Negative Vote Buying and the Secret Ballot^{*}

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Abstract

We offer a model of "negative vote buying"—paying voters to abstain. While negative vote buying is feasible under the open ballot, it is never optimal. In contrast, a combination of positive and negative vote buying is optimal under the secret ballot: Lukewarm supporters are paid to show up at the polls, while lukewarm opponents are paid to stay home. Surprisingly, the imposition of the secret ballot *increases* the amount of vote buying—a greater fraction of the electorate votes insincerely than under the open ballot. Moreover, the secret ballot may *reduce* the costs of buying an election.

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1 Introduction

The use of voting to make collective decisions inevitably brings with it the possibility of vote buying. Thus, a crucial aspect of designing voting procedures is to ensure that election outcomes reflect the will of the electorate, rather than the wallets of interest groups. An important reform along these lines was the introduction of the secret ballot. Prior to its introduction, vote buying was common and, often times, quite open. For instance, Seymour (1915) reports that, in English parliamentary elections of the 19th century, the price of a vote was often posted openly outside the polling station and updated numerous times over the course of the day, much as a stock price. Some 50 years later little had changed, at least in Texas. For instance, Caro (1982) recounts the following vote buying scheme organized by Lyndon Johnson on behalf of Maury Maverick's 1934 Congressional campaign:

Johnson was sitting at a table in the center of the room—and on the table there were stacks of five-dollar bills. "That big table was just *covered* with money—more money than I had ever seen," Jones says. (...) Mexican American men would come into the room one at a time. Each would tell Johnson a number—some, unable to speak English, would indicate the number by holding up their fingers—and Johnson would count out that number of five-dollar bills, and hand them to him. "It was five dollars a vote," Jones realized. "Lyndon was checking each name against a list someone had furnished him with. These Latin people would come in, and show how many eligible votes they had in the family, and Lyndon would pay them five dollars a vote."

Of course, the workability of this scheme depended crucially on the observability of votes cast. The Maverick campaign was in an enviable position in this regard, since Texas thoughtfully matched a voter's registration number with the number on his ballot.¹

In an important study, Cox and Kousser (1981) trace the effects of the imposition of the secret ballot on vote buying in New York elections. They find that the secret ballot substantially changed the form but not necessarily the amount of vote buying. Specifically, prior to the imposition of the secret ballot, "regular" vote buying dominated. In contrast, after the imposition of the secret ballot, negative vote

¹In Texas, the ballot was not truly secret until 1949.

buying became the most commonly mentioned vote buying practice in news reports. The rationale for this shift is nicely explained by a turn-of the century New York Democratic state chairman: "Under the new ballot law you cannot tell how a man votes when he goes into the booth, but if he stays home you know that you have got the worth of your money." (Cox and Kousser, p. 656). In fact, if one judges the pervasiveness of vote buying by the number of mentions in newspapers, then Cox and Kousser's study reveals no diminution after the imposition of the secret ballot.

The Cox and Kousser study raises a number of questions. For instance, why was there no negative vote buying prior to the introduction of the secret ballot? Standard marginal economic analysis would suggest the optimality of using *all* available vote buying tools. More fundamentally, did the secret ballot achieve its policy objective of reducing corruption at all? From cox and Kousser's study, it is unclear that vote buying was, in any way, reduced.

To examine these questions, we study a theoretical model of positive and negative vote buying with competing interest groups. Using the Groseclose and Snyder (1996) vote buying framework as a starting point, we enrich their model in three key respects. First, we allow for abstention, thus making negative vote buying possible. Second, we study both the open and the secret ballot. Under the open ballot interest groups can contract on individual votes, while under the secret ballot, contracts can only be contingent on whether a voter shows up at the polls. Third, we endogenize the timing of the interest groups' vote buying.

Our main results are:

- 1. Under the open ballot, negative vote buying is never optimal.
- 2. Under the secret ballot, the optimal contract entails both positive and negative vote buying—lukewarm supporters are paid to show up at the polls, while lukewarm opponents are paid to stay home.
- 3. More voters vote sincerely under the open ballot than under the secret ballot. In other words, the secret ballot *increases* the amount of vote buying.
- 4. In close elections where a policy has many lukewarm supporters, buying the election is cheaper under the secret ballot than under the open ballot.²

 $^{^2\}mathrm{An}$ election is said to be bought if and only if the outcome does not reflect the intrinsic preferences of the majority.

The paper proceeds as follows. In the remainder of this section, we review the extant literature. In section 2, we outline the model. Section 3 characterizes the unique optimal vote buying strategy under the open ballot, while section 4 undertakes the same exercise for the secret ballot. In section 5, we compare the scope of vote buying and the buyability of voting bodies under the open versus the secret ballot. Finally, section 6 concludes. All proofs are in the Appendix.

Related Literature

In their seminal paper, Groseclose and Snyder (1996) (hereafter, GS) showed that buying a supermajority of voters is optimal when interest groups compete. GS study the open ballot. That is, they allow contracts to be based on individual votes. Dal Bo (2007) shows that a richer contract space makes vote buying essentially free for a monopoly interest group. In his model, the introduction of the secret ballot unambiguously raises the cost of buying the election. Felgenhauer and Gruener (2003) extend Dal Bo's framework to allow for competing interest groups and private information. They study the effects of the open versus the secret ballot in a Condorcet type model. Stokes (2005) offers a dynamic model of vote buying with a monopoly interest group. Her model offers a blend between open and secret ballots—there is some probability that a voter's action will be observed ex post. Using folk theorem type arguments, she offers conditions in which vote buying takes place despite imperfect monitoring. Finally, Seidmann (2006) examines the effect of expost rewards by outsiders on votes cast by members of a committee. The possibility of outside rewards creates a divergence between social and private incentives for committee members. The degree of divergence is affected by the openness of the voting rule, since this permits outsiders to draw differing inferences about a committee member's vote.

Our analysis here differs from the previous literature in several key respects. Abstention, and hence the possibility of negative vote buying, is absent from the extant theoretical literature. Second, none of these models endogenize the timing of vote buying. Third, most of this literature (with GS as the notable exception) focus on small elections, where pivotality and, hence, instrumental motives are at the fore. We focus on large elections, where pivotality incentives do not play much of a role. Specifically, we follow GS and assume that voters behave as if they only had expressive preferences. Morgan and Várdy (2008) offer a formal justification for this assumption: As long as there is some uncertainty about the preferences of an arbitrarily small fraction of the electorate, the probability of being pivotal *uniformly* converges to zero as the size of the electorate grows. Hence, when voters have both instrumental and expressive preferences, the incentives from expressive preferences completely dominate in large elections.

In addition to the theoretical literature reviewed above, there is a considerable literature on the practice of vote buying. Shaffer and Schedler (2007), for instance, offer and excellent overview of various vote buying techniques. The seminal paper on negative vote buying is Cox and Kousser (1981). They highlight how the imposition of the secret ballot in upstate New York affected vote buying practices. In particular, prior to its imposition, vote buying was mostly of the "positive" (get out the vote) variety, whereas after its imposition, negative vote buying aiming to suppress turnout became more prominent. Heckelman (1995) finds the same turnout suppressing effect of the secret ballot using a panel data set of US gubernatorial elections from 1870-1910.

Stokes (2005) as well as Nichter (2008) examine vote buying practices of Peronists in Argentina. They find evidence of positive vote buying; however, they differ in their interpretation of the data—Nichter finds evidence to suggest that vote buying is concentrated on mobilizing lukewarm supporters while Stokes sees the data as more consistent with compensating lukewarm opponents in exchange for their votes. Finally, Vincente (2007) uses randomized field experiments in São Tome and Principe to identify vote buying patterns. He finds evidence that vote buying disproportionately benefits well-financed challengers and that voter information campaigns can be effective in reducing vote buying.

2 The Model

Our model is identical to that of GS save for endogenizing the interest groups' order of moves, introducing the option of abstention for voters, and studying both the open and the closed ballot.

There is a continuum of voters with unit mass who can choose between two policies, labeled a and b. In addition, there are two interest groups, A and B, that are trying to affect the voters' policy choice in a simple-majority election. Group Aprefers policy a, while group B prefers policy b. Excluding the cost of buying votes, group B receives a payoff W > 0 when policy b is adopted and zero when policy a is adopted. Conversely, group A receives U > 0 when policy a is adopted and zero when b is adopted. Thus, groups A and B have diametrically opposed policy preferences.

In order to induce voters to vote for their preferred policy, the interest groups can offer (enforceable) contracts prescribing (non-negative) contingent transfers from the interest group to voters. We will vary the contingencies on which these contracts can be based.

Voters' preferences are the same as in GS. Apart from money, voters derive utility solely from expressive motives—the utility derived from voting according to one's convictions—rather than from instrumental motives—utility based on election outcomes. Clearly, this is a simplification, since real-world voters likely derive utility both from expressive and instrumental motives. Instrumental motives, however, only matter to the extent that a voter is pivotal to the election outcome, and, in large elections, this probability becomes vanishingly small. Thus, regardless of the weight placed on instrumental motives, in large elections, voters' optimal behavior is indistinguishable from the case where only expressive motives are present.³

Voting for policy a provides a voter of type θ with a utility $u(a;\theta)$. Likewise, voting for b provides the same voter a utility $u(b;\theta)$. Finally, expressing no view, i.e., abstaining, yields a utility $u_i(\emptyset;\theta)$. Without loss of generality, we can set $u_i(\emptyset;\theta)$ equal to zero for all types θ .

Define

$$v(\theta) = u(a; \theta) - u(b; \theta)$$

to be the difference between the utility a type θ voter receives from voting for policy a versus voting for policy b. We assume that the utility of voting for one's preferred policy relative to abstaining is the same as the disutility of voting for the non-preferred policy, i.e., $u(b; \theta) = -u(a; \theta)$. Then we have that

$$v\left(\theta\right) = 2u\left(a;\theta\right)$$

That is, for a voter of type θ , the difference in utility from voting for a as opposed to abstaining is simply $v(\theta)/2$, while the difference in utility between voting for b and abstaining is $-v(\theta)/2$. We shall refer to type θ voters where $v(\theta) > 0$ as *intrinsic* a supporters and to voters where $v(\theta) < 0$ as *intrinsic* b supporters.

In the Appendix we show that the model is robust to other assumptions about

³This result is immediate in a model with a continuum of voters. Morgan and Várdy (2008) show that the result extends to the discrete case, even in the presence of strategic vote-buying.

voter preferences. Specifically, we analyze the polar case where the disutility from abstention is the same as that from voting for the non-preferred policy. That is, the payoffs of failing to "do the right thing" are the same regardless of whether the voter commits a "sin of commission" (i.e., votes for his non-preferred policy) or commits a "sin of omission" (i.e., abstains).⁴ By continuity, qualitatively similar results are likely to hold for intermediate cases as well. Since preferences become multidimensional for these cases, their analysis is beyond the scope of the present paper. Note that the opposite polar case where abstention produces the same utility as voting for one's preferred policy seems to make little sense.

We make the following regularity assumptions on the $v(\cdot)$ function:⁵

- 1. $v(\theta)$ is continuous, strictly decreasing, and differentiable almost everywhere.
- 2. $v\left(\frac{1}{2}\right) < 0$
- 3. $v(0) = -v(1) = \infty$

The first assumption merely ensures that preferences are "well-behaved" and that voters are ordered in a sensible way. The second assumption ensures that the median voter strictly prefers policy b. Hence, in the absence of interest group interference, policy b will be chosen in the election, while a win for a implies that the election has been bought. Throughout, we assume that U is sufficiently large relative to W such that group A indeed wants to buy the election. The third assumption amounts to a set of "Inada conditions" and merely rules out corner solutions. It is useful to define θ_0 to be the type who is indifferent between voting for A and voting for B. Note that, given our assumptions on $v(\cdot)$, θ_0 exists, is unique and is strictly less than $\frac{1}{2}$. Finally, we assume that voters' preferences are linear in the transfer and additively separable with respect to the act of voting.

We follow GS and assume that the preferences of the voters are commonly known to all parties; thus allowing the interest groups to write contracts specific to the preferences of each voter. While this is clearly not literally true in practice, it seems a reasonable approximation given the availability of observable characteristics such

⁴The alternative preference specification produces essentially identical results to Propositions 1-5. It differs from the main text in that the imposition of the secret ballot always increases the cost of buying the election.

⁵Since $v(\theta)$ is a strictly monotone transformation of $u(a;\theta)$, regularity conditions on $v(\cdot)$ are equivalent to almost identical, analogous assumptions on $u(a; \cdot)$.

as race, gender, and metropolitan statistical area that correlate with preferences in the aggregate.

The extensive form of the game is as follows. In the lead-up to the election, which will occur at time t = 1, interest groups may offer contracts to voters. Time is continuous and interest groups are free to make offers at any point in time $t \in [0, 1]$. Once an interest group makes its contract offer, it can make no further offers and its move is visible to its rival.

An offer consists of a schedule of non-negative contingent transfers to all voters. This includes the possibility of offering some voter types a null contract (the promise of a zero transfer in all contingencies). For technical reasons, it is useful to assume that, if, at time t = 1 an interest group has not made an offer, then it is assumed to have offered a null contract to all voters. Next, each voter opts for one of the two contracts he has been offered and votes. Finally, the policy outcome is determined and payoffs are realized.

We study subgame perfect equilibria of the game. To rule out "nuisance" cases where one of the interest groups makes contract offers under the assumption that none of these will be accepted (owing to a later counteroffer), we use a trembling hand type refinement.⁶ Specifically, we suppose that there is an infinitesimal possibility that no competing interest group is present. That is, with arbitrarily small probability, an interest group is a monopolist.

We adopt the following tie-breaking conventions: (1) If an interest group can do no better than to propose the null contract, we assume that it opts for this strategy. (2) If a voter is indifferent between accepting the contract offered by A and that offered by B, he is assumed to accept A's contract.⁷

3 Least Cost Vote Buying under the Open Ballot

In this section we study vote buying under the open ballot. That is, individual votes are contractible. To develop a benchmark, we first consider the case where abstention is not allowed. When the order of moves is specified exogenously and A moves first,

⁶Because the strategies of interest groups are functions, we cannot directly adopt Selten's trembling hand refinement for extensive form games.

⁷The tie-breaking conventions merely serve to get rid of the usual open set problems in studying subgame perfect equilibria. If we discretized the transfer space, then any tie-breaking convention would produce essentially the same results as the one we have adopted.

this is the model analyzed by GS. Their main result is to show that A will optimally preempt B from making any counteroffer by paying transfers sufficient to induce a supermajority to vote for a in the election.

To more easily characterize their result, it is helpful to introduce the notion of surplus—the difference in a voter's utility, including transfers, from voting for a as compared to voting for b. Formally, suppose that a transfer of t is offered to a voter of type θ in exchange for an a vote. Then the voter's surplus $s \equiv v(\theta) + t$ (since, in equilibrium, B is optimally preempted from making any counteroffer). Since there is a one-to-one relationship between transfers and surplus, it is equivalent to think of A's offers in terms of surplus rather than transfers. To induce a desired surplus s for a θ type voter requires a transfer $t(\theta) = s - v(\theta)$. Under these conditions, GS show that the cheapest possible contract through which Group A can guarantee its preferred policy (i.e. the "least cost successful contract") is given by:⁸

Proposition 1 (GS, 1996) Without abstention, the unique least cost successful contract offered by A is as follows:

In exchange for voting for 'a', all voters with type $\theta < \theta_a^*$ receive transfers $t(\theta) = \max(0, s(\theta_a^*) - v(\theta))$ and earn surplus equal to $\max(s(\theta_a^*), v(\theta))$.

where

$$s\left(\theta_{a}^{*}\right) = \frac{W}{\theta_{a}^{*} - \frac{1}{2}}$$

and, θ_a^* is the unique solution to

$$\arg\min_{\theta_{a}\geq\frac{1}{2}}\int_{0}^{\theta_{a}}t\left(\theta\right)d\theta$$

(Notice that if A moves first and is successful, B does not make any counter offers. Hence, we only describe A's optimal strategy.) The proof of the proposition follows immediately from Propositions 1 and 2 on pp. 307 and 309 in GS. Figure 1 below illustrates the form of the least cost successful contract. All voters in group A's coalition, $[0, \theta_a^*]$, receive surplus of at least $s(\theta_a^*)$ and a supermajority of voters are recruited into the coalition (i.e. $\theta_a^* > \frac{1}{2}$).

⁸Whenever we refer to "unique," it should be understood as being unique up to a measure zero change in strategies. Likewise, any reference to "all" should be understood to mean "almost all."



Figure 1

Now suppose that we endogenize the order of moves in the GS model. Our next proposition shows that the extensive form they analyzed is in fact the unique subgame perfect equilibrium when the timing of bribes is also a strategic decision.

To gain some intuition for why this is the case. Let C_M denote the cost to group A of securing its preferred policy outcome as a monopolist, i.e., in the absence of group B. Notice that A's optimal strategy as a monopolist is very simple: It pays a transfer $-v(\theta)$ to voters with types $\theta_0 \leq \theta \leq \frac{1}{2}$ in exchange for voting for a. Let C denote the cost to A of the least cost successful contract derived in Proposition 1. It may be readily verified that $C > W + C_M$. Thus, it is never in B's interest to move first since, if it did, A could "neutralize" B's offers at a cost of at most W and then get its most preferred policy at additional cost of at most C_M . Hence, by moving first, B only makes it cheaper for A to buy the election. Notice that this intuition does not depend on the particulars of whether voters can abstain or whether the ballot is open or secret. Formally:

Proposition 2 In any subgame perfect equilibrium of the vote buying game with endogenous moves, group A moves first.

Abstention What happens when voters can abstain? Abstention provides a new, and potentially useful tool to the (endogenous) first mover, i.e., group A. Since intrinsic b supporters dislike abstention only half as much as they dislike voting for

a, it would seem that group A could use this to economize on the cost of buying the election. On the other hand, abstention also provides a new tool for the opposing interest group, B, to counter A's offers. And since A needs to anticipate B's response if it wants to be successful, the possibility of abstention might in fact make it more expensive for A to buy the election.

It turns out, however, that this additional tool is completely useless for both interest groups. Indeed, under the optimal contract, no voter is induced to abstain. Specifically, under the open ballot with abstention, the least cost successful contract is *identical* to that in Proposition 1. We now derive this result formally.

The following lemmas provide properties useful in characterizing the least cost successful contract offered by A under the open ballot.

Lemma 1 It is a dominated strategy to offer intrinsic supporters compensation in exchange for abstention.

The intuition is straightforward. Intrinsic supporters require more compensation to abstain than to vote for their most preferred option.

Next, under any least cost successful contract offered by A under the open ballot,

Lemma 2 There exist $\theta_a, \theta_b \in (0, 1)$ such that

- 1. All voters with types $\theta \in [0, \theta_a]$ vote for 'a'.
- 2. All voters with types $\theta \in (\theta_a, \theta_b)$ abstain.
- 3. All voters with types $\theta \in [\theta_b, 1]$ vote for 'b'.

Lemma 2 states that, under the least cost successful contract offered by A, the sets of voters making each choice $\{a, b, \emptyset\}$ are convex. The intuition for the ordering of the sets is that, if group A wants to recruit a given fraction of voters, it is cheapest to recruit from those who are least hostile towards policy a. Thus, a generic least cost successful contract has boundary points θ_a and θ_b as illustrated in Figure 2 below. Voters with types to the left of θ_a are induced to vote for a while those to the right of θ_b vote for b. Voters with types between θ_a and θ_b are induced to abstain. (Later we will show that, in fact, $\theta_a = \theta_b$. Hence, nobody abstains.)



Figure 2

The next lemma describes the form of A's transfers under a least cost successful contract. As for the case where abstention was not allowed, it is helpful to think of transfers in terms of surplus offers *relative to a voter's next best option*. Since abstention is now a possibility, we amend the definition of surplus as follows: To provide a voter of type θ with surplus s from abstaining rather than voting for b, group A would have to offer a transfer $t(\theta) = s - v(\theta)/2$.

Lemma 3 All voters who receive a transfer in exchange for voting for 'a' enjoy the same surplus, s. Similarly, all voters who receive a transfer in exchange for abstaining enjoy the same surplus, s'. Moreover, s = 2s'.

This lemma captures a "no arbitrage" condition resulting from competition between A and B. Intuitively, the cost to group B of shifting the vote total slightly in its favor should be the same irrespective of which voters it picks. If this were not the case, then A is spending too much to "protect" some voters from being poached by B. The second part of the lemma says that the cost for B of poaching an a voter and turning that voter into a b voter should be exactly twice the cost of poaching an abstaining voter and turning that voter into a b voter. The reason is that the change in the vote lead for the first type of switch is two votes (a reduction of one vote for a and an increase of one vote for b), while the change in the vote lead for the latter is only one vote (an increase of one vote for b but no reduction in the number of votes for a). For B's cost of shifting vote share to be equalized across voters requires that a voters receive twice as much surplus as abstainers. This is illustrated graphically in Figure 2.

Next, under any least cost successful contract offered by A under the open ballot,

Lemma 4 Voters with types $\theta \in [0, \frac{1}{2})$ are induced to vote for 'a'.

Lemma 4 states that it is always more cost effective for group A to recruit up to the median voter to vote in favor of a rather than have any one of these voters abstain. Intuitively, if group A attracts a fraction less than half of the voting populace to vote for a, then it must induce some stronger b supporters to abstain. With each additional vote for a, group A can economize on buying abstentions and, at least up to the median voter, this trade-off is always favorable.

Finally,

Lemma 5 Negative vote buying is never used under the open ballot.

Lemma 5 says that, under the open ballot, negative vote buying is never as cost effective as positive vote buying. Why is this? In any least cost successful vote buying scheme, group A divides voters into three groups—those voting for a, those abstaining, and those voting for b—ordered by their intrinsic preference for policy a. Suppose that group A decides to change the mix of abstainers and a voters while preserving the same vote lead over policy b. One way it can do this is to offer additional money to the least hostile abstainer in exchange for him voting for a, while, at the same time, offering the null contract to the voter who until now was the most hostile abstainer, such that the latter now votes for b. This is illustrated in Figure 3. Notice that the additional cost of "flipping" the least hostile abstainer to vote for a is given by the pair of boxes near θ_a^* in the figure. The savings from letting go of the most hostile abstainer is given by the *larger* rectangle near θ_b^* in the figure. Hence, such an alteration of the mix of bribes offered by A is always profitable.



Figure 3

Together the above lemmas imply that:

Proposition 3 Under the open ballot, negative vote buying is always feasible but never optimal.

Specifically, the least cost successful contract identified in Proposition 1 is also optimal with abstention.

Several implications emerge from Proposition 3. First, because negative vote buying is never optimal, one would expect to see little of it under the open ballot. This is consistent with the empirical findings of Cox and Kousser (1981). They report that in New York elections prior to the introduction of the secret ballot, there were few instances of negative vote buying reported in the popular press. It was only with the introduction of the secret ballot that negative vote buying gained prominence.

Second, under the open ballot, the imposition of compulsory voting has *no effect* on *A*'s costs to buy the election. This may be seen by comparing the least cost successful contract under GS where voting is compulsory with the result of Proposition 3 where voting is voluntary.

4 Least Cost Vote Buying under the Secret Ballot

Reflecting the ideals of the Revolution, Article 31 of the French Constitution of 1795 prescribed that all elections were to be held by secret ballot.⁹ About 60 years later, the Anglo-Saxon world started following suit. Motivated by Chartist principles and worried about the corruption endemic to its electoral process, the Australian state of Victoria adopted the secret ballot in general elections in 1856. Britain and many U.S. states introduced the secret ballot soon thereafter.

In this section, we analyze least cost successful vote buying under the secret ballot. Once again, we consider the case where group A goes first followed by group B. This is without loss of generality since, by almost identical arguments, Proposition 2 also holds for the secret ballot.

The imposition of the secret ballot limits the contracting possibilities of the interest groups to payments contingent only on whether a voter goes to the polls. Formally, this amounts to restricting the contracting space to $\{\{A, B\}, \emptyset\}$.

We offer a set of structural properties that are shared by any least cost successful contract. These properties mimic those identified in the previous section under the open ballot.

Under any least cost successful contract offered by A under the secret ballot,

Lemma 6 There exists a $\theta_b \in (0, 1)$ such that

- 1. All voters with types $\theta \in [0, \theta_0]$ vote for 'a'
- 2. All voters with types $\theta \in (\theta_0, \theta_b]$ abstain
- 3. All voters with types $\theta \in (\theta_b, 1]$ vote for b'.

Notice that, compared to Lemma 2 which had two free parameters, here there is only a single parameter, θ_b , characterizing the partition of voter types under the secret ballot. The intuition is that an intrinsic *b* supporter can never be induced to vote for *a*, and vice versa, because votes cannot be monitored. Thus, a least cost successful contract offered by *A* amounts to dividing the intrinsic *b* supporters into abstainers and *b* voters, while sufficiently incenting the intrinsic *a* supporters to deter counteroffers from interest group *B*.

⁹That vote buying was a significant concern can be seen in the very next Article, which imposes exclusion from the political process for twenty years to life for anyone found buying or selling votes.

Under any least cost successful contract offered by A under the secret ballot,

Lemma 7 All voters who are paid to abstain (i.e. voters with types $\theta \in (\theta_0, \theta_b]$) receive the same surplus, s, relative to voting for b.

Lemma 8 Let V be the set of voters receiving non-zero payment in exchange for coming to the polls. All voters in V receive the same surplus, s', relative to abstaining.

Lemma 9 All voters receiving payment from group A earn the same surplus relative to their outside option. Formally, s = s'.

If B makes a counteroffer, it targets voters offered the *least* surplus by A. The lemmas show how group A anticipates this and sets its transfers such that all voters receiving payments are equally costly for B to "flip." Finding a least cost successful contract then consists of pinning down the surplus s offered to abstainers and the size of the abstaining coalition sufficient to dissuade group B from counter attacking. Intrinsic a supporters must also receive surplus s, or more, *relative to their next best* option, i.e., abstaining. Hence, group A offers voters with type $\theta \in [0, \theta_b]$ a transfer of $t(\theta) = \max\{0, s - \frac{1}{2}v(\theta)\}$, where s is a constant to be determined below. Voters with type $\theta \in [0, \theta_0]$, i.e. intrinsic a supporters, are paid for coming to the polls, while voters with type $\theta \in [\theta_0, \theta_b]$, i.e. intrinsic b supporters, are paid to stay away.

To counter A's offer, B would have to induce abstainers to vote for b and a supporters to abstain, such that the total mass of voters it recruits is at least $\theta_b - (1 - \theta_0)$. In both cases, the cost per voter is equal to s by construction. Therefore, B's total cost of recruiting a minimal winning coalition is $s \times (\theta_b - (1 - \theta_0))$. Hence, for A to achieve deterrence, it must be that

$$s \ge \frac{W}{\theta_b - (1 - \theta_0)} \tag{1}$$

Let $s(\theta_b)$ describe a surplus offer satisfying equation (1) with equality. The problem of determining a least cost successful contract now reduces to choosing a cost minimizing value for θ_b . Note that group A's cost as a function of θ_b is

$$C(\theta_b) = \int_0^{\theta_b} t(\theta) d\theta$$

=
$$\int_{v^{-1}(2s(\theta_b))}^{\theta_b} \left(s(\theta_b) - \frac{1}{2}v(\theta) \right) d\theta$$

It may be readily verified that $C(\cdot)$ is strictly convex in θ_b and, hence, there exists a unique θ_b^* that minimizes A's cost. We may now conclude:

Proposition 4 Under the secret ballot with abstention, the unique least cost successful contract offered by A is as follows:

Voters with type $\theta \in [0, \theta_0]$ vote for 'a'; voters with type $\theta \in (\theta_0, \theta_b^*]$ abstain; voters with type $\theta \in (\theta_b^*, 1]$ vote for 'b'.

All voters with type $\theta \leq \theta_b^*$ receive transfers $t(\theta) = \max\left\{0, s(\theta_b^*) - \frac{1}{2}v(\theta)\right\}$ and earn surplus equal to $\max\left\{s(\theta_b^*), v(\theta), \frac{1}{2}v(\theta) + s(\theta_b^*)\right\}$

where

$$s\left(\theta_{b}^{*}\right) = \frac{W}{\theta_{b}^{*} - (1 - \theta_{0})}$$

and θ_b^* is the unique solution to

$$\arg\min_{\theta_{b}\geq 1-\theta_{0}}\int_{0}^{\theta_{b}}t\left(\theta\right)d\theta$$

A typical least cost successful contract under the secret ballot is shown in Figure 4 below. A key difference between this figure and Figure 1 is the upward sloping surplus for intrinsic *a* supporters with types just to the left of θ_0 . The reason is that these voters must receive constant surplus *relative to their outside option*—which is abstention—and the value of this option changes with a voter's type.



Figure 4

In contrast to the open ballot, negative vote buying takes on considerable prominence under the secret ballot. It may take many different forms, and policy changes meant to reduce electoral fraud can sometimes have the perverse effect of making negative vote buying a cheaper and more effective tool. In some elections, indelible ink is used to mark the fingers of voters in order to prevent them from voting multiple times. Negative vote buying then consists of paying voters in opposition districts to dip their fingers in ink and thereby prevent them from going to the polls. Similarly, leading up to the 1998 general elections in Guyana, the government instituted a system of voter identification cards to curtail vote fraud. Voters needed to present these cards at the polling stations in order to vote. Ironically, these cards, combined with readily observable racial identifiers, served to make the process of negative vote buying cheap and easy for the ruling party. The ruling party, which was favored by most Indo-Guyanans, set about buying the identification cards of Afro-Guyanans, who were the main opposition. See Shaffer (2002).

Another vivid example of an alleged negative vote buying and demobilization campaign occurred in the New Jersey gubernatorial race of 1993, when Christine Todd Whitman won a narrow and unexpected victory over the Democratic incumbent Jim Florio. After the election, Whitman's campaign manager, Ed Rollins, told the *The New York Times* that the key to victory was a combination of negative vote buying and neutralization of the Democratic Party's money offers. Specifically, it was alleged that the GOP paid African-American pastors *not* to encourage voters to turn out in the election. Also, local Democratic Party workers were allegedly asked what they were paid to get out the vote on election day and then offered an identical amount to "stay home and watch TV."¹⁰

Finally, in the Philippines, negative vote buying and demobilization campaigns often take the form of offering opposition supporters coach trips to interesting resorts with lots of booze on the day of the election (Quimpo, 2002).

Our model predicts voter turn out to decrease when the secret ballot is introduced. This is consistent with Heckelman (1995) who found that the introduction of the secret ballot accounted for a seven percentage point drop in turnout in U.S. gubernatorial elections during 1870–1910. In addition to negative vote buying, the optimal contract under the secret ballot also entails positive vote buying—lukewarm intrin-

¹⁰Faced with a barrage of negative press and possible legal action, Rollins back-pedaled from his claims. Inquiries produced no definitive evidence that the alleged vote-buying had, in fact, occurred.

sic a supporters are compensated in exchange for coming to the polls. In practical terms, this might take the from of "get out the vote" campaigns by ward heelers offering transportation to the voting station. In the 2008 Democratic primaries, "street money" paid to ward heelers in Philadelphia was indeed a source of controversy. The predicted changes in vote buying under the secret ballot also resemble the empirical findings of Cox and Kousser (1981). As mentioned above, there was a lot of positive vote buying but little negative vote buying prior to the introduction of the secret ballot. After its introduction, the amount of negative vote buying went up dramatically, but parties continued to engage in positive vote buying as well.

5 The Secret Ballot and the Buyability of Voting Bodies

A key justification for the introduction of the secret ballot was the curtailment of vote buying. This raises the question how effective it is in this regard. Obviously, resolving this question empirically is difficult given that vote buying is generally illegal. Our model offers an opportunity for a theoretical answer.

It is worthwhile to distinguish between two separate metrics of vote buying. The first measure is the number of people paid in exchange for not voting according to their intrinsic preferences. The second measure is how much it costs group A to buy the election. It might seem apparent that the secret ballot unambiguously improves the situation on both counts (i.e., reduces the number of people bribed and increases the cost of buying influence). After all, it would seem that depriving interest group A of a key tool—the ability to contract on individual votes—would make it harder to bribe voters and, hence, raise the cost of influencing the election. However, this ignores the competition between interest groups. By the imposition of the secret ballot, rival interest group B is deprived of the same key tool. Could it be that, as a result of having A's rival "disarmed" in this fashion, interest group A can actually more cheaply exert influence under the secret ballot? Moreover, perhaps owing to the bluntness of the vote buying tools available under the secret ballot, could a "shotgun" approach to vote buying become optimal, such that the amount of insincere voting actually increases?

5.1 Insincere Voting

Let us first examine the amount of insincere voting, including abstention, under the secret ballot as compared to the open ballot. That is, we compare the number of people voting against their intrinsic preferences under the two regimes. To do so, we take advantage of the fact that the structure of least cost successful vote buying contracts derived in Propositions 3 and 4 allow us to characterize optimal contracts only in terms of s and θ .

Specifically, under the open ballot, the strategy of interest group A amounts to choosing a surplus level s and a fraction of voters θ_a to induce to vote for policy a, subject to the constraint that s and θ_a are jointly sufficient to deter B. That is, group A chooses s and θ_a to minimize

$$C = \int_{v^{-1}(s)}^{\theta_a} \left(s - v\left(t\right)\right) dt$$

subject to $s \ge \frac{W}{\theta_a - \frac{1}{2}}$. We may then think of the problem in price theory terms. An iso-cost curve for group A satisfies

$$0 = (s - v(\theta_a)) d\theta_a + (\theta_a - v^{-1}(s)) ds$$

The slope of the interest group's iso-cost curve, which we shall refer to as its marginal rate of substitution, or MRS_{open} , is

$$MRS_{open} = \frac{ds}{d\theta_a} = -\frac{(s - v(\theta_a))}{(\theta_a - v^{-1}(s))}$$

Similarly, under the secret ballot, the slope of the iso-cost curve is

$$MRS_{secret} = -\frac{\left(s - \frac{1}{2}v\left(\theta_{b}\right)\right)}{\left(\theta_{b} - v^{-1}\left(2s\right)\right)}$$

A useful feature of this formulation is that we can order the marginal rates of substitution under the open and secret ballot at any point (θ, s) .

Lemma 10 For all (θ, s) , $|MRS_{open}| > |MRS_{secret}|$

The lemma is intuitive. Under the open ballot, the cost of buying the marginal voter consists of compensating him for voting against his intrinsic preference. Under the secret ballot, the cost of buying the marginal voter consists of compensating him for abstaining. Since abstaining is less abhorrent to him than voting against his preferred policy, the required change in surplus to maintain cost neutrality is lower.

We now use the iso-cost curves to identify an optimal contract. A necessary condition for an optimal, i.e. least cost successful, contract is that (θ, s) form a tangency point between an iso-cost curve and the deterrence constraint, which under the open ballot is given by

$$s = \frac{W}{\theta_a - \frac{1}{2}}$$

Thus, under the open ballot, a least cost successful contract (θ_a^*, s_a^*) satisfies

$$MRS_{open}\left(\theta_{a}, s_{a}\right) = -\frac{W}{\left(\theta_{a} - \frac{1}{2}\right)^{2}}$$

$$\tag{2}$$

while also satisfying the determined constraint. Similarly, under the secret ballot, a necessary condition for a least cost successful contract characterized by (θ_b^*, s_b^*) is that

$$MRS_{secret}\left(\theta_{b}, s_{b}\right) = -\frac{W}{\left(\theta_{b} - \left(1 - \theta_{0}\right)\right)^{2}} \tag{3}$$

Next, we show that we can order the slopes of the determined constraints under the open and secret ballot at any point θ .

Lemma 11 For all θ , the slope of the determinance constraint is flatter under the open ballot than under the secret ballot. Formally, $-\frac{W}{\left(\theta-\frac{1}{2}\right)^2} > -\frac{W}{\left(\theta-(1-\theta_0)\right)^2}$ for all θ .

The proof follows immediately from the fact that $\theta_0 < \frac{1}{2}$. Intuitively, group A's savings from exceeding a minimal winning coalition decrease as the size of the coalition grows. The slope of the feasibility constraint simply expresses the speed of this decline. In the case of the open ballot, the size by which a coalition exceeds the minimal winning coalition is $\theta - \frac{1}{2}$, while under the secret ballot the size is given by $\theta - (1 - \theta_0)$, where $1 - \theta_0 > \frac{1}{2}$. Thus, for each value of θ , the marginal savings from expanding the supermajority are smaller under the open ballot than under the secret ballot.

Together, the orderings of the marginal rates of substitution and the deterrence constraints allow us to make an unambiguous statement about the fraction of voters paid to change their voting behavior under the open ballot as compared to the secret ballot. (See Figure 5 for a graphical representation.)

Proposition 5 The introduction of the secret ballot raises the fraction of voters not voting sincerely. Formally, $\theta_a^* \leq \theta_b^*$.

The proposition shows that the secret ballot leads group A to bribe more pervasively than under the open ballot. There are two economic forces driving the result. First, the marginal benefit of increasing θ is higher under the secret ballot, since the effective supermajority is smaller and hence the feasible surplus reduction greater. Second, the marginal cost of increasing θ is lower under the secret ballot since the marginal voter, who is an intrinsic b supporter, need only be compensated for abstaining, as opposed to voting for a under the open ballot. Both forces push the optimal contract in the direction of buying more voters under the secret ballot.



Figure 5

5.2 Buyability

We saw that more voters are influenced by group A under the secret ballot than under the open ballot. This would seem to suggest that the secret ballot is in fact successful in reducing outside influence by raising A's cost of buying the election. However, this simple intuition ignores that the surplus promised to voters also differs under the secret versus the open ballot. Thus, even though more voters are bought under the secret ballot, it could be that the transfers paid are sufficiently small that, in fact, the secret ballot is cost-reducing for A. To examine this possibility, we now compare the cost of successful vote buying under the open versus the secret ballot.

We begin by analyzing the case where the preferences of voters are a linear function of their type.¹¹ This restriction corresponds to uniformly distributed preferences over some interval. Specifically, let

$$v\left(\theta\right) = \alpha - \beta\theta$$

where $\beta > 2\alpha$, such that the median voter strictly prefers b. Under the open ballot, the marginal rate of substitution is simply the slope of $v(\theta)$; that is, $-\beta$. Under the secret ballot, the marginal rate of substitution is half this amount. Since the marginal rate of substitution is independent of s and θ , characterization of the optimal contract under the open and secret ballot is straightforward. Under the open ballot, substituting the MRS into equation (2), we obtain

$$\beta = \frac{W}{\left(\theta_a - \frac{1}{2}\right)^2}$$
$$\theta_a^* = \frac{1}{2} + \sqrt{\frac{W}{\beta}}$$

or

The term $\sqrt{\frac{W}{\beta}}$ represents the size of the supermajority recruited by A. Notice that the size of the optimal supermajority is increasing in W, the value of policy b to group B, while it is decreasing in the (absolute value) of the slope of the preference function. The associated surplus is $s^*_{Open} = \sqrt{W\beta}$

Similarly, under the secret ballot we obtain

$$\theta_b^* = (1 - \theta_0) + \sqrt{\frac{2W}{\beta}}$$

Again, this condition is intuitive. The minimal group of voters that must be induced to abstain is $[\theta_0, 1 - \theta_0]$, which translates into $\theta_b = 1 - \theta_0$. The size of the "supermajority" of abstentions that are optimally induced again depends positively on W and negatively on the slope of the preference function. The associated surplus is

¹¹Strictly speaking, linear v functions do not satisfy the "Inada conditions," i.e., $v(0) = -v(1) = \infty$. However, as long as we restrict attention to interior solutions, this is irrelevant.

 $s_{Secret}^* = \sqrt{\frac{W\beta}{2}}.$

Because preferences are linear in θ , it is also straightforward to calculate the cost of optimal vote buying. Under the open ballot, it is

$$C_{open} = \frac{1}{2} \left(\theta_a^* - v^{-1} \left(s_{Open}^* \right) \right) \left(s_{Open}^* - v \left(\theta_a^* \right) \right)$$
$$= \frac{1}{2} \left(\frac{1}{2} + 2\sqrt{\frac{W}{\beta}} - \frac{\alpha}{\beta} \right) \left(2\sqrt{W\beta} - \alpha + \frac{\beta}{2} \right)$$

while under the secret ballot it is

$$C_{secret} = \frac{1}{2} \left(\theta_b^* - v^{-1} \left(2s_{secret}^* \right) \right) \left(s_{secret}^* - \frac{1}{2} v \left(\theta_b^* \right) \right)$$
$$= \frac{1}{2} \left(1 - 2\theta_0 + 2\sqrt{\frac{2W}{\beta}} \right) \left(2\sqrt{\frac{W\beta}{2}} - \alpha + \frac{1}{2}\beta \right)$$

Comparing the costs under the open and secret ballot we get

$$C_{secret} - C_{open} = \frac{1}{8\beta} \left(\beta - 2\alpha\right) \left(\beta - 2\alpha + 8\sqrt{\beta W} \left(\sqrt{2} - 1\right)\right)$$

$$> 0$$

$$(4)$$

since $\beta > 2\alpha$. Equation (4) reveals that, when the median voter strictly prefers policy b, vote buying costs are strictly higher under the secret ballot than under the open ballot. Interestingly, when the median voter is indifferent between a and b, then the secret ballot does nothing to affect vote buying costs—the increase in the fraction of voters receiving payments under the secret ballot is exactly offset by the reduction in the surplus paid to each of these voters. To summarize, we have shown:

Proposition 6 When preferences are linear in θ and interior solutions obtain, the imposition of the secret ballot raises the costs of vote buying.

Proposition 6 goes in the expected direction—the introduction of the secret ballot does indeed raise vote buying costs and, consequently, reduces the possibility of buying the election.

When preferences are linear in θ , the marginal rate of substitution is also linear. This means that group A faces exactly the same cost trade-off anywhere in (θ, s) space. Clearly, this is a special property of the linear case. Moreover, we obtained a neutrality result when the median voter was indifferent between policies a and b.

To examine the role of linearity in θ and indifference of the median voter more carefully, we consider a class of preferences where the median voter is indifferent between policies a and b, but where the preferences of intrinsic a supporters may be nonlinear in θ . Specifically, suppose that

$$v(\theta) = \begin{cases} (1-2\theta)^{\rho} & \theta \le \frac{1}{2} \\ \beta\left(\frac{1}{2}-\theta\right) & \theta > \frac{1}{2} \end{cases}$$
(5)

Notice that, when $\rho = 1$ and $\beta = 2$, this class of preferences includes one version of the linear preferences analyzed above.

While the assumption that the median voter is indifferent between a and b fails to satisfy $\theta_0 < \frac{1}{2}$, it simplifies the comparison of optimal contracting under the open and secret ballot considerably since, for this case, the deterrence constraints becomes *identical* under the two regimes. As we will show below, it allows us to obtain closed-form solutions for this class of voter preferences and, consequently, to offer conditions where the secret ballot reduces the buyability of the voting body as well as circumstances where precisely the opposite is true—the secret ballot actually makes the voting body *more* vulnerable to outside manipulation. Since we derive strict inequalities between the two regimes, it follows from continuity that for smaller values of θ_0 close to $\frac{1}{2}$ the same ordering applies.

Using the necessary conditions for optimality, i.e. equations (2) and (3), we obtain the following characterization of the optimal contract under the open ballot:

$$\theta_a^* = \frac{1}{2} + \left(\frac{1}{2}\right)^{\frac{1+\rho}{3\rho+1}} (2W)^{\frac{1}{3\rho+1}} \left(\frac{W}{\beta}\right)^{\frac{\rho}{3\rho+1}}$$

with associated surplus

$$s_{Open}^* = \left(2\beta W^2\right)^{\frac{\rho}{3\rho+1}}$$

Similarly, the optimal contract under the secret ballot is characterized by:

$$\theta_b^* = \frac{1}{2} + (2W)^{\frac{1}{3\rho+1}} \left(\frac{W}{\beta}\right)^{\frac{p}{3\rho+1}}$$

with associated surplus

$$s_{Secret}^{*} = \frac{1}{2}^{\frac{1}{3\rho+1}} \left(\beta W^{2}\right)^{\frac{\rho}{3\rho+1}}$$

With these expressions in hand, we are now in a position to state the main result of this section:

Proposition 7 For the class of preference functions in equation (5), the imposition of the secret ballot **reduces** the cost of buying the election if and only if the preferences of intrinsic 'a' supporters are convex in θ . Formally,

- 1. If $\rho < 1$, $C_{open} < C_{secret}$
- 2. If $\rho > 1, C_{open} > C_{secret}$
- 3. If $\rho = 1, C_{open} = C_{secret}$

Proposition 7 illustrates that the effect of the imposition of the secret ballot on the buyability of an election crucially depends on the structure of preferences. Roughly, the proposition says that in close elections where much of the intrinsic support for policy a is lukewarm, the imposition of the secret ballot makes it easier for group A to achieve its desired policy. The reason is that, owing to the inability of B to contract on votes directly, group A is able to economize on payments to lukewarm supporters of its preferred policy, while still deterring B from making any counter offers. (Proposition 7 also illustrates that the cost neutrality result obtained in Proposition 6 generalizes for the case of kinked linear preferences where the median voter is indifferent.) In short, despite the common intuition that the imposition of the secret ballot offers an antidote to vote buying, the theory suggests this may not be true.



Figure 6

6 Conclusion

We have analyzed the effects of the secret ballot on vote buying when voters have to option to abstain. First, we derived the optimal vote buying strategy when interest groups and voters can contract on votes directly, i.e. under the open ballot. Here, we found that, although negative vote buying was feasible, it was never optimal. Indeed, we showed that the option of abstention has no effect at all on vote buying under the open ballot.

Next, we studied optimal vote buying under the secret ballot. In this case, interest groups and voters can only contract on whether to show up at the polls—not on actual votes. We showed that this changes optimal vote buying significantly: Interest groups make extensive use of negative vote buying to induce lukewarm opponents of their preferred policy to stay home on election day. Positive vote buying is also used: Interest groups optimally pay lukewarm supporters of their preferred policy to show up at the polls. Our results are consistent with the empirical findings of Cox and Kousser (1981) who observed little evidence of negative vote buying prior to the imposition of the secret ballot in New York, but considerable evidence of the practice thereafter.

We then compared both the scope and the cost of successful vote buying under the secret versus the open ballot. We found that the imposition of the secret ballot always increases the scope of vote buying—more people vote insincerely under the secret ballot than under the open ballot. We also found circumstances where, paradoxically, the imposition of the secret ballot makes it easier for interest groups to wield influence. In particular, for close elections where the bulk of the supporters of an interest group's desired policy are lukewarm, it is cheaper for that interest group to buy the election under the secret ballot than under the open ballot. Taken together, this suggests that the common intuition about the effectiveness of the secret ballot as a robust deterrent to electoral corruption needs to be revisited.

A useful implication of our analysis is that a particular *combination* of policy reforms is likely to be effective at reducing electoral corruption; combining the secret ballot with mandatory voting removes all scope for vote buying in our model. Thus, one should see less vote buying in countries such as Belgium that have both the secret ballot and mandatory voting than in countries such as the U.S. where voting is not mandatory. That this is not merely a theoretical possibility is suggested by the case of the aborigines in Australia. Unlike the rest of the country, voting was not mandatory for this group from the time they got the vote in 1962 until 1984. In those years, free-flowing alcohol was used extensively and successfully to lure aborigines away from the polls. (See Orr, 2004.)

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A Proofs

Proof of Lemma 2

Suppose not. Clearly, neither interest group acting cannot be an equilibrium, as the vote would then go to b. In that case, by assumption, group A would want to deviate by offering contracts. Hence, group B must be moving first.

Define V to be the set of voters that would accept B's contract if A stayed out. Since we have assumed that there is an infinitesimal possibility that B is a monopolist, the sum of the transfers it offers to voters in V must be smaller or equal to W. In that case, A could respond as follows: First, A "negates" all of B's offers to voters in V and then, in addition, A offers the monopoly contract. Formally, A negates B's contracts by making a counteroffer that leaves the incentives of each voter identical to the case where no contracts are offered by either party. Specifically, if B offers a voter an amount x to vote for b, then A offers the same voter an amount x for not voting for b. (Likewise, if abstention is allowed, if B offers a voter an amount x to abstain, A offers this same voter an amount x for not abstaining.)

The negation contracts leave A almost in the position of acting as a monopolist. The only difference is the possible presence of contracts offered by B to voters who would refuse them even if A stayed out. Note that all such contracts must have been offered to intrinsic A supporters. Else, they would not have been refused. But because the contracts are not sufficiently attractive to sway these intrinsic A supporters, group A can, in fact, safely ignore them. Hence, offering the monopoly contract on top of the negation contracts is sufficient to ensure that policy a is adopted.

Finally, note that the cost of the negation contracts is, at most, W, while the cost of the monopoly contract is C_M . Hence, A's total expenditure on vote buying following a move by B is at most $W + C_M < C$; therefore this is a profitable response by A. As a result, B secures no advantage by going first and, hence, will leave it up to A to make the first move.

Proof of Lemma 1

First, consider the case of interest group *B*. Suppose that it costs *s* to induce an intrinsic *b* supporter of type θ to vote for *b*. Then, it costs $s - \frac{1}{2}v(\theta) > s$ to induce this same voter to abstain. Since abstention is less preferred by *B* and more costly, offering such a contract is a dominated strategy. An analogous argument holds for interest group *A*.

Proof of Lemma 3

Define V to be the set of voter types who receive payments in exchange for voting for a. Define V' be the set of voter types who receive payments in exchange for abstaining.

Recall that a supermajority of the voters intrinsically support policy b. Moreover, note that a best response for group B entails recruiting a *minimal* winning coalition at the lowest possible cost, and that the size of a minimal winning coalition is strictly smaller than $V \cup V'$. This implies that, given a best response by B, there exists a strictly positive fraction of voters recruited by A who will not receive a counter offer from B.

Order all voters in V(V') such that their surplus in the least cost successful contract is non-decreasing. First, if there is no positive measure of voters in V(V')who get a counter offer, then all voters in V(V') must receive the same surplus. Else, all voters in V(V') who receive more than some other voter can be given as little as the infimum of the surpluses of the voters in V(V'). (*B* did not want to recruit any voter in V(V') before; he still will not want to recruit any of them now.) Hence, in this case, all voters in V(V') receive the same surplus.

Next, if there is a positive measure of voters in V(V') who do get a counteroffer from B, then all voters in V(V') who do not get a counter must receive weakly greater surplus than those who do get a counter offer. Else, B would not be minimizing his cost. Moreover, notice that the voters who do not get a counter offer cannot have strictly greater surplus than the supremum of the surpluses of the voters in V(V')who do get a counter offer. Else, A could pay the former strictly less. Hence, also when there are voters in V(V') who receive a counter offer, the surplus of the nonreceivers in V(V'), if they exist, must be "flat."

What about the "counter offer receivers" in V(V')? Irrespective of whether there are also non-receivers, if not all receivers receive the same surplus, then A can give all of them their average surplus, without materially affecting B's problem. Note that this average surplus must be strictly smaller than the surplus of the non-receivers, if they exist.

Suppose that there are non-receivers in both V and V'. If the surplus of non-receivers in V(V') is strictly greater than 2 times $(\frac{1}{2} \text{ time})$ the surplus of non-receivers in V'(V), then the surplus of non-receivers in V(V') can be marginally reduced. If the surplus of the non-receivers in V(V') is exactly equal to 2 times $(\frac{1}{2} \text{ time})$ the

surplus of the non-receivers in V'(V), then the surplus of non-receivers in both V(V') can be marginally reduced simultaneously.

Next, suppose that there are only non-receivers in V(V') but not in V'(V). In that case, all voters in V'(V)—who are all receivers—get the same surplus. If the surplus of non-receivers in V(V') is strictly greater than 2 times $(\frac{1}{2} \text{ time})$ the surplus of the receivers in V'(V), then the non-receivers' surplus in V(V') can be marginally reduced. If the surplus of non-receivers in V(V') is strictly smaller than 2 times $(\frac{1}{2} \text{ time})$ the surplus of the receivers in V'(V), then B is not optimizing. Finally, if the surplus of the non-receivers in V(V') is exactly equal to 2 times $(\frac{1}{2} \text{ time})$ the surplus of the receivers in V'(V), then, either the receivers in V get the same surplus as the non-receivers in V, in which case the lemma holds, or they get strictly less. In that case, some surplus can be transferred in a "budget neutral" fashion from the receivers in V' to the receivers in V, without materially altering B's problem. But, after this transfer, the non-receivers in V now get strictly more than 2 times the surplus of receivers in V'. As we saw previously, this generates a profitable deviation for A.

This implies that all voters in V(V') receive the same surplus s(s'), and that s = 2s'.

Proof of Lemma 2

First, since $v(0) = -v(1) = \infty$ it follows immediately that voters with types $[0, \varepsilon)$ vote for a while those with types $(1 - \varepsilon, 1]$ vote for b, for ε sufficiently small. Thus, we need only prove that the sets of a voters, abstainers, and the union of a voters and abstainers is convex to obtain the lemma.

To establish part 1 of the lemma, suppose to the contrary that there is a set O consisting of a measure $\mu > 0$ of voters who do something other than vote for a and a set M consisting of the same measure, μ , of voters all of whom vote for a. Suppose further that for all $\theta \in O$ and $\theta' \in M$ it is the case that $\theta < \theta'$. By Lemma 1, the voters in O and M consist entirely of intrinsic B supporters.

Now consider the following deviation: Voters in O are paid to vote for A while voters in M are paid to do whatever the former O voters did. By Lemma 1, we need only consider the costs to B of switching abstainers or a voters to b voters. By construction, these costs are unchanged; hence the deviation contract is successful. Furthermore, the change in the cost of the contract is at most

$$\Delta C = \frac{1}{2} \left(\int_{\theta \in M} v(\theta) \, d\theta - \int_{\theta \in O} v(\theta) \, d\theta \right)$$

< 0

since $v(\cdot)$ is strictly decreasing and strictly negative, which contradicts the notion that the original contract was least cost. An identical argument can be used to show parts 2 and 3 of the lemma.

Proof of Lemma 4

Suppose not. Suppose that under the optimal scheme $\theta_a < \frac{1}{2}$. In that case, because the contract is successful, a positive mass of voters must also be paid to abstain; i.e. $\theta_b > \theta_a$. Let \bar{s} be the amount of surplus paid to abstainers in equilibrium.

Now, consider a deviation by group A where it recruits a mass ε of additional voters to vote for a rather than to abstain. At the same time, the mass of abstaining voters is reduced by the same amount. By Lemma ??, we know that such voters intrinsically prefer b, while from Lemma 3 we know that the transfer required for the new a voters is $2\overline{s}$. Hence, the incremental cost of this deviation is

$$\Delta C = \bar{s}\varepsilon - \frac{1}{2} \int_{\theta_a}^{\theta_a + \varepsilon} v(\theta) d\theta$$
$$- \left(\bar{s}\varepsilon - \frac{1}{2} \int_{\theta_b - \varepsilon}^{\theta_b} v(\theta) d\theta \right)$$
$$= \frac{1}{2} \left(\int_{\theta_b - \varepsilon}^{\theta_b} v(\theta) d\theta - \int_{\theta_a}^{\theta_a + \varepsilon} v(\theta) d\theta \right)$$
$$< 0$$

since $v(\cdot)$ is strictly decreasing and negative for all voters who intrinsically prefer B. This is a contradiction.

Proof of Lemma 5

From Lemma 4 we know that A has all voters up to $\frac{1}{2}$ voting for a. Suppose that, contrary to the Lemma, the least cost successful contract entails buying a mass of negative votes as well; i.e. $\theta_b > \theta_a$ under Lemma 2. Let the surplus of the abstainers be \bar{s} . From Lemma 3 we know that these voters must be offered surplus $2\bar{s}$ to vote for a.

Consider a deviation whereby voters $\left[\theta_a, \frac{\theta_a + \theta_b}{2}\right]$ are paid to vote for *a* while the remaining voters are not paid at all. The net change in the surplus associated with this deviation is zero and, moreover, *B* remains deterred following the deviation. The change in the costs to *A* are

$$\Delta C = \frac{1}{2} \left(\int_{\frac{\theta_a + \theta_b}{2}}^{\theta_b} v(\theta) \, d\theta - \int_{\theta_a}^{\frac{\theta_a + \theta_b}{2}} v(\theta) \, d\theta \right)$$

< 0

Hence, this is a profitable deviation.

Proof of Lemma 6

To prove part 1, first notice that in any successful contract B makes no counter offers. Next, note that, in the absence of counter offers, only types $\theta \in [0, \theta_0]$ will ever vote for a. Further, it costs group A more to induce these voters to abstain, and this is clearly worse for that group than voting for a. Hence, voters with types $[0, \theta_0]$ vote for a.

Thus, we need only consider the interval $[\theta_0, 1]$, which consists of intrinsic *b* supporters. Under the secret ballot, these voters can only be induced to abstain or to vote for *b*. They will never vote for *a* under any contract. Suppose, contrary to part 2 of the Lemma, that there exists a set *O* containing a positive measure of voters voting for *b* (and hence not paid by *A*) and a set *M* containing a positive measure of voters induced to abstain, such that, for all $\theta \in O$ and $\theta' \in M$, it is the case that $\theta < \theta'$. It is without loss of generality to assume that *O* and *M* contain equal mass. Consider the following deviation: Voters in *O* are paid to abstain while voters in *M* are not paid at all. Clearly, the net surplus is unchanged by this deviation. The change in the cost of the contract is at most

$$\Delta C = \frac{1}{2} \left(\int_{\theta \in M} v(\theta) \, d\theta - \int_{\theta \in O} v(\theta) \, d\theta \right)$$

< 0

since $v(\cdot)$ is strictly decreasing and strictly negative, which contradicts the notion that the original contract was least cost.

Part 3 of the lemma follows immediately from parts 1 and 2.

Proof of Lemma 7

First, notice that any best response by B entails making counteroffers to only a subset of voters in $(\theta_0, \theta_b]$. Furthermore, under any best response, counteroffers will only be made to voters receiving the least surplus. Suppose, contrary to the lemma, that a positive measure of voters in this set receive different surplus amounts.

Consider the case where a positive measure of voters in $(\theta_0, \theta_b]$ receive a counter offer. In that case, the surpluses to a positive measure of voters not receiving a counter offer may be lowered without affecting *B*'s incentives while still reducing *A*'s costs. This contradicts the notion of a least cost contract.

Next, consider the case where none of the voters in $(\theta_0, \theta_b]$ receive a counter offer. In that case, for a positive measure of voters receiving the highest surplus, group A can lower their transfers infinitesimally while not affecting B's incentives. This strictly lowers A's costs and hence is a contradiction.

Proof of Lemma 8

For the cases where a positive measure of voters in V do not receive a counteroffer, the proof is identical to that given in Lemma 7 with the observation that the only credible counter offer B can make to intrinsic a supporters induces them to abstain.

It remains to consider the case where all voters in V receive a counter offer. First, for this to constitute a best response by B, it must be the case that the highest surplus (compared to abstention) received by any voter in V is smaller than s, the surplus received by all abstaining voters. Consider the following deviation: Suppose that all voters in V are paid the average surplus (relative to abstention). Clearly, this amount is strictly smaller than the surplus paid to the abstaining voters and, hence, does not affects B's incentives. Define C' to be the set of abstaining voters receiving counter offers from B. As above, $C' \subset [\theta_0, \theta_b]$. Next, suppose that A marginally reduces the surplus paid to the abstainers by an amount ε and transfers ε times the measure of C' on a per capita basis to all voters in V. This again has no effect on the incentives of B, since it costs exactly the same for B to induce $V \cup C'$ to switch their votes and no other coalition of the same measure is cheaper. Further, this deviation strictly reduces A's costs. This is a contradiction.

Proof of Lemma 9

Suppose not. Then either s < s' or vice versa. Suppose that s' < s. In that case, a best response for B is to bribe as many voters in V as are needed to form a winning coalition. If there are insufficient voters in V, then B should bribe voters who abstain.

Regardless, a positive measure of voters who abstain will be left without a counter offer. Consider the following deviation by A. Define C' to be the set of abstaining voters receiving counter offers from B. As above, $C' \subset [\theta_0, \theta_b]$. Next, suppose that Amarginally reduces the surplus paid to the abstainers by an amount ε and transfers ε times the measure of C' on a per capita basis to all voters in V. This has no effect on the incentives of B since it costs exactly the same for B to induce $V \cup C'$ to switch their votes and no other coalition of the same measure is cheaper. Further, this deviation strictly reduces A's costs. This is a contradiction.

The proof for the case where s' > s is analogous.

Proof of Lemma 10

Recall that

$$MRS_{open} = -\frac{s - v(\theta)}{\theta - v^{-1}(s)}$$
$$MRS_{secret} = -\frac{s - \frac{1}{2}v(\theta)}{\theta - v^{-1}(2s)}$$

and note that $s - v(\theta) > s - \frac{1}{2}v(\theta)$, whilst $\theta - v^{-1}(s) < \theta - v^{-1}(2s)$. Hence, $|MRS_{open}| > |MRS_{secret}|.$

Proof of Proposition 5

Suppose to the contrary that $\theta_a^* > \theta_b^*$. Consider a solution to

$$MRS_{secret} = -\frac{W}{\left(\theta - \frac{1}{2}\right)^2}$$

That is, find a solution where the marginal rate of substitution under the secret ballot is tangent to the deterrence line under the open ballot. Call this solution θ'_b and, be Lemma 10, it follows immediately that $\theta'_b > \theta^*_a$. Next, notice that, evaluated at θ'_b ,

$$MRS_{secret} > -\frac{W}{\left(\theta_b' - (1 - \theta_0)\right)^2}$$

by Lemma 11. Furthermore, since the absolute value of the slope of the deterrence constraint is decreasing, it then follows that $\theta_b^* > \theta_b'$, but this is a contradiction.

Proof of Proposition 7

Under the open ballot, the costs are

$$\begin{split} C_{open} &= \int_{v^{-1}(s)}^{\frac{1}{2}} \left(s - v\left(\theta\right)\right) d\theta + \left(\theta - \frac{1}{2}\right) s + \frac{1}{2} \left(\theta - \frac{1}{2}\right) \left(-v\left(\theta\right)\right) \\ &= \int_{\frac{1}{2}\left(1 - s^{\frac{1}{\rho}}\right)}^{\frac{1}{2}} \left(s - (1 - 2t)^{\rho}\right) dt + W + \frac{\beta}{2} \left(\theta - \frac{1}{2}\right)^{2} \\ &= \frac{1}{2} \frac{\rho}{1 + \rho} s^{\frac{1 + \rho}{\rho}} + W + \frac{\beta}{2} W^{\frac{2}{3\rho + 1}} \left(\frac{W^{2}}{(2\beta)^{2}}\right)^{\frac{\rho}{3\rho + 1}} \\ &= \frac{1}{2} \frac{\rho}{1 + \rho} \left(2\beta W^{2}\right)^{\frac{1 + \rho}{3\rho + 1}} + W + \frac{\beta}{2} W^{\frac{2}{3\rho + 1}} \left(\frac{W^{2}}{(2\beta)^{2}}\right)^{\frac{\beta}{3\rho + 1}} \\ &= \frac{1}{2} \frac{\rho}{1 + \rho} \left(W^{2}\right)^{\frac{1}{3\rho + 1}} \left(W^{2}\right)^{\frac{\rho}{3\rho + 1}} \left(2\beta\right)^{\frac{1 + \rho}{3\rho + 1}} + W + \frac{\beta}{2} \left(\frac{1}{(2\beta)^{2}}\right)^{\frac{\beta}{3\rho + 1}} \left(W^{2}\right)^{\frac{1}{3\rho + 1}} \left(W^{2}\right)^{\frac{\rho}{3\rho + 1}} \\ &= \left(W^{2}\right)^{\frac{1}{3\rho + 1}} \left(W^{2}\right)^{\frac{\rho}{3\rho + 1}} \left(\frac{1}{2} \frac{\rho}{1 + \rho} \left(2\beta\right)^{\frac{1 + \rho}{3\rho + 1}} + \frac{\beta}{2} \left(\frac{1}{(2\beta)^{2}}\right)^{\frac{\beta}{3\rho + 1}} + W^{\frac{\rho - 1}{3\rho + 1}}\right) \\ &= W^{\frac{2(1 + \rho)}{3\rho + 1}} \left(\frac{1}{2} \frac{\frac{3\rho}{3\rho + 1}} \left(\frac{\rho}{1 + \rho} + \frac{1}{2}\right) + W^{\frac{\rho - 1}{3\rho + 1}}\right) \end{split}$$

while under the secret ballot

$$\begin{split} C_{secret} &= \int_{v^{-1}(2s)}^{\frac{1}{2}} \left(s - \frac{1}{2}v\left(t\right) \right) dt + \left(\theta - \frac{1}{2} \right) s + \frac{1}{2} \left(\theta - \frac{1}{2} \right) \left(-\frac{1}{2}v\left(\theta\right) \right) \\ &= \int_{v^{-1}(2s)}^{\frac{1}{2}} \left(s - \frac{1}{2} \left(1 - 2t \right)^{\rho} \right) dt + W + \frac{\beta}{4} \left(\theta - \frac{1}{2} \right)^{2} \\ &= \frac{1}{2} \frac{\rho}{1+\rho} 2^{\frac{1}{\rho}} s^{\frac{1+\rho}{\rho}} + W + \frac{\beta}{4} \left(2^{\frac{1}{3\rho+1}} W^{\frac{1+\rho}{3\rho+1}} \left(\frac{1}{\beta} \right)^{\frac{\rho}{3\rho+1}} \right)^{2} \\ &= \frac{1}{2} \frac{\rho}{1+\rho} 2^{\frac{1}{\rho}} \left(\frac{1}{2} \frac{1}{3\rho+1} \left(\beta W^{2} \right)^{\frac{\rho}{3\rho+1}} \right)^{\frac{1+\rho}{\rho}} + W + \frac{1}{2} \left(\frac{1}{2} \right)^{\frac{3\rho-1}{3\rho+1}} W^{\frac{2+2\rho}{3\rho+1}} \left(\beta \right)^{\frac{\rho+1}{3\rho+1}} \\ &= W^{\frac{2(1+\rho)}{3\rho+1}} \left(\frac{1}{2} \beta^{\frac{\rho+1}{3\rho+1}} \left(\frac{\rho}{1+\rho} 2^{\frac{2\rho}{(3\rho+1)\rho}} + \left(\frac{1}{2} \right)^{\frac{3\rho-1}{3\rho+1}} \right) + W^{\frac{\rho-1}{3\rho+1}} \right) \end{split}$$

Comparing the voting costs under the two regimes reveals:

$$\begin{split} C_{secret} - C_{open} &= W^{\frac{2(1+\rho)}{3\rho+1}} \left(\frac{1}{2} \beta^{\frac{\rho+1}{3\rho+1}} \left(\frac{\rho}{1+\rho} 2^{\frac{2\rho}{(3\rho+1)\rho}} + \left(\frac{1}{2} \right)^{\frac{3\rho-1}{3\rho+1}} \right) + W^{\frac{\rho-1}{3\rho+1}} \right) \\ &- W^{\frac{2(1+\rho)}{3\rho+1}} \left(\frac{1}{2} \frac{\frac{2\rho}{3\rho+1}}{\beta^{\frac{2}{3\rho+1}}} \beta^{\frac{1+\rho}{3\rho+1}} \left(\frac{\rho}{1+\rho} + \frac{1}{2} \right) + W^{\frac{\rho-1}{3\rho+1}} \right) \\ &= W^{\frac{2(1+\rho)}{3\rho+1}} \beta^{\frac{\rho+1}{3\rho+1}} \left(\left(\frac{1}{2} \frac{\rho}{1+\rho} 2^{\frac{2\rho}{(3\rho+1)\rho}} - \frac{1}{2} \frac{\frac{2\rho}{3\rho+1}}{1+\rho} \right) \right) \\ &+ \left(\frac{1}{2} \left(\frac{1}{2} \right)^{\frac{3\rho-1}{3\rho+1}} - \frac{1}{2} \frac{\frac{2\rho}{3\rho+1}}{1+\rho} \right) \\ &+ \left(\left(\frac{1}{2} \right)^{\frac{2\rho}{3\rho+1}} \left(\frac{1}{2} \right)^{\frac{\rho+1}{3\rho+1}} \left(\frac{1}{2} \right)^{\frac{2\rho}{3\rho+1}} \left(\frac{1}{2} \right)^{\frac{\rho+1}{3\rho+1}} \left(\frac{1}{2} \right)^{\frac{3\rho-1}{3\rho+1}} - \frac{1}{2} \frac{\frac{2\rho}{3\rho+1}}{1+\rho} \right) \\ &= W^{\frac{2(1+\rho)}{3\rho+1}} \beta^{\frac{\rho+1}{3\rho+1}} \left(\frac{1}{2} \right)^{\frac{2\rho}{3\rho+1}} \left(\frac{1}{2} \right)^{\frac{\rho+1}{3\rho+1}} \left(\frac{1}{2} \right)^{\frac{3\rho-1}{3\rho+1}} - \frac{1}{2} \frac{2\rho}{3\rho+1} \frac{1}{2} \right) \\ &= W^{\frac{2(1+\rho)}{3\rho+1}} \beta^{\frac{\rho+1}{3\rho+1}} \left(\frac{1}{2} \right)^{\frac{2\rho}{3\rho+1}} \left(\frac{\rho}{1+\rho} \left(\left(\frac{1}{2} \right)^{\frac{\rho+1}{3\rho+1}} 2^{\frac{2\rho}{3\rho+1}} - 1 \right) \right) \\ &+ \left(\left(\frac{1}{2} \right)^{\frac{3\rho+1}{3\rho+1}} - \frac{1}{2} \right) \end{pmatrix} \end{split}$$

and this expression is positive iff $\rho < 1$, negative iff $\rho > 1$ and equals zero at $\rho = 1$.

B Alternative Preference Specifications

In this section, we suppose that for all $\theta \leq \theta_0$, $u(b; \theta) = 0$, while for $\theta > \theta_0$, $u(a; \theta) = 0$. We show that essentially the same results for equilibrium vote buying arise for this preference specification as in the model presented in the main text. However, in this case, where buying an intrinsic opponent's abstention is equally expensive as buying his vote, unsurprisingly, the secret ballot always raises the cost of buying the election.

Proposition 8 Under the open ballot, negative vote buying is always feasible but never optimal.

Proof. Suppose, to the contrary, that there exists a least-cost vote buying scheme in which a positive measure of voters are paid to abstain. In that case, group A can deter B by deviating and offering a fraction $1 - \varepsilon$ of these voters the same contract in exchange for voting for a while offering the remaining fraction ε of the voters the null contract. This strictly reduces A's costs and hence contradicts the notion that the original scheme was least cost.

Proposition 9 Under the secret ballot with abstention, the unique least cost successful contract is as follows:

Voters with type $\theta \in [0, \theta_0]$ vote for 'a'; voters with type $\theta \in (\theta_0, \theta_b^*]$ abstain; voters with type $\theta \in (\theta_b^*, 1]$ vote for 'b'.

All voters with type $\theta \leq \theta_b^*$ receive transfers $t(\theta) = \max\{0, s(\theta_b^*) - v(\theta)\}$ and earn surplus equal to $\max\{s(\theta_b^*), v(\theta)\}$

where

$$s\left(\theta_{b}^{*}\right) = \frac{W}{\theta_{b}^{*} - (1 - \theta_{0})}$$

and θ_b^* is the unique solution to

$$\arg\min_{\theta_{b}\geq 1-\theta_{0}}\int_{0}^{\theta_{b}}t\left(\theta\right)d\theta$$

Proof. The argument is identical to that leading up to Proposition 4.

Proposition 10 The introduction of the secret ballot raises the fraction of voters not voting sincerely. Formally, $\theta_a^* < \theta_b^*$.

Proof. Under these preferences, it is easily verified that for all (s, θ) , $|MRS_{open}| = |MRS_{secret}|$. Next, notice that the deterrence constraints are independent of voter preferences over abstention. It then follows that the ordering given in Lemma 11 is unchanged. Together, these two orderings orderings imply that $\theta_a^* < \theta_b^*$.

Proposition 11 The introduction of the secret ballot raises the cost of buying the election

Proof. First, notice that the determence constraints imply that, for a fixed θ , the surplus paid under the secret ballot always exceeds that under the open ballot. Formally,

$$s_{open} = \frac{W}{\theta - \frac{1}{2}} < \frac{W}{\theta - (1 - \theta_0)} = s_{secret}$$

Next, notice that, if we substitute for s using the deterrence constraint, then the costs of buying the election are only a function of θ . Moreover, since $s_{open} < s_{secret}$, it then immediately follows that for all θ ,

$$C_{open}\left(\theta\right) < C_{secret}\left(\theta\right)$$

Finally, since θ_a^* minimizes costs under the open ballot, it then follows that

$$C_{open}\left(\theta_{a}^{*}\right) < C_{open}\left(\theta_{b}^{*}\right) < C_{secret}\left(\theta_{b}^{*}\right)$$

which establishes the result. \blacksquare