}} \right)$, and

(3) discloses both draws with probability $\mu \in (0, 1)$, if $v_0 = v + ln \left(\frac{e^{\frac{2\alpha\delta}{4(1-\alpha)+\alpha(2-\sigma)}} - 1}{e^{\delta} - e^{\frac{\alpha\delta}{4(1-\alpha)+\alpha(2-\sigma)}}} \right)$.

Also, $\hat{v}_0 > \tilde{v}_0$, and in the entire $(\tilde{v}_0, \hat{v}_0)$ range, a $\mu \in (0, 1)$ can be found in an equilibrium in addition to the disclosure and silence equilibria.

Case 1 and result (1) in Case 4 above establish Lemma 1.

Proof of Lemma 2 (Competitive Information Disclosure)

The proof and the analysis for the competitive information revelation setup is similar to that of the unilateral information revelation case presented above, and therefore we describe only the main steps and results here.

Consider a candidate competitive revelation symmetric equilibrium in which firms' strategies are described as follows. An informed firm i discloses both valuation draws $m_i = \{\delta_i, \delta_j\}$ with (1) probability λ', if $\delta_i = \delta$ and $\delta_j = \delta$, (2) probability 1, if $\delta_i = \delta$ and $\delta_j = -\delta$, (3) probability zero, if $\delta_i = -\delta$ and $\delta_j = \delta$, and (4) probability μ', if $\delta_i = -\delta$ and $\delta_j = -\delta$. An uninformed firm i stays silent. As in the unilateral disclosure case, any partial disclosure induces the most unfavorable consumer inference for the undisclosed information and therefore cannot be payoff improving.

If both firms are silent, consumers' ex-post beliefs of valuation draws are derived using Bayes' rule and are given by

$$E \left(\delta_i | m_i = \phi, m_j = \phi \right) = E \left(\delta_j | m_i = \phi, m_j = \phi \right) = \frac{2\alpha \left(1 - \alpha\right) \left(\mu' - \lambda'\right) + \alpha^2 \left[\left(1 - \lambda'\right)^2 - \left(1 - \mu'\right)^2 \right]}{\alpha^2 \left[\left(1 - \lambda'\right)^2 - \left(1 - \mu'\right)^2 \right] + 2\alpha \left(1 - \alpha\right) \left(3 - \mu' - \lambda'\right) + 4 \left(1 - \alpha\right)^2} \delta.$$

Next, we consider each of the four information states.

Case 1. Suppose firm i is informed and $\{\delta_i, \delta_j\} = \{\delta, \delta\}$. As in the previous lemma, it can be shown that only $\lambda' = 1$ can be an equilibrium, because the firms have profitable deviations for all values of $\lambda' \in [0, 1)$. Firm i's profit corresponding to $\lambda' = 1$ is

$$\pi_i = \frac{e^{v+\delta}}{e^{v+\delta} + e^{v+\delta} + e^{v_0}}.$$

Cases 2&3. Firm i discloses both draws if $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$ and stays silent if $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$. The proof is similar to the unilateral information disclosure case.

Case 4. Suppose $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$ and firm i is informed. Disclosing both draws ($\mu' = 1$) is not an equilibrium strategy for firm i, because it can make higher profits by deviating and staying silent. Also, no $\mu' \in (0, 1)$ can be part of the equilibrium because staying silent is more profitable. No profitable deviations from the equilibrium strategy of staying silent ($\mu' = 0$) exist in this information state. Therefore, in the equilibrium, if firm i is informed and $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, it stays silent.

All four cases presented above can be summarized as follows. In the equilibrium, if firm i is informed and $\delta_i = \delta$, it discloses both its own and its rival's valuation draws. Otherwise, it stays silent.

Proof of Proposition 1

A comparison of firm i's equilibrium disclosure strategies in the unilateral and competitive information revelation cases (described above) reveals the strategies are identical if $\{\delta_i, \delta_j\}$ are $\{\delta, \delta\}$, $\{\delta, -\delta\}$, or $\{-\delta, \delta\}$.

If $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, an informed firm i stays silent in the competitive information case. However, in the unilateral information revelation case, in the equilibrium, firm i discloses both draws if $v_0 < \hat{v}_0$. Also, if $v_0 \in (\tilde{v}_0, \hat{v}_0)$, there exists a mixed-strategy equilibrium in which firm i randomizes between disclosing both valuation draws and staying silent.

Therefore, if $v_0 < \hat{v}_0$, firm i is less likely to disclose information in the competitive information revelation case compared to the unilateral information revelation case. If $v_0 > \tilde{v}_0$, an equilibrium exists in which firm i stays silent in the unilateral information disclosure case and the likelihood of disclosure is the same in the two configurations.

Proof of Lemma 3

(a) Suppose $m_i = \delta_j = -\delta$ and $m_j = \phi$. Because $m_i \neq \phi$, $s_i = 1$. The draw δ_i remains undisclosed. We derive consumers' beliefs about δ_i in this case.

Three possible realizations of valuation draws and information can result in this situation:

(1) The first is $\{\delta_i, \delta_j\}$ =$\{-\delta, -\delta\}$, $s_j = 0$. This realization can happen with probability $\frac{\alpha(1-\alpha)}{4}$.

(2) The second is $\{\delta_i, \delta_j\}$ =$\{\delta, -\delta\}$, $s_j = 0$. This realization can happen with probability $\frac{\alpha(1-\alpha)}{4}$.

(3) The third is $\{\delta_i, \delta_j\}$ =$\{\delta, -\delta\}$, $s_j = 1$, and this realization happens with probability $\frac{\alpha^2}{4}$.

Because firm i's relevant equilibrium disclosure strategy is to disclose its rival's draw with probability 1 in the $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$ state and with probability $(1 - \rho)$ in the $\{\delta_i, \delta_j\}$ =$\{\delta, -\delta\}$ state, the probability of the consumers observing $m_i = \delta_j$ and $m_j = \phi$ is simply

$$prob\,(m_i = \delta_j, m_j = \phi) = \frac{\alpha\,(1-\alpha)}{4} + \frac{\alpha\,(1-\alpha)}{4}\,(1-\rho) + \frac{\alpha^2}{4}\,(1-\rho) = \frac{\alpha\,(2-\alpha-\rho)}{4}.$$

The probabilities of realizations conditional on observed disclosures can be derived using Bayes' rule as

$$prob\left(\{\delta_i,\delta_j\}=\{-\delta,-\delta\},s_j=0|m_i=\delta_j,m_j=\phi\right)=\tfrac{1-\alpha}{2-\alpha-\rho},$$
$$prob\left(\{\delta_i,\delta_j\}=\{\delta,-\delta\},s_j=0|m_i=\delta_j,m_j=\phi\right)=\tfrac{(1-\alpha)(1-\rho)}{2-\alpha-\rho},$$
$$prob\left(\{\delta_i,\delta_j\}=\{\delta,-\delta\},s_j=1|m_i=\delta_j,m_j=\phi\right)=\tfrac{\alpha(1-\rho)}{2-\alpha-\rho}.$$

Therefore, the consumers' inference about δ_i is

$$E\left(\delta_i\right)=\frac{1-\alpha}{2-\alpha-\rho}\left(-\delta\right)+\frac{\left(1-\alpha\right)\left(1-\rho\right)}{2-\alpha-\rho}\delta+\frac{\alpha\left(1-\rho\right)}{2-\alpha-\rho}\delta=\frac{\alpha-\rho}{2-\alpha-\rho}\delta.$$

(b) Suppose $m_i=\delta_i=\delta$ and $m_j=\phi$. Because $m_i\neq\phi$, $s_i=1$. We derive consumers' ex-post belief about the draw δ_j that remains undisclosed. The possible realization of draws and information that can result in this situation are

(1) $\{\delta_i,\delta_j\}=\{\delta,\delta\},s_j=0$ and $prob\left(\{\delta_i,\delta_j\}=\{\delta,\delta\},s_j=0\right)=\frac{\alpha(1-\alpha)}{4}$,

(2) $\{\delta_i,\delta_j\}=\{\delta,-\delta\},s_j=0$ and $prob\left(\{\delta_i,\delta_j\}=\{\delta,-\delta\},s_j=0\right)=\frac{\alpha(1-\alpha)}{4}$,

(3) $\{\delta_i,\delta_j\}=\{\delta,-\delta\},s_j=1$ and $prob\left(\{\delta_i,\delta_j\}=\{\delta,\delta\},s_j=1\right)=\frac{\alpha^2}{4}$.

Note, firm i's relevant equilibrium disclosure strategy is to disclose $m_i=\delta_i$ with probability 1 if $\{\delta_i,\delta_j\}=\{\delta,\delta\}$, and with probability ρ if $\{\delta_i,\delta_j\}=\{\delta,-\delta\}$.

Therefore,

$$prob\left(m_i=\delta_i,m_j=\phi\right)=\frac{\alpha\left(1-\alpha\right)}{4}+\frac{\alpha\left(1-\alpha\right)}{4}\rho+\frac{\alpha^2}{4}\rho=\frac{\alpha\left(1-\alpha+\rho\right)}{4},$$

and

$$prob\left(\{\delta_i,\delta_j\}=\{\delta,\delta\},s_j=0|m_i=\delta_i,m_j=\phi\right)=\tfrac{1-\alpha}{1-\alpha+\rho},$$
$$prob\left(\{\delta_i,\delta_j\}=\{\delta,-\delta\},s_j=0|m_i=\delta_i,m_j=\phi\right)=\tfrac{(1-\alpha)\rho}{1-\alpha+\rho},$$
$$prob\left(\{\delta_i,\delta_j\}=\{\delta,-\delta\},s_j=1|m_i=\delta_i,m_j=\phi\right)=\tfrac{\alpha\rho}{1-\alpha+\rho}.$$

Therefore, the consumers' ex-post beliefs about δ_j can be written as

$$E\left(\delta_j\right)=\frac{1-\alpha}{1-\alpha+\rho}\delta+\frac{\left(1-\alpha\right)\rho}{1-\alpha+\rho}\left(-\delta\right)+\frac{\alpha\rho}{1-\alpha+\rho}\left(-\delta\right)=\frac{1-\alpha-\rho}{1-\alpha+\rho}\delta.$$

(c) Suppose $m_i=\phi$ and $m_j=\phi$. Note both firms will be silent if one of following three situations happens. First, $s_i=s_j=0$, which can happen with probability $\left(1-\alpha\right)^2$. Second, $s_i=0$, $s_j=1$, and $\{\delta_i,\delta_j\}=\{\delta,-\delta\}$, which can happen with probability $\frac{\alpha(1-\alpha)}{4}$. Third, $s_i=1$, $\{\delta_i,\delta_j\}=\{-\delta,\delta\}$, and $s_j=0$, which can also happen with probability $\frac{\alpha(1-\alpha)}{4}$. Following the same procedure as parts (a) and (b) above, it is straightforward to show that, in this case, along the equilibrium path, consumers' ex-post beliefs are $E\left(\delta_i\right)=E\left(\delta_j\right)=0$.

Proof of Proposition 2

(1) Suppose $s_i = 1$ and $\{\delta_i, \delta_j\} = \{\delta, \delta\}$. In this case, firm i's payoff from disclosing its own positive valuation draw $m_i = \delta_i = \delta$ is given by equation (2) in the text. Firm i may deviate by disclosing its rival's draw or by staying silent. Clearly, firm i's deviation to $m_i = \delta_j$ is not profitable and this would result in consumers' worst possible off-equilibrium belief about δ_i to be $-\delta$ leading to the lowest possible payoffs for firm i.'s

Suppose firm i deviates and stays silent $m_i = \phi$. If firm j is informed (with probability α), $m_j = \delta_j$, and therefore $E(\delta_i) = \frac{1-\alpha-\rho}{1-\alpha+\rho}$. However, if firm j is uninformed (with probability $1-\alpha$), $m_j = \phi$, and therefore $E(\delta_i) = E(\delta_j) = 0$. Firm i's payoff in this deviation is

$$\pi_i^{D2} = \alpha \frac{e^{v+\frac{1-\alpha-\rho}{1-\alpha+\rho}\delta}}{e^{v+\frac{1-\alpha-\rho}{1-\alpha+\rho}\delta} + e^{v+\delta} + e^{v_0}} + (1-\alpha)\frac{e^v}{e^v + e^v + e^{v_0}}.$$

A comparison of π_i^{D2} with the equilibrium payoff given in equation (2) reveals the deviation is not more profitable. Therefore, in the equilibrium, an informed firm i discloses $m_i = \delta_i = \delta$ if $\{\delta_i, \delta_j\} = \{\delta, \delta\}$.

(2) Next, suppose $s_i = 1$ and $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$. Firm i's strategy could be to (i) disclose its rival's negative draw $m_i = \delta_j$ (i.e., $\rho = 0$), (ii) disclose its own draw with probability $\rho \in (0, 1)$, or (iii) disclose own positive draw $m_i = \delta_i$ (i.e., $\rho = 1$). Below, we examine each of the three possible strategies.

(i) If $\rho = 0$, consumers' ex-post beliefs simplify to $E(\delta_i | m_i = \delta_j, m_j = \phi) = \frac{\alpha}{2-\alpha}\delta$ and $E(\delta_j | m_i = \delta_i, m_j = \phi) = \delta$. If both firms are silent, expected draws for both firms are zero. Firm i's payoff corresponding to $\rho = 0$ is given by

$$\pi_i = \frac{e^{v+\frac{\alpha}{2-\alpha}\delta}}{e^{v+\frac{\alpha}{2-\alpha}\delta} + e^{v-\delta} + e^{v_0}}. \tag{4}$$

Suppose firm i deviates by $m_i = \delta_i$ (i.e., $\rho = 1$). Firm i's profit under this deviation

$$\pi_i^{D3} = \frac{e^{v+\delta}}{e^{v+\delta} + e^{v+\delta} + e^{v_0}}. \tag{5}$$

However, if firm i deviates by staying silent,

$$\pi_i^{D4} = \frac{e^v}{e^v + e^v + e^{v_0}}, \tag{6}$$

and if firm i deviates by playing a mixed strategy $\rho \in (0, 1)$, firm i's profit $\pi_i^{D5} = (1-\rho)\pi_i + \rho\pi_i^{D3}$. Because $\pi_i^{D3} > \pi_i^{D4}$, for the existence, it suffices to show that $\pi_i > \pi_i^{D3}$ or

$$\frac{e^{v+\frac{\alpha}{2-\alpha}\delta}}{e^{v+\frac{\alpha}{2-\alpha}\delta} + e^{v-\delta} + e^{v_0}} > \frac{e^{v+\delta}}{e^{v+\delta} + e^{v+\delta} + e^{v_0}}.$$

Simplifying the above inequality, we get

$$v - v_0 > ln\left(\frac{e^{\frac{-\alpha\delta}{2-\alpha}} - e^{-\delta}}{1 - e^{\frac{-2\delta}{2-\alpha}}}\right). \tag{7}$$

Therefore, if the above condition is satisfied, disclosing the rival's negative draw in the $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$ state can constitute an equilibrium.

(ii) Next, we check if firm i disclosing its own draw with probability $\rho \in (0, 1)$ can be part of the equilibrium. The indifference condition is given by

$$\frac{e^{v+\delta}}{e^{v+\delta} + e^{v+\frac{1-\alpha-\rho}{1-\alpha+\rho}\delta} + e^{v_0}} = \frac{e^{v+\frac{\alpha-\rho}{2-\alpha-\rho}\delta}}{e^{v+\frac{\alpha-\rho}{2-\alpha-\rho}\delta} + e^{v-\delta} + e^{v_0}}.$$

The above equation can be satisfied only if $\rho < \frac{1}{2}$. Also, the mixed-strategy equilibrium exists in the space where $\rho = 0$ is an equilibrium.

(iii) It is straightforward to show no profitable deviations exist for firm i from $\rho = 1$. However, we do not present the proof here, because in part (4) below, we establish that $\rho = 1$ cannot be part of the equilibrium.

(3) Next, suppose $s_i = 1$ and $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$. Firm i's strategy is $m_i = \phi$.

Firm i can deviate from staying silent by disclosing its own negative draw, disclosing its rival's positive draw, and mixing between the two. However, because in all of these deviations, consumers will know firm i has deviated and will assign the worst possible out-of-equilibrium beliefs for undisclosed draws, all the deviations result in the same profit of

$$\pi_i^{D6} = \frac{e^{v-\delta}}{e^{v-\delta} + e^{v+\delta} + e^{v_0}}.$$

Because this deviation profit is the lowest possible profit firm i can make in this setup, a deviation cannot be more profitable regardless of what profits firm i makes in this state along the equilibrium path.

(4) Last, suppose $s_i = 1$ and $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$. Firm i's strategy is $m_i = \delta_j$. Firm j will be informed with probability α (in which case, $m_j = \delta_i$) and will be uninformed with probability $1 - \alpha$ (in which case, $m_j = \phi$). Therefore, firm i's profit is

$$\pi_i = \alpha\frac{e^{v-\delta}}{e^{v-\delta} + e^{v-\delta} + e^{v_0}} + (1-\alpha)\frac{e^{v+\frac{\alpha-\rho}{2-\alpha-\rho}\delta}}{e^{v+\frac{\alpha-\rho}{2-\alpha-\rho}\delta} + e^{v-\delta} + e^{v_0}}.$$

It is trivial that deviating and disclosing $m_i = \delta_i = -\delta$ cannot be more profitable. Firm i may also deviate by staying silent. Firm i's profit under this deviation is given by

$$\pi_i^{D7} = \alpha\frac{e^{v-\delta}}{e^{v-\delta} + e^{v+\frac{\alpha-\rho}{2-\alpha-\rho}\delta} + e^{v_0}} + (1-\alpha)\frac{e^{v}}{e^{v} + e^{v} + e^{v_0}}.$$

A comparison of firm i's profits along the equilibrium path and under deviation to staying silent reveals that for $\rho = 1$, $\pi_i < \pi_i^{D7}$. However, for $\rho \in \left[0, \frac{1}{2}\right)$, $\pi_i > \pi_i^{D7}$.

From parts $(1)-(4)$ above, we establish the following negative-communication equilibrium.

If $v - v_0 > ln\left(\frac{e^{\frac{-\alpha\delta}{2-\alpha}} - e^{-\delta}}{1-e^{\frac{-2\delta}{2-\alpha}}}\right)$, a negative-communication equilibrium exists in which, if $s_i = 1$ and (1) $\{\delta_i, \delta_j\} = \{\delta, \delta\}$, $m_i = \delta_i$, (2) $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$, $m_i = \delta_j$, (3) $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$, $m_i = \phi$, and (4) $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, $m_i = \delta_j$.

A mixed-strategy negative-communication equilibrium also exists in which firm i discloses its own positive valuation draw with probability $\rho \in \left(0, \frac{1}{2}\right)$.

The existence condition in equation (6) can be written as

$$v_0 < \bar{v}_0 \equiv v - ln\left(\frac{e^{\frac{-\alpha\delta}{2-\alpha}} - e^{-\delta}}{1 - e^{\frac{-2\delta}{2-\alpha}}}\right).$$

Also, $\bar{v}_0|_{\alpha=0} = v$ and $\frac{\partial \bar{v}_0}{\partial \alpha} > 0 \, \forall \alpha \in (0, 1)$.

Proof of Lemma 4

(a) Suppose $m_i = \delta_j = -\delta$ and $m_j = \phi$. These disclosures are possible only if $s_i = 1$ and $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$. If $\rho' = 1$, the disclosure confirms an out-of-equilibrium action and results in $E(\delta_i) = -\delta$. However, if $\rho' \neq 1$, the disclosure can be made along the equilibrium path and $E(\delta_i) = \delta$.

(b) Suppose $m_i = \delta_i = \delta$ and $m_j = \phi$. Because the proof proceeds in the manner as the proof of Lemma 3 part (b), we only provide the result here. Consumers' ex-post belief about firm j's undisclosed draw is given by

$$E(\delta_j) = \frac{1 - \alpha - \rho'}{1 - \alpha + \rho'}\delta.$$

(c) Suppose $m_i = m_j = \phi$. The proof is similar to Lemma 3 part (c). Consumers' ex-post belief about undisclosed draws is given by

$$E(\delta_i) = E(\delta_j) = \frac{-\alpha}{2 - \alpha}\delta.$$

Proof of Proposition 3

First, we establish that $\rho' = 1$ constitutes an equilibrium and then show that a $\rho' \neq 1$ cannot be part of any equilibrium.

Suppose $\rho' = 1$.

If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{\delta, \delta\}$, $m_i = \delta_i$. In this case, firm i's profit

$$\pi_i = \alpha \frac{e^{\delta}}{e^{\delta} + e^{\delta} + e^{v_0 - v}} + (1-\alpha) \frac{e^{\delta}}{e^{\delta} + e^{\frac{-\alpha}{2-\alpha}\delta} + e^{v_0 - v}}.$$

Firm i may deviate by $m_i = \delta_j$ or by $m_i = \phi$. In both deviations, firm i makes a lower profit than equilibrium profits given above.

If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$, $m_i = \delta_i$. Also, $m_j = \phi$ regardless of whether firm j is informed. In this case, firm i's profit

$$\pi_i = \frac{e^{\delta}}{e^{\delta} + e^{\frac{-\alpha}{2-\alpha}\delta} + e^{v_0 - v}}.$$

Firm i may deviate by revealing $m_i = \delta_j$, $m_i = \phi$, or by disclosing $m_i = \delta_i$ with probability $\rho' \in (0, 1)$. It is straightforward to show that in none of the three possible deviations is firm i better off.

If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$, it is trivial to show $m_i = \phi$ is an equilibrium strategy. No profitable deviations exist.

If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, firm i stays silent. In this case, firm i's profit

$$\pi_i = \frac{e^{\frac{-\alpha}{2-\alpha}\delta}}{e^{\frac{-\alpha}{2-\alpha}\delta} + e^{\frac{-\alpha}{2-\alpha}\delta} + e^{v_0 - v}}.$$

Firm i may deviate by revealing $m_i = \delta_i = -\delta$ or $m_i = \delta_j = -\delta$. Because $m_i = \delta_i = -\delta$ induces out-of-equilibrium belief, profit is lower under this deviation. If $m_i = \delta_j = -\delta$, $E(\delta_i) = -\delta$ and the deviation profit is $\frac{1}{2+e^{v_0-v+\delta}}$, which is lower than equilibrium π_i given above.

Therefore, the equilibrium specified in Proposition 3 exists.

Non-existence of $\rho' = 0$ equilibrium

Suppose $s_i = 1$ and $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$. In this case, both firms stay silent and make profit of $\frac{1}{2+e^{v_0-v+\delta}}$. Consider a deviation in which firm i reveals $m_i = \delta_j = -\delta$. Consumers' ex-post belief about δ_i is $E(\delta_i) = \delta$. Firm i's profit in this deviation is $\frac{1}{1+e^{-2\delta}+e^{v_0-v-\delta}}$, which is greater than $\frac{1}{2+e^{v_0-v+\delta}}$. Therefore, $\rho' = 0$ cannot be part of the equilibrium.

Non-existence of $\rho' \in (0, 1)$ equilibrium

The indifference condition

$$\frac{e^{\delta}}{e^{\delta} + e^{\frac{1-\alpha-\rho'}{1-\alpha+\rho'}\delta} + e^{v_0 - v}} = \frac{e^{\delta}}{e^{\delta} + e^{-\delta} + e^{v_0 - v}}$$

holds only for $\alpha = 1$ and cannot be satisfied for any $\alpha \in [0, 1)$.

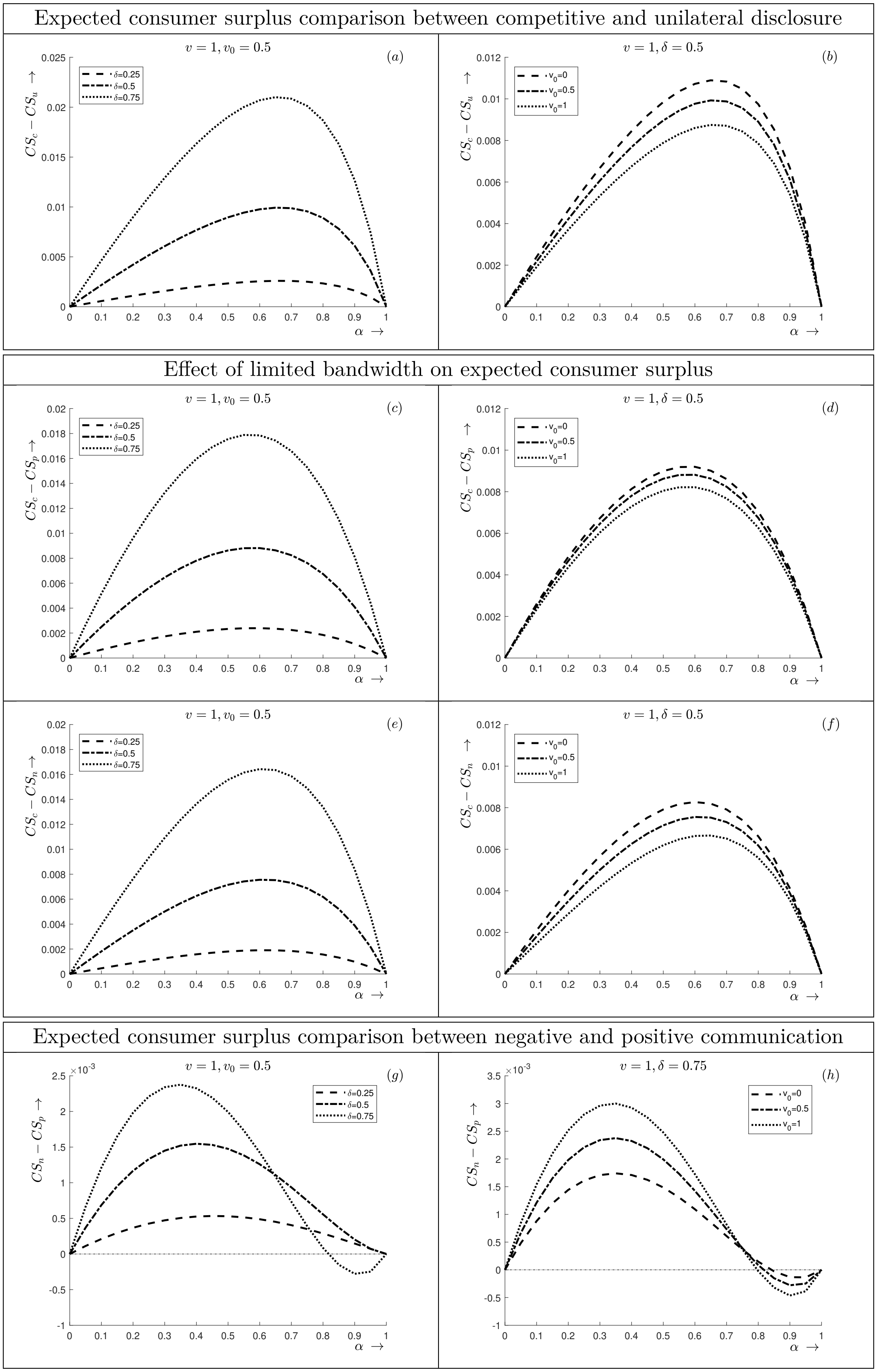


Figure 3: Consumer surplus comparisons (Additional details are in the Online Appendix.)

Proof of Proposition 4

If $|m_i| \leq 2$, from Lemma 2 (competitive information disclosure), the probability that firm i discloses information

$$prob\,(m_i \neq \phi) = \frac{\alpha}{2}.$$

Similarly, if $|m_i| \leq 1$, from Proposition 3 (positive-communication equilibrium), the probability that firm i discloses information

$$prob\,(m_i \neq \phi) = \frac{\alpha}{2}.$$

Note this probability is the same as the competitive information disclosure case without bandwidth limitation. Also, if $|m_i| \leq 1$, from Proposition 2 (negative-communication equilibrium), the probability of disclosing information

$$prob\,(m_i \neq \phi) = \frac{3\alpha}{4}.$$

Note this probability is higher than the probability of disclosure in the competitive information disclosure with $|m_i| \leq 2$.

Proof of Proposition 5

[**a**] Consider the candidate equilibrium in which firm i discloses $m_i = \{\delta_i, \delta_j\}$, when $s_i = 1$ and its own information is positive ($\delta_i = \delta$). Otherwise, $m_i = \phi$. Partial disclosure induces the out-of-equilibrium belief that undisclosed information is unfavorable to the disclosing firm. It is straightforward to show that if both firms are silent (i.e., if $m_i = m_j = \phi$), $E(\delta_i) = E(\delta_j) = \frac{-\alpha\delta}{2-\alpha}$.

Next, we establish the existence of this candidate equilibrium.

(i) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{\delta, \delta\}$, $m_i = \{\delta_i, \delta_j\}$ and $\pi_i = \frac{e^{v+\delta}+ke^{v_0}}{2e^{v+\delta}+e^{v_0}}$. If firm i deviates to $m_i^D = \phi$ its expected profit becomes $\pi_i^D = \alpha\left(\frac{e^{v+\delta}+ke^{v_0}}{2e^{v+\delta}+e^{v_0}}\right) + (1-\alpha)\left(\frac{e^{v-\frac{\alpha\delta}{2-\alpha}}+ke^{v_0}}{2e^{v-\frac{\alpha\delta}{2-\alpha}}+e^{v_0}}\right)$. A comparison of firm i's profits under equilibrium and deviation strategies reveals that $\pi_i \geq \pi_i^D$ if $k \leq \frac{1}{2}$.

(ii) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$, $m_i = \{\delta_i, \delta_j\}$ and there are no profitable deviations.

(iii) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$, $m_i = \phi$ and there are no profitable deviations.

(iv) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, $m_i = \phi$ and $\pi_i = \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}+ke^{v_0}}{2e^{v-\frac{\alpha\delta}{2-\alpha}}+e^{v_0}}$. If firm i deviates to $m_i^D = \{\delta_i, \delta_j\}$ its expected profit becomes $\pi_i^D = \frac{e^{v-\delta}+ke^{v_0}}{2e^{v-\delta}+e^{v_0}}$. A comparison of firm i's profits under equilibrium and deviation strategies reveals that $\pi_i \geq \pi_i^D$ if $k \leq \frac{1}{2}$.

Therefore, if $k \leq \frac{1}{2}$, then $m_i = \{\delta_i, \delta_j\}$ is an equilibrium strategy if firm i is informed and $\delta_i = \delta$.

[**b**] Next, consider the candidate equilibrium in which firm i discloses $m_i = \{\delta_i, \delta_j\}$, when $s_i = 1$ and the rival's information is negative ($\delta_j = -\delta$). Otherwise, $m_i = \phi$. As before, any partial information disclosure induces the out-of-equilibrium belief that undisclosed information is unfavorable for firm i. It is straightforward to show that if both firms are silent (i.e., if $m_i = m_j = \phi$), $E(\delta_i) = E(\delta_j) = \frac{\alpha\delta}{2-\alpha}$. Next, we establish the existence of this candidate equilibrium.

(i) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{\delta, \delta\}$, $m_i = \phi$ and $\pi_i = \frac{e^{v+\frac{\alpha\delta}{2-\alpha}}+ke^{v_0}}{2e^{v+\frac{\alpha\delta}{2-\alpha}}+e^{v_0}}$. If firm i deviates to $m_i^D = \{\delta_i, \delta_j\}$ its expected profit is given by $\pi_i^D = \frac{e^{v+\delta}+ke^{v_0}}{2e^{v+\delta}+e^{v_0}}$. A comparison of π_i and π_i^D reveals that $\pi_i \geq \pi_i^D$ if $k \geq \frac{1}{2}$.

(ii) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$, $m_i = \{\delta_i, \delta_j\}$ and there are no profitable deviations.

(iii) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$, $m_i = \phi$ and there are no profitable deviations.

(iv) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, $m_i = \{\delta_i, \delta_j\}$ and $\pi_i = \frac{e^{v-\delta}+ke^{v_0}}{2e^{v-\delta}+e^{v_0}}$. If firm i deviates to $m_i^D = \{\delta_i, \delta_j\}$ its expected profit $\pi_i^D = \alpha\left(\frac{e^{v-\delta}+ke^{v_0}}{2e^{v-\delta}+e^{v_0}}\right) + (1-\alpha)\left(\frac{e^{v-\frac{\alpha\delta}{2-\alpha}}+ke^{v_0}}{2e^{v-\frac{\alpha\delta}{2-\alpha}}+e^{v_0}}\right)$. A comparison of firm i's profits under equilibrium and deviation strategies reveals that $\pi_i \geq \pi_i^D$ if $k \geq \frac{1}{2}$.

Therefore, if $k \geq \frac{1}{2}$, then $m_i = \{\delta_i, \delta_j\}$ is an equilibrium strategy if firm i is informed and $\delta_j = -\delta$. Otherwise, it stays silent.

Two other equilibria (one in which the informed firm i discloses both draws except in the $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$ state and the other in which the informed firm i stays silent except in the $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$ state) also exist in the knife-edge case of $k = \frac{1}{2}$.

Proof of Proposition 6

Because proof is similar to that of Lemmas 3 and 4, and Propositions 2 and 3, we provide only its sketch.

[a] Consider the candidate positive-communication equilibrium strategies (under limited communication bandwidth, $|m_i| = 1$) specified in Proposition 6 [a]. It is straightforward to derive consumers' beliefs of undisclosed valuation draws:

(a) $E(\delta_i | m_i = \delta_j = -\delta, m_j = \phi) = -\delta$,

(b) $E(\delta_j | m_i = \delta_i = \delta, m_j = \phi) = -\frac{\alpha\delta}{2-\alpha}$,

(c) $E(\delta_i | m_i = \phi, m_j = \phi) = E(\delta_j | m_i = \phi, m_j = \phi) = -\frac{\alpha\delta}{2-\alpha}$.

Next, we establish the existence of this candidate equilibrium.

(i) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{\delta, \delta\}$, $m_i = \delta_i$ and there are no profitable deviations.

(ii) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$, $m_i = \delta_i$ and $\pi_i = \frac{e^{v+\delta}+ke^{v_0}}{e^{v+\delta}+e^{v-\frac{\alpha\delta}{2-\alpha}}+e^{v_0}}$. Firm i can deviate by disclosing δ_j or by staying silent. If firm i deviates by disclosing δ_j its expected profit is $\pi_i^D = \frac{e^{v-\delta}+ke^{v_0}}{2e^{v-\delta}+e^{v_0}}$. A comparison of firm i's profits under equilibrium ($m_i = \delta_i$) and deviation ($m_i = \delta_j$) strategies reveals that $\pi_i \geq \pi_i^D$ if $v_0 \leq v + ln\left(\frac{1-e^{\frac{-2\delta}{2-\alpha}}}{-e^{\delta}+e^{-\delta}+k\left(-2e^{-\delta}+e^{\delta}+e^{\frac{-\alpha\delta}{2-\alpha}}\right)}\right)$. It is useful to note that the condition is always satisfied if $k \leq \frac{1}{2}$. The deviation to staying silent is not more profitable.

(iii) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$, $m_i = \phi$ and there are no profitable deviations.

(iv) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, $m_i = \phi$ and $\pi_i = \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}+ke^{v_0}}{2e^{v-\frac{\alpha\delta}{2-\alpha}}+e^{v_0}}$. A deviation to disclose $m_i = \delta_i$ induces unfavorable out-of-equilibrium beliefs and is therefore not desirable. If firm i deviates to $m_i^D = \delta_j$ its expected profit $\pi_i^D = \frac{e^{v-\delta}+ke^{v_0}}{2e^{v-\delta}+e^{v_0}}$. A comparison of firm i's profits under equilibrium and deviation to $m_i^D = \delta_j$ reveals that $\pi_i \geq \pi_i^D$ if $k \leq \frac{1}{2}$.

Therefore, if $k \leq \frac{1}{2}$, a positive communication equilibrium with the disclosure strategies specified in

Proposition 6 [a] exists.

[b] Next, consider the candidate negative-communication equilibrium strategies (with $|m_i| = 1$) specified in Proposition 6 [b]. It is straightforward to derive consumers' beliefs of undisclosed valuation draws:

(a) $E(\delta_i | m_i = \delta_j = -\delta, m_j = \phi) = \frac{\alpha\delta}{2-\alpha}$,

(b) $E(\delta_j | m_i = \delta_i = \delta, m_j = \phi) = \delta$,

(c) $E(\delta_i | m_i = \phi, m_j = \phi) = E(\delta_j | m_i = \phi, m_j = \phi) = 0$.

Next, we establish the existence of this candidate equilibrium.

(i) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{\delta, \delta\}$, $m_i = \delta_i$ and $\pi_i = \frac{e^{v+\frac{\alpha\delta}{2-\alpha}} + ke^{v_0}}{e^{v+\frac{\alpha\delta}{2-\alpha}} + e^{v-\delta} + e^{v_0}}$. A deviation to disclose $m_i = \delta_j$ induces unfavorable out-of-equilibrium beliefs and is therefore not desirable. A deviation to staying silent leads to $\pi_i^D = \alpha\left(\frac{e^{v+\delta}+ke^{v_0}}{2e^{v+\delta}+e^{v_0}}\right) + (1-\alpha)\left(\frac{e^{v}+ke^{v_0}}{2e^{v}+e^{v_0}}\right)$. A comparison of firm i's profits under equilibrium and deviation to $m_i^D = \phi$ reveals that $\pi_i \geq \pi_i^D$ if $k \leq \frac{1}{2}$.

(ii) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$, $m_i = \delta_j$ and $\pi_i = \frac{e^{v+\delta}+ke^{v_0}}{e^{v+\delta} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}$. Firm i can deviate by disclosing $m_i^{D1} = \delta_i$ and make $\pi_i^{D1} = \frac{e^{v+\delta}+ke^{v_0}}{2e^{v+\delta}+e^{v_0}}$. A comparison of firm i's profits under equilibrium ($m_i = \delta_j$) and deviation ($m_i^{D1} = \delta_i$) strategies reveals that $\pi_i \geq \pi_i^{D1}$ if $v_0 \leq v + \hat{v}_k$,where $\hat{v}_k \equiv ln\left(\frac{1-e^{\frac{-2\delta}{2-\alpha}}}{e^{\frac{-\alpha\delta}{2-\alpha}} - e^{\delta} + k\left(-2e^{\frac{-\alpha\delta}{2-\alpha}} + e^{-\delta} + e^{-2\delta-\frac{-\alpha\delta}{2-\alpha}}\right)}\right)$. It is useful to note that the condition is always satisfied if $k \geq \frac{e^{\frac{-\alpha\delta}{2-\alpha}} - e^{-\delta}}{2e^{\frac{-\alpha\delta}{2-\alpha}} - e^{-\delta} - e^{-2\delta-\frac{-\alpha\delta}{2-\alpha}}}$. In addition, $\frac{e^{\frac{-\alpha\delta}{2-\alpha}} - e^{-\delta}}{-2e^{\frac{-\alpha\delta}{2-\alpha}} + e^{-\delta} + e^{-2\delta-\frac{-\alpha\delta}{2-\alpha}}} < \frac{1}{2}$.

Firm i can also deviate by staying silent $m_i^{D2} = \phi$ and make $\pi_i^{D2} = \frac{e^{v}+ke^{v_0}}{2e^{v}+e^{v_0}}$. However, because $\pi_i^{D1} > \pi_i^{D2}$ given $k \leq \frac{1}{2}$ (which is a requirement for equilibrium from case (1) above), this deviation does not result in higher profits if $\pi_i \geq \pi_i^{D1}$.

(iii) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$, $m_i = \phi$ and there are no profitable deviations.

(iv) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, $m_i = \delta_j$ and there are no profitable deviations.

Therefore, if $k \leq \frac{1}{2}$ and $v_0 \leq v + \hat{v}_k$, a negative communication equilibrium with the disclosure strategies specified in Proposition 6 [b] exists.

[c] Finally, consider the candidate "extreme" negative-communication equilibrium strategies (with $|m_i| = 1$) specified in Proposition 6 [c]. It is straightforward to derive consumers' beliefs of undisclosed valuation draws:

(a) $E(\delta_i | m_i = \delta_j = -\delta, m_j = \phi) = \frac{\alpha\delta}{2-\alpha}$,

(b) $E(\delta_j | m_i = \delta_i = \delta, m_j = \phi) = \delta$ (out-of-equilibrium belief, by assumption),

(c) $E(\delta_i | m_i = \phi, m_j = \phi) = E(\delta_j | m_i = \phi, m_j = \phi) = \frac{\delta}{3-2\alpha}$.

Next, we establish the existence of this candidate equilibrium.

(i) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{\delta, \delta\}$, $m_i = \phi$ and $\pi_i = \frac{e^{v+\frac{\delta}{3-2\alpha}} + ke^{v_0}}{2e^{v+\frac{\delta}{3-2\alpha}} + e^{v_0}}$. If the firm deviates to $m_i^D = \delta_i$, $\pi_i^D = \frac{e^{v+\delta}+ke^{v_0}}{2e^{v+\delta}+e^{v_0}}$. In this case, $\pi_i \geq \pi_i^D$ if $k \geq \frac{1}{2}$. Firm i may also deviate by disclosing rival's positive information. However, because this deviation induces unfavorable out-of-equilibrium belief it is not desirable.

(ii) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$, $m_i = \delta_j$ and $\pi_i = \frac{e^{v+\frac{\alpha\delta}{2-\alpha}} + ke^{v_0}}{e^{v+\frac{\alpha\delta}{2-\alpha}} + e^{v-\delta} + e^{v_0}}$. It is straightforward to show that if $k \geq \frac{1}{2}$, both deviations ($m_i^{D1} = \delta_i$ and $m_i^{D2} = \delta_j$) result in profits that are lower than π_i.

(iii) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$, $m_i = \phi$ and there are no profitable deviations.

(iv) If $s_i = 1$ and $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, $m_i = \delta_j$ and $\pi_i = \alpha\left(\frac{e^{v-\delta}+ke^{v_0}}{2e^{v-\delta}+e^{v_0}}\right) + (1-\alpha)\left(\frac{e^{v+\frac{\alpha\delta}{2-\alpha}}+ke^{v_0}}{e^{v+\frac{\alpha\delta}{2-\alpha}}+e^{v-\delta}+e^{v_0}}\right)$. It is straightforward to show that if $k \geq \frac{1}{2}$, the deviation to staying silent results in lower profits. In addition, a deviation to $m_i^D = \delta_i$ induces unfavorable out-of-equilibrium belief and therefore is not desirable.

Therefore, if $k \geq \frac{1}{2}$, an "extreme" negative-communication equilibrium with the disclosure strategies specified in Proposition 6 [c] exists.

Proof of Proposition 7

Because the proof is similar to that of Lemma 2, Lemma 3, and Proposition 2, we only present formal statements of the equilibria here.

No Bandwidth Limitation ($|m_i| \leq 2$): If $s_i = s_j = 1$ and

(1) $\{\delta_i, \delta_j\} = \{\delta, \delta\}$, $m_i = m_j = \{\delta_i, \delta_j\}$,

(2) $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$, $m_i = \{\delta_i, \delta_j\}$, $m_j = \phi$,

(3) $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$, $m_i = \phi$, $m_j = \{\delta_i, \delta_j\}$, and

(4) $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, $m_i = m_j = \phi$ or $m_i = m_j = \{\delta_i, \delta_j\}$.

Bandwidth Limitation ($|m_i| \leq 1$): If $s_i = s_j = 1$ and

(1) $\{\delta_i, \delta_j\} = \{\delta, \delta\}$, $m_i = \delta_i$,

(2) $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$, firm i discloses own δ_i with probability $\hat{\rho} \in [0, 1]$ and rival's δ_j with probability $1 - \hat{\rho}$,

(3) $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$, $m_i = \phi$, and

(4) $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, $m_i = \delta_j$.

Firm j's strategies are symmetric.

As usual, in both $|m_i| \leq 1$ and $|m_i| \leq 2$ cases, an uninformed firm stays silent.

In the $|m_i| \leq 2$ case, consumers' ex-post beliefs may be different from true realized draws. In other words, all the information may not be transmitted to consumers. However, in the $|m_i| \leq 1$ case, either both draws are directly disclosed or $E(\delta_i)$ and $E(\delta_j)$ are exactly same as realized draws.

Equilibrium and Proof of Results Presented in Section 5.2

The proof is similar to that of Lemmas 1-3 and Propositions 1-4 presented above. We present formal statements of equilibria and a sketch of the proof here.

If firm i is uninformed, it stays silent. However, if it is partially informed about the state of the world (i.e., knows only δ_i), it discloses $m_i = \delta_i$ if $\delta_i = \delta$ and $m_i = \phi$ if $\delta_i = -\delta$.

If firm i is informed about both valuation draws (δ_i and δ_j), the equilibrium disclosure strategy depends of whether it is facing a strategic rival and if the disclosure bandwidth is limited. We describe a fully informed firm i's equilibrium strategies and consumer beliefs in various cases of interest below.

Unilateral Information Disclosure In the equilibrium, if $s_i = 1$ and (1) $\{\delta_i, \delta_j\} = \{\delta, \delta\}$, $m_i = \delta_i$, (2) $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$, $m_i = \{\delta_i, \delta_j\}$, (3) $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$, $m_i = \phi$, and (4) $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, $m_i = \{\delta_i, \delta_j\}$ if $v_0 < \hat{v}_0'$ and $m_i = \phi$ if $v_0 > \tilde{v}_0'$, where $\hat{v}_0' = v + ln\left[\left(e^{\frac{\alpha\delta}{4-3\alpha-2\gamma}} - e^{\frac{-(\alpha+2\gamma)}{4-3\alpha-2\gamma}}\right)/\left(e^{\frac{4(1-\alpha-\gamma)\delta}{4-3\alpha-2\gamma}} - 1\right)\right]$ and $\tilde{v}_0' = v + ln\left[\left(1 - e^{\frac{-(\alpha+\gamma)\delta}{2-\alpha-\gamma}}\right)/\left(e^{\frac{2\delta}{2-\alpha-\gamma}} - 1\right)\right]$. Firm j's strategies are symmetric. Consumers update their beliefs as follows. If $m_i = \delta_i = \delta$ and $m_j = \phi$, $E(\delta_j) = \frac{\alpha\delta}{\alpha+2\gamma}$; if $m_i = \delta_j = -\delta$ and $m_j = \phi$, $E(\delta_i) = -\delta$ (out-of-equilibrium belief, by assumption); and if $m_i = m_j = \phi$, $-\delta < E(\delta_i) < 0$ and $\delta > E(\delta_j) > 0$.

Competitive Information Disclosure ($|m_i| \leq 2$) In the equilibrium, if $s_i = 1$ and (1) $\{\delta_i, \delta_j\} = \{\delta, \delta\}$, $m_i = \delta_i$, (2) $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$, $m_i = \{\delta_i, \delta_j\}$, and (3) $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$ or $\{-\delta, -\delta\}$, $m_i = \phi$. Consumer beliefs: If $m_i = \delta_i = \delta$ and $m_j = \phi$, $0 < E(\delta_j) < \delta$; if $m_i = \delta_j = -\delta$ and $m_j = \phi$, $E(\delta_i) = -\delta$ (out-of-equilibrium belief, by assumption); and if $m_i = m_j = \phi$, $-\delta < E(\delta_i) = E(\delta_j) < 0$.

It is straightforward to see that a firm is less likely to disclose information in the competitive disclosure setting than in the unilateral disclosure setting. Proposition 1 continues to hold.

Competitive Information Disclosure ($|m_i| \leq 1$) Similar to the model analysis presented in section 3, in the presence of a limited bandwidth, a negative communication equilibrium (in which firms disclose their rival's negative information shock whenever possible) and a positive-communication equilibrium (in which firms disclose their own positive information shock whenever possible) exist.

If $v - v_0 > ln\left[\left(e^{\frac{-\alpha\delta}{2-\alpha}} - e^{-\delta}\right)/\left(e^{\delta_0-\delta} - e^{\frac{-2\delta}{2-\alpha}}\right)\right]$, where $\delta_0 = \frac{[\alpha(1-\alpha-\gamma)-(\alpha+\gamma)\gamma]\delta}{(1-\alpha-\gamma)(\alpha+2\gamma)+(\alpha+\gamma)\gamma}$, a negative-communication equilibrium exists in which, if firm i is informed and (1) $\{\delta_i, \delta_j\} = \{\delta, \delta\}$, $m_i = \delta_i$, (2) $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$, $m_i = \delta_j$, (3) $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$, $m_i = \phi$, and (4) $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, $m_i = \delta_j$. Consumers update their beliefs as follows. If $m_i = \delta_i = \delta$ and $m_j = \phi$, $E(\delta_j) = \frac{[\alpha(1-\alpha-\gamma)-(\alpha+\gamma)\gamma]\delta}{(1-\alpha-\gamma)(\alpha+2\gamma)+(\alpha+\gamma)\gamma}$; if $m_i = \delta_j = -\delta$ and $m_j = \phi$, $E(\delta_i) = \frac{\alpha\delta}{2-\alpha}$; and if $m_i = m_j = \phi$, $E(\delta_i) = E(\delta_j) = \frac{-\left[2\gamma(1-\alpha-\gamma)+\gamma^2\right]\delta}{[(2-\alpha)(1-\alpha-\gamma)+\gamma^2]} < 0$.

A positive-communication equilibrium also exists in which, if firm i is informed and (1) $\{\delta_i, \delta_j\} = \{\delta, \delta\}$ or $\{\delta, -\delta\}$, $m_i = \delta_i$, and (2) $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$ or $\{-\delta, -\delta\}$, $m_i = \phi$. Consumer beliefs: If $m_i = \delta_i = \delta$ and $m_j = \phi$, $E(\delta_j) < \delta$; if $m_i = \delta_j = -\delta$ and $m_j = \phi$, $E(\delta_i) = -\delta$ (out-of-equilibrium belief, by assumption); and if $m_i = m_j = \phi$, $E(\delta_i) = E(\delta_j) = \frac{-(\alpha+\gamma)\delta}{2-\alpha-\gamma} < 0$.

A comparison of equilibrium conditions with and without bandwidth limitation cases reveals that Proposition 4 continues to hold in this setup as well.

Equilibrium and Proof of Results Presented in Section 5.3

Here we present formal statements of results presented in Asymmetric Valuations section. Consumers' beliefs (and their proofs) for all the equilibria are same as those described in sections 2 and 3. Therefore, we do not repeat them here. Other proofs are similar to those for lemmas 1-2 and propositions 1-3. We only consider pure stategy equilibria. In all cases, if $s_i = 0$, $m_i = \phi$. In addition, strategies of firm j are symmetric.

Unilateral Information Disclosure: If $s_i = 1$ and the realized state (1) $\{\delta_i, \delta_j\} = \{\delta, \delta\}$, $m_i = \{\delta_i, \delta_j\}$, (2) $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$, $m_i = \{\delta_i, \delta_j\}$, (3) $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$, $m_i = \phi$, (4) $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, $m_i = \{\delta_i, \delta_j\}$

if $v_0 < v + ln\left(\frac{e^{-\triangle_v}\left(e^{\frac{2\alpha\delta}{4-3\alpha}}-1\right)}{e^{\delta}-e^{\frac{\alpha\delta}{4-3\alpha}}}\right)$, and $m_i = \phi$, otherwise. It is straightforward that the disclosure of $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$ state happens over a smaller region if $\triangle_v$ is larger.

Competitive Information Disclosure: The statement of equilibrium in the competitive information disclosure case is the same as Lemma 2.

Negative Communication: If $v_0 < v + ln\left(\frac{e^{-\triangle_v}\left(1-e^{\frac{-2\delta}{2-\alpha}}\right)}{e^{-\frac{\alpha\delta}{2-\alpha}}-e^{-\delta}}\right)$ a negative communication equilibrium exists in which, if $s_i = 1$ and the realized state (1) $\{\delta_i, \delta_j\} = \{\delta, \delta\}$, $m_i = \{\delta_i\}$, (2) $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$, $m_i = \{\delta_j\}$, (3) $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$, $m_i = \phi$, (4) $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, $m_i = \{\delta_j\}$. An increase in $\triangle_v$ makes firm i less likely to disclose $m_i = \{\delta_j\}$ in the $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$ state.

Positive Communication: The statement of equilibrium is the same as Proposition 3.

Incentives for Information Acquisition (Section 5.4)

We solve the normal-form game in Table 2.

Firm j

Firm i		$\alpha_j = 0$	$\alpha_j = \alpha$
	$\alpha_i = 0$	$\frac{e^v}{2e^v+e^{v_0}}$, $\frac{e^v}{2e^v+e^{v_0}}$	$\pi_i\|_{\alpha_i=0,\alpha_j=\alpha}$, $\pi_j\|_{\alpha_i=0,\alpha_j=\alpha}$
	$\alpha_i = \alpha$	$\pi_i\|_{\alpha_i=\alpha,\alpha_j=0}$, $\pi_j\|_{\alpha_i=\alpha,\alpha_j=0}$	$\pi_i\|_{\alpha_i=\alpha_j=\alpha}$, $\pi_j\|_{\alpha_i=\alpha_j=\alpha}$

Table 2: Information acquisition game

If both firms do not acquire information (i.e., $\alpha_i = \alpha_j = 0$), $E(\delta_i) = E(\delta_j) = 0$ and, therefore, $\pi_i = \pi_j = \frac{e^v}{2e^v+e^{v_0}}$.

Assume that v_0 is small enough such that an informed firm prefers to disclose information (than pretend ignorance) when the realized state is $\{\delta_i = -\delta, \delta_j = -\delta\}$.

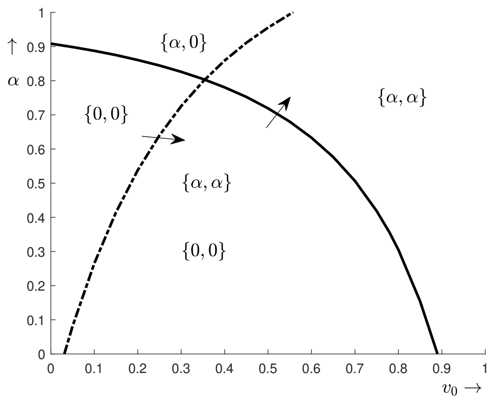
If one of the two firms acquires information (for example, if $\alpha_i = \alpha$ and $\alpha_j = 0$), the information-disclosure game is the one that is presented in the section 2.1 (Unilateral Information Disclosure). Following the proof of Lemma 1, the expected profit of firm i is given by $\pi_i|_{\alpha_i=\alpha,\alpha_j=0} = \frac{\alpha}{4}(\pi_{i1} + \pi_{i2} + \pi_{i3} + \pi_{i4}) + (1-\alpha)\pi_{i5}$, where $\pi_{i1} = \frac{e^{v+\delta}}{2e^{v+\delta}+e^{v_0}}$, $\pi_{i2} = \frac{e^{v+\delta}}{e^{v+\delta}+e^{v-\delta}+e^{v_0}}$, $\pi_{i3} = \frac{e^{v-\frac{\alpha\delta}{4-3\alpha}}}{e^{v-\frac{\alpha\delta}{4-3\alpha}}+e^{v+\frac{\alpha\delta}{4-3\alpha}}+e^{v_0}}$, $\pi_{i4} = \frac{e^{v-\delta}}{e^{v-\delta}+e^{v-\delta}+e^{v_0}}$, and $\pi_{i5} = \pi_{i3}$. The expected profit for firm j can be written as $\pi_j|_{\alpha_i=\alpha,\alpha_j=0} = \frac{\alpha}{4}(\pi_{j1} + \pi_{j2} + \pi_{j3} + \pi_{j4}) + (1-\alpha)\pi_{j5}$, where $\pi_{j1} = \frac{e^{v+\delta}}{2e^{v+\delta}+e^{v_0}}$, $\pi_{j2} = \frac{e^{v-\delta}}{e^{v-\delta}+e^{v+\delta}+e^{v_0}}$, $\pi_{j3} = \frac{e^{v+\frac{\alpha\delta}{4-3\alpha}}}{e^{v+\frac{\alpha\delta}{4-3\alpha}}+e^{v-\frac{\alpha\delta}{4-3\alpha}}+e^{v_0}}$, $\pi_{j4} = \frac{e^{v-\delta}}{e^{v-\delta}+e^{v-\delta}+e^{v_0}}$, and $\pi_{j5} = \pi_{j3}$. The expected profits for the case of $\alpha_i = 0$ and $\alpha_j = \alpha$ can be written in a similar way.

If both firms acquire information (i.e., $\alpha_i = \alpha_j = \alpha$), expected profits depend on whether firms engage in negative or positive communication.

Suppose firms engage in negative communication. Following the proof of results presented in section 3, we can write firm i's expected profit as $\pi_i^n|_{\alpha_i=\alpha_j=\alpha} = \frac{\alpha}{4}(\pi_{i1}^n + \pi_{i2}^n + \pi_{i3}^n + \pi_{i4}^n) + (1-\alpha)\pi_{i5}^n$, where $\pi_{i1}^n = \frac{e^{v+\delta}}{2e^{v+\delta}+e^{v_0}}$, $\pi_{i2}^n = \frac{e^{v+\frac{\alpha\delta}{2-\alpha}}}{e^{v+\frac{\alpha\delta}{2-\alpha}}+e^{v-\delta}+e^{v_0}}$, $\pi_{i3}^n = \frac{\alpha e^{v-\delta}}{e^{v-\delta}+e^{v+\frac{\alpha\delta}{2-\alpha}}+e^{v_0}} + \frac{(1-\alpha)e^{v}}{2e^{v}+e^{v_0}}$, $\pi_{i4}^n = \frac{\alpha e^{v-\delta}}{2e^{v-\delta}+e^{v_0}} + \frac{(1-\alpha)e^{v+\frac{\alpha\delta}{2-\alpha}}}{e^{v+\frac{\alpha\delta}{2-\alpha}}+e^{v-\delta}+e^{v_0}}$, and $\pi_{i5}^n = \frac{\alpha}{4}\left(\frac{e^{v+\delta}}{2e^{v+\delta}+e^{v_0}} + \frac{2e^{v-\delta}}{e^{v-\delta}+e^{v+\frac{\alpha}{2-\alpha}\delta}+e^{v_0}}\right) + \left(1-\frac{3\alpha}{4}\right)\frac{e^v}{2e^v+e^{v_0}}$. In addition, $\pi_j^n|_{\alpha_i=\alpha_j=\alpha} = \pi_i^n|_{\alpha_i=\alpha_j=\alpha}$.

Similarly, if firms engage in positive communication, following the proof of proposition 3, firm i's expected profit is given by $\pi_i^p|_{\alpha_i=\alpha_j=\alpha} = \frac{\alpha}{4}\left(\pi_{i1}^p + \pi_{i2}^p + \pi_{i3}^p + \pi_{i4}^p\right) + (1-\alpha)\,\pi_{i5}^p$, where $\pi_{i1}^p = \frac{\alpha e^{v+\delta}}{2e^{v+\delta}+e^{v_0}} + \frac{(1-\alpha)e^{v+\delta}}{e^{v+\delta}+e^{v-\frac{\alpha\delta}{2-\alpha}}+e^{v_0}}$, $\pi_{i2}^p = \frac{e^{v+\delta}}{e^{v+\delta}+e^{v-\frac{\alpha\delta}{2-\alpha}}+e^{v_0}}$, $\pi_{i3}^p = \frac{\alpha e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}}+e^{v+\delta}+e^{v_0}} + \frac{(1-\alpha)e^{v-\frac{\alpha\delta}{2-\alpha}}}{2e^{v-\frac{\alpha\delta}{2-\alpha}}+e^{v_0}}$, $\pi_{i4}^p = \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{2e^{v-\frac{\alpha\delta}{2-\alpha}}+e^{v_0}}$, and $\pi_{i5}^p = \frac{\alpha}{2}\left(\frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}}+e^{v+\delta}+e^{v_0}}\right) + \left(1-\frac{\alpha}{2}\right)\frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{2e^{v-\frac{\alpha\delta}{2-\alpha}}+e^{v_0}}$. In addition, $\pi_j^p|_{\alpha_i=\alpha_j=\alpha} = \pi_i^p|_{\alpha_i=\alpha_j=\alpha}$.

Because analytical comparisons of the expected-profit expressions are intractable, we derive the equilibrium of the game numerically. We fix $v = 1$ and vary the remaining parameters (v_0, α, and δ). We look for pure strategy equilibria in which i) both firms acquire information, i.e., the $\{\alpha, \alpha\}$ equilibrium, (2) only one firm acquires information, i.e., the $\{\alpha, 0\}$ equilibrium, and (3) no firm acquires information, i.e., the $\{0, 0\}$ equilibrium, and the parameter values where they exist. Figure 4 indicates the parameter space in which different equilibria exist.



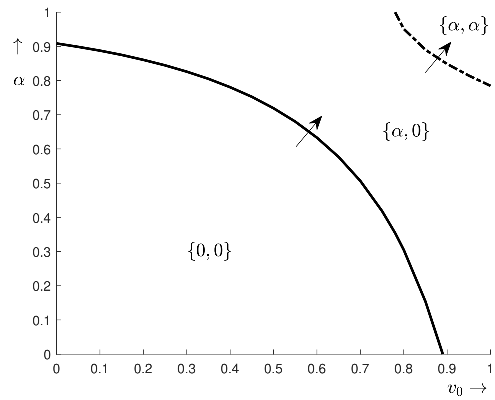


Figure 4: Information acquisition equilibria for positive (left) and negative (right) communication: $\{0,\, 0\}$, $\{0,\, \alpha\}$, $\{\alpha,\, \alpha\}$ represent investment in information acquisition by no, one, and both firms, respectively. Arrows indicate the direction in which curves move with increase in δ.

Figure 4 indicates the following insights. (1) Firms have higher incentives to acquire information in the positive communication equilibrium than in the negative communication equilibrium. (2) Competing firms are more likely to acquire information, if v_0 is large. They are also more likely to acquire information, if δ is small.

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Online Appendix for "Persuasion Contest: Disclosing Own and Rival Information"

Consumer Information and Welfare (Section 3.4)

Expanded consumer-surplus expressions for different equilibria

As shown in section 3.4, the expected consumer surplus is given by

$$CS_m = E_{\{\delta_i,\delta_j\}}\left[E_{\{s_i,s_j\}}\left(E\left(U_{i^*}\right)\right)\right].$$

Here, we provide expressions for consumer utilities for all possible realizations of state $\{\delta_i, \delta_j\}$ and firm information $\{s_i, s_j\}$. Recall, given firms are independently informed about the realized state with probability α, (1) $s_i = 1$ and $s_j = 1$ with probability α^2, (2) $s_i = 1$ and $s_j = 0$ with probability $\alpha(1-\alpha)$, (3) $s_i = 0$ and $s_j = 1$ with probability $\alpha(1-\alpha)$, and (4) $s_i = 0$ and $s_j = 0$ with probability $(1-\alpha)^2$. In addition, the probability that a particular $\{\delta_i, \delta_j\}$ state is realized is $\frac{1}{4}$. We take the expectation over all these realizations to calculate the expected consumer surplus for each of the four equilibria.

1. Unilateral communication

Consider the unilateral disclosure equilibrium.

(1) If $\{\delta_i, \delta_j\} = \{\delta, \delta\}$ and $s_i = 1$, consumer utility is

$$U_1 = \ln\left[e^{v+\delta} + e^{v+\delta} + e^{v_0}\right].$$

Note, in this case $d_i = d_j = 0$. In addition, d_0 is always zero.

(2) If $\{\delta_i, \delta_j\} = \{\delta, \delta\}$ and $s_i = 0$, consumer utility is

$$\begin{aligned} U_2 &= \ln\left[e^{v-\frac{\alpha\delta}{4-3\alpha}} + e^{v+\frac{\alpha\delta}{4-3\alpha}} + e^{v_0}\right] \\ &\quad + \frac{e^{v-\frac{\alpha\delta}{4-3\alpha}}}{e^{v-\frac{\alpha\delta}{4-3\alpha}} + e^{v+\frac{\alpha\delta}{4-3\alpha}} + e^{v_0}}\left(\delta + \frac{\alpha\delta}{4-3\alpha}\right) + \frac{e^{v+\frac{\alpha\delta}{4-3\alpha}}}{e^{v-\frac{\alpha\delta}{4-3\alpha}} + e^{v+\frac{\alpha\delta}{4-3\alpha}} + e^{v_0}}\left(\delta - \frac{\alpha\delta}{4-3\alpha}\right). \end{aligned}$$

(3) If $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$ and $s_i = 1$, consumer utility is

$$U_3 = \ln\left[e^{v+\delta} + e^{v-\delta} + e^{v_0}\right].$$

Note, in this case $d_i = d_j = 0$.

(4) If $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$ and $s_i = 0$, consumer utility is

$$\begin{aligned} U_4 &= \ln\left[e^{v-\frac{\alpha\delta}{4-3\alpha}} + e^{v+\frac{\alpha\delta}{4-3\alpha}} + e^{v_0}\right] \\ &+ \frac{e^{v-\frac{\alpha\delta}{4-3\alpha}}}{e^{v-\frac{\alpha\delta}{4-3\alpha}} + e^{v+\frac{\alpha\delta}{4-3\alpha}} + e^{v_0}}\left(\delta + \frac{\alpha\delta}{4-3\alpha}\right) + \frac{e^{v+\frac{\alpha\delta}{4-3\alpha}}}{e^{v-\frac{\alpha\delta}{4-3\alpha}} + e^{v+\frac{\alpha\delta}{4-3\alpha}} + e^{v_0}}\left(-\delta - \frac{\alpha\delta}{4-3\alpha}\right). \end{aligned}$$

(5) If $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$ and $s_i = 1$, consumer utility is

$$\begin{aligned} U_5 &= \ln\left[e^{v-\frac{\alpha\delta}{4-3\alpha}} + e^{v+\frac{\alpha\delta}{4-3\alpha}} + e^{v_0}\right] \\ &+ \frac{e^{v-\frac{\alpha\delta}{4-3\alpha}}}{e^{v-\frac{\alpha\delta}{4-3\alpha}} + e^{v+\frac{\alpha\delta}{4-3\alpha}} + e^{v_0}}\left(-\delta + \frac{\alpha\delta}{4-3\alpha}\right) + \frac{e^{v+\frac{\alpha\delta}{4-3\alpha}}}{e^{v-\frac{\alpha\delta}{4-3\alpha}} + e^{v+\frac{\alpha\delta}{4-3\alpha}} + e^{v_0}}\left(\delta - \frac{\alpha\delta}{4-3\alpha}\right). \end{aligned}$$

(6) If $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$ and $s_i = 0$, consumer utility is

$$\begin{aligned} U_6 &= \ln\left[e^{v-\frac{\alpha\delta}{4-3\alpha}} + e^{v+\frac{\alpha\delta}{4-3\alpha}} + e^{v_0}\right] \\ &+ \frac{e^{v-\frac{\alpha\delta}{4-3\alpha}}}{e^{v-\frac{\alpha\delta}{4-3\alpha}} + e^{v+\frac{\alpha\delta}{4-3\alpha}} + e^{v_0}}\left(-\delta + \frac{\alpha\delta}{4-3\alpha}\right) + \frac{e^{v+\frac{\alpha\delta}{4-3\alpha}}}{e^{v-\frac{\alpha\delta}{4-3\alpha}} + e^{v+\frac{\alpha\delta}{4-3\alpha}} + e^{v_0}}\left(\delta - \frac{\alpha\delta}{4-3\alpha}\right). \end{aligned}$$

(7) If $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$ and $s_i = 1$, consumer utility is

$$U_7 = \ln\left[e^{v-\delta} + e^{v-\delta} + e^{v_0}\right].$$

Note, in this case $d_i = d_j = 0$.

(8) If $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$ and $s_i = 0$, consumer utility is

$$\begin{aligned} U_8 &= \ln\left[e^{v-\frac{\alpha\delta}{4-3\alpha}} + e^{v+\frac{\alpha\delta}{4-3\alpha}} + e^{v_0}\right] \\ &+ \frac{e^{v-\frac{\alpha\delta}{4-3\alpha}}}{e^{v-\frac{\alpha\delta}{4-3\alpha}} + e^{v+\frac{\alpha\delta}{4-3\alpha}} + e^{v_0}}\left(-\delta + \frac{\alpha\delta}{4-3\alpha}\right) + \frac{e^{v+\frac{\alpha\delta}{4-3\alpha}}}{e^{v-\frac{\alpha\delta}{4-3\alpha}} + e^{v+\frac{\alpha\delta}{4-3\alpha}} + e^{v_0}}\left(-\delta - \frac{\alpha\delta}{4-3\alpha}\right). \end{aligned}$$

Expected consumer surplus is

$$CS_u = \frac{\alpha}{4}U_1 + \frac{1-\alpha}{4}U_2 + \frac{\alpha}{4}U_3 + \frac{1-\alpha}{4}U_4 + \frac{\alpha}{4}U_5 + \frac{1-\alpha}{4}U_6 + \frac{\alpha}{4}U_7 + \frac{1-\alpha}{4}U_8.$$

2. Competitive Disclosure

Next, consider the competitive disclosure equilibrium.

(1) If $\{\delta_i, \delta_j\} = \{\delta, \delta\}$ and $\{s_i, s_j\} = \{0, 1\}$ or $\{1, 0\}$ or $\{1, 1\}$, consumer utility is

$$C_1 = \ln\left[e^{v+\delta} + e^{v+\delta} + e^{v_0}\right].$$

Note, in this case $d_i = d_j = 0$. In addition, d_0 is always zero.

(2) If $\{\delta_i, \delta_j\} = \{\delta, \delta\}$ and $\{s_i, s_j\} = \{0, 0\}$, consumer utility is

$$\begin{aligned} C_2 &= \ln\left[e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}\right] \\ &\quad + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(\delta + \frac{\alpha\delta}{2-\alpha}\right) + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(\delta + \frac{\alpha\delta}{2-\alpha}\right). \end{aligned}$$

(3) If $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$ and $\{s_i, s_j\} = \{1, 0\}$ or $\{1, 1\}$, consumer utility is

$$C_3 = \ln\left[e^{v+\delta} + e^{v-\delta} + e^{v_0}\right].$$

Note, in this case $d_i = d_j = 0$.

(4) If $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$ and $\{s_i, s_j\} = \{0, 1\}$ or $\{0, 0\}$, consumer utility is

$$\begin{aligned} C_4 &= \ln\left[e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}\right] \\ &\quad + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(\delta + \frac{\alpha\delta}{2-\alpha}\right) + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(-\delta + \frac{\alpha\delta}{2-\alpha}\right). \end{aligned}$$

(5) If $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$ and $\{s_i, s_j\} = \{0, 1\}$ or $\{1, 1\}$, consumer utility is

$$C_5 = \ln\left[e^{v-\delta} + e^{v+\delta} + e^{v_0}\right].$$

Note, in this case $d_i = d_j = 0$.

(6) If $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$ and $\{s_i, s_j\} = \{1, 0\}$ or $\{0, 0\}$, consumer utility is

$$\begin{aligned} C_6 &= \ln\left[e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}\right] \\ &\quad + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(-\delta + \frac{\alpha\delta}{2-\alpha}\right) + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(\delta + \frac{\alpha\delta}{2-\alpha}\right). \end{aligned}$$

(7) If $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, consumer utility is

$$\begin{aligned} C_7 &= \ln\left[e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}\right] \\ &\quad + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(-\delta + \frac{\alpha\delta}{2-\alpha}\right) + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(-\delta + \frac{\alpha\delta}{2-\alpha}\right). \end{aligned}$$

Expected consumer surplus is

$$CS_c = \frac{1}{4}\left(2\alpha - \alpha^2\right)C_1 + \frac{1}{4}\left(1-\alpha\right)^2 C_2 + \frac{\alpha}{4}C_3 + \frac{1-\alpha}{4}C_4 + \frac{\alpha}{4}C_5 + \frac{1-\alpha}{4}C_6 + \frac{1}{4}U_7.$$

3. Negative Communication Equilibrium

Next, consider the negative communication equilibrium.

(1) If $\{\delta_i, \delta_j\} = \{\delta, \delta\}$ and $\{s_i, s_j\} = \{0, 1\}$ or $\{1, 0\}$ or $\{1, 1\}$, consumer utility is

$$N_1 = \ln\left[e^{v+\delta} + e^{v+\delta} + e^{v_0}\right].$$

Note, in this case $d_i = d_j = 0$. In addition, d_0 is always zero.

(2) If $\{\delta_i, \delta_j\} = \{\delta, \delta\}$ and $\{s_i, s_j\} = \{0, 0\}$, consumer utility is

$$N_2 = \ln\left[e^{v} + e^{v} + e^{v_0}\right] + \frac{e^{v}}{e^{v} + e^{v} + e^{v_0}}\delta + \frac{e^{v}}{e^{v} + e^{v} + e^{v_0}}\delta.$$

(3) If $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$ and $\{s_i, s_j\} = \{1, 0\}$ or $\{1, 1\}$, consumer utility is

$$N_3 = \ln\left[e^{v+\frac{\alpha\delta}{2-\alpha}} + e^{v-\delta} + e^{v_0}\right] + \frac{e^{v+\frac{\alpha\delta}{2-\alpha}}}{e^{v+\frac{\alpha\delta}{2-\alpha}} + e^{v-\delta} + e^{v_0}}\left(\delta - \frac{\alpha\delta}{2-\alpha}\right).$$

Note, in this case $d_j = 0$.

(4) If $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$ and $\{s_i, s_j\} = \{0, 1\}$ or $\{0, 0\}$, consumer utility is

$$N_4 = \ln\left[e^{v} + e^{v} + e^{v_0}\right] + \frac{e^{v}}{e^{v} + e^{v} + e^{v_0}}\delta + \frac{e^{v}}{e^{v} + e^{v} + e^{v_0}}(-\delta).$$

(5) If $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$ and $\{s_i, s_j\} = \{0, 1\}$ or $\{1, 1\}$, consumer utility is

$$N_5 = \ln\left[e^{v-\delta} + e^{v+\frac{\alpha\delta}{2-\alpha}} + e^{v_0}\right] + \frac{e^{v+\frac{\alpha\delta}{2-\alpha}}}{e^{v-\delta} + e^{v+\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(\delta - \frac{\alpha\delta}{2-\alpha}\right).$$

Note, in this case $d_i = 0$.

(6) If $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$ and $\{s_i, s_j\} = \{1, 0\}$ or $\{0, 0\}$, consumer utility is

$$N_6 = \ln\left[e^{v} + e^{v} + e^{v_0}\right] + \frac{e^{v}}{e^{v} + e^{v} + e^{v_0}}(-\delta) + \frac{e^{v}}{e^{v} + e^{v} + e^{v_0}}\delta.$$

(7) If $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$ and $\{s_i, s_j\} = \{1, 1\}$, consumer utility is

$$N_7 = \ln\left[e^{v-\delta} + e^{v-\delta} + e^{v_0}\right].$$

Note, in this case $d_i = d_j = 0$.

(8) If $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$ and $\{s_i, s_j\} = \{1, 0\}$, consumer utility is

$$N_8 = \ln\left[e^{v+\frac{\alpha\delta}{2-\alpha}} + e^{v-\delta} + e^{v_0}\right] + \frac{e^{v+\frac{\alpha\delta}{2-\alpha}}}{e^{v+\frac{\alpha\delta}{2-\alpha}} + e^{v-\delta} + e^{v_0}}\left(-\delta - \frac{\alpha\delta}{2-\alpha}\right).$$

(9) If $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$ and $\{s_i, s_j\} = \{0, 1\}$, consumer utility is

$$N_9 = \ln\left[e^{v-\delta} + e^{v+\frac{\alpha\delta}{2-\alpha}} + e^{v_0}\right] + \frac{e^{v+\frac{\alpha\delta}{2-\alpha}}}{e^{v-\delta} + e^{v+\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(-\delta - \frac{\alpha\delta}{2-\alpha}\right).$$

(10) If $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$ and $\{s_i, s_j\} = \{0, 0\}$, consumer utility is

$$N_{10} = \ln\left[e^{v} + e^{v} + e^{v_0}\right] + \frac{e^{v}}{e^{v} + e^{v} + e^{v_0}}(-\delta) + \frac{e^{v}}{e^{v} + e^{v} + e^{v_0}}(-\delta).$$

Expected consumer surplus is

$$\begin{aligned} CS_n = {} & \frac{\alpha^2 + 2\alpha(1-\alpha)}{4}N_1 + \frac{1}{4}(1-\alpha)^2 N_2 + \frac{\alpha}{4}N_3 + \frac{(1-\alpha)}{4}N_4 \\ & + \frac{\alpha}{4}N_5 + \frac{(1-\alpha)}{4}N_6 + \frac{\alpha^2}{4}N_7 + \frac{\alpha(1-\alpha)}{4}N_8 + \frac{\alpha(1-\alpha)}{4}N_9 + \frac{(1-\alpha)^2}{4}N_{10}. \end{aligned}$$

4. Positive Communication Equilibrium

Finally, consider the positive communication equilibrium.

(1) If $\{\delta_i, \delta_j\} = \{\delta, \delta\}$ and $\{s_i, s_j\} = \{1, 1\}$, consumer utility is

$$P_1 = \ln\left[e^{v+\delta} + e^{v+\delta} + e^{v_0}\right].$$

Note, in this case $d_i = d_j = 0$. In addition, d_0 is always zero.

(2) If $\{\delta_i, \delta_j\} = \{\delta, \delta\}$ and $\{s_i, s_j\} = \{1, 0\}$, consumer utility is

$$P_2 = \ln\left[e^{v+\delta} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}\right] + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v+\delta} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(\delta + \frac{\alpha\delta}{2-\alpha}\right).$$

Note, in this case $d_i = 0$.

(3) If $\{\delta_i, \delta_j\} = \{\delta, \delta\}$ and $\{s_i, s_j\} = \{0, 1\}$, consumer utility is

$$P_3 = \ln\left[e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v+\delta} + e^{v_0}\right] + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v+\delta} + e^{v_0}}\left(\delta + \frac{\alpha\delta}{2-\alpha}\right).$$

Note, in this case $d_j = 0$.

(4) If $\{\delta_i, \delta_j\} = \{\delta, \delta\}$ and $\{s_i, s_j\} = \{0, 0\}$, consumer utility is

$$
\begin{aligned}
P_4 &= \ln\left[e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}\right] \\
&\quad + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(\delta + \frac{\alpha\delta}{2-\alpha}\right) + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(\delta + \frac{\alpha\delta}{2-\alpha}\right).
\end{aligned}
$$

(5) If $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$ and $\{s_i, s_j\} = \{1, 0\}$ or $\{1, 1\}$, consumer utility is

$$
P_5 = \ln\left[e^{v+\delta} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}\right] + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v+\delta} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(-\delta + \frac{\alpha\delta}{2-\alpha}\right).
$$

Note, in this case $d_i = 0$.

(6) If $\{\delta_i, \delta_j\} = \{\delta, -\delta\}$ and $\{s_i, s_j\} = \{0, 1\}$ or $\{0, 0\}$, consumer utility is

$$
\begin{aligned}
P_6 &= \ln\left[e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}\right] \\
&\quad + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(\delta + \frac{\alpha\delta}{2-\alpha}\right) + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(-\delta + \frac{\alpha\delta}{2-\alpha}\right).
\end{aligned}
$$

(7) If $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$ and $\{s_i, s_j\} = \{1, 1\}$ or $\{0, 1\}$, consumer utility is

$$
P_7 = \ln\left[e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v+\delta} + e^{v_0}\right] + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v+\delta} + e^{v_0}}\left(-\delta + \frac{\alpha\delta}{2-\alpha}\right).
$$

Note, in this case $d_j = 0$.

(8) If $\{\delta_i, \delta_j\} = \{-\delta, \delta\}$ and $\{s_i, s_j\} = \{1, 0\}$ or $\{0, 0\}$, consumer utility is

$$
\begin{aligned}
P_8 &= \ln\left[e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}\right] \\
&\quad + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(-\delta + \frac{\alpha\delta}{2-\alpha}\right) + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(\delta + \frac{\alpha\delta}{2-\alpha}\right).
\end{aligned}
$$

(9) If $\{\delta_i, \delta_j\} = \{-\delta, -\delta\}$, consumer utility is

$$
\begin{aligned}
P_9 &= \ln\left[e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}\right] \\
&\quad + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(-\delta + \frac{\alpha\delta}{2-\alpha}\right) + \frac{e^{v-\frac{\alpha\delta}{2-\alpha}}}{e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v-\frac{\alpha\delta}{2-\alpha}} + e^{v_0}}\left(-\delta + \frac{\alpha\delta}{2-\alpha}\right).
\end{aligned}
$$

Expected consumer surplus is

$$
\begin{aligned}
CS_p &= \frac{1}{4}\left(\alpha^2 P_1 + \alpha\left(1-\alpha\right) P_2 + \alpha\left(1-\alpha\right) P_3 + \left(1-\alpha\right)^2 P_4\right) \\
&\quad + \frac{\alpha}{4} P_5 + \frac{1-\alpha}{4} P_6 + \frac{\alpha}{4} P_7 + \frac{1-\alpha}{4} P_8 + \frac{1}{4} P_9.
\end{aligned}
$$

Consumer surplus comparisons

We now numerically compare these expected consumer surplus expressions. For the numerical analysis, we set v at 1 and vary α from 0 to 1. We then choose values of v_0 such that the respective existence conditions (specified in Lemma 1 and Proposition 2) are satisfied. In addition, we vary δ between 0 and 1. For presentation purposes in Figure 3 in the main Appendix, we have chosen values of v_0 at 0, 0.5 and 1, and values of δ at 0.25, 0.5, and 0.75.