A Simple Model of Advertising and Subscription Fees

Michael R. Baye
Indiana University
Email: mbaye@indiana.edu
Phone: (812) 855-2779
Fax: (812) 855-3354

John Morgan
Princeton University
Email: rjmorgan@princeton.edu
Phone: (609) 258-4842
Fax: (609) 258-2809

Final Version, May 2000

Abstract

Traditional and modern mass media – such as television, newspapers, magazines, and Internet sites – typically derive the bulk of their revenues from advertisements rather than subscriptions. We present a simple model that explains this phenomenon. **(JEL Numbers: D4, D8, M3, L13. Keywords: Price Dispersion, Advertising)**

1 Introduction

Internet content providers tend to derive the bulk of their revenues from information transmitters (advertisers) rather than information receivers (subscribers). For instance, Cnet.com charges firms who wish to advertise prices on their site, but permits consumers to access the menu of prices for free. Internet "portal" sites and search engines such as Yahoo! and AltaVista likewise derive the bulk of their revenues from banner ads and other informational postings at their site rather than from fees charged to consumers for the information provided. Indeed, the few Internet sites, such as Slate, attempting to derive significant revenues from consumers have been notable failures.

Differences in the fees associated with acquiring versus transmitting information are not unique to new media. Established types of media, such as newspapers and magazines, also tend to derive the bulk of their revenues from advertising. This is illustrated in Figure 1, which shows the distribution of advertising revenues as a percentage of total (advertising plus subscription) revenues for the top 300 magazines as reported in 1997. As the figure makes clear, almost two-thirds of these magazines derived 60 percent or more of their revenues from advertisements. At the aggregate level, advertising revenues account for 63.31 percent of their total revenues.

This paper shows that a simple duopoly version of Baye and Morgan (2000) explains this phenomenon. In contrast to the general approach in Baye and Morgan (2000), the present approach permits us to obtain closed-form expressions for the subscription and advertising fees set by a profit-maximizing gatekeeper.

2 The Model

We begin by sketching a duopoly version of the Baye and Morgan (2000) model; the interested reader should see their paper for details. A product market consists of a continuum of risk-neutral consumers (normalized to have unit measure) and two identical risk-neutral firms who are known to produce identical products at zero cost. Consumers do not know the price charged by either firm, and have unit demand up to the monopoly price, r $(0 < r < \infty)$.

Advertising permits firms to transmit price information to consumers who subscribe to the gatekeeper's outlet. To be concrete, we shall refer to the gatekeeper as the "newspaper," although as the introduction makes clear, the model applies to a wide variety of media outlets. The newspaper is the sole source of price information available to consumers, and the information contained in it depends upon the advertising decisions of firms. By paying an advertising fee of $\phi > 0$, firm i can advertise its price, $p_i \in [0, r]$, in the paper. A consumer who has paid a subscription fee of $\kappa \geq 0$ can read the paper and determine where to shop. Both subscribers and non-subscribers have the option of visiting their local store at a cost of ε; for the sequel, we assume that ε is negligible and can be ignored. Half of the customers are located near firm 1, and the rest are located near firm 2.

Let $\mu > 0$ denote the fraction of subscribers, $(p_1, p_2) \in [0, r] \times [0, r]$ denote firm prices, and $(a_1, a_2) \in \{A, N\} \times \{A, N\}$ denote the advertising decisions of firms. Here, A represents the event where a firm chooses to advertise its price in the newspaper, and N is the event where a firm does not place an ad. Propositions 1 and 2 in Baye and Morgan (2000) imply that the duopolists payoff functions are:

$$\pi_i(p_1, p_2, a_1, a_2) = \begin{cases} \mu p_i + \frac{(1-\mu)}{2} p_i - \phi & \text{if} \quad p_i < p_j, a_i = A, a_j = A \\ \frac{\mu}{2} p_i + \frac{(1-\mu)}{2} p_i - \phi & \text{if} \quad p_i = p_j, a_i = A, a_j = A \\ \frac{(1-\mu)}{2} p_i - \phi & \text{if} \quad p_i > p_j, a_i = A, a_j = A \\ \mu p_i + \frac{(1-\mu)}{2} p_i - \phi & \text{if} \quad a_i = A, a_j = N \\ \frac{1-\mu}{2} p_i & \text{if} \quad a_i = N, a_j = A \\ \frac{1}{2} p_i & \text{if} \quad a_i = N, a_j = N \end{cases}$$

3 Equilibrium

Let α be the probability that a firm places an advertisement in the newspaper. Then, given $\mu \in (0,1]$ and $\phi \in \left(0, \frac{r\mu}{2}\right)$, Proposition 3 of Baye and Morgan (2000) implies that there exists a symmetric equilibrium in which each firm advertises with probability

$$\alpha^* (\mu, \phi) = 1 - \frac{2\phi}{r\mu}, \tag{1}$$

and the cumulative distribution of advertised prices is

$$F^*(p; \mu, \phi) = \frac{[p(\mu+1) - r(1-\mu) - 4\phi]\, r}{2p(r\mu - 2\phi)} \text{ on } \left[\underline{p}, r\right], \tag{2}$$

where $\underline{p} = \frac{r(1-\mu)+4\phi}{\mu+1}$. Furthermore, firms that do not advertise charge the monopoly price, r.

Our first proposition establishes that, for the duopoly case at hand, this symmetric equilibrium is the only equilibrium.

Proposition 1 *Suppose $\mu \in (0,1]$ and $\phi \in \left(0, \frac{r\mu}{2}\right)$. Then the firms' advertising decisions are uniquely determined by (α^*, F^*) in equations (1) and (2).*

Proof. Let (F_i, α_i) $i = 1, 2$ denote candidate mixed-strategy equilibria for the two firms (note that pure strategies are covered as the special case when these strategies are degenerate). If we view α_i as a parameter of the game instead of a choice variable for firms, we have a simple pricing game with discontinuous payoffs (see Dasgupta and Maskin, 1986). Applying standard arguments to this parameterized game (cf. Baye, Kovenock, and de Vries, 1992) and letting $\pi_i^*(\alpha_1, \alpha_2)$ denote firm i's equilibrium profits when it advertises yields the following:

Lemma 1 *Suppose $0 < \alpha_2 \leq \alpha_1 \leq 1$ and $\mu > 0$. Then in any Nash equilibrium of the parameterized game:*

$$\pi_1^*(\alpha_1, \alpha_2) = \frac{1}{2} r (1 + \mu - 2\alpha_1 \mu) - \phi$$
$$\pi_2^*(\alpha_1, \alpha_2) = r\left(\frac{1-\mu}{2} + (1-\alpha_1)\mu\right) - \phi.$$

Furthermore, if $\alpha_1 = \alpha_2 = \alpha > 0$, the equilibrium distributions of advertised prices are symmetric and unique, and

$$\pi_i^*(\alpha, \alpha) = r\left(\frac{1-\mu}{2} + (1-\alpha)\mu\right) - \phi.$$

In light of Lemma 1, it is sufficient to show that $0 < \alpha_1 = \alpha_2$ in any equilibrium. Lemma 2 below establishes first strict inequality, while Lemma 3 establishes the second equality.

Lemma 2 *In any Nash equilibrium,* $\alpha_i > 0$ *for* $i = 1, 2$.

Proof: By way of contradiction, suppose $\alpha_2 = 0$. Since firm 2 does not advertise, it is clear that firm 1's optimal advertised a price is r. The profits associated with this price are $\frac{r}{2}(1+\mu)-\phi$ which exceed firm 1's expected profits of $\frac{r}{2}$ from not advertising. Hence, when $\alpha_2 = 0$, the best response by firm 1 is to set $\alpha_1 = 1$ and charge r with probability 1. Notice, however, that firm 2 can profitably deviate by advertising and charging a price slightly below r. This contradicts the hypothesis that $\alpha_2 = 0$. QED

Lemma 3 *In any Nash equilibrium,* $\alpha_1 = \alpha_2$.

Proof: By way of contradiction, suppose $\alpha_2 < \alpha_1$.

Case 1: $0 < \alpha_2 < \alpha_1 < 1$. For this to be an equilibrium, each firm must be indifferent between advertising and not, so

$$r\left(\frac{1-\mu}{2} + (1-\alpha_1)\mu\right) - \phi = \frac{1}{2}r(1-\alpha_1\mu)$$

and

$$\frac{1}{2}r(1+\mu-2\alpha_1\mu) - \phi = \frac{1}{2}r(1-\alpha_2\mu).$$

Solving these two equations for $\alpha_1$and α_2 yields:

$$\alpha_1 = \alpha_2 = 1 - \frac{2\phi}{r\mu},$$

which contradicts the hypothesis that $\alpha_2 < \alpha_1$.

Case 2: $0 < \alpha_2 < \alpha_1 = 1$. Since $\alpha_2 \in (0,1)$, firm 2 must be indifferent between advertising and not, so

$$\frac{1}{2}r(1-\mu) - \phi = \frac{1}{2}r(1-\mu).$$

This is a contradiction, since $\phi > 0$. QED

Since Lemmas 2 and 3 imply that $0 < \alpha_1 = \alpha_2 = \alpha$, it follows from Lemma 1 that (α^*, F^*) in equations 1 and 2 uniquely define the firms' advertising decisions. ■

Equilibrium in this model also requires that each consumer's subscription decision be determined optimally given the newspaper's fee-setting decisions and the advertising/pricing decisions of firms. With this in mind, we now turn to optimal fee-setting decisions on the part of the newspaper.

The newspaper's expected profits consist of expected advertising and subscription revenues (we assume costs are zero):

$$E\Pi \equiv 2\alpha\phi + \mu\kappa.$$

To maximize expected profits, the newspaper must take into account the impact of changes in advertising and subscription fees (ϕ, κ) on firms' advertising decisions (α^*) and consumers' subscription decisions (μ). In particular, the direct effect of an increase in ϕ is to reduce firms' propensities to advertise in equation (1) and to change the distribution of advertised prices in equation (2). This, in turn, changes the benefit a consumer derives from subscribing to the paper; therefore, a change in advertising fees indirectly affects subscription decisions. Similarly, an increase in κ directly affects consumer subscription decisions, and indirectly affects firm advertising decisions since the number of subscribers affects firms' advertising decisions.

The following proposition shows that when these effects are taken into account, the newspaper maximizes its expected profits by setting subscription fees low enough that all consumers subscribe. Advertising fees, in contrast, are set above subscription fees.

Proposition 2 *In the equilibrium that maximizes the newspaper's profits:*
(a) *all consumers subscribe to the paper* $(\mu^* = 1)$*;*
(b) *each firm is charged* $\phi^* = \frac{r}{2e}$ *to advertise its price in the paper (where* $e \equiv \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$ *);*
(c) *consumers are charged a subscription fee of* $\kappa^* = \frac{r}{e^2}$*; and*
(d) *each firm advertises its price with probability* $\alpha^* = 1 - \frac{1}{e}$*.*

Proof. Let $\beta(\mu, \phi)$ denote the expected (gross) benefit to a consumer from subscribing to the paper when firms employ the strategies given by equations (1) and (2). A consumer's gross benefit from subscribing to the paper is the amount saved by having access to the list of prices in the paper versus not having that information. The expected value of these savings can be written as

$$\beta(\mu, \phi) = \alpha^*(\mu, \phi)\left(\alpha^*(\mu, \phi)\left[E_1^{(2)}(\mu, \phi) - r\right] + r - M_1(\mu, \phi)\right)$$

where $E_1^{(2)}(\mu, \phi)$ denotes the expectation of the maximum order statistic of two draws from the distribution F^*, and $M_1(\mu, \phi)$ denotes the mean price a firm offers conditional on advertising.

For any choice of $\kappa > 0$ and $\phi > 0$, the gatekeeper clearly earns zero expected profits in any equilibrium where $\mu = 0$. When a fraction $\mu > 0$ of consumers subscribe to the paper, the unique equilibrium is the one described by (α^*, F^*). Hence the profit-maximizing subscription rate is $\kappa = \beta(\mu, \phi)$. The equilibrium that maximizes the gatekeeper's expected profits is the solution to the programing problem

$$\max_{\mu,\phi} \Pi = \mu\beta(\mu, \phi) + 2\phi\alpha^*(\mu, \phi).$$

We first solve for the optimal ϕ for a given μ, temporarily ignoring the constraints that $\mu \in (0,1]$ and $\phi \in \left(0, \frac{r\mu}{2}\right)$. The first-order condition is

$$\frac{\partial}{\partial \phi}\left(\mu\beta(\mu,\phi) + 2\phi\left(1 - \frac{2\phi}{r\mu}\right)\right) = -2\frac{\mu + \ln(r - r\mu + 4\phi) - \ln(\mu+1) - \ln r}{\mu} = 0,$$

which implies

$$\phi(\mu) = \frac{1}{4}r\frac{-e^{\mu} + e^{\mu}\mu + \mu + 1}{e^{\mu}}.$$

It is routine to verify that $\mu\beta(\mu,\phi) + 2\phi\alpha^*(\mu,\phi)$ is strictly concave in ϕ for a given μ; hence $\phi(\mu)$ is a maximand.

We now show that $\frac{\partial}{\partial\mu}(\mu\beta(\mu,\phi) + 2\phi\alpha^*(\mu,\phi))$ is everywhere positive when evaluated at $\phi = \phi(\mu)$. Since

$$\frac{\partial}{\partial\mu}(2\phi\alpha^*)\,|_{\phi=\phi(\mu)} = \frac{4(\phi(\mu))^2}{\mu^2 r}$$

is positive for all $\mu > 0$, it is sufficient to show that $\frac{\partial}{\partial\mu}\mu\beta(\mu,\phi)\,|_{\phi=\phi(\mu)} > 0$.

This is equivalent to showing that

$$\frac{1}{2}r^2(\mu+1)\left((\mu+1)^2\left(1 - e^{-2\mu}\right) - 2\mu e^{-\mu}(2\mu+1)\right) > 0. \tag{3}$$

Inequality (3) follows from the fact that

$$\left(e^{\mu} - \frac{1}{e^{\mu}}\right) > 2\mu\frac{2\mu+1}{(\mu+1)^2} \tag{4}$$

for $\mu > 0$. To see this, note that

$$\frac{\partial}{\partial\mu}\left(e^{\mu} - \frac{1}{e^{\mu}}\right) = e^{\mu} + e^{-\mu} > 0; \tag{5}$$

$$\frac{\partial}{\partial\mu}\left(2\mu\frac{2\mu+1}{(\mu+1)^2}\right) = 2\frac{3\mu+1}{(\mu+1)^3} > 0; \tag{6}$$

$$\frac{\partial^2}{\partial\mu^2}\left(2\mu\frac{2\mu+1}{(\mu+1)^2}\right) = -12\frac{\mu}{(\mu+1)^4} < 0; \text{ and}$$

$$\frac{\partial^2}{\partial\mu^2}\left(e^{\mu} - \frac{1}{e^{\mu}}\right) = \left(1 - e^{-2\mu}\right)e^{\mu} > 0,$$

so (5) attains its minimum at $\mu = 0$ and (6) obtains its maximum at $\mu = 0$. For any $\mu > 0$, we may write

$$\left(e^{\mu} - \frac{1}{e^{\mu}}\right) = \int_0^{\mu}\frac{\partial}{\partial\mu}\left(e^{\mu} - \frac{1}{e^{\mu}}\right)d\mu.$$

Likewise,

$$2\mu\frac{2\mu+1}{(\mu+1)^2} = \int_0^\mu \frac{\partial}{\partial\mu}\left(2\mu\frac{2\mu+1}{(\mu+1)^2}\right)d\mu.$$

The facts that at $\mu = 0$, $\frac{\partial}{\partial\mu}\left(e^\mu - \frac{1}{e^\mu}\right) = \frac{\partial}{\partial\mu}\left(2\mu\frac{2\mu+1}{(\mu+1)^2}\right)$, that (5) attains its minimum at $\mu = 0$, and that (6) attains its maximum at $\mu = 0$, imply the inequality in (4).

Thus, $\frac{\partial}{\partial\mu}\left(\mu\beta(\mu,\phi) + 2\phi\alpha^*(\mu,\phi)\right)$ is everywhere positive when evaluated at $\phi = \phi(\mu)$. It follows that, given the constraint $\mu \in (0,1]$, the gatekeeper's expected profits are maximized in an equilibrium where $\mu = 1$. This establishes part (a) of the proposition. Substituting μ^* into $\phi(\mu)$ and $\beta(\mu,\phi(\mu))$ yields parts (b) and (c) of the proposition. Substituting μ^* and ϕ^* into equation (1) yields part (d). Finally, one may readily verify that the constraint $\phi \in \left(0, \frac{r\mu}{2}\right)$ is also satisfied. ■

Proposition 2 explains the stylized facts presented in the introduction. First, $\phi^* > \kappa^*$ in equilibrium; that is, the newspaper has an incentive to set advertising fees above subscription fees. The reason is simple: To grow a market for information, the newspaper must reduce the rents extracted from subscribers by a sufficient amount to prevent them from attempting to "free ride" on the generally lower prices that prevail as a result of price advertisements. Second, a profit-maximizing newspaper earns the bulk of its revenues from advertisements. In particular, the model predicts that the equilibrium ratio of advertising revenues to total revenues is

$$\begin{aligned}\frac{\text{Advertising Revenues}}{\text{Advertising + Subscription Revenues}} &= \frac{2\alpha^*\phi^*}{2\alpha^*\phi^* + \kappa^*\mu^*} \\ &= 1 - \frac{1}{e},\end{aligned}$$

or about 63 percent.

References

[1] Baye, Michael R., Dan Kovenock, and Casper de Vries, "It Takes Two to Tango: Equilibria in a Model of Sales," *Games and Economic Behavior* (1992) 4, pp. 493-510.

[2] Baye, Michael R. and John Morgan, "Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets," *American Economic Review*, forthcoming, 2000.

[3] Dasgupta, Partha and Eric Maskin, "The Existence of Equilibrium in Discontinuous Economic Games, I: Theory," *Review of Economic Studies* (1986) LIII, pp. 1-26.

Dummy page